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Guanghua Cambridge School

Two Approximation Methods on the Period of Compound Pendulum with Liquid Damping

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Guanghua Cambridge International School

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INTRODUCTION

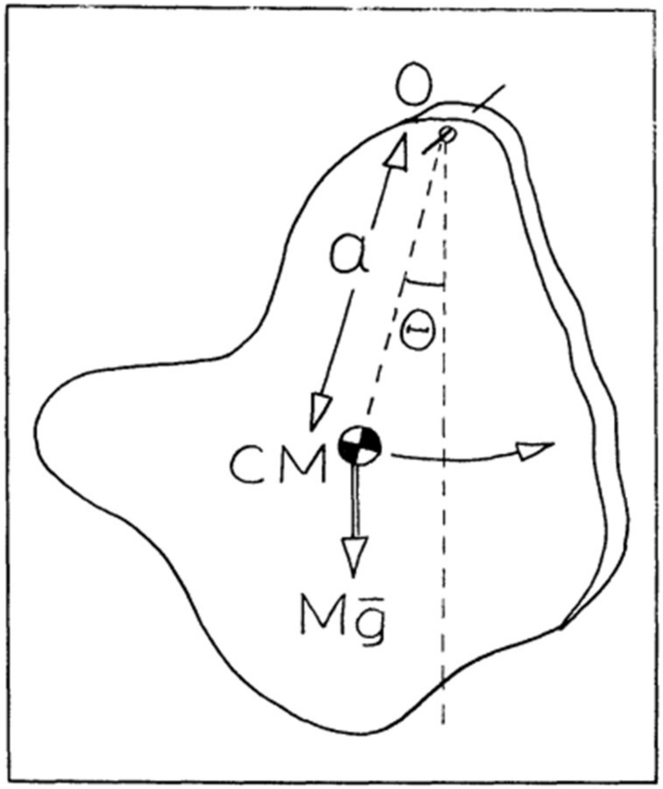

Fig. 1. For an extended object of moment of inertia $I = M(a^2 + k^2)$ about the pivot O, the period of small oscillation depends only on the distance a from the pivot to the center of mass, and the gyradius k.

Fig 1.

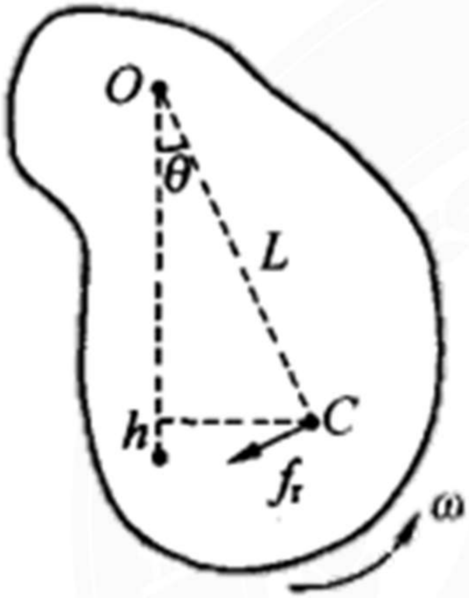

Fig 2.

← Compound Pendulum

INTRODUCTION

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Instead…

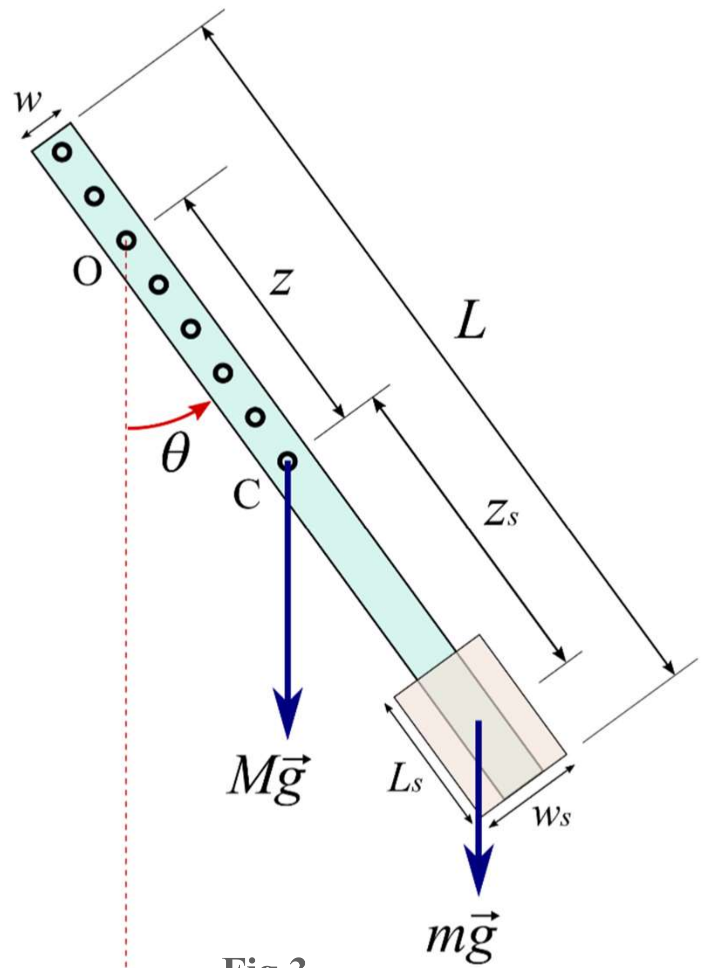

Fig 3.

TIMELINE

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2023.07-08: Determine research topic, make the first step to the question, have a basic understanding

2023.09-10: Collect references, work out the first version of formula about the pendulum

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ADJUSTMENTS

Previous topic: The Effects on the period …

Current topic: Two Approximation Methods…

时,常采用 2 种简化模型[3-5],一是假设振动物体所受的阻力与物体的速度成正比,另一种是假设阻力与速度的平方成正比. 本文分别采用 2 种空气阻力模型,推导出考虑空气阻力时复摆的振动周期公式,并结合实验测量结果,分析比较 2 种情况下空气的阻力对复摆周期的影响.

Fig 4. (air friction)

ADJUSTMENTS

Reasons:

- Easy to **determine variables**
  - --viscous constant / inertia / length ...
- Hard to find **accurate period**
  - **Nonlinear motion** under friction and driving conditions [1]

ADJUSTMENTS

Methods:[2]

- $f \propto v$
- $f \propto v^{2}$

RESEARCH PROGRESS

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From analogue ***Newton's II Law of motion*** in rotation:

$$m \to I,\ a \to \alpha,\ F \to M$$

$$\boldsymbol{ma} = \sum \boldsymbol{F} \ \rightarrow \ \boldsymbol{I\alpha} = \sum \boldsymbol{M}$$

Sep. 16th

RESEARCH PROGRESS

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$$I\ddot{\theta} = F \cdot d$$

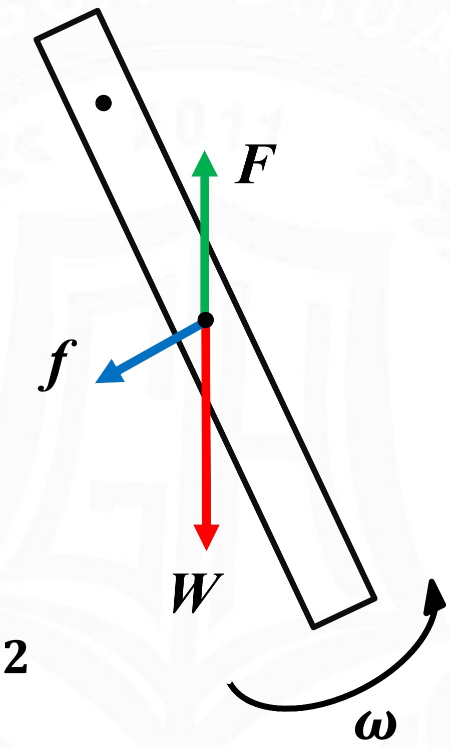

Fig 5.

Force list:

1. Weight
2. Buoyance
3. Viscous force

$$W = mg$$

$$F_{up} = \rho_w g V$$

$f = kv$ and $f = kv^2$

Sep. 25th

RESEARCH PROGRESS

$$I\ddot{\theta} = WR - fR - FR$$

1. $f = kR\omega = kR\dot{\theta}$

$$I\ddot{\theta} = (m - \rho V)gR\theta - kR^2\dot{\theta}$$

For $\theta \ll 5$, $\sin\theta \approx \theta$ [3]

里，ω_0=0.7。显然，当幅角较小时，两者的振动周期、振幅都比较接近，当幅角更小时，复摆的振动可看作简谐振动。而实际的复摆，幅角小于 5° 的

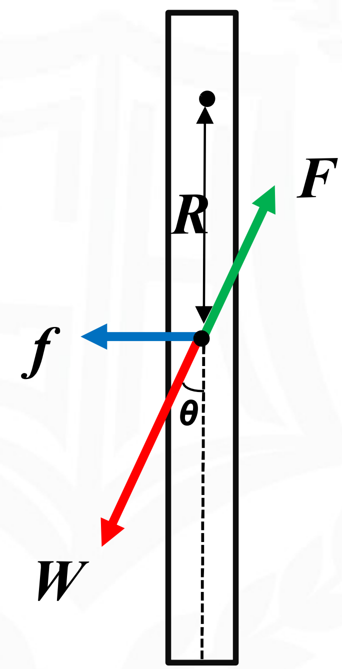

Fig 6.

Oct. 4th

RESEARCH PROGRESS

$$I\ddot{\theta} = WR - fR - FR$$

1. $f = kR\omega = kR\dot{\theta}$

$$\ddot{\boldsymbol{\theta}} + \frac{\boldsymbol{kR^2}}{\boldsymbol{I}}\dot{\boldsymbol{\theta}} - \frac{(\boldsymbol{m} - \boldsymbol{\rho V})\boldsymbol{gR}}{\boldsymbol{I}}\boldsymbol{\theta} = \mathbf{0}$$

Homogeneous second-order
linear ordinary differential equation[4]

RESEARCH PROGRESS

$$I\ddot{\theta} = WR - fR - FR$$

1. $f = kR\omega = kR\dot{\theta}$

$$\ddot{\boldsymbol{\theta}} + \frac{\boldsymbol{kR^2}}{\boldsymbol{I}}\dot{\boldsymbol{\theta}} - \frac{(\boldsymbol{m} - \boldsymbol{\rho V})\boldsymbol{gR}}{\boldsymbol{I}}\boldsymbol{\theta} = \boldsymbol{0}$$

Homogeneous second-order linear ordinary differential equation[4]

RESEARCH PROGRESS

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$$I\ddot{\theta} = WR - fR - FR$$

1. $f = kR\omega = kR\dot{\theta}$

$$\boldsymbol{\theta = ce^{-\frac{kR^2}{I}t}\cos\left(\frac{\sqrt{\frac{4(mg-\rho gV)R}{I} - \frac{k^2R^4}{I^2}}}{2}t\right)}$$

$$\boldsymbol{\omega = \frac{\sqrt{\frac{4(mg-\rho gV)R}{I} - \frac{k^2R^4}{I^2}}}{2}}$$

Oct. 18th

RESEARCH PROGRESS

$$I\ddot{\theta} = WR - fR - FR$$

2. $\boldsymbol{f = kR^2\omega^2 = kR^2\dot{\theta}^2}$

$$\boldsymbol{\ddot{\theta} + \frac{kR^3}{I}\dot{\theta}^2 - \frac{(m-\rho V)gR}{I}\theta = 0}$$

Let $x = \left(\frac{d\theta}{dt}\right)^2 = \dot{\theta}^2$, so $\ddot{\theta} = \frac{d^2\theta}{dt^2} = \frac{dx}{d\theta} = \dot{x}$

RESEARCH PROGRESS

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$$I\ddot{\theta} = WR - fR - FR$$

2. $\boldsymbol{f = kR^2\omega^2 = kR^2\dot{\theta}^2}$

$$\dot{\boldsymbol{x}} + \frac{\boldsymbol{kR^3}}{\boldsymbol{I}}\boldsymbol{x} - \frac{\boldsymbol{(m-\rho V)gR}}{\boldsymbol{I}}\boldsymbol{\theta} = \boldsymbol{0}$$

RESEARCH PROGRESS

$$T = 2\pi\sqrt{\frac{I^2}{IR(m-\rho V)g - 1/4\,(k^2R^4)}} \qquad f = kv$$

$$T = 2\pi\sqrt{\frac{I^2 - 2kR^3\theta(I - kR^3)}{gR(m-\rho V)(I - 2kR^3\theta)}} \qquad f = kv^2$$

Oct. 25th

PROBLEMS & SOLUTIONS

Big angles**?** **Small** angles**?**

Directly proportional**?** **Squared** proportional**?**

- Small angles can occur in air damping[2]
- Same in liquid? **Bigger damping force**

Oct. 10th

RESEARCH PROGRESS

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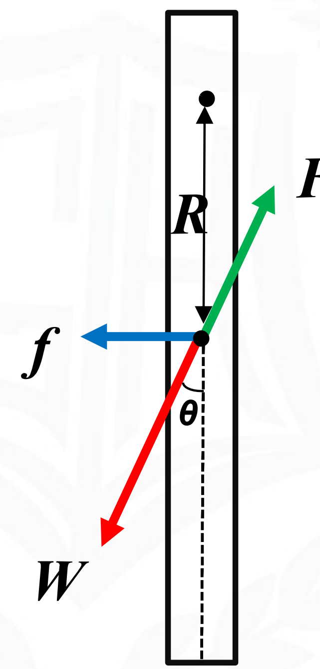
$$I\ddot{\theta} = WR - fR - FR$$

1. $f = kR\omega = kR\dot{\theta}$

$$I\ddot{\theta} = (m - \rho V)gR\theta - kR^2\dot{\theta}$$

For θ ≪ 5, sin θ ≈ θ[3]



A ROUGH EXPERIMENT

- Does not need to verify the formula’s correctness
- Does need to verify **small angle is observable**

Nov. 10th

PROBLEMS & SOLUTIONS

Fig 7.

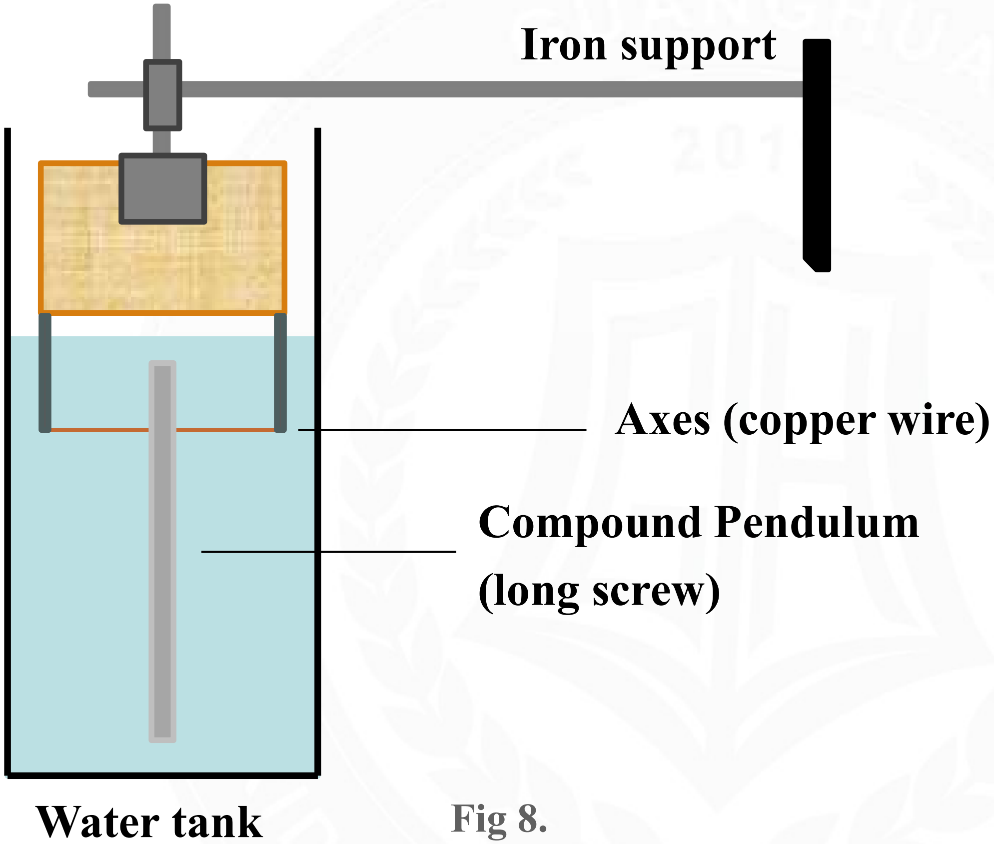


Fig 8.

Nov. 19th

PROBLEMS & SOLUTIONS

Data Collection

- ❑ Use phone camera – slow motion mode
- ❑ Did it for four times
- ❑ Use frame counter to calculate time in tiny units
  - ❑ 1 frame = 1 / 240 second
- ❑ Sum up and calculate average value

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Conclusion:

CAN swing **FOR A LONG TIME** (~40s, at least 60 periods directly by eyes) under **SMALL ANGLE CONDITION** & **LIQUID DAMPING**

No.	Frame/f	10T/s	T/s
1	740	6.167	0.614
2	736	6.133	
3	732	6.100	
4	738.5	6.154	

Data Table

PROBLEMS & SOLUTIONS

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$k = \frac{1}{2} c\rho A = 1.17 \times 1.431 \times 10^{-3} \times 1000 \times 0.5 = 0.837\ kg\ s^{-1}$

$I = 5.934 \times 10^{-5}\ kg\ m^2 \quad W - F = 304.11mN \quad R = 25.7mm$

No.	Frame/f	10T/s	T/s
1	740	6.167	0.614
2	736	6.133	
3	732	6.100	
4	738.5	6.154	

Data Table

Measured Results:

Period $\approx \mathbf{0.614}\ seconds$

Theoretical Results:

1. $f \propto v,\ T = 2\pi\sqrt{\frac{I^2}{IR(m-\rho V)g-1/4(k^2R^4)}}$, Period $\approx \mathbf{0.579}\ seconds$ $\quad \delta \approx -5.7\%$
2. $f \propto v^2, T = 2\pi\sqrt{\frac{I^2-2kR^3\theta(I-kR^3)}{gR(m-\rho V)(I-2kR^3\theta)}}$, Period $\approx \mathbf{0.551}\ seconds$ $\quad \delta \approx -10.3\%$

Nov. 20th

PROBLEMS & SOLUTIONS

WHY?

1. More **friction** than ideal –not only liquid damping
2. Did **not fixed** axes and pendulum
3. Measuring **uncertainty** –especially ***k***

ROUGH EXPERIMENT

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WORK PLAN

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Do ACCURATE EXPERIMENTs

- ✓ **Design**. –3D printer, a model of pendulum.
- ✓ **Produce**. –Prepare a list of buying.
- ✓ **Write**. –Start writing the report.

REFERENCES

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[1] 复摆运动状态的研究
Study on the motion behaviors of the compound pendulum
[2] 空气阻力对复摆振动周期的影响
Influence of air resistance on vibration period of compound pendulum
[3] 任意摆角的复摆振动规律研究
Research on Vibrations of Compound Pendulum for Arbitrary Angles
[4]
https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/17%3A_Second-Order_Differential_Equations/17.01%3A_Second-Order_Linear_Equations#mjx-eqn-super
Fig 1. Practical Applications of the Compound Pendulum
Fig 2. 空气阻力对复摆振动周期的影响
Influence of air resistance on vibration period of compound pendulum
Fig 3. Experimental analysis of a physical pendulum with variable suspension point
Fig 4. 空气阻力对复摆振动周期的影响
Influence of air resistance on vibration period of compound pendulum
Fig 6. 任意摆角的复摆振动规律研究
Research on Vibrations of Compound Pendulum for Arbitrary Angles

Thank you for your listening