



Inversion Homework #1

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Appendix A

Exercise 4

Let

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 1 & 3 \\ 4 & 6 & 7 & 11 \end{bmatrix} \quad (1)$$

Find bases for $\mathcal{N}(\mathbf{A})$, $\mathcal{R}(\mathbf{A})$, $\mathcal{N}(\mathbf{A}^T)$, and $\mathcal{R}(\mathbf{A}^T)$. What are the dimensions of the four subspaces?

Solution:

To find $\mathcal{N}(\mathbf{A})$, we solve the system of equations $\mathbf{A}\mathbf{x} = \mathbf{0}$,

$$\mathbf{A}\mathbf{x} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 1 & 3 \\ 4 & 6 & 7 & 11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (2)$$

Put the system of equations into an augmented matrix,

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 2 & 2 & 1 & 3 & 0 \\ 4 & 6 & 7 & 11 & 0 \end{array} \right] \quad (3)$$

and then find the reduced row echelon form (**RREF**),

$$\left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & \frac{5}{2} & \frac{5}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \quad (4)$$

So, we can find that

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2x_3 + x_4 \\ -\frac{5}{2}x_3 - \frac{5}{2}x_4 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ -\frac{5}{2} \\ 1 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 1 \\ -\frac{5}{2} \\ 0 \\ 1 \end{bmatrix} x_4 \quad (5)$$

Exercise 10

Show that if $\mathbf{x} \perp \mathbf{y}$, then

$$\|\mathbf{x} + \mathbf{y}\|_2^2 = \|\mathbf{x}\|_2^2 + \|\mathbf{y}\|_2^2. \quad (6)$$

Solution:

Exercise 11

In this exercise, we will derive the formula (A.88) for the 1-norm of a matrix. Begin with the optimization problem

$$\|\mathbf{A}\|_1 = \max_{\|\mathbf{x}\|_1=1} \|\mathbf{A}\mathbf{x}\|_1. \quad (7)$$

(a) Show that if $\|\mathbf{x}\|_1 = 1$, then

$$\|\mathbf{Ax}\|_1 \leq \max_j \sum_{i=1}^m |A_{i,j}|. \quad (8)$$

Solution:

(b) Find a vector \mathbf{x} such that $\|\mathbf{x}\|_1 = 1$, and

$$\|\mathbf{Ax}\|_1 = \max_j \sum_{i=1}^m |A_{i,j}|. \quad (9)$$

Solution:

(c) Conclude that

$$\|\mathbf{A}\|_1 = \max_{\|\mathbf{x}\|_1=1} \|\mathbf{Ax}\|_1 = \max_j \sum_{i=1}^m |A_{i,j}|. \quad (10)$$

Solution:

Appendix B

Exercise 6

Suppose that $\mathbf{X} = (X_1, X_2)^T$ is a vector composed of two random variables with a multivariate normal distribution with expected value μ and covariance matrix \mathbf{C} , and that \mathbf{A} is a 2 by 2 matrix. Use properties of expected value and covariance to show that $\mathbf{y} = \mathbf{Ax}$ has expected value $\mathbf{A}\mu$ and covariance \mathbf{ACA}^T .

Solution:

Exercise 9

Using MATLAB, repeat the following experiment 1000 times. Generate five exponentially distributed random numbers from the exponential probability density function (B.10) with means $\mu = 1/\lambda = 10$. You may find the library function **exp rand** to be useful here. Use (B.74) to calculate a 95% confidence interval for the 1000 mean determinations. How many times out of the 1000 experiments did the 95% confidence interval cover the expected value of 10? What happens if you instead generate 50 exponentially distributed random numbers at time? Discuss your results.

Solution:

Appendix C

Exercise 1

Let

$$f(\mathbf{x}) = x_1^2 x_2^2 - 2x_1 x_2^2 + x_2^2 - 3x_1^2 x_2 + 12x_1 x_2 - 12x_2 + 6. \quad (11)$$

Find the gradient, $\nabla f(\mathbf{x})$, and Hessian, $\mathbf{H}(f(\mathbf{x}))$. What are the critical points of f ? Which of these are minima and maxima of f ?

Solution:

Exercise 2

Find a Taylor's series approximation for $f(\mathbf{x} + \Delta\mathbf{x})$, where

$$f(\mathbf{x}) = e^{-(x_1+x_2)^2} \tag{12}$$

is near the point

$$\mathbf{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}. \tag{13}$$

Solution: