Ch4.

standard optimization problem.
$$\min f_0(x) \quad \text{s.t. } f_i(x) \le 0 \quad i=1,\cdots,m. \quad h_i(x)=0 \quad i=1,\cdots,p.$$

feasible solution if $\begin{cases} f_i(x) \le 0 & i=1,\cdots,m. \\ h_i(x)=0 & i=1,\cdots,p. \end{cases} \Rightarrow$ feasible set.

optimal value $p^* = \inf\{f_0(x) \mid f_i(x) \le 0, i=1,\cdots,m;\ h_i(x)=0, i=1,\cdots,p\} = \begin{cases} \infty & \text{infeasible } \{\} = \emptyset. \\ -\infty & \text{unbdd below.} \end{cases}$

optimal point: ~~pt~~ feasible x^* s.t. $f_0(x^*) = p^*$.

ε ~~opt~~ suboptimal x : x feasible ⊕ $f_0(x) \le p^* + \varepsilon$.

问题的等价变换: ① Slack variable s. $f_i(x) \le 0 \Rightarrow f_i(x) + s_i = 0$

② optimize over some variables. 部分变量优化. ~~$f_i(x)$~~ $\tilde{f}(x) = \inf_y f(x,y)$. $\inf_{x,y} f(x,y) = \inf_x \tilde{f}(x)$.

③ epigraph problem form. $\min t$ s.t. $f_0(x) - t \le 0$, $f_i(x) \le 0$, $h_i(x) = 0$

standard convex optimization problem:
$$\min f_0(x) \rightarrow \text{convex}$$
$$\text{s.t. } f_i(x) \le 0 \quad i=1\cdots m \rightarrow \text{convex}$$
$$a_i^T x = b_i \quad i=1\cdots p. \rightarrow \text{affine.}$$

$\Rightarrow$ feasible set convex.

$\Rightarrow$ local optimal is global optimal.

$\Rightarrow$ Thm: feasible set X ⊕ f_0 differentiable in $X \Rightarrow x$ optimal $\Leftrightarrow \nabla f_0(x)^T(y-x) \ge 0.\ \forall y \in X.$

$\Leftrightarrow \nabla f_0(x) = 0$ if 无约束.

$\Leftrightarrow \exists \nu$ s.t. $\nabla f_0(x) + A^T \nu = 0$ if only equation constraint. $Ax=b$.

$\Leftrightarrow x \succeq 0,\ \nabla f_0(x) \succeq 0,\ \nabla f_0(x)^T x = 0$ if constrain $x \succeq 0$.

Linear Programming 线性规划. $\min c^T x + d$ s.t. $Gx \preceq h$, $Ax = b$ $\Rightarrow$ feasible set is a polyhedron.

linear fractional programming 线性分式规划. $\min f_0(x) = \dfrac{c^T x + d}{e^T x + f}$ s.t. $Gx \preceq h$, $Ax = b$ can be transfired to LP.

Quadratic Programming 二次优化. $\min \frac{1}{2} x^T P x + q^T x + r$ s.t. $Gx \preceq h.$ $Ax = b$ $\Rightarrow$ feasible set is a polyhedron.

$P \in S_+^n.\ G \in R^{m \times n}.\ A \in R^{p \times n}.$

Second-order cone programming $\min f^T x$ s.t. $\|A_i x + b_i\|_2 \le c_i^T x + d_i$ $i=1,\cdots,m.$ $\leftarrow$ second-order cone constraint. $Fx = g$

$A_i \in R^{n_i \times n}$ $F \in R^{p \times n}.$

examples of SOCP: ① robust linear programming

② linear programming with random constraints

Geometric Programming 几何规划

monomials 单项式: $f(x) = c x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}$ $(c > 0,\ a_i \in R)$ so not necessarily convex.

posynomial 多项式 $f(x) = \sum_{k=1}^{K} c_k x_1^{a_{1k}} x_2^{a_{2k}} \cdots x_n^{a_{nk}}$

$\min\ f_0(x)$

s.t. $f_i(x) \le 1 \quad i = 1, \cdots, m.$ } posynomial

domain. $x \in R_{++}^n$

$h_i(x) = 1 \quad i = 1, \cdots, p.$ monomial.

$\Rightarrow$ can be transformed to convex optimization by using log-exp.