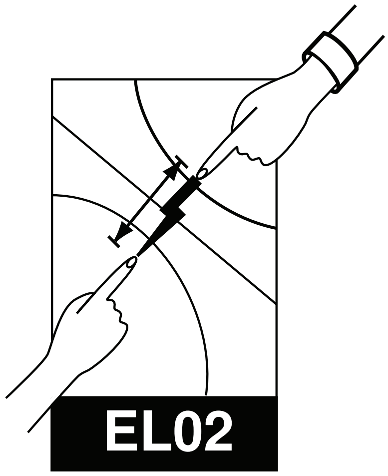
Circuits excited by harmonic voltages



GDP, JAJ, updated RN October 2019

Before coming to the laboratory read sections 3.1 to 3.7 in the Electronics Manual[1] (hereafter referred to as the Manual), start to learn the voltage drop expressions in section 3.2 , and work through the theory exercises in sections 4.3 and 5.3 of this script.

1 Introduction and objectives

You will continue the investigation, begun in EL01 — *circuits excited by stepped voltages*[2], of how well the behaviour of some real circuits is modelled by the predictions of simple equivalent circuits. Here the circuits are excited by voltages which are sine or cosine functions of time (harmonic voltages). Note that strictly speaking we should talk about harmonic *EMFs*, but we have used the (incorrect) almost ubiquitous practice of referring to an EMF as a voltage. You will use complex algebra to work out the predictions of the models (again useful for Prelims), and gain more experience in making measurements and using Python to fit models to data.

2 Preliminaries

Look back at your record of EL01 and check that the serial number of the board you used for EL01 matches the one on the bench in front of you now. Plug the BNC 'T' adapter into the socket on (**CH1**) of your oscilloscope and then use a BNC to BNC cable to connect one input of the 'T' to the oscillator output. The second arm of the 'T' adapter should be connected to your circuit board. The input of your board has a resistive divider on it to reduce the effective output impedance of the oscillator to approximately $4.5\,\Omega$. This arrangement reduces the effect of the oscillator's internal resistance on measurements you make. It also means that the signal you see on the scope in this configuration is not the input signal to the circuit under test, but a multiple of it (approximately $\times 11$). If you need to use the scope channel with a probe for a particular measurement, you can simply disconnect the 'T' from the scope, leaving the other cables connected, which leaves the input signal to the circuit unaffected. Note that if you switch from the 'T' type input to the scope to a scope probe, then you need to change the probe attenuation setting on the scope. This is done by selecting the channel (yellow or blue buttons), choosing the "Probe $1\times$ Voltage" menu and selecting the relevant attenuation. It is $10\times$ for our probes.

As in EL01, when a circuit diagram refers to an "adapted" oscillator, it is referring to the combination of the oscillator and the resistive divider network on the board. This is equivalent to a signal source which has an output impedance of approximately $4.5\,\Omega$ (this is different to the value of $4.6\,\Omega$ for the EL01 configuration.)

Note that we will often drop the word angular when talking about the angular frequency ω. Also note that the DMM that could not be used for measuring voltages in EL01 comes into its own here in providing accurate and convenient measurements of the rms values of harmonic voltages.

[1] www-teaching.physics.ox.ac.uk/practical_course/ElectronicsManual.pdf
[2] www-teaching.physics.ox.ac.uk/practical_course/scripts/srv/local/rscripts/trunk/Electronics/EL01/EL01.pdf

3 Why we make measurements

One of the reasons why physicists make measurements is to identify inadequacies in a model. If any are found the next steps are to improve the model and then subject it to more tests. There are two possible outcomes from a test, either:

(a) the measurements are not good enough to find a problem with the model but simply define the level of accuracy to which the model can be considered perfect (this is still useful if the measurements are better than all previous ones);

or (usually more interesting),

(b) the measurements are good enough to identify a problem.

In this practical you will look for defects in models of two circuits. The models are called equivalent circuits.

4 A resistor and capacitor circuit

The circuit you will be investigating is shown in figure 1. The 'Adapted Oscillator' voltage is taken from the divider chain on your board, marked 'A' in the diagram, and also marked 'A' on the circuit board.

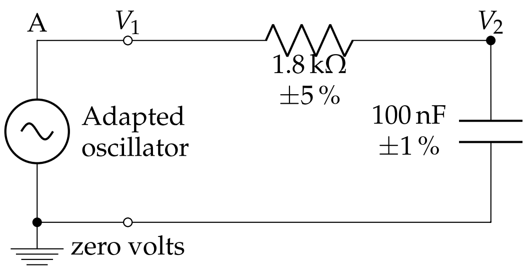


Figure 1: A simple RC oscillator circuit

4.1 The equivalent circuit and voltage transmittance

The assumed equivalent circuit (just a resistance and a capacitance) and the consequent predictions of the form of the complex voltage transmittance

$$T = V_2/V_1 = \frac{1}{1 + j\omega CR} \tag{1}$$

are set out in section 3.3.1 of your Manual.

▶ **Discuss any aspects that are unclear with a demonstrator.**

This expression says that the transmittance tends to 1 and becomes real (so no phase shift) as ω tends to zero, has a magnitude of $1/\sqrt{2}$ and a phase lag of $\pi/4$ at $\omega CR = 1$, and tends to zero with a phase lag of $\pi/2$ as ω tends to infinity.

4.2 A quick look at the behaviour of the circuit

> Remember that you are responsible for your own safety when working in the laboratory. Always talk to a demonstrator or technician if you are concerned that anything may be unsafe.

Disconnect the 'T' adapter from (**CH1**), leaving the other cables connected to the adapter. Connect up the voltage from the oscillator divider, the capacitor, the correct resistor, as shown in figure 1 above and use probes on (**CH1**) and (**CH2**) to measure v_1 and v_2 respectively.

Set both scope channels to DC, trigger from (**CH1**), and set the amplitude controls to 100 mV per division. On the AFG select a sinewave output, set the frequency to 30 Hz, and turn the voltage output up so you read around 500 mV peak to peak on the oscilloscope. You should see that the amplitudes of V_2 and V_1 are similar and that the phase shift between them is a small fraction of a period. See what happens as the frequency is raised to 1 kHz and then to 30 kHz. Sketch what you see. Measure as carefully as you can the phase difference at the frequency at which the magnitude of the transmittance is $\frac{1}{\sqrt{2}}$ using time differences between zero crossings (method (a) in the Manual section 8.4). If we define one complete cycle of a sinewave as 360°, then a phase difference is the number of degrees two sinewaves are displaced with respect to each other.

▶ **How do you tell that V_2 lags V_1?**

4.3 Testing the model

Your quick look should convince you that the expression for the complex transmittance is plausible. The test could be improved by taking more points and using the DMM rather than the oscilloscope to measure the magnitudes of the voltages. However, it would still be necessary to measure phase shifts for which the accuracy that can be achieved using the scope is relatively poor. To make a better test we need the model to be expressed in terms only of quantities we can measure with our most accurate instruments. These are the rms magnitudes of voltages, $|V_i|$, measured using the DMM, and the frequency obtained from the oscillator settings. Show that the transmittance expression can be cast into the form

$$|T|^2 = \frac{|V_2|^2}{|V_1|^2} = \frac{1}{1 + 4\pi^2 C^2 R^2 f^2} \tag{2}$$

This is the relationship you will test.

4.4 Gathering data

Disconnect the (**CH2**) scope probe to eliminate its effect on the circuit (although in practice it is negligible in this case). It is good practice to monitor the oscillator output so keep (**CH1**) connected. Set your DMM to the 2 V ac range and connect it to measure $|V_1|$. In your logbook create a data table with three columns; frequency f, $|V_1|$ and $|V_2|$. Set $|V_1|$ to about 180 mV at 100 Hz and measure its actual value at frequencies of 100 Hz, 160 Hz, 260 Hz, 360 Hz, 500 Hz, 700 Hz, 1 kHz, 1.6 kHz, 2.6 kHz, 3.6 kHz, 5.0 kHz, 7.0 kHz and 10 kHz (Make sure the DMM display has settled before recording the voltages.) Then measure $|V_2|$ at the same frequencies.

4.5 Entering your data into Python and deriving a new variable

Go to a computer and launch Python via Jupyter Notebooks or Spyder. Create a new directory `EL02` and save your raw frequency and voltage data (in millivolts) into a file `RC.csv`, for example by typing the data into an Excel spreadsheet with suitable column headings and then saving it in `.csv` format. The result should look something like:

```
freq,   mv1,   mv2
100, 139.6, 138.5
160, 139.5, 127.1
...
```

Read the data from `RC.csv` using a method you learned in lab DA01. One way is to type:

```
import numpy as np
import pandas as pd
df = pd.read_csv("RC.csv")
```

Before you can use the data you need to assign columns of the data to variables using the command (for example for frequency):

```
f = df.freq.values
```

where `df` is the dataset name and `freq` is the relevant column name. Repeat this for all the variables.

Plot $V1$ and $V2$ against f as a quick check that you have entered the data correctly; remember the command is

```
import matplotlib.pyplot as plt
plt.scatter(x,y)
```

Now derive from your raw data values of a new variable, the magnitude of the transmittance squared with the command

```
T2 = (mv2/mv1)**2
```

and plot T^2 against f. It is best to use a semi-log plot by using the command `plt.xscale('log')` before using the final plot command (for example, `plt.show()`). Give your plot a title and label the axes appropriately. Save the graph.

4.6 Errors

The design of the oscillator is such that the truncation error in a frequency setting is negligible (a setting of 1000 Hz can be considered to be a setting of 1000.000 Hz) so the error in the frequency is just the calibration error of 0.002 % which is a systematic error. Fitting routines generally deal with random errors only so we will ask you to allow for the calibration error in some other way later.

When the transmittances are calculated the calibration errors cancel out leaving just the truncation errors which are random. The errors in the T2 values can then be derived using the rules to combine errors given in AD26 — *error guide in first year electronics*[3] (see if you can understand why):

$$\sigma(V_2^2/V_1^2) = 2\sigma_T \frac{V_2^2}{V_1^2}\sqrt{\frac{1}{V_1^2} + \frac{1}{V_2^2}}, \text{ where } \sigma_T = \frac{(LSB)}{\sqrt{12}} \text{ is the LSB truncation error.} \tag{3}$$

[3]`www-teaching.physics.ox.ac.uk/practical_course/scripts/srv/local/rscripts/trunk/Admin/AD26/AD26.pdf`

This can be recast as

$$\sigma(V_2^2/V_1^2) = 2\sigma_T\sqrt{\frac{V_2^4}{V_1^6} + \frac{V_2^2}{V_1^4}}, \tag{4}$$

and evaluated with the command:

```
dT2=(2/(12**0.5))*(mv2**4/mv1**6+mv2**2/mv1**4)**0.5)
```

where the LSB has been assumed to be 1 mV.

There are various other ways of writing this, which you could use if you prefer.

4.7 Fitting

The equivalent circuit predicts that

$$T^2 = \frac{1}{a + cf^2}, \text{ where } a = 1 \text{ and } c = 4\pi^2C^2R^2 \tag{5}$$

You can fit your data to equation 5 using the following commands:

```
from scipy.optimize import curve_fit
def RCfit1(f,a,c)):       # define the function to fit to
     return 1/(a+c*f**2)
p0=1,1.3e-6              # define the starting values of the fitted variables
popt,pcov=curve_fit(RCfit1, f, T2, p0, sigma=dT2)
```

The fit is assisted by giving starting values for the parameters `a` and `c`; we have chosen the starting values using theory and the nominal component values. The weights are the error values `dT2`. The values of the parameters that give the best fit can be displayed with the commands:

```
print('a:  ' + str(popt[0]) + 'c:  ' + str(popt[1]))
print('Errors (a, c):  ', str(np.sqrt(pcov.diagonal())))
```

and confidence intervals can be found with

```
print(popt+1.959964*np.sqrt(pcov.diagonal()))
print(popt-1.959964*np.sqrt(pcov.diagonal()))
```

These are pairs of values between which there is a 95 % probability that the actual value lies.

You will need a copy of these parameters for your logbook, but do not print it yet: wait until you have all the parameters and you can cut and paste the output into a single file for printing. You can also combine multiple graphs into a single plot by using the command `plt.subplot(***)` before the `plt.scatter()` command in Python: `plt.subplot(221)` will draw a 2x2 grid with the graph to be plotted in the top left; `plt.subplot(224)` places the graph in the bottom right of a 2x2 grid; `plt.subplot (211)` puts the graphs in a 2x1 grid, with the graph at the top.

Next it is useful to overlay the fitted curve onto the experimental data. To do this you need to calculate predicted y values, and then plot these values as a smooth line. The line will look smoother if you calculate the predicted values for more points than you have data, and this can be done using the `arange` function from `numpy`, where you specify the starting value, end value, and step size:

```
newx=np.arange(100,10000,1)
yfit=1/(popt[0]+popt[1]*newx**2)
plt.plot(newx,yfit,'r',label='fit')
```

You should find that the line *appears* to pass exactly through the experimental points, with the residuals being too small to see. To examine the residuals it is necessary to extract them from the fit and plot them directly:

```
fitatx=RCfit1(f,*popt)
res=T2-fitatx
plt.scatter(f,res,marker='o',label='data')
```

Remember to use a logarithmic scale on the x-axis. You may need to adjust the limits on the y-axis using the command `plt.ylim(min,max)` to zoom in on the data. Add a title and axis labels as before, and then save and print out both plots.

This procedure finds the best values of the parameters assuming the model relationship. This is not quite what we want. Better would be to set limits on departures from the model relationship. You can begin to explore this by fitting to

$$y = \frac{1}{a + bf + cf^2} \tag{6}$$

giving a starting value of 10^{-6} for b. Try other fits, including different small deviations from the ideal model, noting that in some cases the fit or the calculation of confidence intervals may not succeed.

▶ **What do the data and residuals plots tell you?**

▶ **Are the values of a consistent with the theoretical value of 1?**

▶ **How might the calibration error in the frequency settings be allowed for?**

▶ **Are the fitted values of c consistent with the value of $4\pi^2C^2R^2$ derived from your measurements of the component values? (The percentage error in $\pi^2C^2R^2$ is the sum of twice the percentage error in C and twice the percentage error in R.)**

▶ **Note that the input of the DMM consists of a $10\,\mathrm{M\Omega}$ resistance in parallel with a $100\,\mathrm{pF}$ capacitance. Do these need to be allowed for and if so how?**

If the model's parameter values are consistent with the theoretical values and the uncertainties in any additional parameters are much greater than their fitted values your measurements have not been able to identify a problem with the model. All you can say is that over the range of frequencies investigated, and with the measurement accuracy achieved, the assumed equivalent circuit is 'ideal'.

When you have finished, log off to free up the machine — they are in demand.

5 An inductor and capacitor circuit

This circuit exhibits a resonance and can be used to select a narrow range of frequencies (it behaves as a bandpass filter). Its investigation will follow broadly the same pattern as that in section 4.

5.1 The circuit

The circuit connected to the signal generator consists of a capacitor, an inductor, and a resistor (nominally $47\,\Omega$) in series. In our model we assume that the capacitor and the resistor are perfect (i.e., can be represented by just a capacitance and a resistance) and that all the imperfections in the inductor can be represented by a resistance r in series with its inductance L. The equivalent circuit is therefore as shown in figure 2. (The effects of connecting the measuring instruments have been ignored, and the resistor shown immediately above the oscillator in the diagram represents the output impedance of the adapted oscillator, as explained in section 2, which is about 4.5 Ω.)

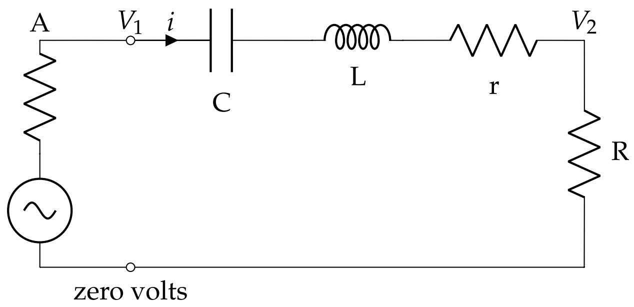


Figure 2: A simple LC circuit

Use the method of combining impedances and the potential divider formula to write down an expression for the complex voltage transmittance $T_v = V_2/V_1$.

Hence, writing $1/(LC) = 4\pi^2 f_0^2$ show that T_v is given by

$$T_v = \frac{V_2}{V_1} = \frac{1}{1 + \dfrac{r}{R} + \dfrac{j}{2\pi f_0 CR}\left(\dfrac{f}{f_0} - \dfrac{f_0}{f}\right)} \tag{7}$$

which predicts that T_v has a maximum value of $R/(R+r)$ and is real (no phase shift) at $f = f_0$, tends to zero with a phase lead of $\pi/2$ at low frequencies, and tends to zero with a phase lag of $\pi/2$ at high frequencies.

5.2 Quick look at the behaviour

Connect up the circuit and attach the scope channels to display V_1 and V_2 . Trigger from V_1. Have a look at how the amplitude of V_2 and its phase shift from V_1 vary with frequency at and around the peak in the transmittance which occurs between 800 Hz and 1 kHz. Measure carefully and record the frequency at which the peak in the transmittance occurs.

▶ **Write a short account of what you see.**

5.3 Deriving an expression to test

From the complex transmittance derive the expression:

$$\frac{|V_2|^2}{|V_1|^2} = \frac{1}{\left(1 + \dfrac{r}{R}\right)^2 + \dfrac{1}{(2\pi f_0 CR)^2}\left(\dfrac{f^2}{f_0^2} + \dfrac{f_0^2}{f^2} - 2\right)} \tag{8}$$

which can be written as

$$y = \frac{1}{(1+c)^2 + \dfrac{b^2}{a^2}\left(\dfrac{f^2}{a^2} + \dfrac{a^2}{f^2} - 2\right)} \qquad \text{where } a = f_0,\ b = 1/(2\pi CR) \text{ and } c = r/R\,. \tag{9}$$

This is the expression you will test.

5.4 Gathering and entering data

Measure the values of R and r with your DMM, remembering to check also the reading with the DMM leads connected together. In the frequency column of your data table create a series of about 25 frequencies in ascending order between 500 and 2000 Hz with one at the frequency of the peak, five on either side of the peak with a spacing increasing from 2 Hz to 4 Hz, and the rest with a gradually increasing spacing as the frequencies get further from the peak.

As for the transmission measurements in section 4.4 use a $|V_1|$ of about 180 mV rms. Without adjusting the amplitude setting on the oscillator record the actual values of $|V1|$ at your chosen frequencies. Give the DMM readings time to settle. Then move the DMM leads to measure $|V_2|$ at each frequency.

Log on to Jupyter Notebook or Spyder again and create a new datafile named `LCR.csv`. Enter your data using the same procedure and the same names as in section 4.5.

5.5 Plotting data and fitting to the model

Plot the experimental data as before, and fit it using the commands:

```
def RCfit2(f,a,b,c)):        # define the function to fit to
return 1/((1+c)**2+(b/a)**2*((f/a)**2+(a/f)**2-2))
p0=1,1,1 # define the starting values of the fitted variables, set to something suitable
popt,pcov=curve_fit(RCfit2, f, T2, p0, sigma=dT2)
```

Starting values for a can be taken from the frequency of the maximum transmittance in your raw data, for b can be obtained from the nominal component values, and for c from the resistance measurements.

Set up a parameter table and a residuals plot as in section 4.

5.6 Discussion

Considering your measured values of C, and R, and the fitted parameter b, examine the consistency of the relation:

$$2\pi b = \frac{1}{RC} \tag{10}$$

Derive a value of L from $L = \frac{1}{4\pi^2 a^2 C}$. Comment on the residuals plot.

6 Comparison with the results from EL01

In your study of the RC circuit you have seen that the transient behaviour (which we have called the time domain view) and the steady state behaviour (the frequency domain view) are related by a time constant $R_{total}C$.

The views of the LCR circuit are related by a resonant frequency and a loss resistance R_{total}. In practice resonances are often discussed in terms of a quantity

$$Q = \frac{\omega_0 L}{R_{total}} = \frac{1}{\omega_0 C R_{total}} \tag{11}$$

Q characterises the decay of oscillations following excitation by a step voltage, and the width of the peak in a plot of the magnitude of the transmittance against frequency when the circuit is excited by a harmonic voltage. The higher the Q, the slower is the decay and the narrower the peak. Q is called the “quality factor” because it is a measure of the sharpness of the resonance, and at one time it was important to be able to make narrow band filters using this circuit.

It is convenient to study resonance in circuits, but resonance is important throughout physics, not just in electronics. A system-independent definition of Q can be given as

$$Q = \frac{\text{resonant angular frequency} \times \text{stored energy}}{\text{average power dissipation}} \tag{12}$$

It is easy to see what this becomes in the special case of an LCR circuit. When the current has its peak value I_{max} the stored energy is all in the inductor and is $\frac{1}{2}LI_{max}^2$. The average power dissipation in the inductor is $\frac{1}{2}I_{max}^2 R_{total}$, so Q is $\frac{\omega_0 L}{R_{total}}$, in agreement with our earlier definition.

Compare your values for L and r with the best values you obtained in EL01.

▶ **Write a brief summary of what you have learned from this practical and discuss it with a demonstrator.**

Don't forget to obtain a copy of the script for EL03 — *introduction to digital data processing circuits*[4] so that you can do the theory exercises before you come back to the laboratory.

[4] `www-teaching.physics.ox.ac.uk/practical_course/scripts/srv/local/rscripts/trunk/Electronics/EL03/EL03.pdf`