Continuum effects for the shell-model calculation near the drip line oxygen isotopes

Koshiro Tsukiyama[1] Takaharu Otsuka[123] Rintaro Fujimoto[4]

[1]Department of Physics, The University of Tokyo

[2]Center for Nuclear Study, The University of Tokyo

[3]RIKEN

[4]Hitachi Corporation

JPS annual meeting, September 23rd 2007, Hokkaido

Outline

1. From stable line to drip line
2. Continuum effect
3. Calculation and Results
   - The first excited state of ^{24}O
   - The first excited state of ^{23}O
   - The ground state of ^{25}O
   - Density
4. Summary
5. Compliment
   - The ground state of ^{26}O(preliminary)
   - Convergence

Outline

From β-stable to exotic

Near the β-stable line is well described with shell-model in which s.p. states are treated as bound state, and the wave functions are described as the eigenfunction of H.O potential.

Near the drip line, excitation strength distribution is explained by the continuum coupling in which not only the resonant state but also the whole continuum states are mixed through the residual interaction.

shf

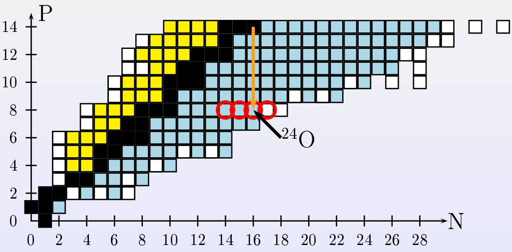
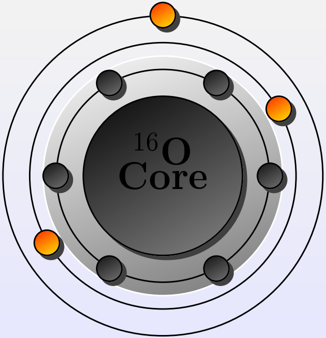


Fig 2.1: Nuclear chart in small mass region

Outline

The drip line of oxygen isotopes

- Many peaks are observed near the particle emission threshold.
- New magic numbers $\boldsymbol{N = 16, 14}$ seem to be confirmed. (Stanoiu,et al., PRC **69** 2004)
- $\boldsymbol{0d_{3/2}}$ orbit of neutron is lying at almost unbound level.



Binding energy of oxygen isotopes relative to ^{16}O by shell-model calculation with the lowest configuration which reproduce the full calculation

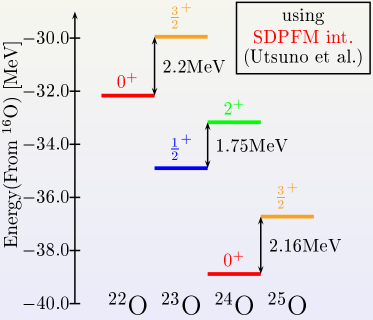


Fig 3.1: The lowest configuration

We reproduce the experimentally measured one neutron separation energy of ^{23}O with full shell-model calculation, modifying

$$\delta < s1/2, d5/2|V|s1/2, d5/2 >^{T=1}_{\text{mono pole}}$$

$$= -0.03\text{MeV}(< 5\%\text{modification}).$$

And then reproduce the levels of $^{23-25}$O relative to ^{22}O in terms of the filling configuration, modifying both s.p.e and 2-body matrix elements

Return to o24 Return to o25 Return to o23

Generating the continuum $d_{3/2}$ states

$$\mathcal{H}_0 = T + U_{WS} + V_{wall}$$
$$R = 1.09A^{1/3}, \text{diff} = 0.67,$$
$$V_{ls} = -0.44V_0$$

V_0 is determined so that W.S. potential satisfies the relation bellow.

$$< 0d_{3/2}|T+U_{WS}|0d_{3/2} >= 2.22\text{MeV}$$

Note: $0d_{3/2}$ is H.O. wave function

The condition that the place of wall L should at least meet

1. t_r =time scale corresponding the excitation energy $E(\sim 0.1 - 1\text{MeV})$
   t_c =classical time scale of which outgoing wave comes back to the origin after reflected at the wall.
   $t_r < t_c \Rightarrow L > O(10^1)$[fm]
2. For the calculation done later, the spectrum obtained should be adequately dense.

$\Rightarrow$ We set $L = 1000$[fm]

Outline

Diagonalization of the hamiltonian with residual interaction

J^+ states of ^{24}O can be constructed with these bases

$$\left|iJ^+\right\rangle = |[1s_{1/2} \otimes id^{c}_{3/2}]; J^+M >$$

$$(i = 1, \cdots, n_{\text{max}}, \text{ corresponding to } 0 - 20\text{MeV})$$

then we diagonalize the hamiltonian

o25.int.

$$\mathcal{H} = \sum_i \epsilon^i a_i^\dagger a_i + \frac{1}{4} \sum_{ijkl} \bar{v}_{res}^{ij,kl} a_i^\dagger a_j^\dagger a_l a_k$$

Type1^o $\quad V_{res}(r_i, r_j) = g_d \delta(r_{ij})$, and

Type2^o $\quad V_{res}(r_i, r_j) = g_1(1 + a_1 \sigma_1 \cdot \sigma_2) \exp(-r_{ij}^2/\mu_1^2)$

$$+ g_2(1 + a_2 \sigma_1 \cdot \sigma_2) \exp(-r_{ij}^2/\mu_2^2)$$

where $\mu_1 = 1.415, \quad \mu_2 = 0.7$

Diagonalization of the hamiltonian with residual interaction

The parameters g_d, g_1, g_2, a_1, a_2 are determined so as to meet the relations(a_2 is given by hand, and can be different)

$$\bar{v}_{res}^{sd,sd,J=2,T=1} = -0.4165\text{MeV}, \qquad \bar{v}_{res}^{sd,sd,J=1,T=1} = 0.6246\text{MeV},$$

and $\bar{v}_{res}^{ss,ss,J=0,T=1} = -1.1967\text{MeV}.$

$$\Rightarrow \quad g_1 = -21.0, g_2 = 440\text{MeV}, \qquad a_1 = -5.0, a_2 = 0.2$$

$$g_d = -301\text{MeV} \cdot \text{fm}$$

Where, $1s_{1/2}, 0d_{3/2}$ is H.O.wave function. The parameters are determined so that they can reproduce the shell-model matrix elements of SDPFM.

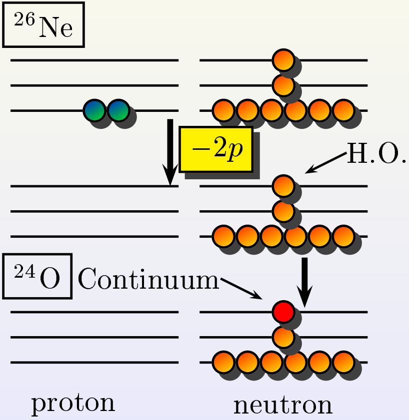
$$\left|J_k^+\right\rangle = \sum_j^{n_{\max}} C_{(k)}^j \left|\left[1s_{1/2} \otimes jd_{3/2}^c; J^+\right]\right\rangle \qquad (k = 1, \cdots, n_{\max})$$

The neutron emission probability in ^{24}O

We assume the low-lying states in ^{24}O is obtained through the reaction ^{9}Be(^{26}Ne,^{24} O)X. After that, a neutron is emitted if the low-lying states are unbound. In this case, the emission probability is considered to be proportional to the overlap between these states bellow

$$\text{Prob.} \propto \left| \left\langle 1s_{1/2} 0d_{3/2}; J^+ \middle| J_k^+ \right\rangle \right|^2$$

$$\propto \left| \sum_j C_{(k)}^j \left\langle 0d_{3/2} \middle| j d_{3/2}^c \right\rangle \right|^2$$



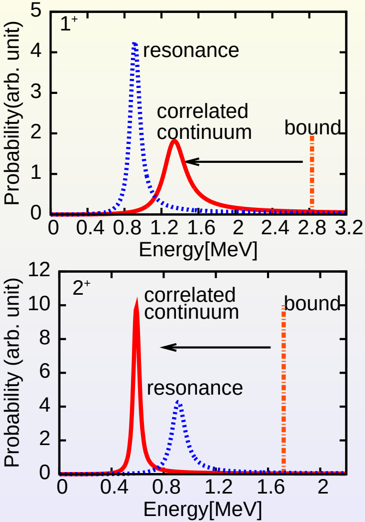


Fig 4.1: Emission probability in ^{24}O*

Experimental data (C.Hoffman, private communication) is now preliminary, but roughly

$$E(2^+) \sim 670\text{keV}$$
$$E(1^+) \sim 1.35\text{MeV}$$

Our result

$$E(2^+) \sim 600\text{keV}$$
$$E_{(}1^+) \sim 1.35\text{MeV}$$

Figure

The neutron emission probability in ^{23}O

- The spin parity of the ground state is $\mathbf{1/2^+}$(Sauvan et al,PLB,2000)
- The excited states(resonance), measured by using the reaction ^{22}O(d,p)^{23}O , is **4.0**MeV(**$3/2^+$**),**5.3**MeV(**$5/2^+$**) (Elekes et al, PRL**98**,102502,2007)

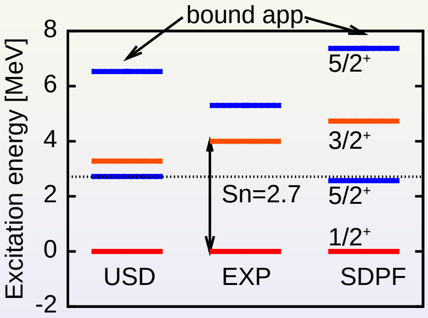


Fig 4.2: Excited states of ^{23}O. Both ends are the shell-model calculation with two deferent interactions.

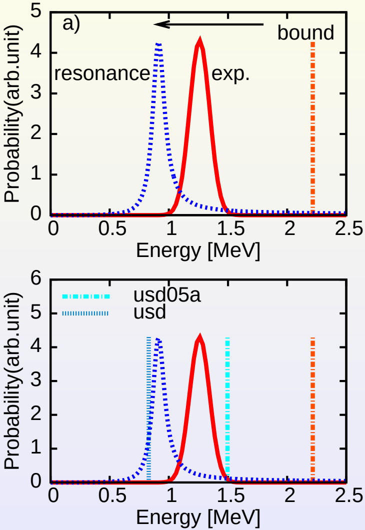


Fig 4.3: overlap value for ^{23}O ◂ Figure

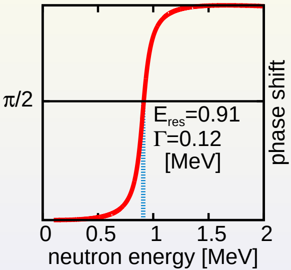
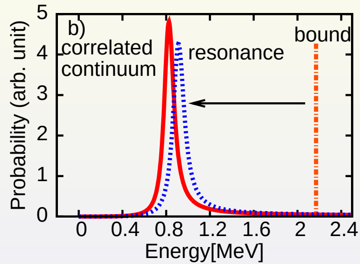
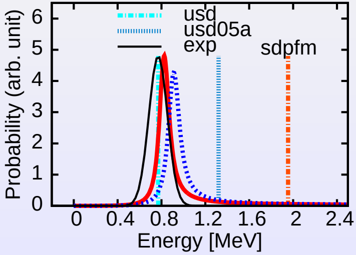


Fig 4.4: one particle phase shift for the W.S. potential given by the ^{22}O core
'resonance' corresponds to the one particle resonant state.

Neutron emission probability ^{25}O





Assuming ^{9}Be(^{26}F,^{25}O)X
Experimental data

$$E_{\mathrm{decay}} = 770(20)\mathrm{keV}$$
$$\Gamma \sim 140\mathrm{keV}$$

Our result

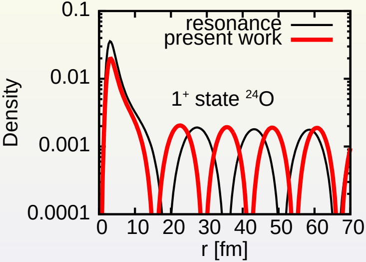
$$E_{d3/2} = 820\mathrm{keV}$$
$$\Gamma \sim 65\mathrm{keV}$$

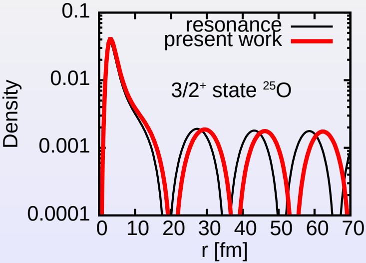
Note: USD seems to be the best, but it has another problem. Using this interaction, ^{26}O is bound for the two neutron separation.

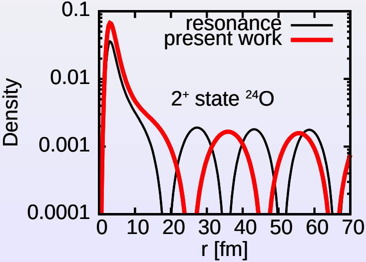
Density of the neutrons is

$$\begin{aligned}\rho_k(r) &= \langle\Psi_k|\,\hat{\rho}(r)\,|\Psi_k\rangle \\ &= n_s|\phi_{1s}(r)|^2 \\ &\quad + \sum_{ij} c^{i*}_{(k)} c^{j}_{(k)} \phi^{*}_{id^c}(r)\phi_{jd^c}(r)\end{aligned}$$

$$= \rho_s(r) + \rho_{d,k}(r)$$







Outline

Result and discussion

- The peak denoted by ’resonance’ is corresponding to the one-particle resonance of W.S. potential($\sim \mathbf{0.9}$[MeV]).
- The coupling to the continuum states lowers the energy of the 1st excited $\mathbf{2^+}$ state of ^{24}O by about $\mathbf{1}$MeV, but $\mathbf{2^+}$ state is still unbound. $\mathbf{1^+}$ state is pushed up from the resonance.
- In the case of ^{25}O, the ground state is lowered by because of the coupling between bound and continuum states, but is still unbound, and it is consistent with the recent experiment.
- With the residual interaction, wave function changes from that of the resonance in and near the nuclear surface.

Outline

We diagonalize the hamiltonian in $M = 0(J = 0, 2)$ space. Wave function is written as

$$|\Psi_k; M = 0 > = \sum_{i,j} c^{ij}_{(k)} \left| s_{1/2,1/2} s_{1/2,-1/2} \, {}_i d^c_{3/2,3/2} \, {}_j d^c_{3/2,-3/2} \right\rangle$$

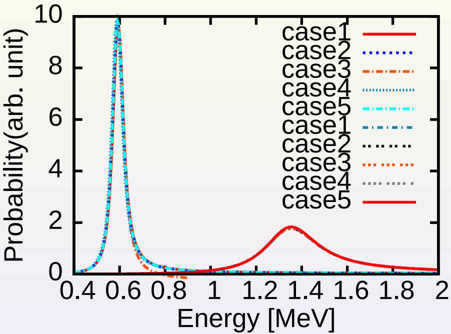
$$= \sum_{i,j} d^{ij}_{(k)} \left| s_{1/2,1/2} s_{1/2,-1/2} \, {}_i d^c_{3/2,1/2} \, {}_j d^c_{3/2,-1/2} \right\rangle$$

$$=: \sum_{i,j} c^{ij}_{(k)} |\Psi^{ij}_1 > + d^{ij}_{(k)} |\Psi^{ij}_2 >$$

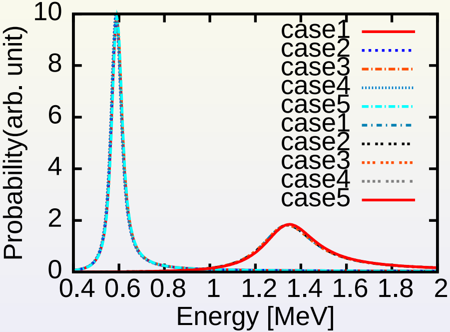
$$H(M = 0) \doteq \begin{pmatrix} A + B^+ + C^+ & B^- + D \\ B^- + D & A + B^+ + C^- \end{pmatrix}$$

Where

$$
\begin{aligned}
A(ij,i'j') &= E(^{24}O) + (\epsilon_{id} + \epsilon_{jd})\delta_{ii'}\delta_{jj'} \\
&+ \frac{1}{N_{ij}N_{i'j'}}\left(\frac{5}{2} < sid|V|si'd >_{J=2} + \frac{3}{2} < sid|V|si'd >_{J=1}\right)\delta_{jj'} \\
&+ \frac{1}{N_{ij}N_{i'j'}}\left(\frac{5}{2} < sjd|V|sj'd >_{J=2} + \frac{3}{2} < sjd|V|sj'd >_{J=1}\right)\delta_{ii'} \\
B^{\pm}(ij,i'j') &= \frac{1}{4N_{ij}N_{i'j'}}(< ij|V|i'j' >_{J=2} \pm < ij|V|i'j' >_{J=0}) \\
C^{\pm}(iji'j') &= \frac{1}{20N_{ij}N_{i'j'}}\left(\left\{\begin{array}{c} 9 \\ 1 \end{array}\right\} < ij|V|i'j' >_{J=1}\right. \\
&\left. + \left\{\begin{array}{c} 1 \\ 9 \end{array}\right\} < ij|V|i'j' >_{J=3}\right)(1-\delta_{ij})(1-\delta_{i'j'}) \\
D(ij,i'j') &= \frac{3}{20N_{ij}N_{i'j'}}(< ij|V|i'j' >_{J=3} - < ij|V|i'j' >_{J=1})
\end{aligned}
$$


10
8
6
4
2
0
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Probability(arb. unit)
Energy [MeV]
case1
case2
case3
case4
case5
case1
case2
case3
case4
case5

(d) $A = 24, nc = 300$


10
8
6
4
2
0
0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
Probability(arb. unit)
Energy [MeV]
case1
case2
case3
case4
case5
case1
case2
case3
case4
case5

(e) $A = 24, nc = 300$