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Guanghua Cambridge School

Two Approximation Methods on the Period of Compound Pendulum with Liquid Damping

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INTRODUCTION



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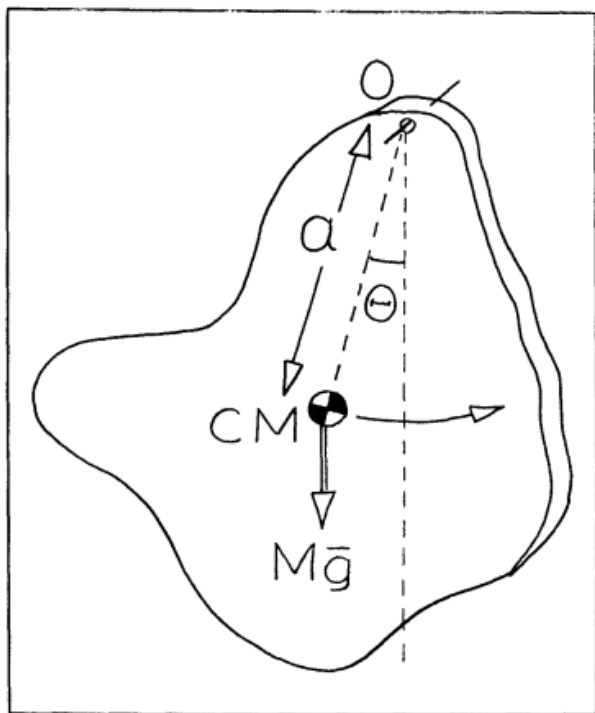


Fig. 1. For an extended object of moment of inertia $I = M(a^2 + k^2)$ about the pivot O, the period of small oscillation depends only on the distance a from the pivot to the center of mass, and the gyradius k .

Fig 1.

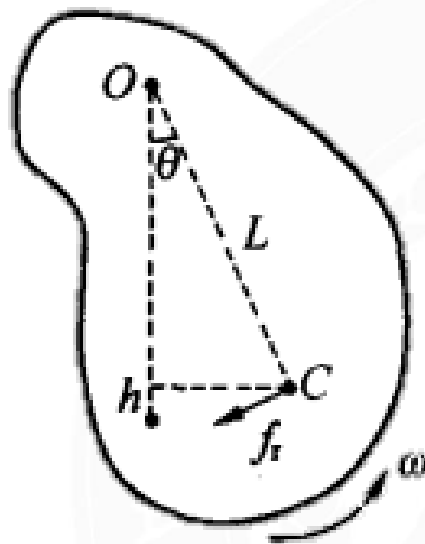


Fig 2.

← **Compound Pendulum**

INTRODUCTION



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Instead...

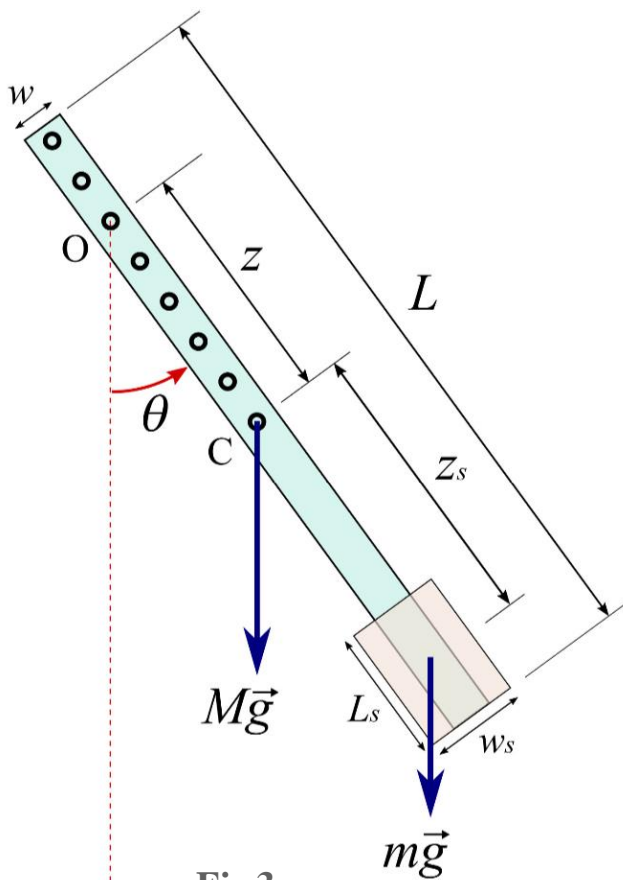


Fig 3.



TIMELINE

2023.09-10:

Collect references,
work out the first
version of formula
about the pendulum

2023.12-24.01:

Design experiment,
make theoretical
expectations, calculate
various answers

2024.03-04:

Analyze data, finish
the essay, submit to
tutor for final revision

2023.07-08:

Determine research
topic, make the first
step to the question,
have a basic
understanding

2023.11-12:

Study how to plot graph
and how to meet the
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python+3d design)

2024.01-02:

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check if any mistakes,
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ADJUSTMENTS



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Previous topic: The Effects on the period ...

Current topic: Two Approximation Methods...

时,常采用 2 种简化模型^[3-5],一是假设振动物体所受的阻力与物体的速度成正比,另一种是假设阻力与速度的平方成正比. 本文分别采用 2 种空气阻力模型,推导出考虑空气阻力时复摆的振动周期公式,并结合实验测量结果,分析比较 2 种情况下空气的阻力对复摆周期的影响.

Fig 4.

ADJUSTMENTS



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Reasons:

- Easy to **determine variables**
 - --viscous constant / inertia / length ...
- Hard to find **accurate period**
 - **Nonlinear motion** under friction and driving conditions [1]

ADJUSTMENTS



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Methods:^[2]

- $f \propto v$
- $f \propto v^2$

RESEARCH PROGRESS



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From analogue ***Newton's II Law of motion*** in rotation:

$$I \rightarrow m, a \rightarrow \alpha, F \rightarrow M$$

$$ma = \sum F \rightarrow I\alpha = \sum M$$

Sep. 16th

RESEARCH PROGRESS



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$$I\ddot{\theta} = F \cdot d$$

Force list:

1. Weight
2. Buoyance
3. Viscous force

$$W = mg$$

$$F_{up} = \rho_w g V$$

$$f = kv \text{ and } f = kv^2$$

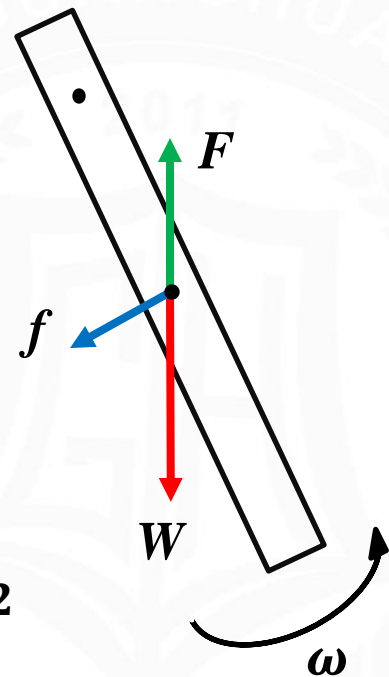


Fig 5.

Sep. 25th

RESEARCH PROGRESS



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$$I\ddot{\theta} = WR - fR - FR$$

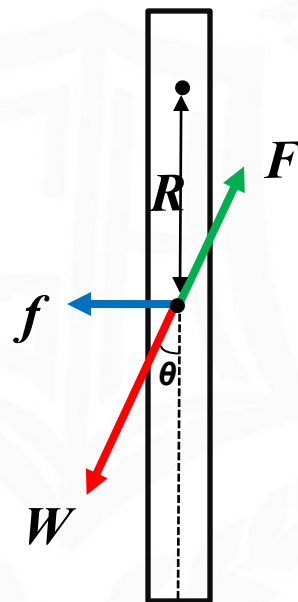
$$1. f = kR\omega = kR\dot{\theta}$$

$$I\ddot{\theta} = (m - \rho V)gR\theta - kR^2\dot{\theta}$$

For $\theta \ll 5$, $\sin \theta \approx \theta$ [3]

里, $\omega_0 = 0.7$ 。显然, 当幅角较小时, 两者的振动周期、振幅都比较接近, 当幅角更小时, 复摆的振动可看作简谐振动。而实际的复摆, 幅角小于 5° 的

Fig 6.



Oct. 4th

RESEARCH PROGRESS



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$$I\ddot{\theta} = WR - fR - FR$$

1. $f = kR\omega$

$$\ddot{\theta} + \frac{kR^2}{I}\dot{\theta} - \frac{(m - \rho V)gR}{I}\theta = 0$$

Homogeneous second-order
linear ordinary differential equation^[4]

RESEARCH PROGRESS



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$$I\ddot{\theta} = WR - fR - FR$$

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RESEARCH PROGRESS



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$$I\ddot{\theta} = WR - fR - FR$$

$$1. f = kR\omega$$

$$\theta = ce^{-\frac{kR^2}{I}t} \cos\left(\frac{\sqrt{\frac{4(mg - \rho gV)R}{I} - \frac{k^2R^4}{I^2}}}{2} t\right)$$

Oct. 18th

$$\omega = \frac{\sqrt{\frac{4(mg - \rho gV)R}{I} - \frac{k^2R^4}{I^2}}}{2}$$

RESEARCH PROGRESS



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$$I\ddot{\theta} = WR - fR - FR$$

$$2. f = kR^2\omega^2$$

$$\ddot{\theta} + \frac{kR^3}{I}\dot{\theta}^2 - \frac{(m - \rho V)gR}{I}\theta = 0$$

$$\text{Let } x = \left(\frac{d\theta}{dt}\right)^2 = \dot{\theta}^2, \text{ so } \ddot{\theta} = \frac{d^2\theta}{dt^2} = \frac{dx}{d\theta} = \dot{x}$$

RESEARCH PROGRESS



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$$T = 2\pi \sqrt{\frac{I^2}{IR(m - \rho V)g - 1/4(k^2 R^4)}}$$

$$f = kv$$

$$T = 2\pi \sqrt{\frac{I^2 - 2kR^3\theta(I - kR^3)}{gR(m - \rho V)(I - 2kR^3\theta)}}$$

$$f = kv^2$$

Oct. 25th

PROBLEMS & SOLUTIONS



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Big angles? **Small** angles?

Directly proportional? **Squared** proportional?

- Small angles can occur in air damping^[2]
- Same in liquid? **Bigger damping force**

Oct. 10th

RESEARCH PROGRESS



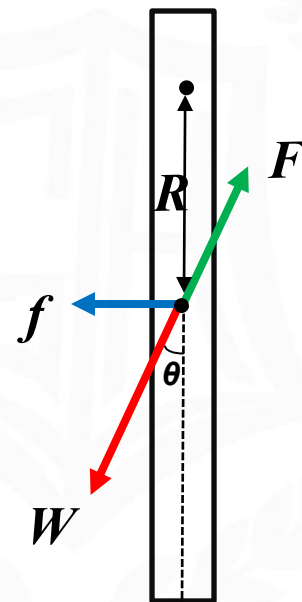
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A **ROUGH** EXPERIMENT

- Does not need to verify the formula's correctness
- Does need to verify **small angle is observable**

PROBLEMS & SOLUTIONS



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Fig 7.

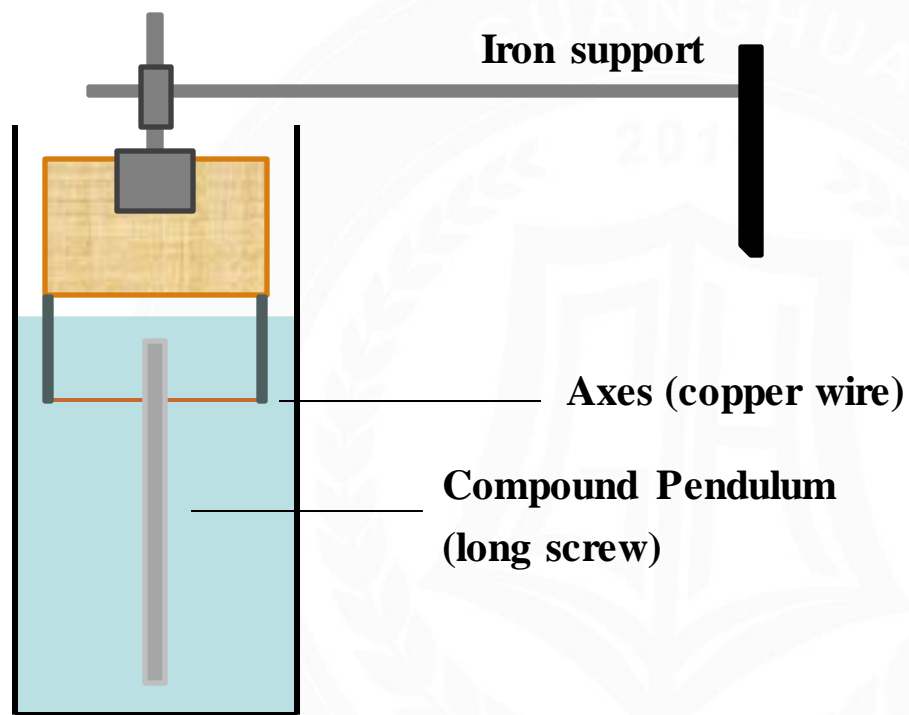


Fig 8.

Nov. 19th

PROBLEMS & SOLUTIONS



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Data Collection

- ☐ Use phone camera – slow motion mode
- ☐ Did it for four times
- ☐ Use frame counter to calculate time in tiny units
 - ☐ 1 frame = 1 / 240 second
- ☐ Sum up and calculate average value

PROBLEMS & SOLUTIONS



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Conclusion:

CAN swing **FOR A LONG TIME** (~40s, at least 60 periods

directly by eyes)

under **SMALL ANGLE**

CONDITION & LIQUID

DAMPING

No.	Frame/f	10T/s	T/s
1	740	6.167	0.614
2	736	6.133	
3	732	6.100	
4	738.5	6.154	

Data Table

PROBLEMS & SOLUTIONS



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Measured Results:

Period ≈ 0.614 seconds

Theoretical Results:

$$T = 2\pi \sqrt{\frac{I^2}{IR(m - \rho V)g - 1/4(k^2 R^4)}},$$

No.	Frame/f	10T/s	T/s
1	740	6.167	0.614
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Data Table

$$k = \frac{1}{2} c \rho A = 1.17 \times 1.412 \times 10^{-3} \times 1000 \times 0.5 = 0.826 \text{ kg s}^{-1}$$

Period ≈ 0.706 seconds difference = 15%

Nov. 20th

PROBLEMS & SOLUTIONS



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WHY?

1. More **friction** than ideal –not only liquid damping
2. Did **not fixed** axes and pendulum
3. Measuring **uncertainty** –especially **k**

ROUGH EXPERIMENT

WORK PLAN



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About 3D printer...





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REFERENCES



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[1] 复摆运动状态的研究

Study on the motion behaviors of the compound pendulum

[2] 空气阻力对复摆振动周期的影响

Influence of air resistance on vibration period of compound pendulum

[3] 任意摆角的复摆振动规律研究

Research on Vibrations of Compound Pendulum for Arbitrary Angles

[4]

[https://math.libretexts.org/Bookshelves/Calculus/Calculus_\(OpenStax\)/17%3A_A_Second-Order_Differential_Equations/17.01%3A_A_Second-Order_Linear_Equations#mjx-eqn-super](https://math.libretexts.org/Bookshelves/Calculus/Calculus_(OpenStax)/17%3A_A_Second-Order_Differential_Equations/17.01%3A_A_Second-Order_Linear_Equations#mjx-eqn-super)

Fig 1. Practical Applications of the Compound Pendulum

Fig 2. 空气阻力对复摆振动周期的影响

Influence of air resistance on vibration period of compound pendulum

Fig 3. Experimental analysis of a physical pendulum with variable suspension point

Fig 4. 空气阻力对复摆振动周期的影响

Influence of air resistance on vibration period of compound pendulum

Fig 6. 任意摆角的复摆振动规律研究

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Thank you for your listening

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