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High Precision Measurement of Moment of Inertia Based on Compound Pendulum

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Abstract. A new method and equipment based on compound pendulum was designed for measuring moment of inertia (MOI). The measurement principle of the system, being quite different from the traditional method which records the swing period of the compound pendulum, records the "Angle-time" curve and conducts a series of computation featuring phase-plane to obtain the MOI of the object being measured. The proposed method could effectively eliminate the friction moment's influence on the measurement results and enhance the precision up to 0.12%. The method is suitable for the heavy and large-scale objects, especially those with large length/diameter ratio.

Introduction

Moment of inertia (MOI) represents the measure of inertia of a rotating rigid body. It is of considerable significance in the design, manufacturing, and ballistic trajectory analyses of ammunitions. Torsion pendulum method and string pendulum method are two methods have been reported for measuring MOI precisely[1-16]. But, because both the methods require the axes of the measured ammunition being placed vertical to the horizontal surface; they are not suitable for large-scale ammunitions, especially those with large length/diameter ratio[17-18]. As we known, to crane, locate and fix a large-scale ammunition upright on a set of giant equipment is always a job of difficulty and danger. Therefore, the author promoted a novel measurement method based on compound pendulum which expected to acquire much more precise measurement results than the traditional compound pendulum method [19]. In this paper, we present the detailed equipment, experimental procedure and experimental results based on the method proposed in [19].

Dynamic Model of Compound Pendulum with Friction Moment

The compound pendulum is a rigid body swings around an fixed axes that is parallel to the horizontal surface, shown in Fig.1, in which C is the centroid of the rigid body, O is the intersection point of the fixed axes and the paper (for convenience, we can also use O to represent the fixed axes). We denote the pendulum's moment of inertia with respect to O by J, the mass of the pendulum by m, the distance from O to C by S, and the included angle between OC and OY by ϕ with anticlockwise direction as positive. OY is in the same direction as the gravity.

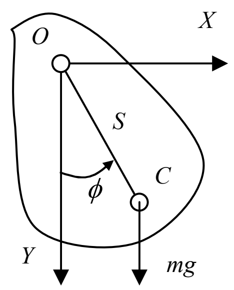

Fig. 1 A Compound Pendulum

We can describe the compound pendulum movement as the equation[19]

$$J \cdot \frac{\mathrm{d}^2\phi}{\mathrm{d}t^2} = -mgS\sin(\phi) - \mathrm{sgn}\left(\frac{\mathrm{d}\phi}{\mathrm{d}t}\right) \cdot M \tag{1}$$

in which, t represents the time, g the acceleration of gravity, M the friction moment of bearing. When a well-lubricated bearing works under medium load and medium rotate speed, M is proportional to the radial load of the bearing, so we have

$$M = \mu \cdot mg \cdot \cos(\phi) \tag{2}$$

in which, μ is a proportional coefficient, relating to the type and the pitch circle's diameter of the bearing. The friction moment trends to decelerate the compound pendulum movement, so we adopt "sgn", the sign function. Eq.1 is a nonlinear differential equation and it is difficult to solve in a direct manner, so we resort to phase-plain analysis. Define

$$x = \phi, \quad y = \frac{\mathrm{d}\phi}{\mathrm{d}t} \tag{3}$$

Substitute Eq.2 and Eq.3 into Eq.1, yields

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-a\sin(x) - b\cos(x)}{y} \text{ for } y \ge 0 \text{ and } \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-a\sin(x) + b\cos(x)}{y} \text{ for } y < 0 \tag{4}$$

Where $a = mgS/J$, $b = \mu mg / J$. Solve the ordinary differential Eq.4, we got

$$y = \sqrt{2\left(a\cos(x) - b\sin(x) + C_1\right)} \text{ for } y \ge 0 \tag{5}$$

$$y = \sqrt{2\left(a\cos(x) + b\sin(x) + C_2\right)} \text{ for } y < 0 \tag{6}$$

Where C_1 and C_2 are integral constants. Eq.5 and Eq.6 are the curves of the "phase plane".

On the experiment, we exert a moment onto the supporting tray(see Fig.2) and then release it suddenly. The deviation from the balance position makes the tray, together with the weights, swings back and forth; that is to say, they are making a compound pendulum movement. The angle encoder recodes the variation of angle with the time and the recorded data are stored in computer. With these data, the "angle-time" curve of the compound pendulum movement could be got. The phase plane curve could be calculated according to the "angle-time" curve (x_i, y_i). With this curve, we conduct following procedure:

(1) Let

$$\hat{y}_i = \sqrt{2\left(a\cos(x_i) - b\sin(x_i) + C_1\right)}$$

$$\Delta y_i = \hat{y}_i - y_i$$

$$Q = \sum \Delta y_i^2$$

(2) Solve the following equations set to get a, b and C_1,

$$\begin{cases} \partial Q/\partial a = 0 \\ \partial Q/\partial b = 0 \\ \partial Q/\partial C_1 = 0 \end{cases} \tag{7}$$

(3)We can obtain the moment of inertia of the compound pendulum

$$J = mgS/a \tag{8}$$

Experiments and Results

Fig.2 shows the mechanical structure of the experiment. The experiment includes the following fives conditions: none weigh(supporting tray only), 1 weight, 2 weights, 3 weights and 4 weights; Sampling rate of the data acquisition system was 100kHz. We use encoder RESM20USA115 made by Renishaw Co., readhead SR005-A, interface module Si-NN-0004-04-0-FN-403-003-3, and NI-6251PCI AD card. Fig.3 shows installation of the angle encoder. Fig.4 shows the work site of the experiment.

The analog signals output from the readhead are simple harmonic waves. Through the interface module, the analog signals are shaped into square wave ones, of which the period represents an interval between two reticules on the outer cylindrical surface of the encoder. By adding the number of the periods, we can get the swing angle of the compound pendulum. Fig.5 shows a part of the "Angle-Time" curve of a certain experiment. Instead of a continuous function, this curve is a step-like. The height between two steps represents the angle of 0.02°. In order to do the differential operation on the "Angle-Time" curve, we resort to polynomial fitting algorithm. Fig.6 is the fitted curve. Fig.7 is the difference between the fitted curve and its original one. Note: in order to illustrate clearer in this paper, Fig.5 and Fig.6 are not selected from the same experiment.

With the fitted "Angle-Time" curve and its derivative, we can easily get "Angle-Angular Rate" curve. This is the phase plane curve, which is shown in Fig.7. With the data on phase plane and Eq.7 and Eq. 8, we obtain the moment of inertia to be measured.

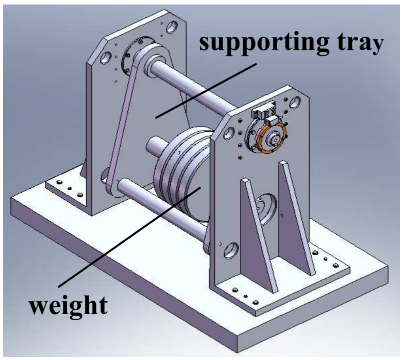

Fig. 2 Mechanical Structure of the Experiment

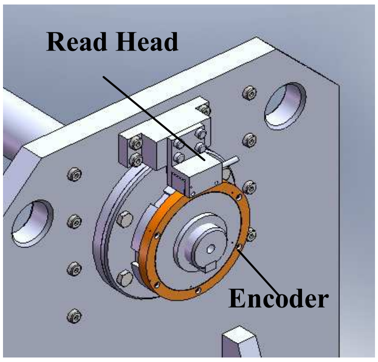

Fig. 3 Installation of the Angle Encoder

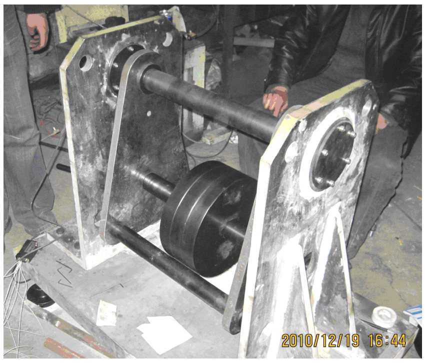

Fig. 4 Work Site of the the Experiment

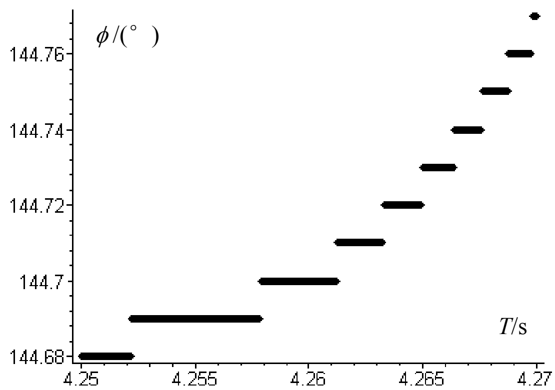

Fig. 5 Angle-Time Curve (Partial)

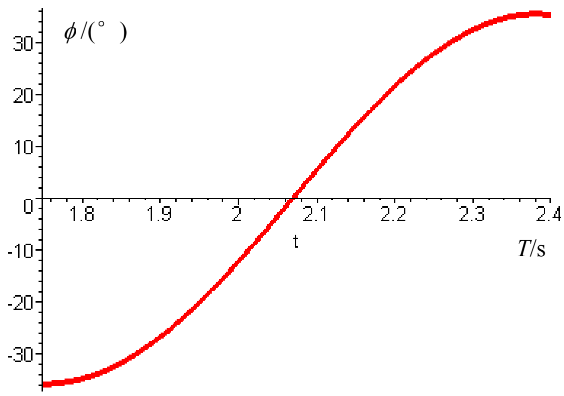

Fig. 6 Fitting Curve of the

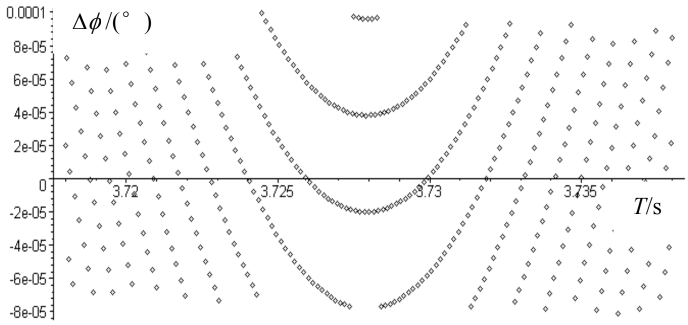

Fig. 7 Fitting Error (Partial)

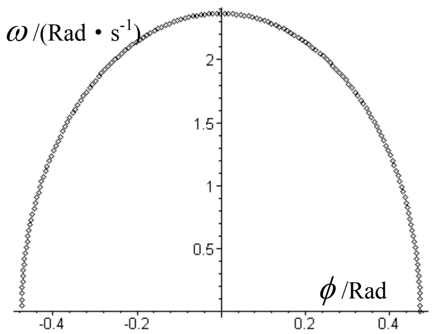

Fig. 8 Phase plane of one Compound Pendulum

Experiments were conducted measuring MOI of both the supporting tray and the weights by using the equipment shown in Fig.2. The comparison of the theoretical value and the experimental results is presented in Table 1.

Conclusions

The equipment based on compound pendulum was designed to measure MOI. The measurement principle of the system is quite different from the traditional compound pendulum based method which records the swing period. The system records the “Angle-time” curve and conducts a series of computation featuring phase-plane. The proposed method could effectively eliminate the friction moment’s influence on the measurement results and enhance the precision up to 0.12%. The method is suitable for the heavy and large-scale objects, especially those with large length/diameter ratio.

Table 1 The Theoretical Value and the Experimental Results [kg·m^2]

	None Weight	One Weight	Two Weights	Three Weights	Four Weights
Theoretical Value	81.53311	100.34332	119.15352	137.96373	156.77394
Test 1	81.62279	100.33685	119.20194	137.97841	156.91043
Test 2	81.61464	100.33791	119.09943	137.95746	156.76704
Test 3	81.46788	100.35210	119.16394	138.08444	156.91487
Test 4	81.52495	100.30172	119.25799	138.08619	156.93071
Test 5	81.55756	100.35339	119.16424	138.05893	156.96206
Relative Error (max)	0.11%	0.10%	0.09%	0.10%	0.12%

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