major ~~than~~ challenges: ① positive definite constraints.

② $\frac{p(p+1)}{2}$ parameters to estimate. grows ~~exponentially~~ quadratically with p.

When $p >> n$. what happens to sample covariance matrix? Stein 1956.

Also when $\frac{p}{n}$ large. ~~Johnst~~ Johnstone 2001

↳ the estimator. distorted: $\frac{\lambda_{max}}{\lambda_{min}} \uparrow$.

solution: shrinkage — shrink eigenvalues of S.
— stein family: eigenvalues of S stay the same.
— shrink S to some pre-speified structure: diagonal / auto regressive ...

when Σ^{-1} is of more interest: $\Sigma \xrightarrow{O(p^3)} \Sigma^{-1}$ and we want sparsity.

Meinshausen & Buhlmann 2006.

solution: fit separate LASSO to each variable ~ other variables.

? Why is this a solution:

consider $Y \sim X_1 + X_2$. $Y = X_1\beta_1 + X_2\beta_2 + \varepsilon$, and X_1, X_2 are r.v.

then why does $\beta_1 = 0$ reprensents?

$Cov(Y, X_1) = Cov(X_1\beta_1 + X_2\beta_2 + \varepsilon, X_1) = \beta_1 Var(X_1) + \beta_2 Cov(X_1, X_2)$.

$Cov(Y, X_1 | X_2) = Cov(X_1\beta_1 + X_2\beta_2 + \varepsilon, X_1 | X_2) = Cov(X_1\beta_1, X_1) = \beta_1 Var(X_1)$

$\beta_1 = 0 \Rightarrow X_1 \perp Y | X_2$. in the context, corresponding entree of $\Sigma^{-1} = 0$.

? How does it deal with symmetry of Σ^{-1}? (need to read)

⇓

this idea inspired: penalized. likelihood. approach with L_1 penalty Yuan & Lin 2007

Banerjee. 2008

Friedman 2008: glasso

↑

read this too. ⟶ Banerjee proved it guaran. to be positive definet.

2000 - 2010: emerging regression-theme on Covariance estimation.

(i) PCA ⟶ regression. Jong & Kotz 1999 / Zou, Hastie & Tibshirani 2006

it's $\boxed{\text{not}}$ permutation invariant for Y. (~~may~~ e.g. time series) why? [Look into]. ⟵ (ii) regression of cholesky decomposition. Pourahmadi 1999. 2001 / Bilmes 2000 / Huang 2006 / Rothman 2010

(iii) / (iv) / (v) something about glasso.

other not ~~per~~ permutation-invariant methods: tapering Furrer & Bengtsson 2007

banding: Bickel & Levina 2004, 2008a. / Wu & Pourahmadi 2003, 2009

permutation-invariant methods: (thresholding individual entrees). Bickel & Levina 2008b / El Karoui 2008a, 2008b / Rothman, L 2009