# Geodynamics Homework #3

**Professor W. Len**

**Jintao Li**

**SA20007037**

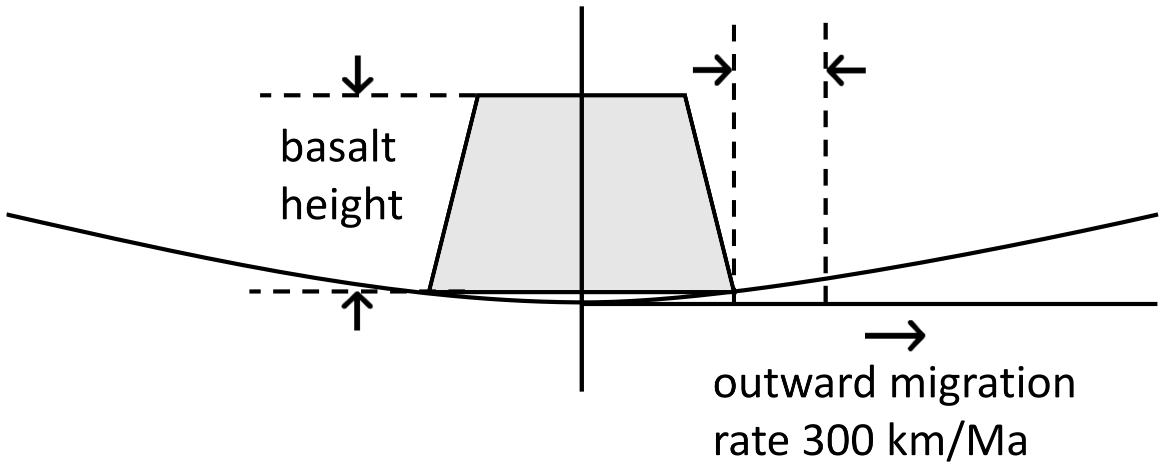
**E-mail: lijintao@mail.ustc.edu.cn**

April 20, 2021

# Chapter 3: Elasticity and Flexure

## Exercise 1

The effect of large scale volcanic eruption on the crustal flexture.




Suppose volcanic eruption lasts for 2 $Ma$, forming basalts accumulation with a height of 2 $km$. The cross-section shape of the basalts are trapezoid, the upper boundary of the trapezoid is 0.8 of the bottom boundary. Basalts spread from the center at a speed of 300 $km/Ma$. Some parameters: Young's modulus 70 $GPa$, poisson's ratio 0.25, density of the basalts 2700 $kg/m^3$, crust density 2900 $kg/m^3$, elastic thickness of the crust 50 $km$.

Solve for the time variation of surface topography at x=150, 300 and 450 $km$ from the eruption center. Discuss the effects of different elastic thickness on the results.

## Solution:

Refering to Brotchie's paper *On Crustal Flexure*, the differential equation for deflection can be represented as:

$$D\nabla^4 w + \left(ET/R^2\right) w + \gamma w = q. \tag{1}$$

And rewriting the formula in plane polar coordinate and the spherical coordinates of the shell, it can be:

$$\nabla^4 w + \left(1/l^4\right) w = q/D, \tag{2}$$

in which $l^4 \equiv D/\left[\left(ET/R^2\right) + \gamma\right]$, $\omega$ is the radial displacementof the shell undernormalloadingof intensityq, D is the flexural stiffness of the shell cross section $\equiv \left[ET^3/12\left(1 - v^2\right)\right]$, $T$ is the thickness of the shell, $E$ is itsmodulusof elasticity, $\upsilon$ is Poisson's ratio for the shellmaterial, $R$ is the radius of its middle surface, $\gamma$ is the density of the enclosed liquid.

We consider this the volcanic loading as *variable loading*. And the deflections of crust for a volcanic eruption of variable thickness are found by superposition using uniform thickness solution. The variable thickness may be approximated by a stepped distribution. The sheet may then be considered to be composed of uniform layers of depth $h$ and radius $a_n$, and we choose step size $h = 5m$.

As to uniform loading, solving the equation 2, we can obtain the solution:

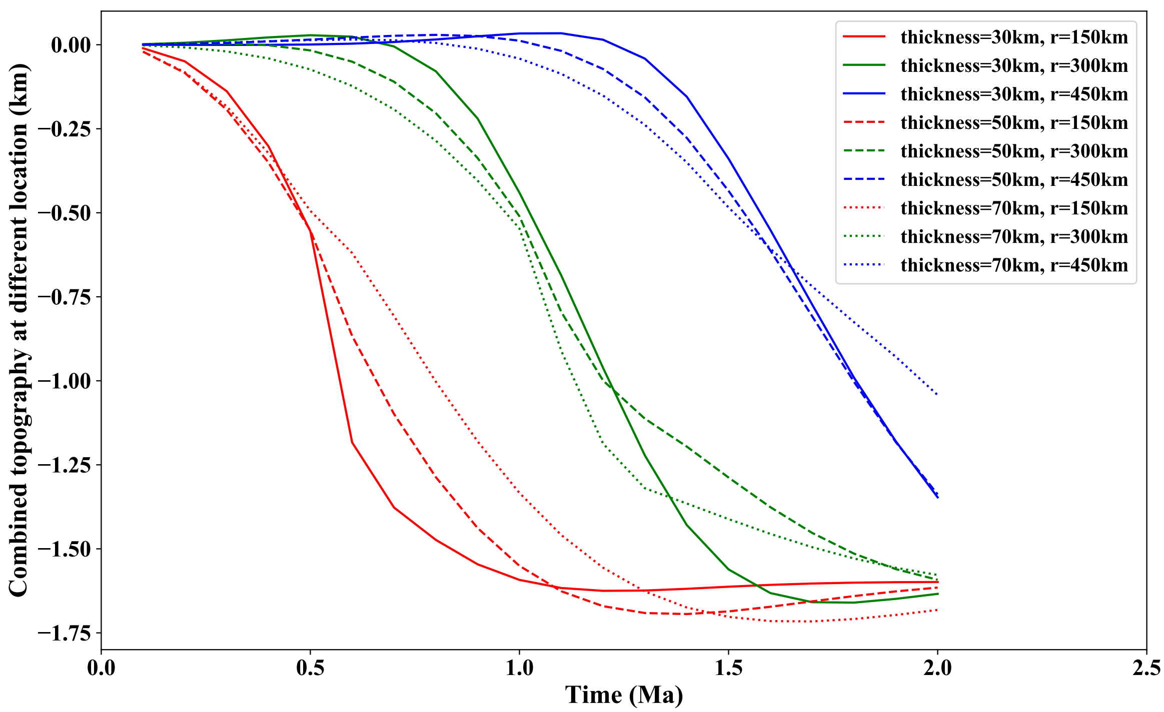
$$w_i = \frac{\gamma_{volcanic} h}{\gamma'} \left( a \operatorname{ker}' a \operatorname{ber} x - a \operatorname{kei}' a \operatorname{bei} x + 1 \right), \tag{3}$$

and, outside the volcanic eruption, deflection $omega_0$ is:

$$w_0 = \frac{\gamma_{volcanic} h}{\gamma'} (a \operatorname{ber}' a \operatorname{ker} x - a \operatorname{bei}' a \operatorname{kei} x), \tag{4}$$

in which $h$ is uniform depth, $\gamma_{volcanic}$ is the density of basalts, $\gamma$ is the density of mantle, and $\gamma' = \gamma + ET/R^2$.

The result is illuminated in blow figure. The red, green and blue lines are represented the results of surface topography at x=150, 300 and 450 km from the eruption center, respectively. And the soild, dashed and dotted lines are the results of surface topography when crust thickness is 30, 50, 70 km, respectively.




As the inset shown, the deflection is lager when the position is closer to the eruption center. And the deflection tends to a stable value as the basalts accumulated except the farthest position, because the accumulation is not enough to bend the farthest position. The deflection declines fastest at 0.5, 1.0, 1.5 Ma, respectively, and its reason is the basalt coverd this position at the time. In general, the thicker the crust, the less deflection there is, and the longer it takes to reach stability.