Inversion Homework #4

Professor H. Yao & H. Zhang

Jintao Li

SA20007037

E-mail: lijintao@mail.ustc.edu.cn

November 17, 2020

Chapter 3

Exercise 2

Another resolution test commonly performed in tomography studies is a **checkerboard test**, which consists of using a test model composed of alternating positive and negative perturbations. Perform a checkerboard test on the tomography problem in Example 3.1 using the test model,

$$\mathbf{m}_{true} = \begin{bmatrix} -1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix} \tag{1}$$

Evaluate the difference between the true (checkerboard) model and the recovered model in your test, and interpret the pattern of differences. Are any block values recovered exactly? If so, does this imply perfect resolution for these model parameters?

Solution:

matlab code

```
clear;clc;close all;

a = ones(5, 1)
% ls tme
```

Exercise 4

A large north-south by east-west–oriented, nearly square plan view, sandstone quarry block (16 m by 16 m) with a bulk compressional wave seismic velocity of approximately 3000 m/s is suspected of harboring higher-velocity dinosaur remains. An ultrasonic tomography scan is performed in a horizontal plane bisecting the boulder, producing a data set consisting of 16 E→W, 16 N→S, 31 NE→SW, and 31 NW→SE travel times (see Figure 1). The travel time data (units of s) have statistically independent errors, and the travel time contribution for a uniform background model (with a velocity of 3000 m/s) has been subtracted from each travel time measurement.

The MATLAB data files that you will need to load containing the travel time data follow: **rowscan.mat**, **colscan.mat**, **diag1scan.mat**, and **diag2scan.mat**. The standard deviations of all data measurements are 1.5×10^{-5} s. Because the travel time contributions for a uniform background model (with a velocity of 3000 m/s) have been subtracted from each travel time measurement, you will be solving for slowness and velocity perturbations relative to a uniform slowness model of 1/3000 s/m. Use a row-by-row mapping between the slowness grid and the model vector (e.g., Example 1.12). The row format of each data file is $(x_1, \gamma_1, x_2, \gamma_2, t)$ where the starting point coordinate of each source is (x_1, γ_1), the end point coordinate is (x_2, γ_2), and the travel time along a ray path between the source and receiver points is a path integral (in seconds).
Parameterize the slowness structure in the plane of the survey by dividing the boulder into a 16×16 grid of 256 1-m-square, north-by-east blocks and construct a linear system for the forward problem (Figure 3.29). Assume that the ray paths through each homogeneous block can be represented by straight lines, so that the travel time expression is

$$\begin{aligned} t &= \int_\ell s(\mathbf{x}) d\ell \\ &= \sum_{\text{blocks}} s_{\text{block}} \cdot \Delta l_{\text{block}} \end{aligned} \tag{2}$$

where Δl_{block} is 1 m for the row and column scans and $\sqrt{2}$ m for the diagonal scans.

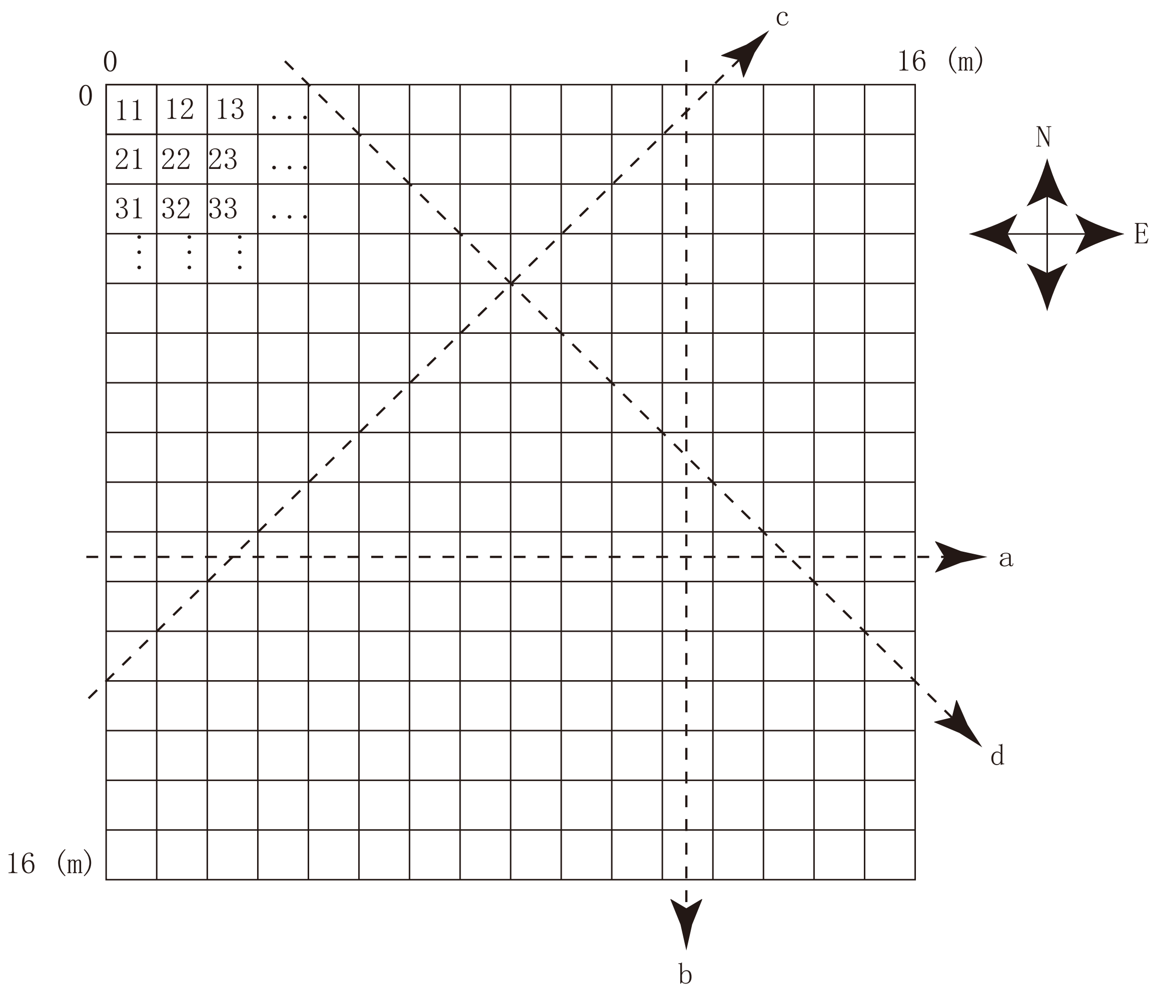

Figure 1: Tomography exercise, showing block discretization, block numbering convention, and representative ray paths going east-west (a), north-south (b), southwest-northeast (c), and northwestsoutheast (d).

Use the SVD to find a minimum-length/least squares solution, $\mathbf{m}_\dagger$, for the 256 block slowness perturbations that fit the data as exactly as possible. Perform two inversions in this manner:
1. Use the row and column scans only.
2. Use the complete data set.
For each inversion:

(a) Note the rank of your $\mathbf{G}$ matrix relating the data and model.

(b) State and discuss the general solution and/or data fit significance of the elements and dimensions of the data and model null spaces. Plot and interpret an element of each space and contour or otherwise display a nonzero model that fits the trivial data set $\mathbf{Gm} = \mathbf{d} = \mathbf{0}$ exactly.

(c) Note whether there are any model parameters that have perfect resolution.

(d) Produce a 16 by 16 element contour or other plot of your slowness perturbation model, displaying the maximum and minimum slowness perturbations in the title of each plot. Interpret any internal structures geometrically and in terms of seismic velocity (in m/s).

(e) Show the model resolution by contouring or otherwise displaying the 256 diagonal elements of the model resolution matrix, reshaped into an appropriate 16 by 16 grid.

(f) Describe how one could use solutions to $\mathbf{Gm} = \mathbf{d} = \mathbf{0}$ to demonstrate that very rough models exist that will fit any data set just as well as a generalized inverse model. Show one such wild model.