Inversion Homework1

Jintao Li

October 3, 2020

Exercise 1 Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 1 & 3 \\ 4 & 6 & 7 & 1 \end{bmatrix} \tag{1}$$

Find bases for $\mathcal{N}(\mathbf{A})$, $\mathcal{R}(\mathbf{A})$, $\mathcal{N}(\mathbf{A}^T)$, and $\mathcal{R}(\mathbf{A}^T)$. What are the dimensions of the four subspaces?

Exercise 2 Show that if $\mathbf{x} \perp \mathbf{y}$, then

$$\|\mathbf{x}+\mathbf{y}\|_2^2 = \|\mathbf{x}\|_2^2 + \|\mathbf{y}\|_2^2. \tag{2}$$

Exercise 3 In this exercise, we will derive the formula (A.88) for the 1-norm of a matrix. Begin with the optimization problem

$$\|\mathbf{A}\|_1 = \max_{\|\mathbf{x}\|_1=1} \|\mathbf{A}\mathbf{x}\|_1. \tag{3}$$

(a) Show that if $\|\mathbf{x}\|_1 = 1$, then

$$\|\mathbf{A}\mathbf{x}\|_1 \leq \max_j \sum_{i=1}^{m} \left|\mathbf{A}_{i,j}\right|. \tag{4}$$

(b) Find a vector $\mathbf{x}$ such that $\|\mathbf{x}\|_1 = 1$, and

$$\|\mathbf{A}\mathbf{x}\|_1 = \max_j \sum_{i=1}^{m} \left|\mathbf{A}_{i,j}\right|. \tag{5}$$

(c) Conclude that

$$\|\mathbf{A}\|_1 = \max_{\|\mathbf{x}\|_1=1} \|\mathbf{A}\mathbf{x}\|_1 = \max_j \sum_{i=1}^{m} \left|\mathbf{A}_{i,j}\right|. \tag{6}$$