Seminar

① Setting.

② Stein's Loss : properties.

③ What does Stein concerns about

not affected. by coordinate we're using
also stablize eigenvalues.

– invariant estimator : $X_i^* = \Lambda X_i \Rightarrow \Sigma^* = \Lambda \Sigma \Lambda^T$, are we still doing same problem with (X_i, Σ) ?

We know : S is invariant $X^* = X\Lambda \Rightarrow S^* = \Lambda S \Lambda^T$

~~we d~~

why ?
coincidence ?

– minimaxity under Stein's Loss , the minimax estimator within a group of transfo

– correct for the distortion of eigenvalues in sample covariance matrix.

1. usually preserved, usually only consider 2 or 3 in an estimator.

Stream : 1961. James & Stein : minimax estimator within triangular transformation group.

they ~~conside~~ also show S is not minimax.

call it original minimax estimator. [though not admissible]

Note : minimax is not alway admissible

minimax : in worst case : θ s.t. $R(\delta, \theta)$ is maximized, δ is better than other δ'

admissible : δ better than other δ' for $\forall \theta$ same fixed θ.

they can't conclude each other.

1984 Takemura improved it by averaging over pxp matrices) [not going to talk about]

↳ dominate the original minimax estimator.

1985 Dipak : improved ~~matrix~~ minimax estimator with orthogonal invariance.

call it improved minimax.

1977 ~~Stein~~ Stein : orthogonally invariant estimator. [correct the dispersion of eigenvalues]

property unknow.

call it Stein shrinkage estimator.

but simulation shows it dominates a wide range of Σ.

S / EB / JS estimators

[Haff(1982) : connection of Stein ~~correcti~~ shrinkage estimator & Bayes rules.]

maybe interesting

not going to talk about.

1961 James & Stein: original minimax.

Loss function: Stein's Loss.

Class of estimator we're considering: $\varphi(S)$ function of S. $\quad \Lambda\begin{pmatrix}1 & & \\ & -1 & \\ & & \ddots\end{pmatrix}$ $\quad \varphi(I) = \Lambda\varphi(I)\Lambda^T$

invariance: $\varphi(\Lambda S \Lambda^T) = \Lambda \varphi(S) \Lambda^T$: $\Rightarrow$ under this setting, ~~then only possible $\varphi(S)$~~ can prove $\varphi(I)$ is diagonal.

method: lower triangular matrix decompositin $S = KK^T \Rightarrow \varphi(S) = K\varphi(I)K^T$

Δ

therefore S is included in the class. by setting $\varphi(I) = I$.

obtain Δ: minimizing risk function. ~~$EL(\Sigma, \hat{\Sigma}, \varphi(S))$~~

~~$= EL(I, \varphi(S))$~~

$$
\begin{aligned}
&E\,L(\Sigma, \varphi(S)) \\
&= EL(TT^T, K\Delta K^T) \\
&= EL(I, T^{-1}K\Delta(T^{-1}K)^T)
\end{aligned}
$$

and $\Delta = \varphi(I)$ should be irrelevant of K or $T^{-1}K$.

$\Rightarrow$ we only need to consider the scenario that $\Sigma = I$.

$$\Delta_i = \frac{1}{n+p-2i+1}$$

minimaxity: Kiefer: under certain conditions, a ~~gro~~ statistical problem invariant under a group of transformations has a minimax solution that is also ~~...~~ invariant under transformation.

[another reason for choosing ~~...~~ triangularly invariant: doesn't hold for all linear transformation]

actually outperform E.B., ~~some times~~ occasionally outperform Stein estimator we're to introduce.

draw back. not admissible.

Calculation fill-in gaps here