# Notes on MIT lectures on Effective Field Theory

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**Abstract**

abstract-text

## 1 Lecture 2

Consider a scalar-field action:

$$S[\phi] = \int d^d x \left[ \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \lambda \frac{1}{4!}\phi^4 - \tau \frac{1}{6!}\phi^6 + \dots \right] \tag{1}$$

where the values have mass dimensions:

$$[\phi] = \frac{d-2}{2} \quad [d^d x] = -d \quad [m^2] = 2 \tag{2}$$

$$[\lambda] = 4 - d \quad [\tau] \quad = 6 - 2d \quad \dots \tag{3}$$

Say we want to study the interaction matrix element $\langle \phi(x_1)\dots\phi(x_n)\rangle$, at a long distance $x^\mu = Sx'^\mu$ where

$$\begin{aligned} S &\to \infty \\ x' &\to \text{fixed} \end{aligned} \tag{4}$$

Let

$$\phi(x) = S^{\frac{2-d}{2}} \phi'(x') \tag{5}$$

$$S'[\phi'] = \int d^d x' \left[ \frac{1}{2}\partial_\mu \phi' \partial^\mu \phi' - \frac{1}{2} m^2 S^2 \phi'^2 - \lambda \frac{S^{4-d}}{4!}\phi'^4 - \tau \frac{S^{6-2d}}{6!}\phi'^6 + \dots \right] \tag{6}$$

Now:

$$\langle \phi(S'x_1)\dots\phi(Sx'_n)\rangle = S^{\frac{n(2-d)}{2}} \langle \phi'(x'_1)\dots\phi'(x'_n)\rangle \tag{7}$$

We know $x'$ is invariant with $S$, so the interaction elements in invariant under $S$. Assume $d = 4$, when $S \to \infty$ we have the following properties:

- The $m^2$-term becomes increasingly important.
- The $\tau$-term becomes less important.
- The $\lambda$-term remains just as important.

Therefore:

- $\phi^2$ is relevant
- $\phi^4$ is marginal
- $\phi^6$ is irrelevant

For finite (large) $S$, the dimension of parameters (or operators) tells us their importance. Use the mass scale of parameters:

$$m^2 \sim \Lambda_{new}^{(2)} \tag{8}$$
$$\lambda \sim \Lambda_{new}^{(0)} \tag{9}$$
$$\tau \sim \Lambda_{new}^{(-2)} \tag{10}$$

Since large distances means small momenta, i.e. $p \ll \Lambda_{new}$.

## 1.1 Divergences

Take now $m = 0$, or small such that $m^2 S^2 \sim 1$.

$$\sim \lambda^2 \int đ^d k \left(\frac{1}{k^2 - m^2 + i\epsilon}\right)\left(\frac{1}{(k+p)^2 - m^2 + i\epsilon}\right) \tag{11}$$

which diverges as $\Lambda^{d-4}$, and $d-4$ is the degree of divergences.

$$d = 4 \sim \int \frac{d^d k}{k^4} \sim \int \frac{dk}{k} \sim \ln(\Lambda) \leftrightarrow \epsilon^{-1} \tag{12}$$

This renormalizes the $\lambda\phi^4$ interaction vertex.

The loop diagrams with a $\lambda$ and $\tau$ vertex is similar to:

$$\lambda\tau \int \frac{đ^d k}{(\ldots)(\ldots)} \tag{13}$$

which renormalizes the $\tau\phi^6$-vertex.
The loop diagrams with two $\tau$ vertices is similar to:

$$\tau^2 \int \frac{đ^d k}{(\ldots)(\ldots)} \tag{14}$$

which renormalizes the $\phi^8$-vertex.
Since $\phi^8$ is not in $S[\phi]$ (unless we consider the $(+\ldots)$ terms), the theory is not renormalized in the traditional sense. However, if $\tau \sim \Lambda_{new}^{(-2)}$ is small ($p^2\tau \ll 1$), then the theory can be renormalized order by order in $(\Lambda_{new}^{(-1)})$. To include all corrections up to $\Lambda_{new}^{(-r)}$ or $S^{-r}$, we include all operators with dimensions $[O] \le d + r$. Here we assume power counting is synonymous with dimensions (i.e. $x_i^\mu = S x_i'^\mu$ where $S_i = S \;\forall\; i$, meaning $S$ is universal). For the standard model (SM), $\mathcal{L}^{(0)}$, all operators are of order $[O] \le 4$, and that it is renormalizable in the traditional sense. For SM correlations, $\mathcal{L}^{(1)}$, we add

$$\mathcal{L}^{(1)} = \frac{C}{\Lambda_{new}} O_5 \quad , \tag{15}$$

where $[O_5] = 5$ and $[C] = 0$, meaning $C \sim 1$, and we have *made* $\Lambda_{new}$ explicit.
Since nothing in $\mathcal{L}^{(0)}$ constrains $\Lambda_{new}$, we're to take[1] $\Lambda_{new} \gg m_t, m_W$ by as much as we want.

[1] Here, $m_t$ and $m_W$ are the top quark and $W$-boson masses, respectively.

## 1.2 Corrections to $\mathcal{L}^{(0)}$

$$\mathcal{L}^{(0)} = \underset{\Lambda_{new}^{(0)}}{\mathcal{L}^{(0)}} + \underset{\Lambda_{new}^{(1)}}{\mathcal{L}^{(1)}} + \underset{\Lambda_{new}^{(2)}}{\mathcal{L}^{(2)}} + \ldots \quad \text{for} \quad p^2 \sim m_t^2,\ m_t \ll \Lambda_{new} \tag{16}$$

- Assume the Lorentz and gauge invariances are unbroken. Each $\mathcal{L}^{(i)}$ is invariant under Lorentz and $SU(3) \times SU(2) \times U(1)$ transforms.
- Construct $\mathcal{L}^{(i)}$ from the same degrees of freedom as $\mathcal{L}^{(0)}$ and assume the Higgs vacuum expectation value ($v = 246$ GeV) holds.
- Assume no new particle produced at $p$, only at $\Lambda_{new}$.

Equation 1:

$$\mathcal{L}^{(1)} = \frac{C_5}{\Lambda_5} \epsilon_{ij} \bar{L}_L^{c_j} H^j \epsilon_{kl} L_L^k H^l \tag{17}$$

where $\bar{L}_L^{c_j} \equiv \left(\bar{L}_L^j\right)^T C$, $H = \begin{pmatrix} h^+ \\ h^- \end{pmatrix}$, and $L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$. This term is the *only* dimension $-5$ operator consistent with symmetries.

Replacing $H \to \begin{pmatrix} 0 \\ \nu \end{pmatrix}$ gives Majorana mass terms for observed $\nu$:

$$\frac{1}{2} m_\nu \nu_L^a \nu_L^b \epsilon_{ab} + h.c. \quad \text{where} \quad m_\nu = \frac{C_5 v^2}{2\Lambda_{new}} \tag{18}$$

Knowing that $m_\nu \leq 0.5$ eV leads us to believe $\Lambda_{new} \gtrsim 6 \times 10^{14}$ GeV for $C_5 \sim 1$.

Equation 2: Dimension $-6$ operators exists that violate baryon number.

Equation 3: With lepton and baryon number imposed, there are 80 operators with dimension $-6$:

$$\mathcal{L}^{(2)} = \sum_{i=1}^{80} \frac{c_i}{\Lambda_{new}^2} O_i^{(6)} \tag{19}$$

There are two points to remark:

1. For any observable, only a "few" terms contribute.
2. For any new theory at $\Lambda_{new}$, a particular pattern of $c_i$'s are expected.