Current results of Gamow HF and Gamow shell model with filling approximation

Koshiroh Tsukiyama

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I. GAMOW HARTREE-FOCK BASED ON REALISTIC NN INTERACTION

To construct the Gamow Hartree-Fock potential and the effective interaction for Gamow shell model, one has to calculate the various diagrams in the framework of many-body perturbation theory. Since all the realistic NN interactions have strong repulsive core, which makes the perturbative approach difficult, the short range (high momenta) corelation has to be renormalized. We carried out a renormalization scheme which is based on cutoff in relative momentum of two nucleons. The NN interaction obtained from this procedure is called $\mathrm{V}_{\text{low-k}}$, having the dependence on the momentum cutoff Λ. Typical value of Λ is about $\Lambda \sim 2\mathrm{fm}^{-1}$. Using this renormalized NN interaction, the A-body Hamiltonian is written up to two-body interaction

$$\begin{aligned} H &= T + V \\ &= \frac{1}{2m}\sum_{i=1}^{A} \boldsymbol{k}_i^2 + \sum_{i<j}^{A} V_{\text{low-k}}(i,j) \end{aligned} \tag{1}$$

The spurious center of mass motion is removed as

$$\begin{aligned} T_{\text{in}} = T - T_{\text{c.m.}} &= \frac{1}{2m}\sum_{i=1}^{A} \boldsymbol{k}_i^2 - \frac{\boldsymbol{K}^2}{2mA} = \frac{1}{2m}\sum_{i=1}^{A} \boldsymbol{k}_i^2 - \frac{1}{2mA}\left(\sum_{i=1}^{A} \boldsymbol{k}_i\right)^2 \\ &= \frac{1}{2m}\left(1 - 1/A\right)\sum_{i=1}^{A} \boldsymbol{k}_i^2 - \frac{1}{mA}\sum_{i<j}^{A} \boldsymbol{k}_i \cdot \boldsymbol{k}_j. \end{aligned} \tag{2}$$

Then one can obtain the internal Hamiltonian as

$$\begin{aligned} (H_{\text{in}}) =: H &= T_{\text{in}} + V \\ &= \frac{1}{2m}(1 - 1/A)\sum_{i=1}^{A} \boldsymbol{k}_i^2 + \sum_{i<j}^{A}\left(V_{\text{low-k}}(i,j) - \frac{1}{mA}\boldsymbol{k}_i \cdot \boldsymbol{k}_j\right) && (3) \\ &\equiv \frac{1}{2m}(1 - 1/A)\sum_{i=1}^{A} \boldsymbol{k}_i^2 + H_1 && (4) \end{aligned}$$

HF equation is solved self-consistently with a trial Gamow basis which is generated by arbitral one-body potential, say Woods-Saxon, and denoted by Greeks $\alpha, \beta, \cdots$.

$$\sum_{\beta}\left[\langle\alpha|t|\beta\rangle + \frac{1}{2j_\alpha+1}\sum_{J}\sum_{h_\gamma h_\delta}(2J+1)\langle\alpha h_\gamma; J|H_1|\beta h_\delta; J\rangle\eta^{HF}_{h_\gamma,h_\delta}\right] z^{a-}_{\beta} = \epsilon_\alpha z^{a-}_{\alpha} \quad (5)$$

where, z^{n-}_{α} is a removing amplitude, defined as

$$z^{n-}_{\mu} = \langle\Psi^{N-1}_{n}|a_\mu|\Psi^{N}_{0}\rangle. \quad (6)$$

The HF density matrix η^{HF} is written by this removing amplitude as

$$\eta^{HF}_{\mu\nu} = \langle\Psi^{N}_{0}|a^\dagger_\mu a_\nu|\Psi^{N}_{0}\rangle = \sum_{n}\langle\Psi^{N}_{0}|a^\dagger_\mu|\Psi^{N-1}_{n}\rangle\langle\Psi^{N-1}_{n}|a_\nu|\Psi^{N}_{0}\rangle = \sum_{n} z^{n-}_{\nu} z^{n-}_{\mu} \quad (7)$$

The particle number is written as

$$A = \sum_{\mu}(2j_\mu+1)\eta^{HF}_{\mu\mu} \quad (8)$$

II. SINGLE-PARTICLE ENERGIES FROM GHF

In GHF basis, denoted by alphabet $a, b, \cdots$, the density matrix is diagonal, and single particle energies (s.p.e) are written as,

$$\epsilon_a = \langle a|t|a\rangle + \Sigma^{HF}(a,a) \quad (9)$$

$$\Sigma^{HF}(a,a) = \frac{1}{2j_a+1}\sum_{h}(2J+1)\langle ah; J|H_1|ah; J\rangle. \quad (10)$$

We show the results of GHF for closed shell nuclei, ^{4}He, ^{16}O and ^{22}O in Table I, II and III,

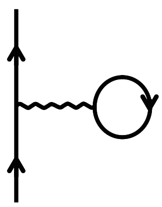


FIG. 1: The HF diagram. The first order self-energy insertion.

respectively. The results for ^{4}He is already obtained by the works of G. Hagen et al. Our results of course reproduce their results.

	^{4}He	
	π	ν
$s_{1/2}$	-23.35	-24.324
$p_{3/2}$	2.932	1.047-0.527i

TABLE I: Single particle states in ^{4}He from Gamow-HF with $\mathrm{V_{lowk}}$ which is renormalized from N3LO with coulomb, $\Lambda = 1.9$ fm^{-1}. Contour L_1^+ and L_2^+ are used.

	^{16}O, N3LO based $\mathrm{V_{lowk}}$							
	$\Lambda = 1.9\mathrm{fm}^{-1}$		$\Lambda = 2.0\mathrm{fm}^{-1}$		$\Lambda = 2.1\mathrm{fm}^{-1}$		$\Lambda = 2.2\mathrm{fm}^{-1}$	
	π	ν	π	ν	π	ν	π	ν
$s_{1/2}$			-62.04+0i	-66.68+0i	-56.70+0i	-61.20+0i	-51.40+0i	-55.76+0i
$p_{3/2}$			-28.25+0i	-32.69+0i	-25.15+0i	-29.46+0i	-22.10+0i	-26.27+0i
$p_{1/2}$			-17.13+0i	-21.39+0i	-15.18+0i	-19.31+0i	-13.28+0i	-17.28+0i
$d_{5/2}$			0.74+0i	-3.43+0i	1.84+0i	-2.18+0i	2.85+0i	-0.990+0i
$s_{1/2}$			1.43+0i	-2.15+0i	2.11+0i	-1.32+0i	2.67+0i	-0.60+0i
$d_{3/2}$			8.17+0i	4.99-1.84$\times10^{-2}i$	8.23+0i	5.06-2.02i	8.29+0i	5.07-2.16i

TABLE II: Single particle states in ^{16}O from Gamow-HF with $\mathrm{V_{lowk}}$ which is renormalized from N3LO with coulomb, Contour L_1^+, L_2^+ is used.

III. THE IMPORTANCE OF THE PARING INTERACTION

In order to investigate the importance of the paring interaction in low-lying resonance states in oxygen isotopes, we vary the strength of the paring channel in the process of the renormalization of realistic NN interaction.

$$\tilde{V}(\alpha) = \alpha \times \langle S=0, L=0; J=0|V|S=0, L=0; J=0\rangle, \qquad \alpha \in [0,1]. \tag{11}$$

Clearly,

$$\alpha = 0 \Rightarrow \text{no paring}$$

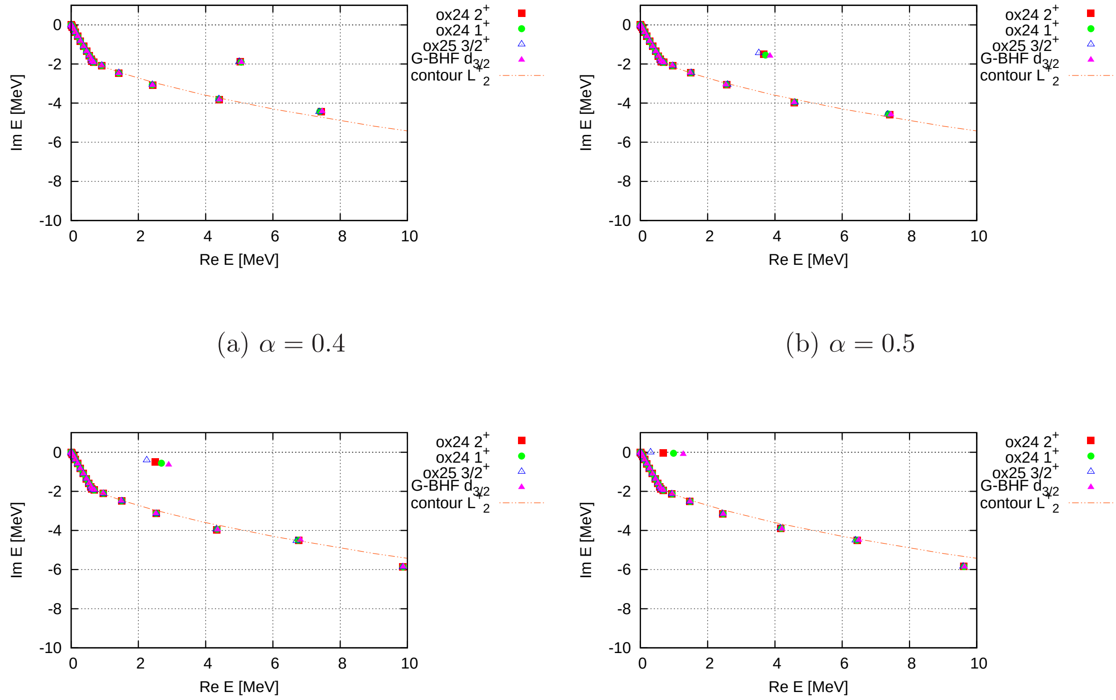
$$\alpha = 1 \Rightarrow \text{full paring}$$

	^{22}O, N3LO based $V_{\rm lowk}$							
	$\Lambda = 1.9{\rm fm}^{-1}$		$\Lambda = 2.0{\rm fm}^{-1}$		$\Lambda = 2.1{\rm fm}^{-1}$		$\Lambda = 2.2{\rm fm}^{-1}$	
	π	ν	π	ν	π	ν	π	ν
$s_{1/2}$			-82.95+0i	-82.91+0i	-75.51+0i	-76.06+0i	-67.93+0i	-69.66+0i
$p_{3/2}$			-44.66+0i	-41.19+0i	-39.88+0i	-37.33+0i	-35.07+0i	-33.41+0i
$p_{1/2}$			-37.89+0i	-35.22+0i	-34.15+0i	-32.15+0i	-30.35+0i	-29.05+0i
$d_{5/2}$			-10.00+0i	-8.94+0i	-7.59+0i	-7.27+0i	-5.29+0i	-5.62+0i
$s_{1/2}$			-6.95+0i	-6.77+0i	-5.11+0i	-5.44+0i	-3.47+0i	-4.27+0i
$d_{3/2}$			-2.12+0i	0.42-0.55$\times10^{-2}i$	-0.93+0i	0.91-0.04i	0.15+0i	1.27-0.09i
1^+ in ^{24}O			$-0.33\times10^{-2} + 0.114 \times 10^{-2}i$		0.57039-0.01295i		0.9852-0.05347i	
2^+ in ^{24}O			$-0.337 + 0.128 \times 10^{-2}i$		0.25126-0.601$\times10^{-3}i$		0.6756-0.02214i	
g.s. in ^{25}O			-0.8748-0i		$-0.1978-0i$		0.3004-0.19446$\times10^{-2}i$	

TABLE III: Single particle states in ^{22}O from Gamow-HF with $V_{\rm lowk}$ which is renormalized from N3LO with coulomb, Contour L_1^+, L_2^+ is used. In addition, the results of Gamow Shell model for the low-lying states in ^{24,25}O are shown.

IV. LOW-LYING STATES IN ^{24}O BY GSM WITH FILLING CONFIGURATION

Since SPE's obtained by Gamow HF have a large cutoff dependence, the low-lying states in oxygen isotopes also have a certain cutoff dependence. We in turn show, In Figs. 3, the results of GSM with filling configuration in which s.p. Gamow states are generated by Woods-Saxon potential. Fig. 3(a), 3(b) are 2^+, and Fig. 3(c), 3(d) are 1^+ states, respectively. The difference between Fig. 3(a) and 3(b) is two-body interaction. Fig. 3(a) is the result with V_{lowk} itself, while Fig. 3(b) is the result with the first order (V_{lowk}) plus core-poralization, particle-particle ladder and hole-hole ladder. The case is the same also for 1^+ states.



(a) $\alpha = 0.4$

(b) $\alpha = 0.5$

(c) $\alpha = 0.8$

(d) $\alpha = 1.0$,Full

FIG. 2: Low-lying states in $^{23-25}$O obtained by Gamow-shell model with different strengths of paring interacton. For realistic NN interaction, N3LO with $\Lambda = 2.2\text{fm}^{-1}$ is used.

V. CORRECTIONS OF S.P.E'S BY MANY-BODY PERTURBATION THEORY

Now we investigate the correction by many-body perturbation theory for the GHF s.p.e's, more presicely for the HF self-energy insertion. In GHF basis, s.p.e's are written as

$$\epsilon(a) = \langle a|t|a\rangle + \Sigma^{HF}(a,a) \tag{12}$$

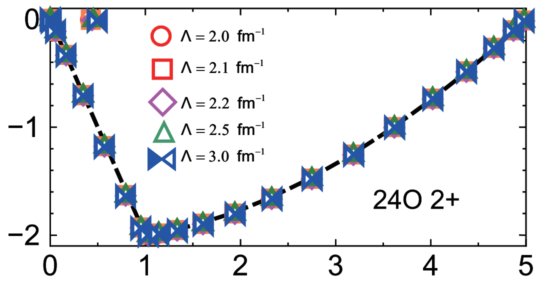
Unfortunately, there are some discrepancies between s.p.e's of GHF and those of experiments. Here we calculate the corrections from the second order perturbation.

$$\epsilon'(a) = \langle a|t|a\rangle + \Sigma^{HF}(a,a) + \Sigma^{2p-1h}(a,a) + \Sigma^{2h-1p}(a,a) \tag{13}$$

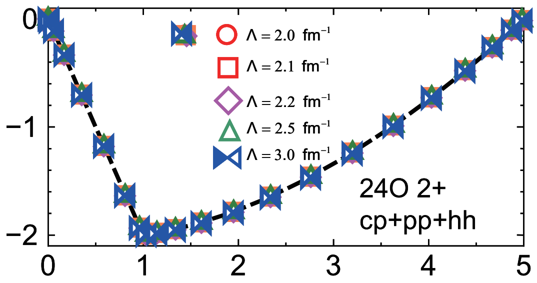
These contributions are shown in Fig. 4, and written as

$$\Sigma^{2p-1h}(a,c) = \frac{1}{2j_a+1}\sum_{p_1p_2hJ}\frac{1}{2}(2J+1)\frac{\langle ah;J|H_1|p_1p_2;J\rangle\langle p_1p_2;J|H_1|ch;J\rangle}{\epsilon_c+\epsilon_h-\epsilon_{p_1}-\epsilon_{p_2}} \tag{14}$$

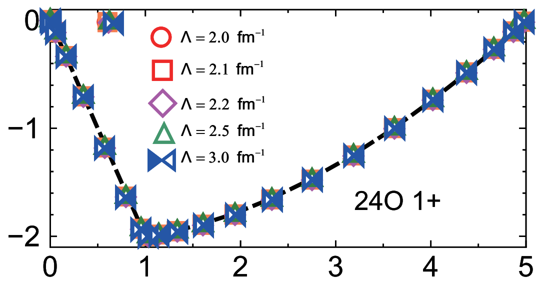
$$\Sigma^{2h-1p}(a,c) = -\frac{1}{2j_a+1}\sum_{ph_1h_2J}\frac{1}{2}(2J+1)\frac{\langle h_1h_2;J|H_1|cp;J\rangle\langle ap;J|H_1|h_1h_2;J\rangle}{\epsilon_{h_1}+\epsilon_{h_2}-\epsilon_a-\epsilon_p} \tag{15}$$



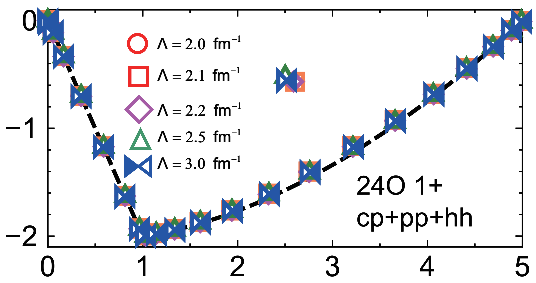
(a) 2^+ state, n3lo-based vlowk 1st oder



(b) 2^+ state, n3lo-based vlowk w/CP PP-,HH-ladder



(c) 1^+ state, n3lo-based vlowk 1st oder



(d) 1^+ state, n3lo-based vlowk w/CP PP-,HH-ladder

FIG. 3: GSM results for the low-lying states in ^{24}O. s. p. Gamow states are generated by Woods-Saxon potential.

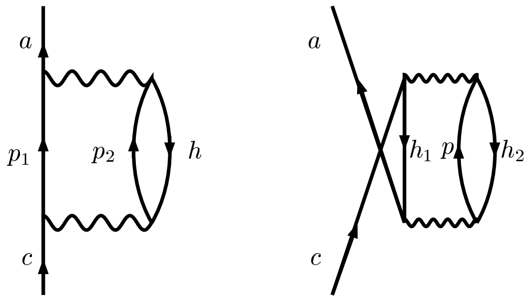


FIG. 4: $2p-1h$ and $2h-1p$ contributions