Geodynamics Homework #3

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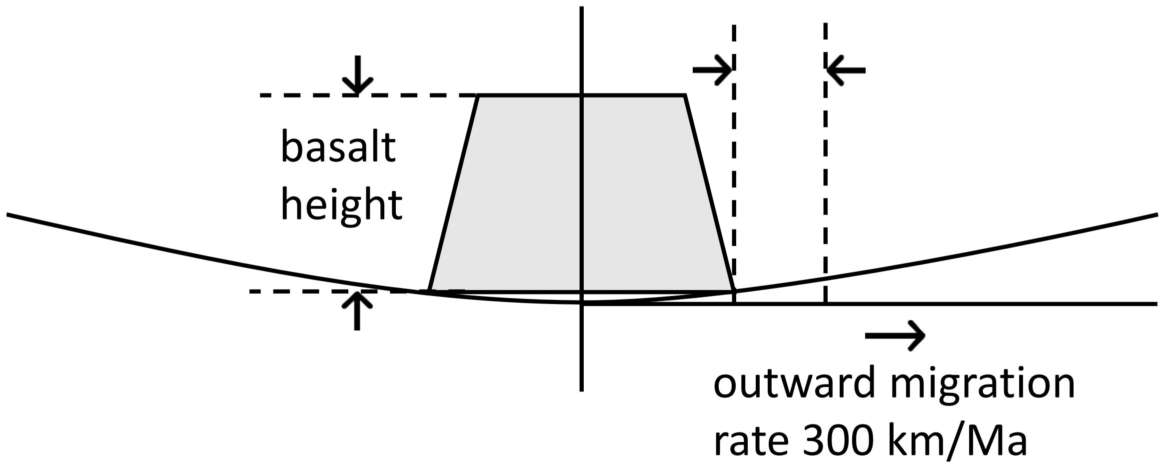
Chapter 3: Elasticity and Flexure

Exercise 1

The effect of large scale volcanic eruption on the crustal flexture.



Suppose volcanic eruption lasts for 2 Ma, forming basalts accumulation with a height of 2 km. The cross-section shape of the basalts are trapezoid, the upper boundary of the trapezoid is 0.8 of the bottom boundary. Basalts spread from the center at a speed of 300 km/Ma. Some parameters: Young's modulus 70 GPa, poisson's ratio 0.25, density of the basalts 2700 kg/m^3, crust density 2900 kg/m^3, elastic thickness of the crust 50 km.

Solve for the time variation of surface topography at x=150, 300 and 450 km from the eruption center. Discuss the effects of different elastic thickness on the results.

Solution:

Refering to Brotchie's paper *On Crustal Flexure*, the differential equation for deflection can be represented as:

$$D\nabla^4 w + \left(ET/R^2\right) w + \gamma w = q. \tag{1}$$

And rewriting the formula in plane polar coordinate and the spherical coordinates of the shell, it can be:

$$\nabla^4 w + \left(1/l^4\right) w = q/D, \tag{2}$$

in which $l^4 \equiv D/\left[\left(ET/R^2\right) + \gamma\right]$, ω is the radial displacementof the shell undernormalloadingof intensityq, D is the flexural stiffness of the shell cross section $\equiv \left[ET^3/12\left(1 - v^2\right)\right]$, T is the thickness of the shell, E is itsmodulusof elasticity, v is Poisson's ratio for the shellmaterial, R is the radius of its middle surface, γ is the density of the enclosed liquid.

We consider this the volcanic loading as *variable loading*. And the deflections of crust for a volcanic eruption of variable thickness are found by superposition using uniform thickness solution. The variable thickness may be approximated by a stepped distribution. The sheet may then be considered to be composed of uniform layers of depth h and radius a_n, and we choose step size $h = 5m$.

As to uniform loading, solving the equation **??**, we can obtain the solution:

$$w_i = \frac{\gamma_{volcanic} h}{\gamma'} \left(a \operatorname{ker}' a \operatorname{ber} x - a \operatorname{kei}' a \operatorname{bei} x + 1 \right), \tag{3}$$

and, outside the volcanic eruption, deflection $omega_0$ is:

$$w_0 = \frac{\gamma_{volcanic} h}{\gamma'} (a \operatorname{ber}' a \operatorname{ker} x - a \operatorname{bei}' a \operatorname{kei} x), \tag{4}$$

in which h is uniform depth, $\gamma_{volcanic}$ is the density of basalts, γ is the density of mantle, and $\gamma' = \gamma + ET/R^2$.