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Guanghua Cambridge School

Comparation of Two Models to Calculate the Period of Compound Pendulum with Liquid Damping

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GENERAL REVIEW

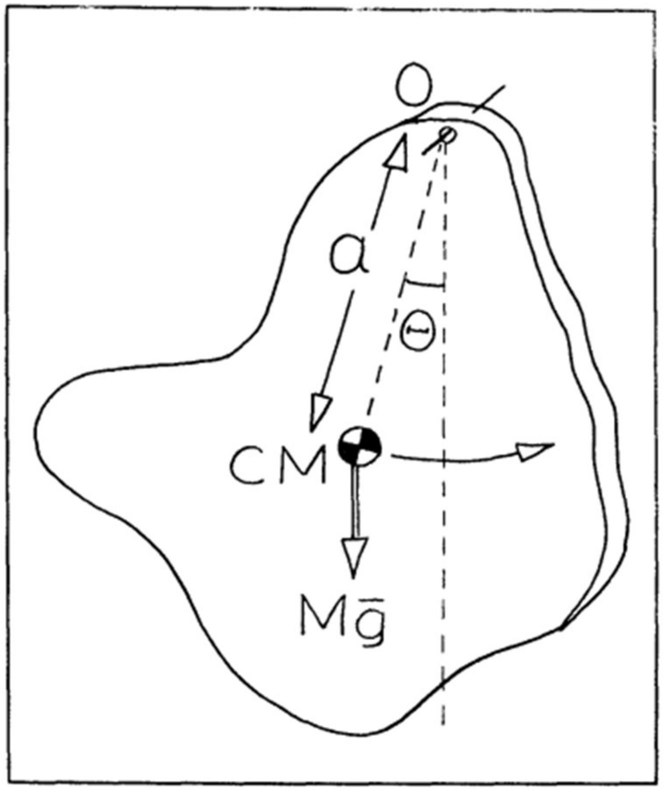

Fig. 1. For an extended object of moment of inertia $I = M(a^2 + k^2)$ about the pivot O, the period of small oscillation depends only on the distance a from the pivot to the center of mass, and the gyradius k.

Fig 1.

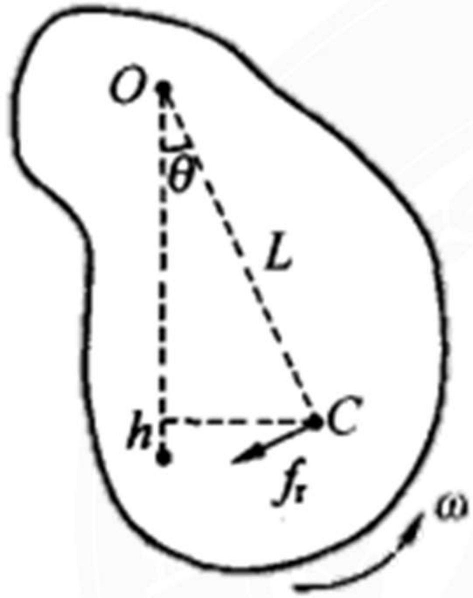

Fig 2.

← Compound Pendulum

GENERAL REVIEW

Methods:

- $f \propto v$ $\quad f = kR\omega = kR\dot{\theta}$ Proportional model
- $f \propto v^{2}$ $\quad f = kR^{2}\omega^{2} = kR^{2}\dot{\theta}^{2}$ Square model

THEORY DERIVATION

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$$T = 2\pi\sqrt{\frac{I^2}{IR(m-\rho V)g - 1/4\,(k^2R^4)}}$$

$$f = kv$$

$$T = 2\pi\sqrt{\frac{I^2 - 2kR^3\theta(I - kR^3)}{gR(m-\rho V)(I - 2kR^3\theta)}}$$

$$f = kv^2$$

ROUGH EXPERIMENT

$$I\ddot{\theta} = WR - fR - FR$$

1. $f = kR\omega = kR\dot{\theta}$

$$I\ddot{\theta} = (m - \rho V)gR\theta - kR^2\dot{\theta}$$

For $\theta \ll 5$, $\sin\theta \approx \theta$ [3]

里，ω_0＝0.7。显然，当幅角较小时，两者的振动周期、振幅都比较接近，当幅角更小时，复摆的振动可看作简谐振动。而实际的复摆，幅角小于5°的

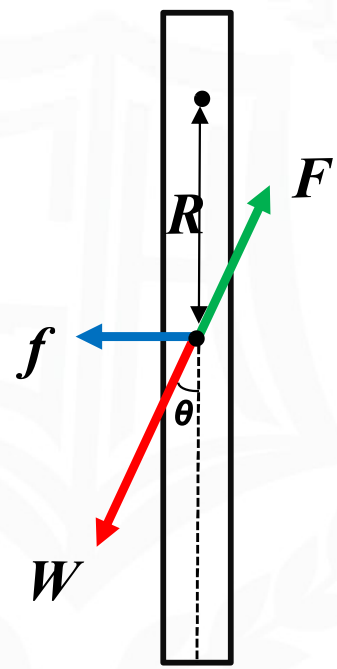

Fig 6.

ROUGH EXPERIMENT

Fig 7.

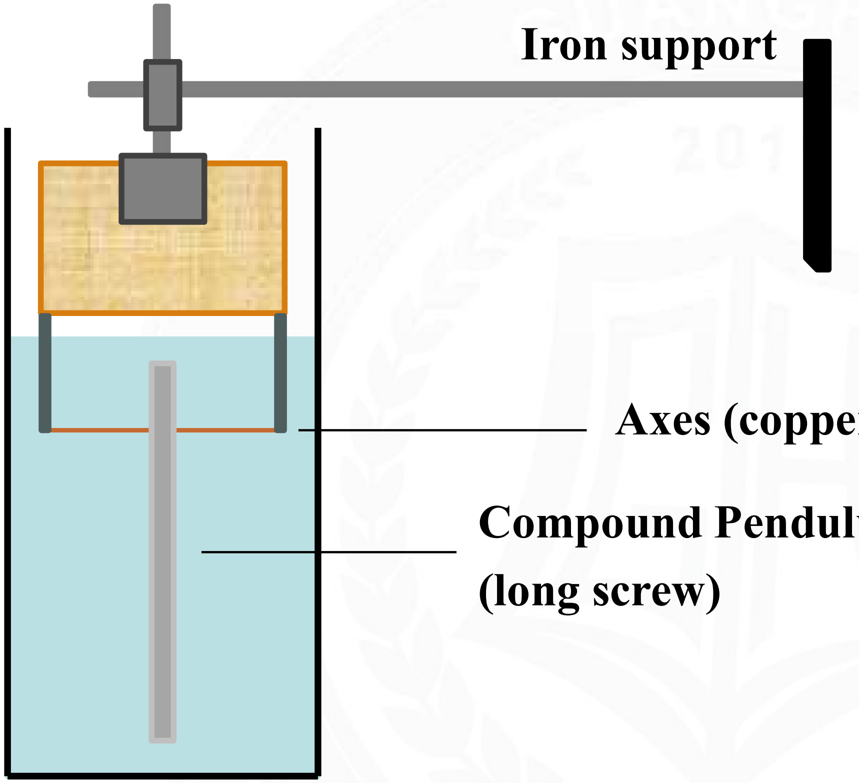

Fig 8.

Nov. 19th

THEORY DERIVATION

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2023.07-08:
Determine research topic, make the first step to the question, have a basic understanding

2023.09-10:
Collect references, work out the first version of formula about the pendulum

2023.11-12:
Study how to plot graph and how to meet the needs of experiments(e.g. python+3d design)

2023.12-24.01:
Design experiment, make theorical expectations, calculate various answers

2024.01-02:
Complete experiment, check if any mistakes, Draw graphs and compare the expectations with reality

2024.03-04:
Analyze data, finish the essay, submit to tutor for final revision

ROUGH EXPERIMENT DATA

Table 2. Data of rough experiments

No.	Frame/f	10T/s	T/s
1	740	6.167	0.614
2	736	6.133	
3	732	6.100	
4	738.5	6.154	

Variable	Value	Unit
k	0.84 ± 0.02	$kg\ s^{-1}$
I	0.60 ± 0.003	$g\ m^2$
m	36.0 ± 0.5	g
V	5.00 ± 0.05	ml
R	25.7 ± 0.1	mm

$$\ddot{\theta} + r_1\dot{\theta} + r_2\theta = 0$$

Let $\delta = \frac{C}{2m}$ for $m\ddot{x} + C\dot{x} + kx = 0$, and $\delta > 1$ is overdamping

$\delta = \frac{r_1}{2} = \frac{kR^2}{2I} = 0.15 < 1$ not over damped

ROUGH EXPERIMENT DATA

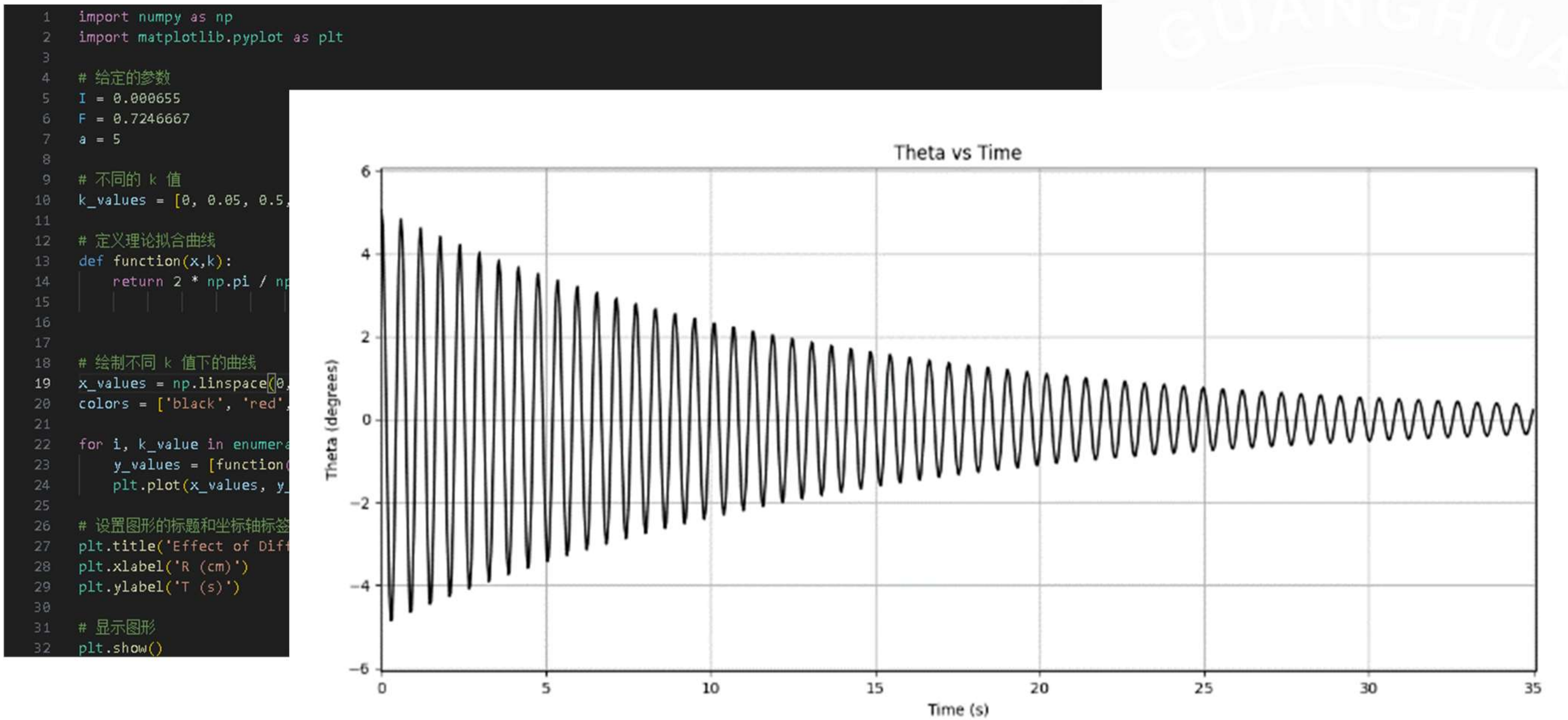

Fig 7. Curves of θ about t [python]

EXPERIMENT

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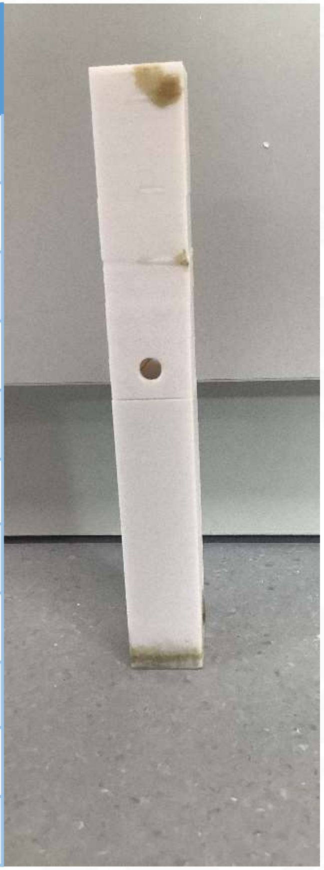
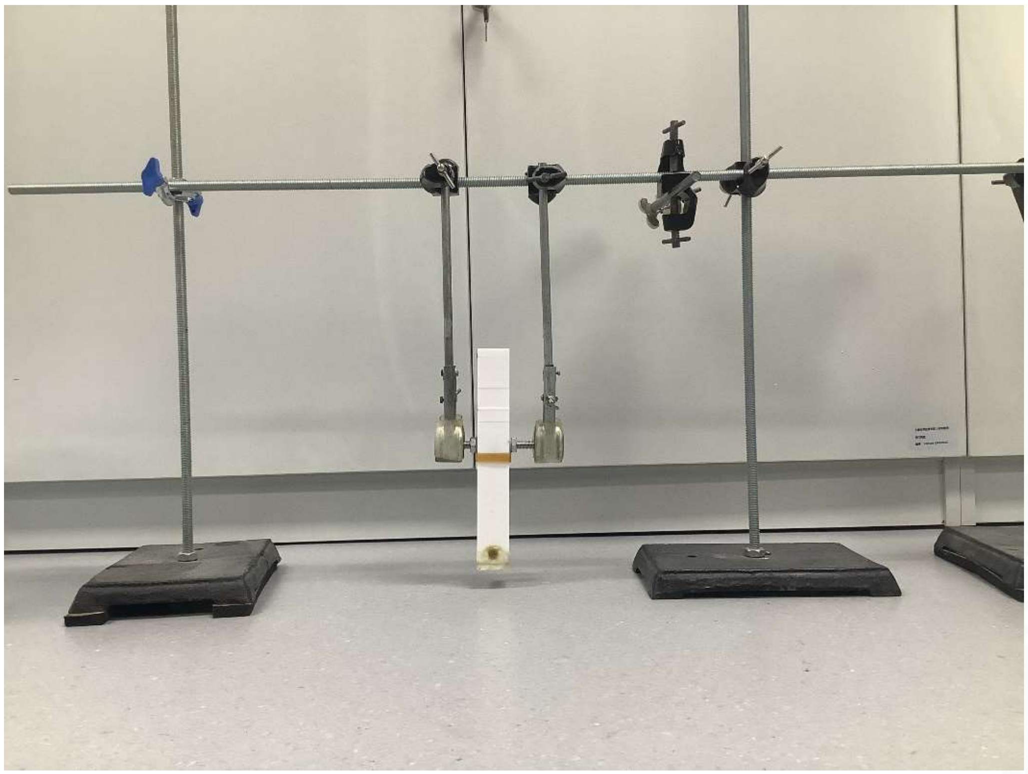
No.	Equipment Name	Device Model	Number
1	Mass block	2cm * 1cm in cross-section	9
2	Iron stand	100cm in length	4
3	Clamps	Iron with rubber	2
4	Rod	M8, 10cm in length	2
5	Bearing	10mm in inner diameter	2
6	Silicon gaskets	M8, 1mm in thickness	2
7	Nut	M8, hexagon	2
8	Rubber band	8cm in original length	1
9	Water bucket	20cm in diameter	1
10	Camera	iPhone 11 *0.125 in slow-motion	1
11	Vanier caliper	15cm max.	1

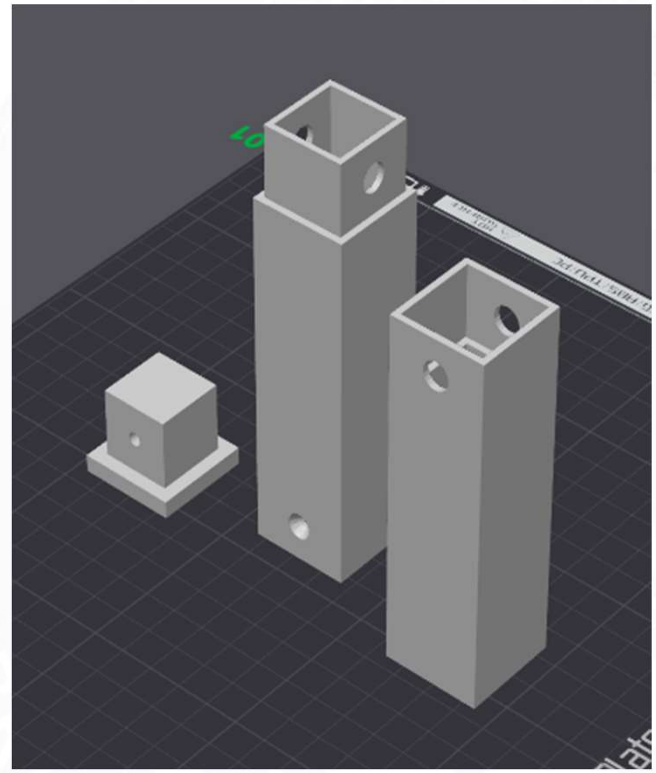
Table 2. Equipment list



Container overview

EXPERIMENT





b. Design display [blender] c. Container ready for experiments

Fig 3. Container and design diagram [blender]

EXPERIMENT

Fig 5. Clamps, water bucket, and equipment

EXPERIMENT

Fig 6. An overview

DATA

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No.	material	A/cm^2	h/cm	V/ml	m1/g	m2/g	m3/g	m/g	density
1	plastic	2.00	2.82	5.64	7.78	7.76	7.76	7.77	1.38
2		2.00	4.00	8.00	10.97	10.98	11.02	10.99	1.37
3		2.00	4.68	9.36	11.47	11.48	11.47	11.47	1.23
4	Al	2.00	3.34	6.68	16.63	16.63	16.63	16.63	2.49
5		2.00	4.32	8.64	22.10	22.08	22.08	22.09	2.56
6		2.00	5.34	10.68	27.64	27.64	27.62	27.63	2.59
7	Fe	2.00	3.14	6.28	46.26	46.25	46.27	46.26	7.37
8		2.00	4.48	8.96	61.89	61.90	61.90	61.90	6.91
9		2.00	5.28	10.56	77.92	77.95	77.95	77.94	7.38

Table 4. Table of masses

DATA

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No.	R/cm	F1/f	F2/f	F3/f	T1/s	T2/s	T3/s	T/s
1	7.96	290	292	284	1.208	1.217	1.183	1.203
2	7.67	226	232	244	0.941	0.965	1.017	0.974
3	5.76	192	209	199	0.802	0.872	0.828	0.834
4	5.00	465	425	448	1.939	1.772	1.866	1.859
5	4.28	227	223	223	0.946	0.929	0.929	0.935
6	3.92	222	219	222	0.925	0.913	0.925	0.921
7	3.50	107	107	105	0.448	0.446	0.438	0.444
8	3.13	258	252	246	1.075	1.050	1.025	1.050
9	2.41	318	295	301	1.325	1.229	1.254	1.269
10	1.50	361	368	378	1.503	1.533	1.575	1.537
11	1.05	439	451	451	1.831	1.879	1.879	1.863

Table 5. Table of Recordings

DATA

- Useful Data (blue dots)
- Wrong Data (red dots)
- Squared Model (purple line)
- Proportional Model (green dotted line)

$$k = 0.5\ kg\ s^{-1}$$
$$V = 185.0\ ml$$
$$L = 20.55\ cm$$
$$\theta = 5°$$

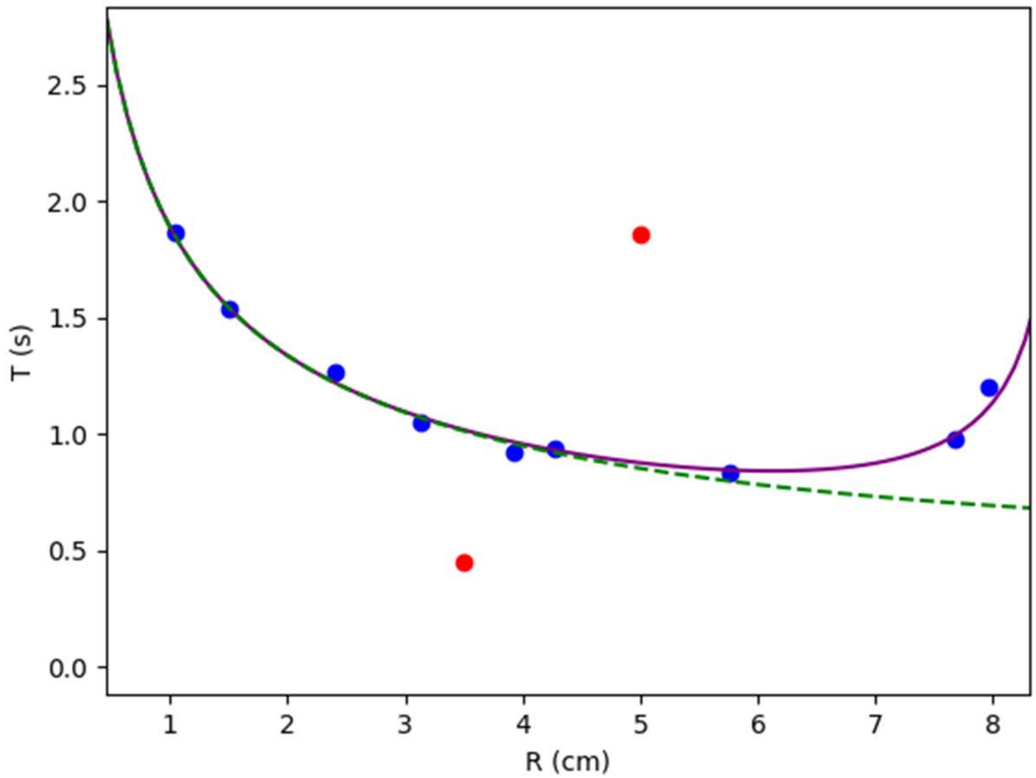


Fig 8. Curves of t about R [python]

ANALYSIS

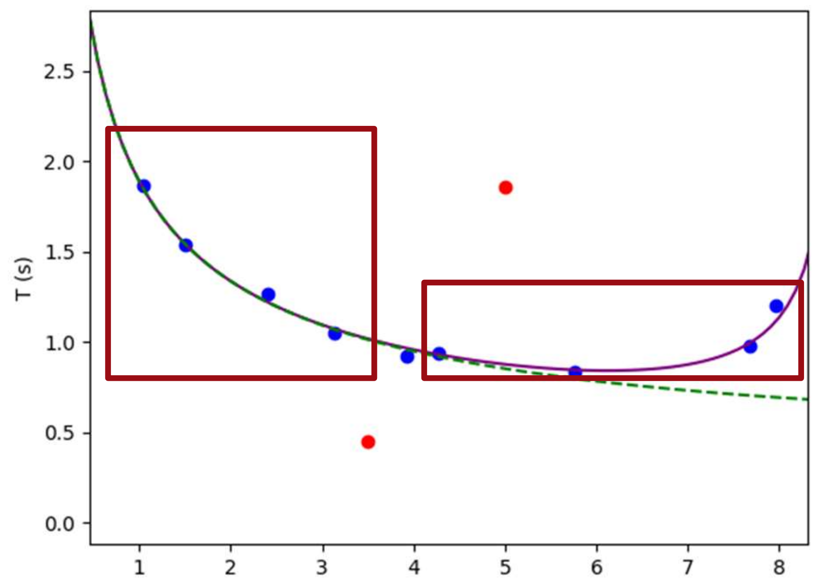

```
Variance of squared model: 0.000756898512469257З
Varience of proportional model: 0.00072711751З7054576
```

Fig 9. Log of the program using the first four sets of data [python]

```
Variance of squared model: 0.0003286635921692313
Varience of proportional model: 0.013896866472079114
```

Fig 10. Log of the program using the last four sets of data [python]

- **Small** radius of rotation
  - **Proportional** model fits better
- **Large** radius of rotation
  - **Square** model fits better



THEORATICAL ANALYSIS

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```
0.0007568985124692573
0.0007271175137054576
```

Fig 9. Log of the program using the first four sets of data [python]

Small radius of rotation

- Almost no difference in variance - 0.0003%

Does not **strongly support** the proportional model

- Further stimulations needed

THEORATICAL ANALYSIS

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$\boldsymbol{k = 0 kg\ s^{-1}}$

$\boldsymbol{k = 0.05 kg\ s^{-1}}$

$\boldsymbol{k = 0.5 kg\ s^{-1}}$

$\boldsymbol{k = 1.0 kg\ s^{-1}}$

$\boldsymbol{k = 2.0 kg\ s^{-1}}$

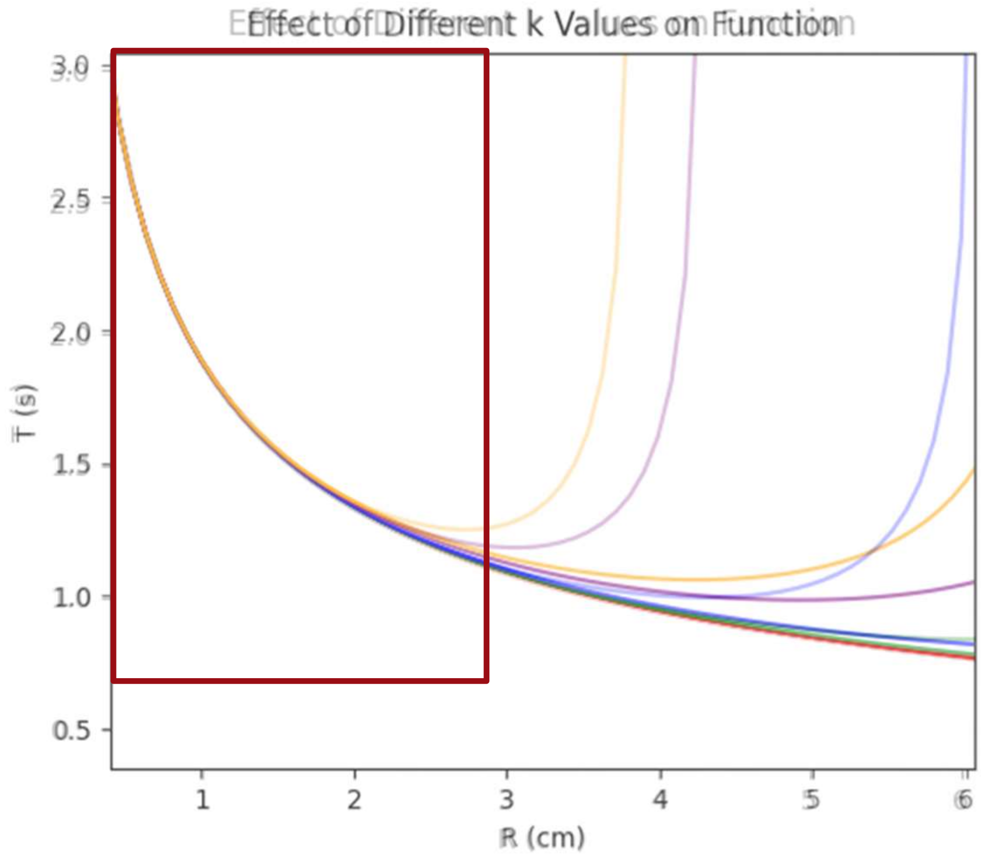
$\boldsymbol{k = 2.5 kg\ s^{-1}}$

$V = 184.95ml$

$m = 258.82g$

$\theta = 5°$

Fig 11. Effects of k on the period about radius [python]



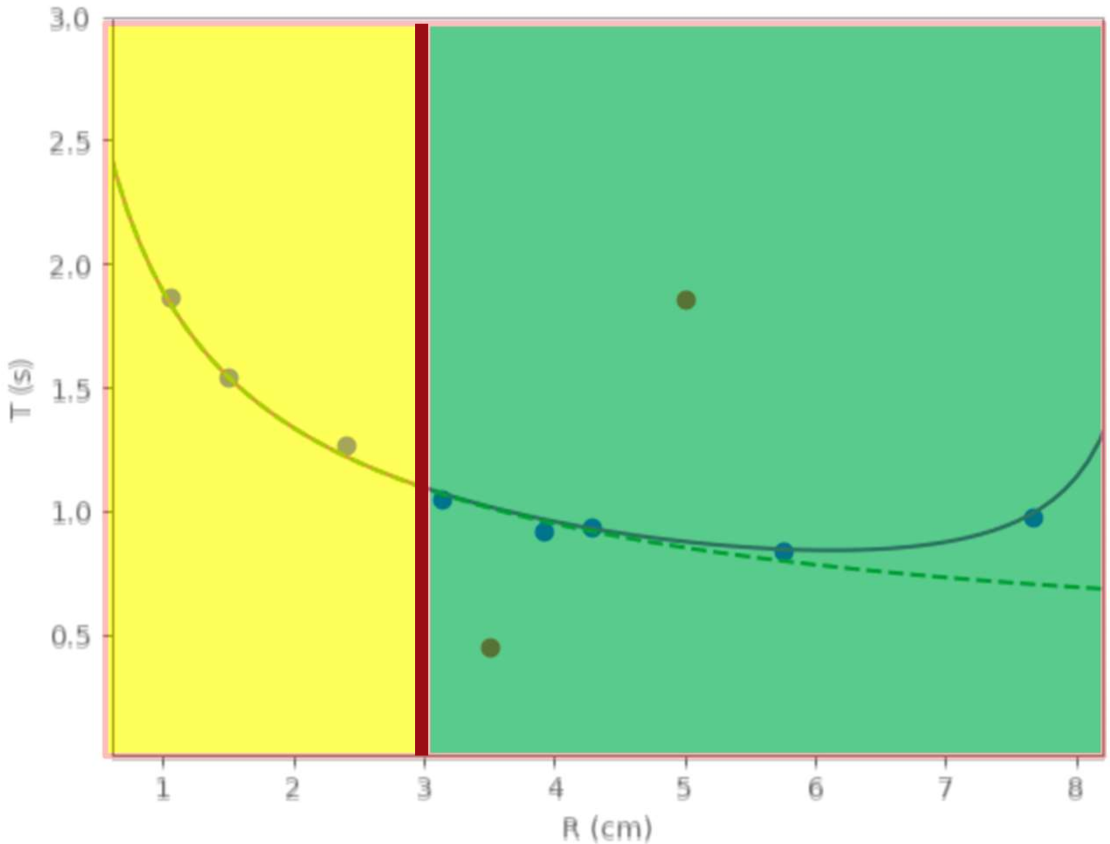
Square model

+

Proportional model

k changing, other variables constant

THEORATICAL ANALYSIS



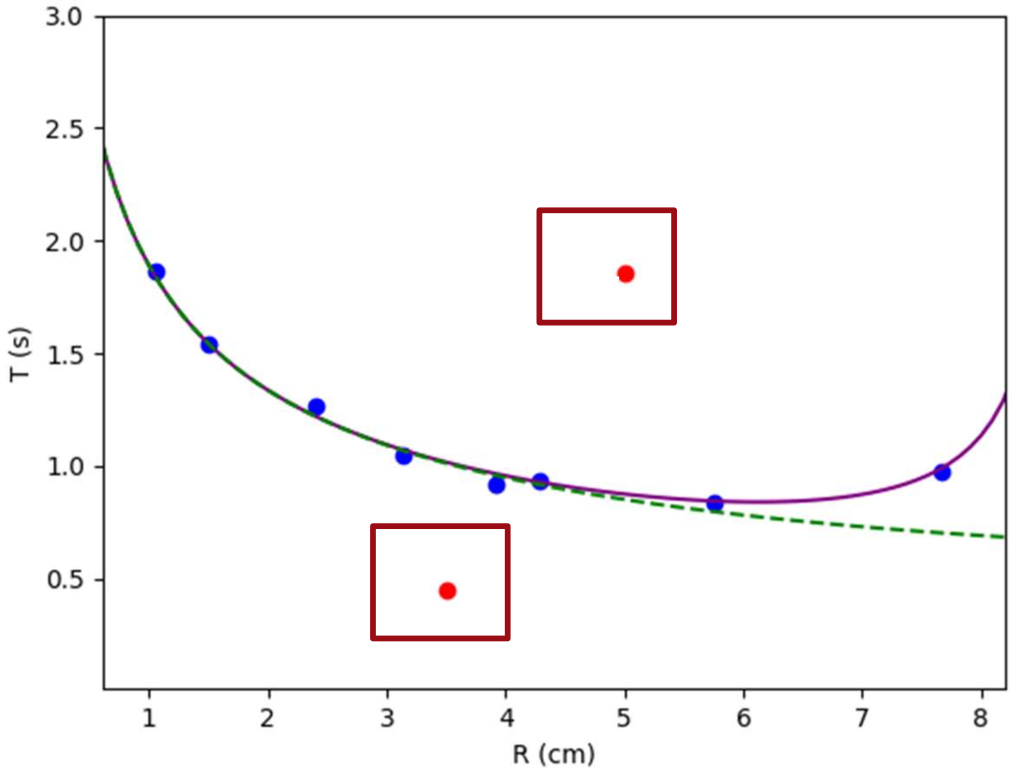
Small radius

- Both - proportional easier

Large radius

- Squared more accurate

REFLECTION



- Two of the eleven sets of experiments are **useless**

REFLECTION

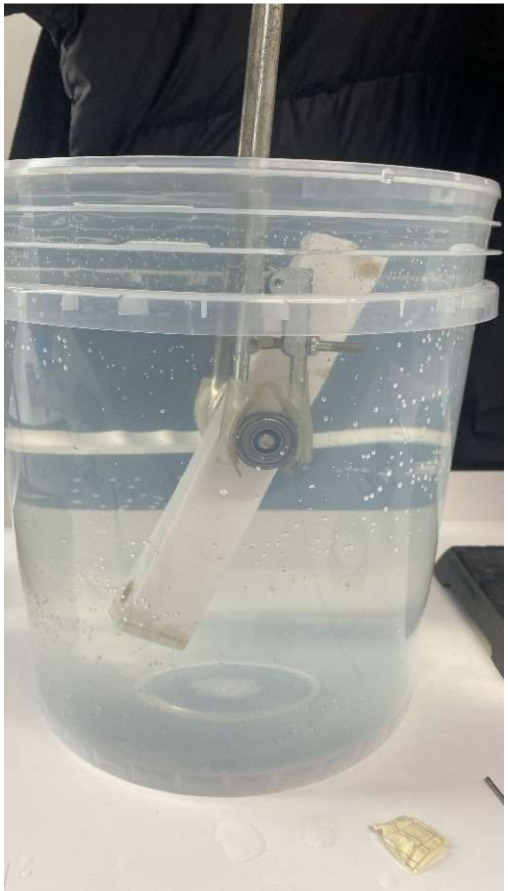
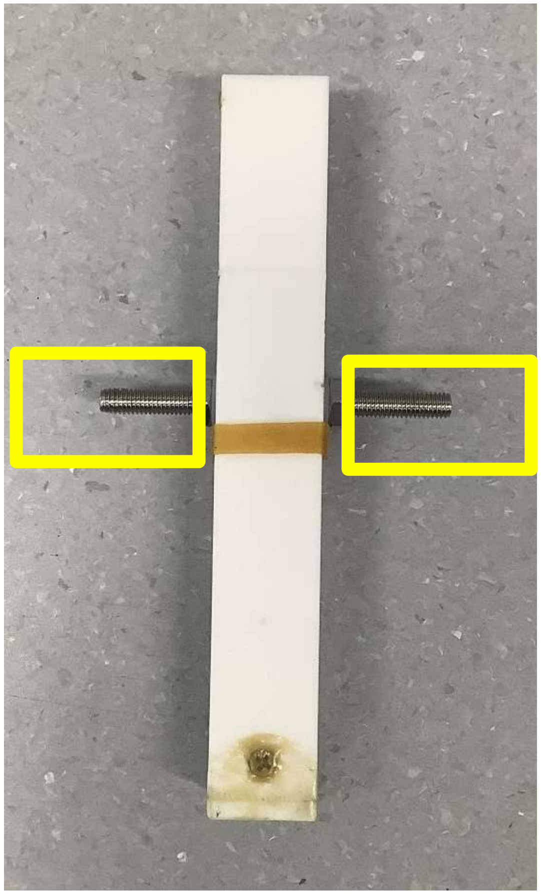


Fig 12. Buoyance higher than weight

- Two of the ten sets of experiments are useless
- Easily making mistakes **affects the efficiency**

REFLECTION



- Two of the ten sets of experiments are useless
- Easily making mistakes affects the efficiency
- M8 screw causing **fluid resistance**

SUMMARIZE

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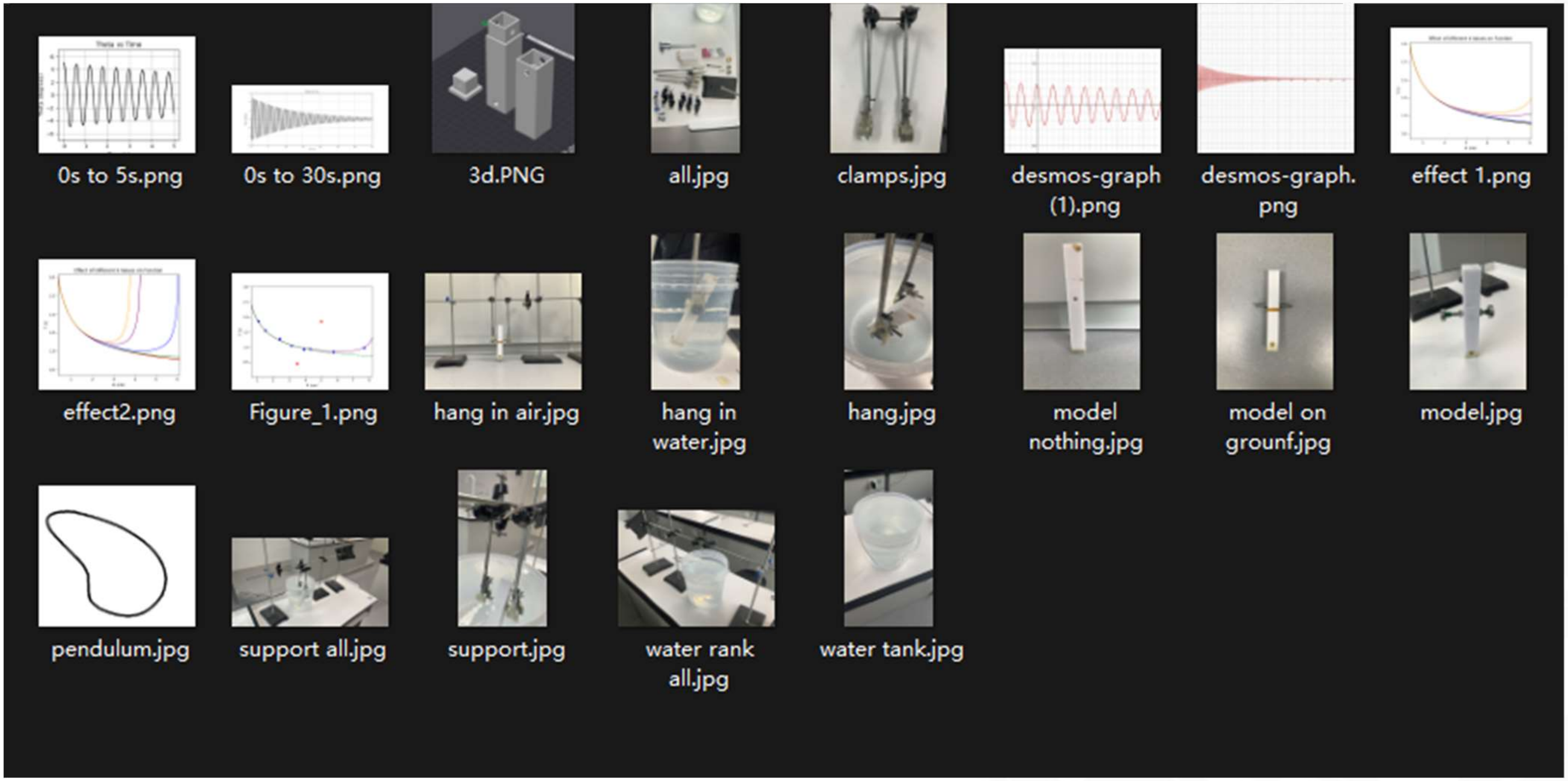
SUMMARIZE



0s to 5s.png
0s to 30s.png
3d.PNG
all.jpg
clamps.jpg
desmos-graph (1).png
desmos-graph.png
effect 1.png
effect2.png
Figure_1.png
hang in air.jpg
hang in water.jpg
hang.jpg
model nothing.jpg
model on grounf.jpg
model.jpg
pendulum.jpg
support all.jpg
support.jpg
water rank all.jpg
water tank.jpg

SUMMARIZE

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This represents a second-order non-linear ordinary differential equation, distinct from our previous solution. A new mathematical approach is required due to the presence of a square on the first derivative. Assume $y = \left(\frac{d\theta}{dt}\right)^2$, where is a function about θ. According to

$$\frac{d}{d\theta}y = \frac{d}{d\theta}\left(\frac{d\theta}{dt}\right)^2 = 2\frac{d^2\theta}{dt^2}$$

(11) can be shifted into another differential equation about y.

$$\frac{dy}{d\theta} + 2r_1 y + 2r_2\theta = 0 \qquad (12)$$

Here (12) is a linear differential equation. By solving y we can get the function of θ through some extent. However, details still need to be listed due to this is not an easy job. Convert (12) into

$$dy + 2(r_1 y + r_2\theta)d\theta = 0 \qquad (13)$$

Let $m = 2(r_1 y + r_2\theta)$, where $dm = 2r_1 dy + 2r_2 d\theta$, and (13) can be transferred into $dy + m d\theta = 0$. According to m, $y = \frac{m}{2r_1} - \frac{r_2}{r_1}\theta$. Differentiate y, $dy = \frac{dm}{2r_1} - \frac{r_2 d\theta}{r_1}$. Use instead into (13).

$$\frac{dm}{2r_1} - \frac{r_2 d\theta}{r_1} + m d\theta = 0$$

Simplify the equation and we get an integrable expression of dm and $d\theta$.

$$\frac{dm}{2r_2 - 2r_1 m} = d\theta$$

Upon integration, the function m is determined as:

$$m = \frac{A}{e^{2r_1\theta}} - \frac{r_2}{r_1}$$

Take into $y = \frac{m}{2r_1} - \frac{r_2}{r_1}\theta$.

$$y = \frac{A}{e^{2r_1\theta}} - \frac{r_2}{r_1}\theta + \frac{r_2}{2(r_1)^2}$$

Where A is integral constant. To calculate A, consider when $\theta = \theta_0$ i.e., maximum amplitude, $\left.\frac{d\theta}{dt}\right|_{\theta=\theta_0} = 0$, along with $y = \left(\frac{d\theta}{dt}\right)^2 = 0$. Through substitution,

$$A = \frac{r_2}{2(r_1)^2}(2r_1\theta_0 - 1)e^{2r_1\theta_0} \qquad (14)$$

Now y and A have been worked out, and it should be followed by deriving θ from y, which

$$\frac{A}{e^{2r_1\theta}} - \frac{r_2}{r_1}\theta + \frac{r_2}{2(r_1)^2} = \left(\frac{d\theta}{dt}\right)^2$$

Take the square root of both LHS and RHS sides. Separate the variables.

$$\sqrt{\frac{A}{e^{2r_1\theta}} - \frac{r_2}{r_1}\theta + \frac{r_2}{2(r_1)^2}} = \frac{d\theta}{dt}$$

Expand and integrate both sides.

$$\int dt = \int\left(\frac{A}{e^{2r_1\theta}} - \frac{r_2}{r_1}\theta + \frac{r_2}{2(r_1)^2}\right)^{-\frac{1}{2}}d\theta \qquad (15)$$

From using Maclaurin series $e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$, the approximate value of e can be simply calculated by the first three terms. And

$$e^{-2r_1\theta} = 1 + \frac{(-2r_1\theta)}{1!} + \frac{(-2r_1\theta)^2}{2!} \qquad (16)$$

Take into (15), RHS can be exchanged into

$$\int\left(\left(A + \frac{r_2}{2(r_1)^2}\right) - \left(2Ar_1 + \frac{r_2}{r_1}\right)\theta + 2Ar_1^2\theta^2\right)^{-\frac{1}{2}}d\theta \qquad (17)$$

By completing the square method, notice that

$$\left(A + \frac{r_2}{2(r_1)^2}\right) - \left(2Ar_1 + \frac{r_2}{r_1}\right)\theta + 2Ar_1^2\theta^2 = \left(\frac{A}{2} - \frac{r_2^2}{4(r_1)^4}\right) + (2A)\left(\left(\frac{1}{2} + \frac{r_2}{4(r_1)^2 A}\right) - r_1\theta\right)^2$$

And (17) is equivalent to

$$\int \frac{\sqrt{-8A}}{\sqrt{\frac{r_2^2}{4(r_1)^4} - A^2}\sqrt{1 - \frac{\left(2Ar_1\theta - \left(A + \frac{r_2}{2(r_1)^2}\right)\right)^2}{\frac{r_2^2}{4(r_1)^4} - A^2}}}d\theta \qquad (18)$$

From $\int\frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}(x)$, take (18) into (15).

$$\Delta t = -\frac{\sqrt{\frac{-2}{A}}}{r_1}\sin^{-1}\left[\frac{2Ar_1\theta - \left(A + \frac{r_2}{2(r_1)^2}\right)}{\sqrt{\frac{r_2^2}{4(r_1)^4} - A^2}}\right] \qquad (19)$$

ACKNOWLEDGEMENTS

- My tutor –Dr. Hou
- Teachers from physics group
- My parents
- My classmates –Tony Zhao
- Other students and friends who support me

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Thank you for your listening