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Two Approximation Methods on the Period of Compound Pendulum with Liquid Damping

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- **Introduction**
- **Adjustments**
- **Research progress**
- **Problems & solutions**
- **Work plan**

INTRODUCTION

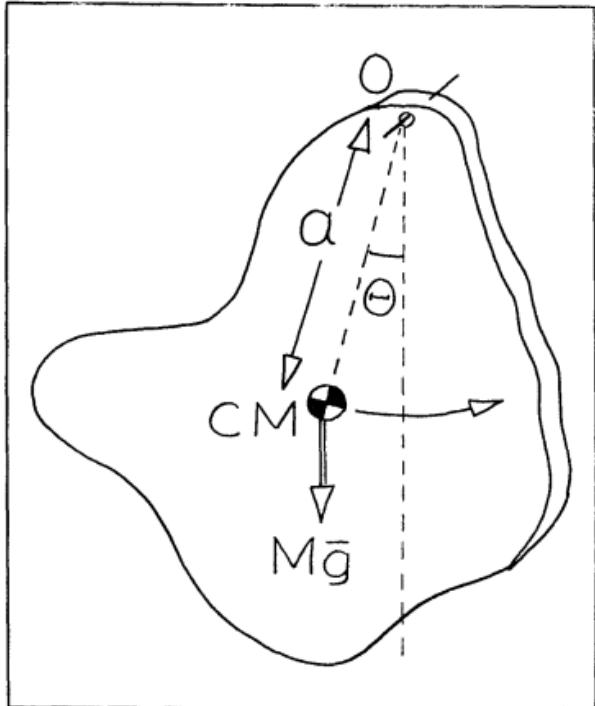


Fig. 1. For an extended object of moment of inertia $I = M(a^2 + k^2)$ about the pivot O, the period of small oscillation depends only on the distance a from the pivot to the center of mass, and the gyroradius k .

Fig 1.

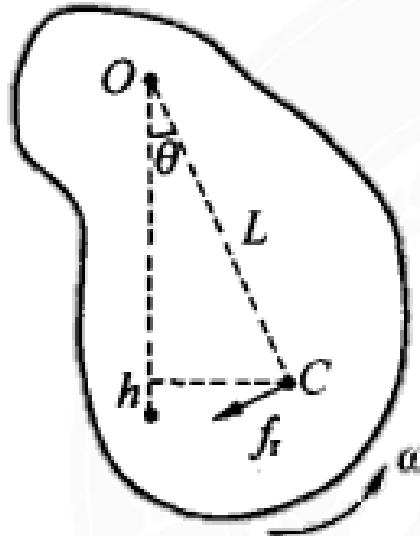
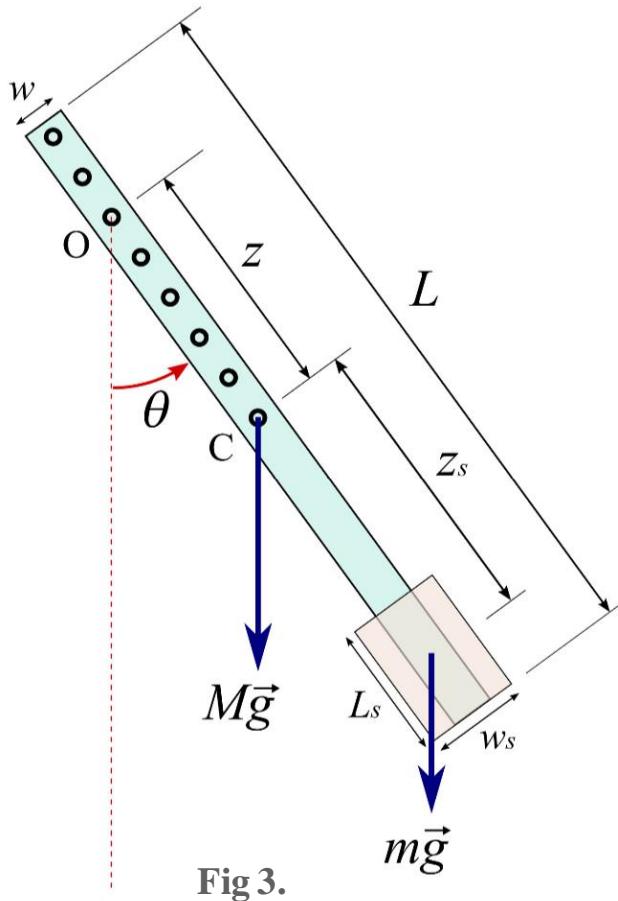


Fig 2.

← Compound Pendulum

INTRODUCTION

Instead...





TIMELINE

2023.09-10:
Collect references,
work out the first
version of formula
about the pendulum

2023.12-24.01:
Design experiment,
make theoretical
expectations, calculate
various answers

2024.03-04:
Analyze data, finish
the essay, submit to
tutor for final revision

2023.07-08:
Determine research
topic, make the first
step to the question,
have a basic
understanding

2023.11-12:
Study how to plot graph
and how to meet the
needs of experiments(e.g.
python+3d design)

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ADJUSTMENTS

Previous topic: The Effects on the period ...

Current topic: Two Approximation Methods...

时,常采用 2 种简化模型^[3-5],一是假设振动物体所受的阻力与物体的速度成正比,另一种是假设阻力与速度的平方成正比.本文分别采用 2 种空气阻力模型,推导出考虑空气阻力时复摆的振动周期公式,并结合实验测量结果,分析比较 2 种情况下空气的阻力对复摆周期的影响.

Fig 4.



ADJUSTMENTS

Reasons:

- Easy to **determine variables**
 - --viscous constant / inertia / length ...
- Hard to find **accurate period**
 - **Nonlinear motion** under friction and driving conditions [1]

ADJUSTMENTS



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Methods:^[2]

- $f \propto v$
- $f \propto v^2$



RESEARCH PROGRESS

From analogue ***Newton's II Law of motion*** in rotation:

$$I \rightarrow m, a \rightarrow \alpha, F \rightarrow M$$

$$ma = \sum F \rightarrow I\alpha = \sum M$$

RESEARCH PROGRESS



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$$I\ddot{\theta} = F \cdot d$$

Force list:

1. Weight
2. Buoyance
3. Viscous force

$$W = mg$$

$$F_{up} = \rho_w g V$$

$$f = kv \text{ and } f = kv^2$$

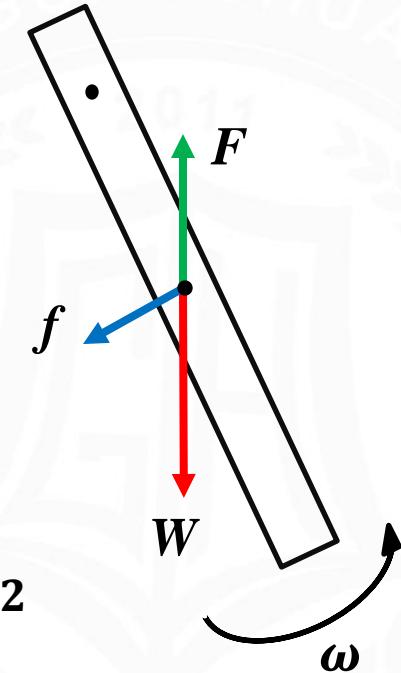


Fig 5.



RESEARCH PROGRESS

$$I\ddot{\theta} = WR - fR - FR$$

$$1. f = kR\omega = kR\dot{\theta}$$

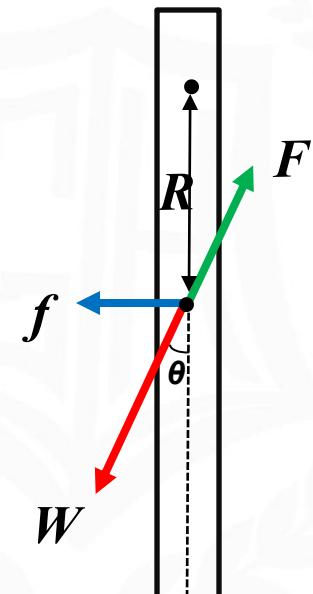
$$I\ddot{\theta} = (m - \rho V)gR\theta - kR^2\dot{\theta}$$



For $\theta \ll 5$, $\sin \theta \approx \theta_{[3]}$

里, $\omega_0 = 0.7$ 。显然, 当幅角较小时, 两者的振动周期、振幅都比较接近, 当幅角更小时, 复摆的振动可看作简谐振动。而实际的复摆, 幅角小于 5° 的

Fig 6.





RESEARCH PROGRESS

$$I\ddot{\theta} = WR - fR - FR$$

$$1. f = kR\omega$$

$$\ddot{\theta} + \frac{kR^2}{I}\dot{\theta} - \frac{(m - \rho V)gR}{I}\theta = 0$$

Homogeneous second-order
linear ordinary differential equation^[4]



RESEARCH PROGRESS

$$I\ddot{\theta} = WR - fR - FR$$

$$1. f = kR\omega$$

$$\ddot{\theta} + \frac{kR^2}{I}\dot{\theta} - \frac{(m - \rho V)gR}{I}\theta = 0$$

Homogeneous second-order
linear ordinary differential equation^[4]



RESEARCH PROGRESS

$$I\ddot{\theta} = WR - fR - FR$$

$$1. f = kR\omega$$

$$\theta = ce^{-\frac{kR^2}{I}t} \cos\left(\frac{\sqrt{\frac{4(mg - \rho g V)R}{I} - \frac{k^2 R^4}{I^2}}}{2} t\right)$$

Oct. 18th

$$\omega = \frac{\sqrt{\frac{4(mg - \rho g V)R}{I} - \frac{k^2 R^4}{I^2}}}{2}$$



RESEARCH PROGRESS

$$I\ddot{\theta} = WR - fR - FR$$

$$2. f = kR^2\omega^2$$

$$\ddot{\theta} + \frac{kR^3}{I}\dot{\theta}^2 - \frac{(m - \rho V)gR}{I}\theta = 0$$

$$\text{Let } x = \left(\frac{d\theta}{dt}\right)^2 = \dot{\theta}^2, \text{ so } \ddot{\theta} = \frac{d^2\theta}{dt^2} = \frac{dx}{d\theta} = \dot{x}$$



RESEARCH PROGRESS

$$T = 2\pi \sqrt{\frac{I^2}{IR(m - \rho V)g - 1/4(k^2 R^4)}} \quad f = kv$$

$$T = 2\pi \sqrt{\frac{I^2 - 2kR^3\theta(I - kR^3)}{gR(m - \rho V)(I - 2kR^3\theta)}} \quad f = kv^2$$

Oct. 25th



PROBLEMS & SOLUTIONS

Big angles? Small angles?

Directly proportional? Squared proportional?

- Small angles can occur in air damping^[2]
- Same in liquid? **Bigger damping force**



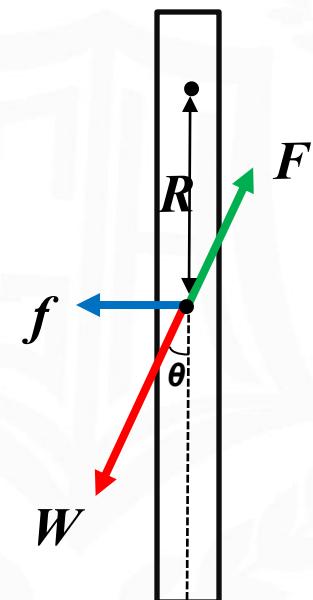
RESEARCH PROGRESS

$$I\ddot{\theta} = WR - fR - FR$$

$$1. f = kR\omega = kR\dot{\theta}$$

$$I\ddot{\theta} = (m - \rho V)gR\theta - kR^2\dot{\theta}$$

For $\theta \ll 5$, $\sin \theta \approx \theta$ ^[3]





PROBLEMS & SOLUTIONS

A **ROUGH EXPERIMENT**

- Does not need to verify the formula's correctness
- Does need to verify **small angle is observable**

Nov. 10th



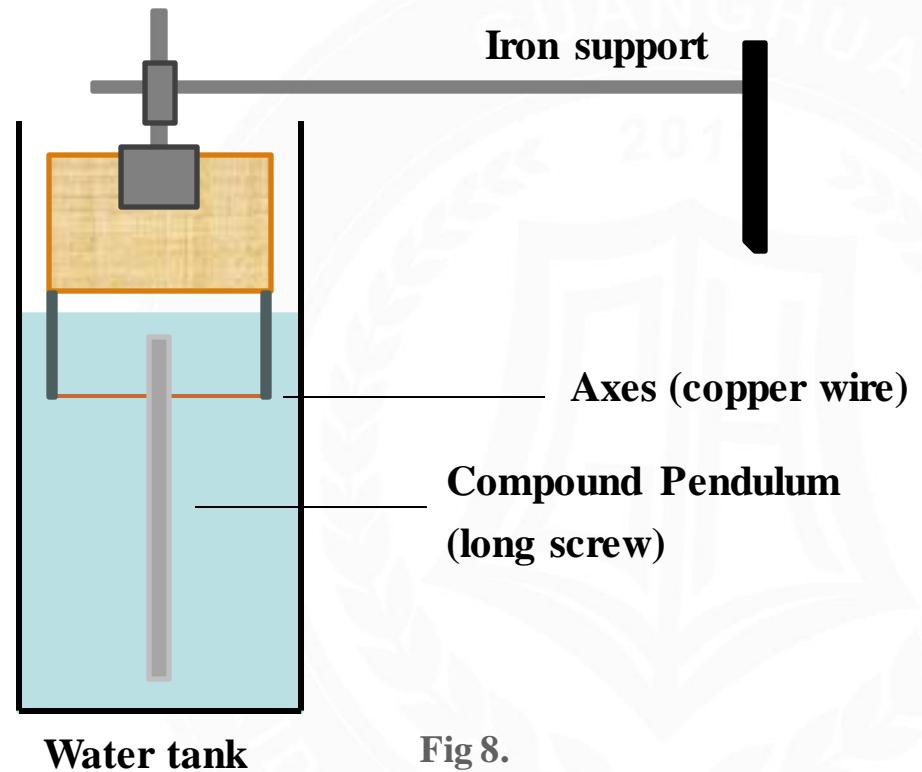
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PROBLEMS & SOLUTIONS



Fig 7.

Nov. 19th



Water tank

Fig 8.



PROBLEMS & SOLUTIONS

Data Collection

- Use phone camera – slow motion mode
- Did it for four times
- Use frame counter to calculate time in tiny units
 - 1 frame = 1 / 240 second
- Sum up and calculate average value



PROBLEMS & SOLUTIONS

Conclusion:

CAN swing **FOR A LONG TIME** (~40s, at least 60 periods directly by eyes)
under **SMALL ANGLE CONDITION & LIQUID DAMPING**

No.	Frame/f	10T/s	T/s
1	740	6.167	0.614
2	736	6.133	
3	732	6.100	
4	738.5	6.154	

Data Table



PROBLEMS & SOLUTIONS

Measured Results:

Period ≈ 0.614 seconds

Theoretical Results:

$$T = 2\pi \sqrt{\frac{I^2}{IR(m-\rho V)g - 1/4(k^2 R^4)}},$$

No.	Frame/f	10T/s	T/s
1	740	6.167	
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Data Table

$$k = \frac{1}{2} c \rho A = 1.17 \times 1.412 \times 10^{-3} \times 1000 \times 0.5 = 0.826 \text{ kg s}^{-1}$$

Period ≈ 0.706 seconds difference =15%

Nov. 20th



PROBLEMS & SOLUTIONS

WHY?

1. More **friction** than ideal –not only liquid damping
2. Did **not fixed** axes and pendulum
3. Measuring **uncertainty** –especially k

ROUGH EXPERIMENT

WORK PLAN



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About 3D printer...



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REFERENCES

[1] 复摆运动状态的研究

Study on the motion behaviors of the compound pendulum

[2] 空气阻力对复摆振动周期的影响

Influence of air resistance on vibration period of compound pendulum

[3] 任意摆角的复摆振动规律研究

Research on Vibrations of Compound Pendulum for Arbitrary Angles

[4]

[https://math.libretexts.org/Bookshelves/Calculus/Calculus_\(OpenStax\)/17%3A_Second-Order_Differential_Equations/17.01%3A_Second-Order_Linear_Equations#mjax-eqn-super](https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/17%3A_Second-Order_Differential_Equations/17.01%3A_Second-Order_Linear_Equations#mjax-eqn-super)

Fig 1. Practical Applications of the Compound Pendulum

Fig 2. 空气阻力对复摆振动周期的影响

Influence of air resistance on vibration period of compound pendulum

Fig 3. Experimental analysis of a physical pendulum with variable suspension point

Fig 4. 空气阻力对复摆振动周期的影响

Influence of air resistance on vibration period of compound pendulum

Fig 6. 任意摆角的复摆振动规律研究

Research on Vibrations of Compound Pendulum for Arbitrary Angles



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Thank you for your listening

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