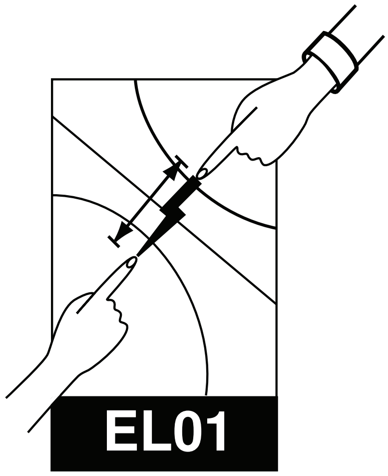
Circuits excited by stepped voltages



GDP, JAJ, updated SWW November 2024

1 Introduction and objectives

Before coming to the laboratory, carefully read chapters 1 and 2 and section 7.3 of the course Electronics Manual[1] (hereafter referred to as the Manual) and derive an expression for the current in section 2.1 of this script. You may also find it helpful to review EL00 — *oscillators, multimeters, and oscilloscopes*. Note that to avoid confusion we have adopted the almost ubiquitous, but incorrect, practice of using the word voltage where we mean an electromotive force. (A Volt is the unit of emf and an Amp is the unit of current).

This practical aims to teach you

(i) how to work out the properties of models of real circuits (called equivalent circuits) when they are excited by voltages which step abruptly from one steady value to another, and

(ii) how to make measurements on real circuits to test the validity of the models.

You will learn how to construct differential equations using Kirchhoff's laws and the definitions of resistance, inductance, and capacitance given in section 1.3 of the Manual. These definitions and Kirchhoff's laws are the starting point for the study of circuits; you will have to know them for prelims so start to learn them now.

If you are doing this practical at the start of Michaelmas term you may find some of the theory quite challenging. Don't worry, ask a demonstrator to help you.

> Remember that you are responsible for your own safety when working in the laboratory. Always talk to a demonstrator or technician if you are concerned that anything may be unsafe.

[1] http://www-teaching.physics.ox.ac.uk/practical_course/ElectronicsManual.pdf

2 A resistor and capacitor circuit

Firstly, record the serial number of your circuit board and the values and tolerances of the two isolated resistors deduced from their coloured bands. Measure their values with your digital multi-meter (DMM) . The capacitor is marked 100nF. Its tolerance is 1%. Check the value with a handheld capacitance meter. See the Manual (chapter 7) for the accuracies of the instruments.

The aim of this section is to investigate the voltage drop across the capacitor in figure 1 (left) following a step in the voltage.

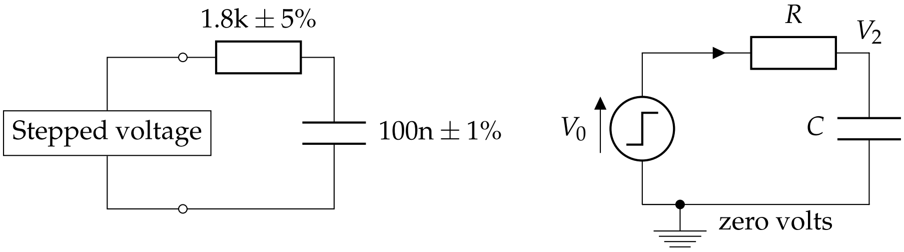


Figure 1: The resistor/capacitor circuit under investigation (left) and the equivalent circuit (right).

2.1 Equivalent circuit

Assuming that the capacitor has only capacitance, the resistor has only resistance, the stepped voltage source has no internal resistance (see the Manual, section 1.4), and that the effects of connecting any measuring instruments (e.g., 'scope channels) can be ignored, a suitable labelled equivalent circuit is depicted in figure 1 (right).

Expressions for V_2 following steps up and down in the applied voltage are derived in the Manual, section 2.5.1. Find an expression for the current I as a function of time following a step up the voltage. All the expressions are characterised by the quantity CR which has dimensions of time and is called *the time constant*.

Show that the model predicts that the time for V_2 to rise from zero to a fraction $n/8$ of its final value is given by

$$t_n = CR \ln \frac{8}{8-n}, \qquad n = 0, 1, 2, \ldots, 7$$

2.2 Qualitative explanation

A capacitor is described as charged when electrons (not necessarily the same ones!) have moved from one plate to the other via an external circuit. The net electrical charge is always zero in a capacitor. When the capacitor is charged, $V = Q/C$ where Q is the charge that moved from one plate to the other.

Kirchhoff's second law tells us that the sum of the voltage drops across the R and C is equal to the source voltage at all times. We assume the steps in the source voltage take negligible time. After a 0 to V step the sum of the two voltage drops is V_0 volts. However, the voltage drop across the capacitor is initially zero because there has been no time for any charges to move from one plate to another. All the V_0 volts are across the resistance so there is an initial current of V_0/R amps through it (and through the voltage source). The current transfers charge from one plate of the capacitor to the other causing the voltage drop across the capacitor to rise (from zero). As time progresses more voltage drop appears across the capacitor, and less across the resistor as the current falls. Eventually all the V_0 volts are dropped across the capacitor, there is no voltage drop across the resistor and therefore no current through it. At the next V_0 to 0 step the capacitor then discharges.

When the driving voltage drops from V_0 to 0 the excess electrons on one plate return to the other via the external circuit. This takes a time determined by the resistance of the circuit. If the time between steps in voltage is long, then all the charges move and the voltage across the capacitor drops to zero.

2.3 Measurements to test the model

Look at the left-hand diagram in figure 2. We could apply a voltage that steps from zero to V_0 and then from V_0 back to zero (a *pulse* voltage) with this circuit by moving the switch abruptly to position 2 and then back again. But to see what happens on our oscilloscopes we need to keep applying voltage steps; we need *a train of pulses*. This can be obtained from the signal generator by selecting square wave output with a 'Low' value of zero, and a 'High' value equal to the pulse-height wanted. (This can also be done by selecting the pulse output, which allows the duty cycle to be controlled). However, the output (internal) resistance is not negligible (50 Ω), so to reduce the effect of this on your measurements, we use a resistive divider network on the board, as shown in the diagram on the right-hand side in figure 2. This reduces the output impedance of the pulse source to close to 4.6 Ω (but at the price of reducing the amplitude of the pulse). The point 'A' in figure 2 is also labelled 'A' on the circuit board.

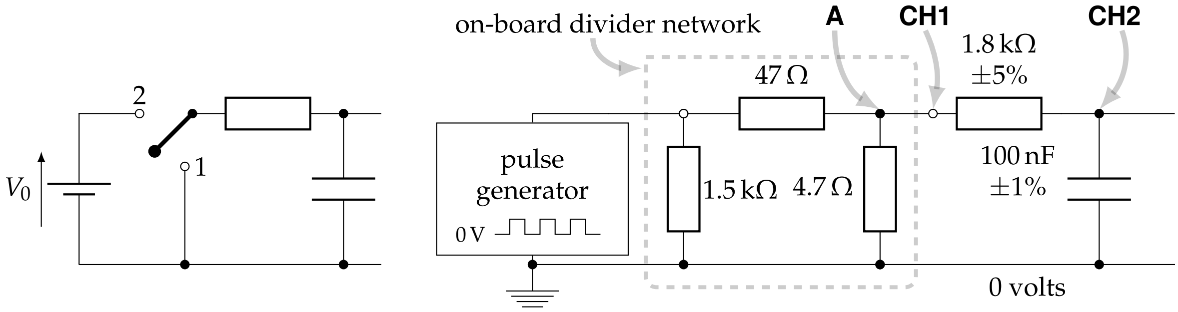


Figure 2: Switching circuit to test the model (left), drawn to help you make up the circuit (right).

Remember to record errors for all measurements. You can refer to AD26 — *Error guide in first year Electronics*[2] for acceptable practice for this lab.

Connect the circuit as shown in figure 2, with the two 'scope channels connected at **CH1** and **CH2** in the diagram. Use 'scope probes and remember to calibrate them, see section 2.5.4 of the Manual. Also recall that all the ground leads from the probes must go to the conductor defined to be 0 V earth. Set the 'scope probe attenuation to ×10 (yellow and blue buttons,"Probe Voltage" menu), trigger from channel 1 and set both inputs to DC coupled. On your AFG make sure that the square wave symbol is illuminated, that the "Low" level is zero and the "High" level is set to 5 V. Choose a pulse length long enough for V_2 to reach a steady value before the next voltage step comes along. Observe and sketch the voltages V_1 (on channel 1) and V_2 (on channel 2).

Ideally we would test the model predictions for both the rising and the falling V_2 curves. To save time we ask you to examine only the rising curve. Measure the times t_n for $n = 1, \ldots, 7$. We suggest that you

(i) Use the CH 2 controls to set the initial and final values of V_2 precisely on the top and bottom graticule lines of the screen.

(ii) Turn up the timebase speed to the maximum that allows all seven intersections of the 'scope trace with horizontal lines corresponding to n to be seen, using the horizontal shift to move the rising edge of V_1 onto the leftmost graticule line, and then read off the values of t_n as carefully as you can. You may wish to use the "Acquire" menu to produce a better trace to read from.

(iii) Keeping the timebase speed as set in (ii) turn up the AFG frequency until a complete cycle of the square wave V_1 can be seen. Use this to calibrate the time base speed of the 'scope against the more accurate signal generator (does this improve your values of t_n?)

[2] `www-teaching.physics.ox.ac.uk/practical_course/scripts/srv/local/rscripts/trunk/Admin/AD26/AD26.pdf`

Calculate two values of CR from your measurements of t_n, and from the component values.

▶ **Are all your measurements equally valuable?**

▶ **Is it all right to ignore the effects of connecting the 'scope channels? (The input of a 'scope channel looks like** $1\,\mathrm{M\Omega}$ **with, including the capacitance of a simple coaxial lead, between** 100 **and** $200\,\mathrm{pF}$ **in parallel. See the Manual, figure 6.3. With the probes this becomes more like** $10\,\mathrm{M\Omega}$ **with** $10\,\mathrm{pF}$ **in parallel — see the Manual, section 2.5.4).**

▶ **From your results, can you tell if there is anything wrong with the model? Discuss your conclusion with a demonstrator.**

3 An inductor and resistor circuit

We now consider figure 3 (left), constructed from the inductor and another of the resistors on your board. In the figure we have included the divider network on the board (which lowers the pulse generator output impedance) to be part of the "adapted pulse generator" as it is effectively part of the signal source, so you do not see the two resistors in the circuit diagram.

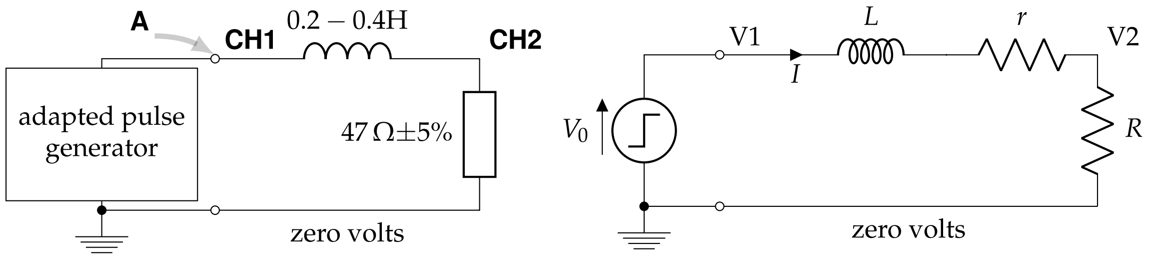


Figure 3: The inductor/resistor circuit under consideration (left) and its equivalent circuit (right).

3.1 Equivalent circuit

The resistor is assumed to have resistance only and the inductor is represented by an inductance L with a resistance r in series (we know that the inductor wire has resistance). Ignoring any effects of connecting the 'scope channels, we guess that a suitable equivalent circuit is that shown on the right of figure 3.

Write down the branch equations and Kirchhoff's second law, eliminate I and V_1 and solve the resulting differential equation for V_2 following a 0 to V_0 step in the applied voltage. The expression you should obtain for the rising V_2 is

$$V_2 = \frac{R}{R+r} V_0 \left(1 - e^{-\frac{R+r}{L}t}\right) \tag{1}$$

which is of the same form as the output of the RC circuit but with time constant $L/(R+r)$.

The expression predicts that the final value of V_2 is a fraction $R/(R+r)$ of the pulse height V_0 and that, as in section 2,

$$t_n = \frac{L}{R+r} \ln \frac{8}{8-n}\,, \qquad n = 0, 1, 2, \ldots, 8 \tag{2}$$

We will now test these predictions.

3.2 Measurements to test the model

Excite the circuit with the pulsed voltage at a repetition rate that allows V_2 to reach a steady value.

You will see straight away that there is a small problem with V_1. According to our model it should have the same step shape as the pulse, but in fact it shows noticeable curvature. This is due to the internal resistance of the adapted pulse generator not being truly negligible. Adding to the equivalent circuit a resistance r_s in series with the voltage to represent the internal resistance does not change the prediction that the ratio of the final values of V_2 and V_1 is $R/(R+r)$. Use this to find a value of r. (Hint: for best accuracy, set the asymptotic value of V_2 to 8 vertical divisions on the 'scope using the "fine" setting for the vertical amplitude controls which you access from the channel 1 menu and vertical position controls. Be sure your choice of repetition rate allows V_2 to reach its asymptotic value. Each vertical division is then one unit of n in equation 2.)

Including r_s does change the form of the expressions for the time constant and for t_n , $R+r$ is replaced wherever it occurs with $R+r+r_s$. You need not prove this.

Using the method of section 2, derive values for the time constant. Finally, using your previously measured values of R and r and taking r_s to be $4.6\,\Omega$ derive a value for L.

▶ **Check your values of L and r with a demonstrator - you will be comparing them with additional measurements in the next section.**

4 An inductor and capacitor circuit

We now consider a circuit constructed from the inductor and the capacitor on your board, as shown in figure 4.

4.1 Equivalent circuit

Making the same assumptions as before about the inductor, the capacitor, and the connection of measuring instruments, the equivalent circuit is shown in figure 4 where $V_0 u\,(t)$ is the pulse waveform of amplitude V_0.

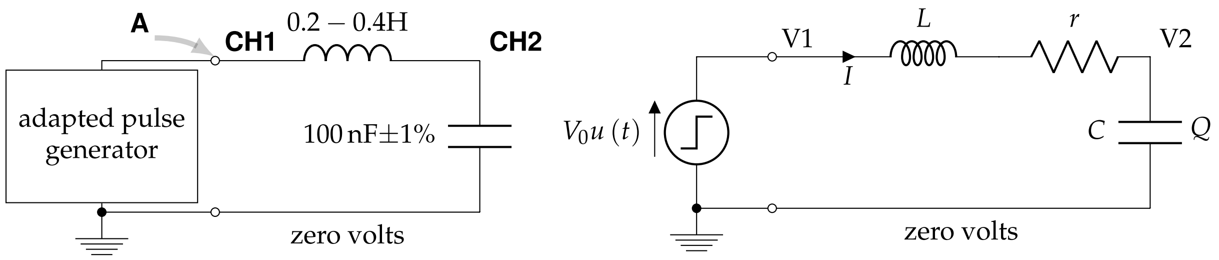


Figure 4: The inductor/resistor circuit under consideration (left) and its equivalent circuit (right).

Write down Kirchhoff's second law and the branch equations. Hence obtain the differential equation for V_2:

$$LC\,\frac{d^2V_2}{dt^2} + rC\,\frac{dV_2}{dt} + V_2 = V_0 u(t) \tag{3}$$

You will be solving second order differential equations like this in your Prelims exams. Here we give you a solution for $t > 0$, with $u\,(t<0) = 0$, $u\,(t \geq 0) = 1$. This is a single step in voltage of amplitude

V_0 at the time $t = 0$. This form is not the form you will work with in general, but is chosen here as it introduces the concept of the quality factor, Q, for a coil. It is

$$V_2 = V_0 \left(1 - \sqrt{1 + \frac{1}{4Q^2}}\, e^{-\frac{\pi t}{QT}} \cos\left(\frac{2\pi t}{T} - \phi \right) \right) \tag{4}$$

where

$$T = 2\pi \sqrt{LC \left(1 + \frac{1}{4Q^2} \right)}$$

$$Q = \frac{2\pi L}{rT} \quad \text{is known as } \textit{the quality factor} \text{ of the coil,}$$

$$\tan\phi = \frac{1}{2Q} \quad \text{hence } \cos\phi = \frac{1}{\sqrt{1 + \frac{1}{4Q^2}}}$$

Looking at equation 4, you should be able to see that V_2 is zero for $t = 0$, and for $t > 0$ has a steady part V_0, and an oscillating part of period T which decays exponentially, with the rate of decay (the damping) depending on the quality factor of the coil. The negative peaks of the oscillation occur at $t = 0$, T, $2T$, $3T$, ... and the positive peaks at $T/2$, $3T/2$, $5T/2$, ... precisely. (The effect of ϕ in moving the peaks to slightly later times is exactly compensated by the exponential decay moving the peaks to earlier times.)

4.2 Measurements to test the model

Connect the circuit and excite it with the pulsed voltage. Make a quick sketch of the waveform of V_2. Measure the time between two positive peaks, one near the left hand side of the screen and one near the right, and obtain a value for the period of the oscillation T.

Measure the voltage difference between a positive peak and the following negative peak at both sides of the screen and obtain a value for Q. Hint, the expression for V_2 predicts that the ratio of the two peak-to-peak amplitudes is $e^{-n\pi/Q}$ where n is the number of periods between the measurements.

Show that with your value of Q the term $1/4Q^2$ in the expression for T is negligible and derive a value for L. Hence derive a value of r.

▶ **Discuss whether your values of L and r found here and in section 3 are consistent, i.e., within the errors. Can you tell from your results if there is anything wrong with either equivalent circuit?**

▶ **Now hand-write a brief summary of the experiment in your logbook. State the aims of the experiment, the method used, quantitative results, and your conclusions, in not more than about half a page. Show this to a demonstrator as part of your assessment.**

Your results will be compared with measurements you will make in next week's experiment. Make sure you do the theory exercises in that script before you return to the laboratory, and when you come back, bring this script with you. **Note your board number**.

4.3 Optional questions

▶ **There are four, single time constant, passive circuits, RC, LR, CR, RL, the first two of which you have looked at. For which ones does the current have the same time dependence as V_2?**

▶ **What would be the effect of removing the wire connected between position 1 of the switch and ground in figure 2 and replacing it with an inductor, L and the switch moved from position 2 to position 1? What would be a mechanical analogue of this circuit? What would happen if instead the inductor was put in series with the battery and the switch moved from position 1 to position 2?**

If you have time left then well done! If you didn't do all the extra measurements in EL00, then do some more now:

(i) **Other trigger sources:** You have been using the signal on CH1 to trigger the time base, but other signals, e.g. on CH2, EXTernal or LINE can be selected. When the trigger source is set to LINE triggering occurs at mains frequency causing any signal at mains frequency to become stationary on the screen. This is useful for identifying interference coming from the mains. Unclip the CH1 probe and leave it lying on the bench. You should pick up a signal from the mains cables around the bench. Notice that this signal becomes larger if you touch the probe input with your finger, but touching the earth lead has little effect.

(ii) **Frequency response:** The sensitivity of the DMM used as an AC voltmeter falls off above 1 kHz whereas the 'scope sensitivity is maintained (with bandwidth limit off) to around 50 MHz. Simultaneously use the 'scope and the DMM to measure the output voltage of the AFG. For the 'scope select the 'measure' menu and then turn on the 'RMS' measurement. Set the AFG output amplitude as 5 V, ensuring that the offset is zero, and record the DMM and 'scope readings at these frequencies: 20, 30, 40, 50, 100, 200, 600 Hz and 1, 2, 4, 10, 20 kHz. Plot the results using a logarithmic frequency scale; use your laptop if you have one, or graph paper is available if not.

(iii) **Aliasing:** A digital oscilloscope effectively works by repeatedly sampling the incoming voltage. It measures the voltage at a given time and records the value in a memory. It then measures it again a short (very) time later and stores the new voltage in a different memory. It may repeat this process (effectively) 10^9 times per second. The recorded values are then manipulated by a small computer in the instrument and displayed on the screen. Set your AFG to generate a 300 kHz signal. Set your oscilloscope time base to one second per division. What is the frequency of the signal visible on the 'scope? Work it out from the trace, not by trying to use the 'scope's frequency measurement option. Explain what you see.

(iv) **Coaxial cable:** The coaxial ("coax") cable used in the lab is referred to as 50 Ω cable. Measure the resistance between the centre conductor and the outer conductor using a DMM. If you don't know why it's called 50 Ω, or don't know why coax is used at all, then ask a demonstrator to explain it.

(v) **XY display:** Connect the 'scope and AFG to measure the phase shift as in section 12 of EL00 and set the AFG frequency to 500 Hz. Press the utility menu button, select the display submenu and then select XY. It is possible to get the phase shift from this display, as described in the Manual; if you are interested, look at how it should be done. Return the 'scope to YT display.