Physics Education COMPOUND PENDULUM

COMPOUND PENDULUM

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Abstract

This is an edited version of the experiment set for the experimental examination conducted at the orientation cum selection camp held at Homi Bhabha Centre for Science Education (TIFR), Mumbai in May 2010. Generally, the compound pendulum studied in undergraduate laboratory is in the form of a uniform bar whose axis of oscillation is varied. In this experiment, a compound pendulum with a fixed axis of oscillation but with a movable mass is used to study the dependence of periodic time on the position of the movable mass and to determine the gravitational field strength.

1. Introduction

The compound pendulum provided for this experiment consists of a rod with a fixed knife-edge, which acts as the axis of oscillation of the oscillating pendulum somewhere along its length. A cylindrical body of mass m_1 is used which can be moved along the length of the rod. Another cylindrical body of mass m_2 is fixed at the lower end of the rod. A plastic washer is used to support the mass m_1 at various positions on the rod.

When the pendulum is suspended with its knife-edge on a rigid platform and set into oscillation, its periodic time of oscillation changes depending on the position of the movable mass. The experiment consists of studying the relationship of the distance of the movable mass from the axis of oscillation with the periodic time of the pendulum.

2. APPARATUS

1) A compound pendulum consisting of a rod with one mass attached at one of its ends, another mass capable of sliding along the rod and a knife edge to be fixed on the rod,
2) An Allen key,
3) A plastic washer for supporting movable mass,
4) An acrylic support with fixed glass slides on which the knife edge is to rest,
5) A G-clamp for clamping the acrylic support to the edge of the table,
6) A stopwatch,
7) A measuring tape,
8) Vernier calipers and
9) A micrometer screw gauge.

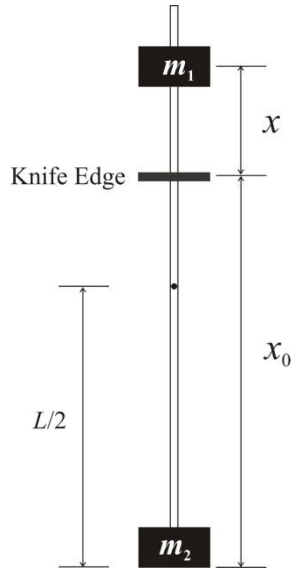


Figure 1.The compound pendulum

3. Description of apparatus:

Figure 2. A rod with a mass m_2 fixed at one end.

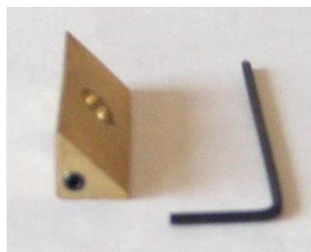


Figure 3. Knife Edge with Allen key

Figure 4. Acrylic support with glass slides fixed on it and the G-clamp

Figure 5. Stopwatch, measuring tape and vernier calipers and micrometer screw gauge

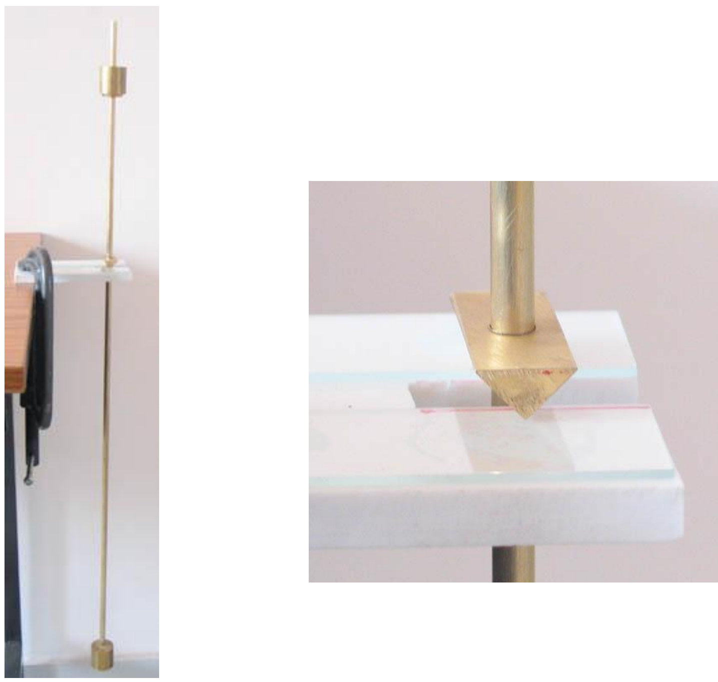


Figure 6. The complete setup

4. Theory

For small oscillations, the periodic time T of compound pendulum, with mass M and moment of inertia I about the axis of oscillation, is given to a good approximation by

$$T = \pi\sqrt{\frac{I}{Mgl}},$$

where, l is the distance between the axis of oscillation and the centre of mass of the pendulum.

5. Experiment

Given data:
Masses of
i) Rod: $m_r = 161 \pm 1$ g
ii) Knife-edge: $m_{ke} = 12 \pm 1$ g
iii) Bodies: $m_1 = m_2 = 99.5 \pm 0.5$ g

1. Express the distance l and the moment of inertia I about the axis of oscillation in terms of the distance x of the movable mass m_1 and other constants of the system.

Use the following symbols in your derivations.
Length of the rod: L.
Distance between the knife-edge and the end of the rod with fixed cylindrical body: x_0
Inner and outer radii of the cylindrical bodies: R_1 and R_2.
Length of the cylindrical bodies with masses m_1 and m_2: h.
Mass of the rod: m_r
Mass of the assembly of knife edge: m_{ke}.

2. Hence express the periodic time T of the pendulum as a function of x. If you have made any assumptions in neglecting any terms in the above derivations mention them with supporting arguments.

3. Make the necessary measurements of physical dimensions of the system forming the compound pendulum. You may use scales, vernier calipers and micrometer screw as required. Tabulate the measured values along with the uncertainties in measurements.

4. Suspend the pendulum from the rigid support and determine its periodic time for different positions of the movable mass by moving it from the top of the rod to the fixed mass at the bottom in suitable steps. [For moving the mass below the knife edge remove the knife edge using the Allen key and after shifting the mass below its position fix it again.] Tabulate your results.

5. Sketch graphs (rough sketches on the plain answer sheet) to show how I and l vary as the mass m_1 is shifted from one end to the other. Plot T versus x and explain the significance of the minimum T in this graph.

6. Reorganize the terms in the equation of T as a function of x and plot a linear graph from which g can be obtained. Determine the slope of the graph, calculate g and estimate the uncertainty in the obtained value.

7. Obtain from the graph the value of T at $x = 0$. Determine the value of g using the formula for T with $x = 0$.

8. If the movable mass is kept at the top end of the pendulum and you are allowed to move the axis of suspension, will it be possible to make T infinite? Explain the conditions under which this is possible. Will it be possible to achieve the condition experimentally? Substantiate your answer with reasons, if necessary.

6. TYPICAL OBSERVATIONS AND CALCULATIONS

1)
<u>Equation for ***I***</u>:

$$I = \frac{m_r}{12}L^2 + m_r\left(x_0 - \frac{L}{2}\right)^2 + \frac{m_1 h^2}{12} + \frac{m_1}{4}\left(R_1^2 + R_2^2\right) + m_1 x^2 + \frac{m_2 h^2}{12} + \frac{m_2}{4}\left(R_1^2 + R_2^2\right)$$
$$+ m_2\left(x_0 - \frac{h}{2}\right)^2 + I_{ke} \qquad (1)$$

<u>Equation for *l*</u>:

$$Ml = m_r\left(x_0 - \frac{L}{2}\right) + m_2\left(x_0 - \frac{h}{2}\right) + m_1 x - m_{ke} \times x_{ke}$$

$$\therefore l = \frac{1}{M}\left[m_r\left(x_0 - \frac{L}{2}\right) + m_2\left(x_0 - \frac{h}{2}\right) + m_1 x - m_{ke} \times x_{ke}\right] \qquad (2)$$

Here, $M = m_r + m_1 + m_2 + m_{ke}$

We can neglect the terms I_{ke} in equation (1) and $m_{ke}\, x_{ke}$ in equation (2) because they would be very small.

2.

$$T = 2\pi\sqrt{\frac{I}{Mgl}}$$

$$T^2 l = 4\pi^2 \frac{I}{Mg}$$

Reorganizing the terms in equation (1) we can write

$$I = m_1 x^2 + \left[m_r\left(\frac{L^2}{12} + \left(x_0 - \frac{L}{2}\right)^2\right) + \frac{(m_1 + m_2)h^2}{12} + \frac{(m_1 + m_2)}{4}\left(R_1^2 + R_2^2\right) + m_2\left(x_0 - \frac{h}{2}\right)^2\right]$$

The terms in the bracket are constant. Representing the constant by A,

$$I = m_1 x^2 + A$$

We can reorganize the terms in equation (2) as

$$l = \frac{m_1}{M}x + \frac{1}{M}\left[m_r\left(x_0 - \frac{L}{2}\right) + m_2\left(x_0 - \frac{h}{2}\right)\right]$$

Again, representing the terms in bracket by a constant B,

$$l = \frac{m_1}{M}x + B$$

$$\therefore T^2\left(B + \frac{m_1}{M}x\right) = \frac{4\pi^2}{Mg}\left(m_1 x^2 + A\right)$$

3.

Measurements of physical dimensions of the system:

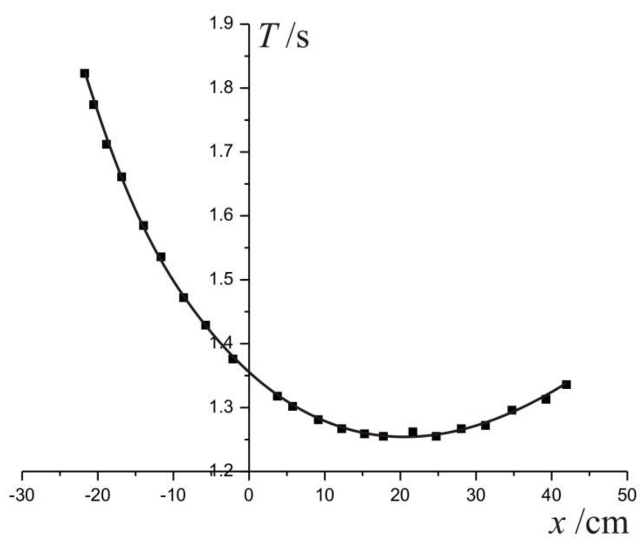
Quantity	Value	Uncertainty
L	69.0 cm	0.1 cm
h	2.500 cm	0.002 cm
x_0	45.9 cm	0.1 cm
R_1	1.265 cm	0.001 cm
R_2	0.300 cm	0.001 cm
Diameter of the rod	0.588 cm	0.002 cm

4.

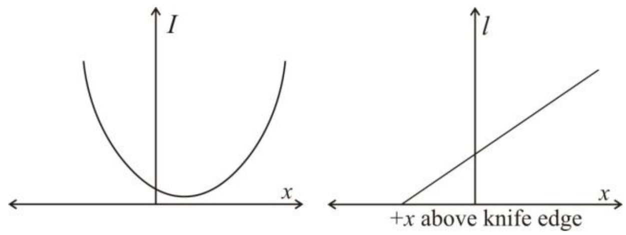
Obs. No.	$(x \pm \Delta x)$/cm	$x + \frac{h}{2}$ /cm	Time for 20 oscillations				T /s
			t_1 /s	t_2 /s	t_3 /s	Mean t /s	
1	-21.75 ± 0.10	-23.0	36.44	36.56	36.40	36.467	1.823 ± 0.008
2	-20.55 ± 0.10	-21.8	35.50	35.56	35.38	35.480	1.774 ± 0.009
3	-18.85 ± 0.10	-20.1	34.22	34.22	34.28	34.240	1.712 ± 0.003
4	-16.85 ± 0.10	-18.1	33.22	33.19	33.21	33.207	1.661 ± 0.002
5	-13.95 ± 0.10	-15.2	31.72	31.75	31.62	31.697	1.585 ± 0.006
6	-11.65 ± 0.10	-12.9	30.69	30.78	30.68	30.717	1.536 ± 0.005
7	-8.65 ± 0.10	-9.9	29.41	29.47	29.43	29.437	1.472 ± 0.003
8	-5.75 ± 0.10	-7.0	28.62	28.47	28.62	28.570	1.429 ± 0.008
9	-2.15 ± 0.10	-3.4	27.53	27.53	27.47	27.510	1.376 ± 0.003
10	3.75 ± 0.10	5.0	26.32	26.47	26.25	26.347	1.318 ± 0.011
11	5.75 ± 0.10	7.0	26.06	26.03	26.00	26.030	1.302 ± 0.003
12	9.15 ± 0.10	10.4	25.62	25.62	25.59	25.610	1.281 ± 0.002
13	12.25 ± 0.10	13.5	25.37	25.37	25.28	25.340	1.267 ± 0.004
14	15.25 ± 0.10	16.5	25.16	25.25	25.16	25.190	1.259 ± 0.004
15	17.75 ± 0.10	19.0	25.00	25.07	25.19	25.087	1.255 ± 0.010
16	21.65 ± 0.10	22.9	25.12	25.28	25.28	25.227	1.262 ± 0.008
17	24.75 ± 0.10	26.0	25.13	25.06	25.06	25.083	1.255 ± 0.004
18	28.05 ± 0.10	29.3	25.37	25.28	25.37	25.340	1.267 ± 0.004
19	31.25 ± 0.10	32.5	25.41	25.50	25.40	25.437	1.272 ± 0.005
20	34.75 ± 0.10	36.0	26.00	25.84	25.89	25.910	1.296 ± 0.008
21	39.25 ± 0.10	40.5	26.28	26.25	26.25	26.260	1.313 ± 0.002
22	41.95 ± 0.10	43.2	26.69	26.68	26.78	26.717	1.336 ± 0.005

5.

a. Plot *T* versus *x*.



b. Rough Sketches:



6.

$$\therefore T^2\left(\frac{m_1}{M}x + B\right) = \frac{4\pi^2}{Mg}\left(m_1 x^2 + A\right)$$

Plot a graph of $\therefore T^2\left(\frac{m_1}{M}x + B\right)$ versus x^2.

Here,

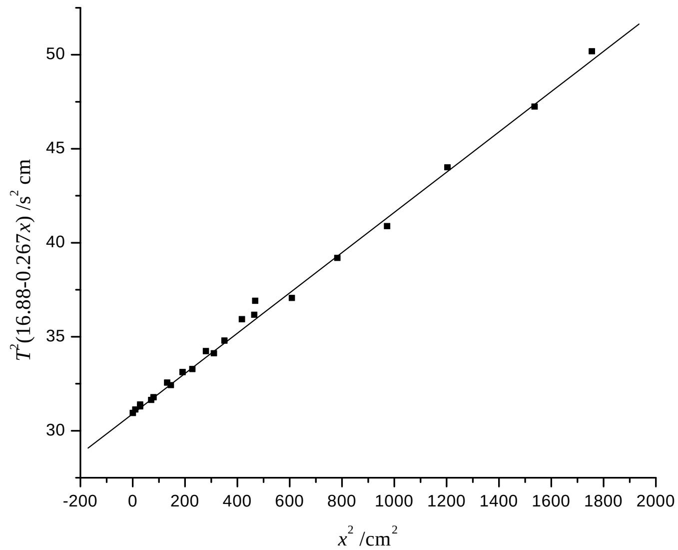
$$B = \frac{1}{M}\left[m_r\left(x_0 - \frac{L}{2}\right) + m_2\left(x_0 - \frac{h}{2}\right)\right] = \frac{1}{372}\left[161\left(45.9 - \frac{69}{2}\right) + 99.5\left(45.9 - \frac{2.5}{2}\right)\right]$$

$$= \frac{1}{372}\left[1835.4 + 4442.7\right] = 16.88$$

Hence, plot a graph of $T^2\left(16.88 + 0.267x\right)$ versus x^2.

	$T^2(16.88 + 0.267x)$/cm s^2	x^2/cm^2
1	36.84	473
2	35.86	422
3	34.72	355
4	34.16	284
5	33.05	195
6	32.49	136
7	31.57	75
8	31.33	33
9	30.87	5
10	31.06	14
11	31.22	33
12	31.71	84
13	32.35	150
14	33.21	233
15	34.05	315
16	36.09	469
17	36.99	613
18	39.12	787
19	40.81	977
20	43.94	1208
21	47.17	1541
22	50.12	1760

Graph:



Slope = 0.0107

$$\text{Slope} = \frac{4\pi^2 m_1}{Mg} = 0.0107$$

$$g = \frac{4\pi^2 \times 99.5}{372 \times 0.0107} = 985.861 \ \text{dynes/g}$$

Uncertainty in g:

$$\frac{\Delta g}{g} = \sqrt{\left(\frac{\Delta m_1 / \sqrt{3}}{m_1}\right)^2 + \left(\frac{\Delta M / \sqrt{3}}{M}\right)^2 + \left(\frac{1}{\sqrt{3}} \frac{\Delta slope}{slope}\right)^2}$$

$$= \sqrt{\left(\frac{0.5 / \sqrt{3}}{99.5}\right)^2 + \left(\frac{1.6 / \sqrt{3}}{372}\right)^2 + \left(\frac{1}{\sqrt{3}} \times 0.0128\right)^2} = 0.00832$$

The expanded uncertainty

$$\Delta g = 0.00832 \times 985.861 \times 2 = 16.4 \cong 17 \ \text{dynes/g}$$

$$g = 986 \pm 17 \ \text{dynes/g}$$

7.

From the graph of T versus x:

$$T \text{ (at } x = 0) = 1.355 \text{ s}$$

$$\therefore T^2\left(B - \frac{m_1}{M} x\right) = \frac{4\pi^2}{Mg}\left(m_1 x^2 + A\right)$$

At $x = 0$

$$\therefore BT^2 = \frac{4\pi^2 A}{Mg}$$

Here

$$A = m_r\left(\frac{L^2}{12} + \left(x_0 - \frac{L}{2}\right)^2\right) + \frac{m_1 h^2}{6} + \frac{m_1}{2}\left(R_1^2 + R_2^2\right) + m_2\left(x_0 - \frac{h}{2}\right)^2$$

$$= 161\times\left(\frac{69^2}{12} + \left(45.9 - \frac{69}{2}\right)^2\right) + \frac{99.5\times 2.5^2}{6} + \frac{99.5}{2}\left(1.27^2 + 0.30^2\right) + 99.5\times\left(45.9 - \frac{2.5}{2}\right)^2$$

$$= 84800 + 104 + 85 + 198365 = 283354 \text{ g}\cdot\text{cm}^2$$

and

$$B = \frac{1}{M}\left[m_r\left(x_0 - \frac{L}{2}\right) + m_2\left(x_0 - \frac{h}{2}\right)\right] = \frac{1}{372}\left[161\left(45.9 - \frac{69}{2}\right) + 99.5\left(45.9 - \frac{2.5}{2}\right)\right]$$

$$= \frac{1}{372}\left[1835.4 + 4442.7\right] = 16.88$$

$$\therefore 16.88\times 1.355^2 = \frac{4\pi^2\times 283354}{372\,g}$$

$$g = 969 \text{ dynes/g or } 9.69 \text{ N/kg}$$

8.

If l is zero, the period will be infinite. This condition can be satisfied if the knife edge is moved and placed at the center of mass with both masses at the two ends of the rod. Another way can be to move the mass m_1 such that the centre of mass of the system coincides with the position of the knife edge.

In either of the case, when the pendulum is displaced from its equilibrium position, it will not return back to the equilibrium position as T is infinite. But the condition of unstable equilibrium will make it unrealizable experimentally.

7. Discussion

In the undergraduate laboratories, bar pendulum is a regular experiment. In that experiment, distance between the knife edge and centre of mass is varied in definite steps.

This experiment explores another way to study the compound pendulum by varying the position of centre of mass by shifting mass m_1 rather than the point of suspension.

The linearization technique in this experiment (to plot the suitable graph) requires a rearrangement of variables which itself is a skill to be developed by students.

8. Acknowledgements

We would like to thank Prof. D. A. Desai for his valuable guidance in the development of this experiment. We also wish to thank the students who helped us in standardizing the apparatus.