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Guanghua Cambridge School

Two Approximation Methods on the Period of Compound Pendulum with Liquid Damping

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INTRODUCTION

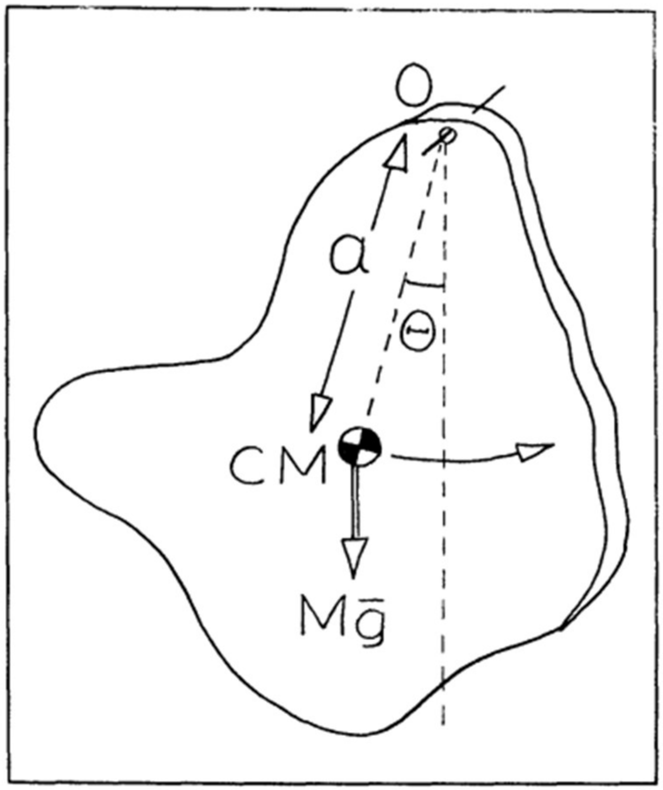

Fig. 1. For an extended object of moment of inertia $I = M(a^2 + k^2)$ about the pivot O, the period of small oscillation depends only on the distance a from the pivot to the center of mass, and the gyradius k.

Fig 1.

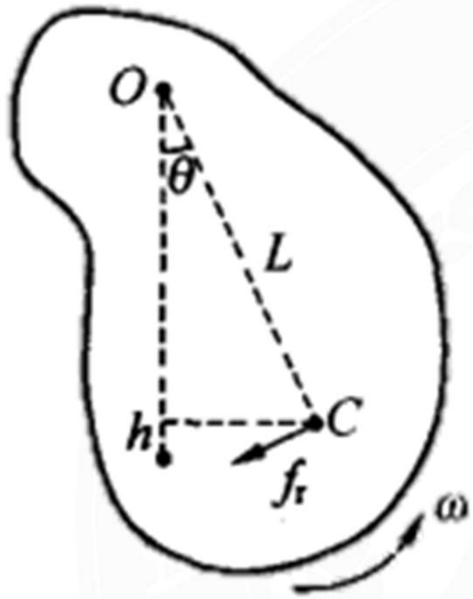

Fig 2.

← Compound Pendulum

INTRODUCTION

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Instead…

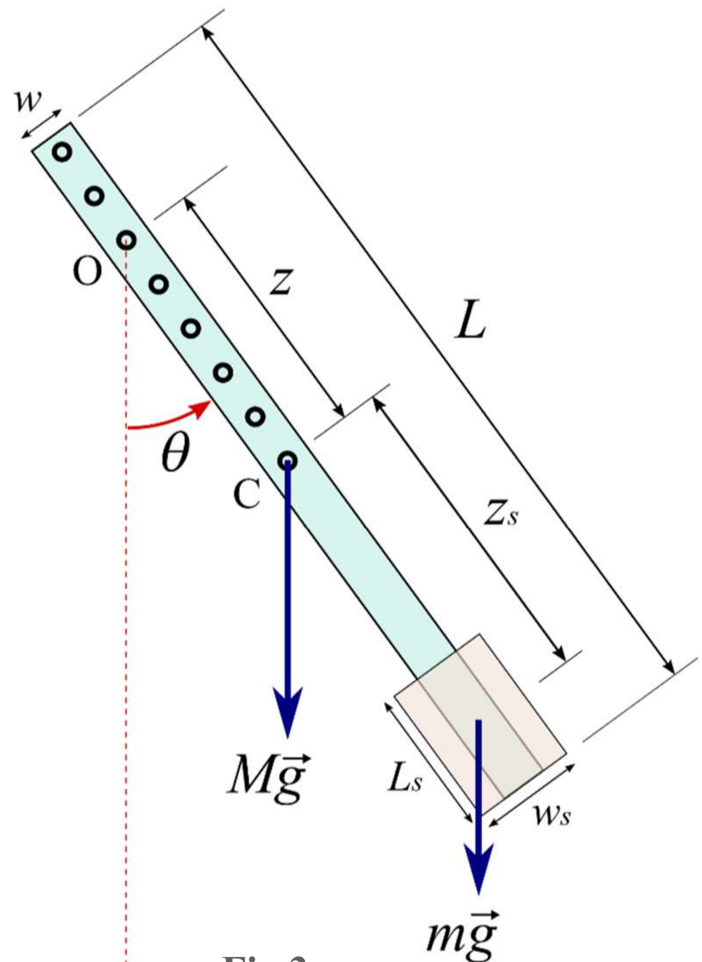

Fig 3.

TIMELINE

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2023.07-08:
Determine research topic, make the first step to the question, have a basic understanding

2023.09-10:
Collect references, work out the first version of formula about the pendulum

2023.11-12:
Study how to plot graph and how to meet the needs of experiments(e.g. python+3d design)

2023.12-24.01:
Design experiment, make theorical expectations, calculate various answers

2024.01-02:
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ADJUSTMENTS

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Previous topic: The Effects on the period …

Current topic: Two Approximation Methods…

时，常采用 2 种简化模型[3-5]，一是假设振动物体所受的阻力与物体的速度成正比，另一种是假设阻力与速度的平方成正比．本文分别采用 2 种空气阻力模型，推导出考虑空气阻力时复摆的振动周期公式，并结合实验测量结果，分析比较 2 种情况下空气的阻力对复摆周期的影响．

Fig 4. (air friction)

ADJUSTMENTS

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Reasons:

- Easy to **determine variables**
  - --viscous constant / inertia / length **…**
- Hard to find **accurate period**
  - **Nonlinear motion** under friction and driving conditions [1]

ADJUSTMENTS

Methods:[2]

- $\boldsymbol{f \propto v}$
- $\boldsymbol{f \propto v^{2}}$

RESEARCH PROGRESS

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From analogue ***Newton's II Law of motion*** in rotation:

$$m \rightarrow I,\ a \rightarrow \alpha,\ F \rightarrow M$$

$$\boldsymbol{ma} = \sum \boldsymbol{F} \ \rightarrow \ \boldsymbol{I\alpha} = \sum \boldsymbol{M}$$

Sep. 16th

RESEARCH PROGRESS

$$I\ddot{\theta} = F \cdot d$$

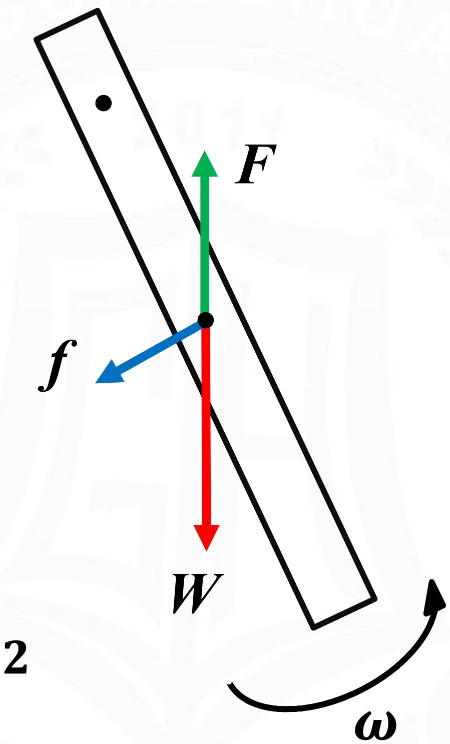

Fig 5.

Force list:

1. Weight $W = mg$
2. Buoyance $F_{up} = \rho_w g V$
3. Viscous force $f = kv$ and $f = kv^2$

Sep. 25th

RESEARCH PROGRESS

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$$I\ddot{\theta} = WR - fR - FR$$

1. $f = kR\omega = kR\dot{\theta}$

$$I\ddot{\theta} = (m - \rho V)gR\theta - kR^2\dot{\theta}$$

For $\theta \ll 5$, $\sin\theta \approx \theta$ [3]

里，$\omega_0=0.7$。显然，当幅角较小时，两者的振动周期、振幅都比较接近，当幅角更小时，复摆的振动可看作简谐振动。而实际的复摆，幅角小于 5^0 的

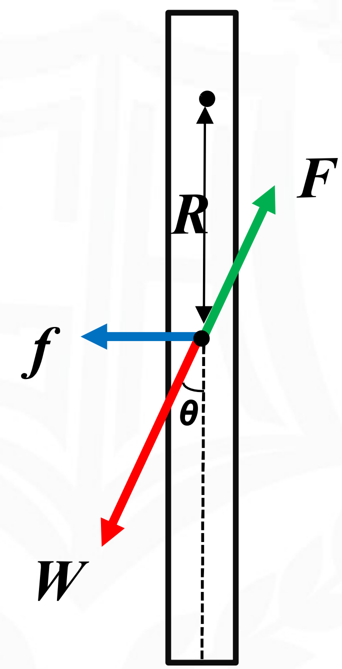

Fig 6.

Oct. 4th

RESEARCH PROGRESS

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$$I\ddot{\theta} = WR - fR - FR$$

1. $f = kR\omega = kR\dot{\theta}$

$$\ddot{\boldsymbol{\theta}} + \frac{\boldsymbol{kR^2}}{\boldsymbol{I}}\dot{\boldsymbol{\theta}} - \frac{(\boldsymbol{m} - \boldsymbol{\rho V})\boldsymbol{gR}}{\boldsymbol{I}}\boldsymbol{\theta} = \mathbf{0}$$

Homogeneous second-order
linear ordinary differential equation[4]

RESEARCH PROGRESS

$$I\ddot{\theta} = WR - fR - FR$$

1. $f = kR\omega = kR\dot{\theta}$

$$\ddot{\boldsymbol{\theta}} + \frac{\boldsymbol{kR^2}}{\boldsymbol{I}}\dot{\boldsymbol{\theta}} - \frac{(\boldsymbol{m} - \boldsymbol{\rho V})\boldsymbol{gR}}{\boldsymbol{I}}\boldsymbol{\theta} = \mathbf{0}$$

Homogeneous second-order
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RESEARCH PROGRESS

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$$I\ddot{\theta} = WR - fR - FR$$

1. $f = kR\omega = kR\dot{\theta}$

$$\boldsymbol{\theta = ce^{-\frac{kR^2}{I}t}\cos\left(\frac{\sqrt{\frac{4(mg-\rho gV)R}{I} - \frac{k^2R^4}{I^2}}}{2}t\right)}$$

$$\boldsymbol{\omega = \frac{\sqrt{\frac{4(mg-\rho gV)R}{I} - \frac{k^2R^4}{I^2}}}{2}}$$

Oct. 18th

RESEARCH PROGRESS

$$I\ddot{\theta} = WR - fR - FR$$

2. $\boldsymbol{f = kR^2\omega^2 = kR^2\dot{\theta}^2}$

$$\boldsymbol{\ddot{\theta} + \frac{kR^3}{I}\dot{\theta}^2 - \frac{(m - \rho V)gR}{I}\theta = 0}$$

Let $x = \left(\frac{d\theta}{dt}\right)^2 = \dot{\theta}^2$, so $\ddot{\theta} = \frac{d^2\theta}{dt^2} = \frac{dx}{d\theta} = \dot{x}$

RESEARCH PROGRESS

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$$I\ddot{\theta} = WR - fR - FR$$

2. $\boldsymbol{f = kR^2\omega^2 = kR^2\dot{\theta}^2}$

$$\boldsymbol{\dot{x} + \frac{kR^3}{I}x - \frac{(m-\rho V)gR}{I}\theta = 0}$$

RESEARCH PROGRESS

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$$T = 2\pi\sqrt{\frac{I^2}{IR(m-\rho V)g - 1/4\,(k^2R^4)}}$$

$$f = kv$$

$$T = 2\pi\sqrt{\frac{I^2 - 2kR^3\theta(I - kR^3)}{gR(m-\rho V)(I - 2kR^3\theta)}}$$

$$f = kv^2$$

Oct. 25th

PROBLEMS & SOLUTIONS

Big angles**?** **Small** angles**?**

Directly proportional**?** **Squared** proportional**?**

- Small angles can occur in air damping[2]
- Same in liquid? **Bigger damping force**

Oct. 10th

RESEARCH PROGRESS

$$I\ddot{\theta} = WR - fR - FR$$

1. $f = kR\omega = kR\dot{\theta}$

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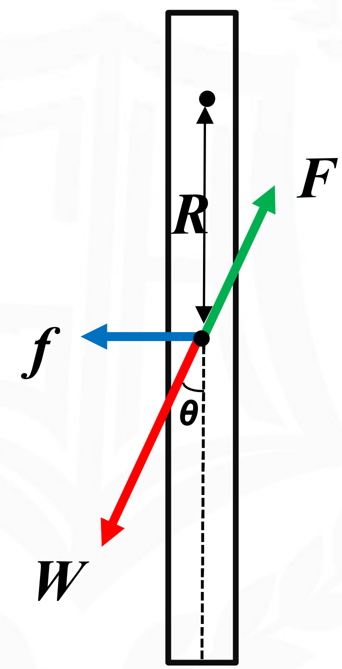

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A ROUGH EXPERIMENT

- Does not need to verify the formula’s correctness
- Does need to verify **small angle is observable**

Nov. 10th

PROBLEMS & SOLUTIONS

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Fig 7.

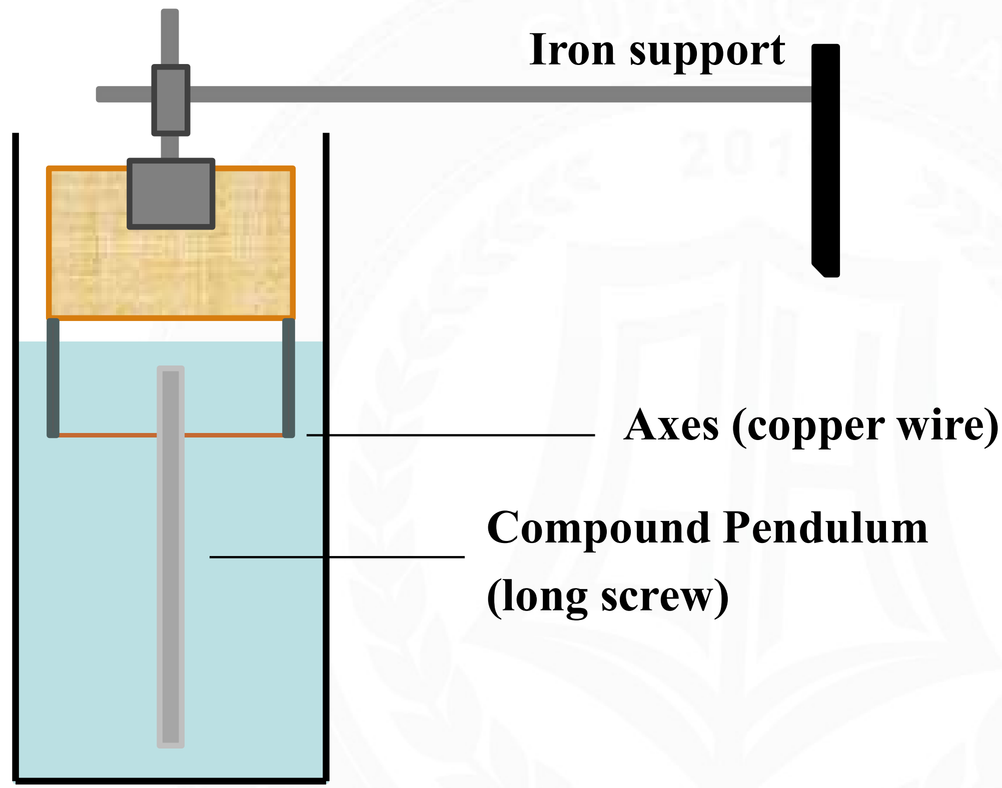

Fig 8.

Nov. 19th

PROBLEMS & SOLUTIONS

Data Collection

- ❑ Use phone camera – slow motion mode
- ❑ Did it for four times
- ❑ Use frame counter to calculate time in tiny units
  - ❑ 1 frame = 1 / 240 second
- ❑ Sum up and calculate average value

Conclusion:

CAN swing **FOR A LONG TIME** (~40s, at least 60 periods directly by eyes) under **SMALL ANGLE CONDITION** & **LIQUID DAMPING**

No.	Frame/f	10T/s	T/s
1	740	6.167	0.614
2	736	6.133	
3	732	6.100	
4	738.5	6.154	

Data Table

PROBLEMS & SOLUTIONS

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$k = \frac{1}{2}c\rho A = 1.17 \times 1.431 \times 10^{-3} \times 1000 \times 0.5 = 0.837\ kg\ s^{-1}$

$I = 5.934 \times 10^{-5}\ kg\ m^2 \quad W - F = 304.11mN \quad R = 25.7mm$

No.	Frame/f	10T/s	T/s
1	740	6.167	0.614
2	736	6.133	
3	732	6.100	
4	738.5	6.154	

Data Table

Measured Results:

Period ≈ $\mathbf{0.614}\ seconds$

Theoretical Results:

1. $f \propto v,\ T = 2\pi\sqrt{\frac{I^2}{IR(m-\rho V)g-1/4(k^2R^4)}}$, Period ≈ $\mathbf{0.579}\ seconds$ $\quad \delta \approx -5.7\%$
2. $f \propto v^2, T = 2\pi\sqrt{\frac{I^2-2kR^3\theta(I-kR^3)}{gR(m-\rho V)(I-2kR^3\theta)}}$, Period ≈ $\mathbf{0.551}\ seconds$ $\quad \delta \approx -10.3\%$

Nov. 20th

PROBLEMS & SOLUTIONS

WHY?

1. More **friction** than ideal –not only liquid damping
2. Did **not fixed** axes and pendulum
3. Measuring **uncertainty** –especially ***k***

ROUGH EXPERIMENT

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WORK PLAN

Do ACCURATE EXPERIMENTs

- ✓ **Design**. –3D printer, a model of pendulum.
- ✓ **Produce**. –Prepare a list of buying.
- ✓ **Write**. –Start writing the report.

REFERENCES

[1] 复摆运动状态的研究
Study on the motion behaviors of the compound pendulum
[2] 空气阻力对复摆振动周期的影响
Influence of air resistance on vibration period of compound pendulum
[3] 任意摆角的复摆振动规律研究
Research on Vibrations of Compound Pendulum for Arbitrary Angles
[4]
https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/17%3A_Second-Order_Differential_Equations/17.01%3A_Second-Order_Linear_Equations#mjx-eqn-super
Fig 1. Practical Applications of the Compound Pendulum
Fig 2.空气阻力对复摆振动周期的影响
Influence of air resistance on vibration period of compound pendulum
Fig 3. Experimental analysis of a physical pendulum with variable suspension point
Fig 4.空气阻力对复摆振动周期的影响
Influence of air resistance on vibration period of compound pendulum
Fig 6.任意摆角的复摆振动规律研究
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Thank you for your listening