Cooling of Neutron Stars

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Formation of Neutron Stars

1. Neutron stars are formed from the collapsed remnant of a massive star, a Type II, Type Ib, or Type Ic supernova.
2. When the core's size exceeds the Chandrasekhar limit of 1.44 solar masses, the pressure of degenerate electrons can no longer support it and the core undergoes a catastrophic collapse to produce a neutron star.
3. If a gravitational collapse is going to occur on a star's core over 3 solar masses, it would inevitably produce a black hole with infinite density.

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Characteristics of Neutron Stars

1. Supported by the pressure of degenerate neutrons:
   - During the collapse of the core, the electrons and protons are pushed closer together, which are then combined to produce neutrons by the inverse β-decay.
   - All the low energy levels available to the neutrons are filled, forcing the neutrons into higher energy states. Consequently, the star becomes colder.
   - The Pauli exclusion principle takes over, creating an effective pressure which prevents further gravitational collapse.
   - Neutron stars can be stable no matter what the internal temperature is, because the pressure that supports the star is independent of temperature.

Characteristics of Neutron Stars

1. Are of exceedingly small size:
   - A typical neutron star has a mass between 1.35 to about 2.1 solar masses ($M_\odot = 1.989 \times 10^{30}$ kg), with a corresponding radius in the range $20 - 10$ km - $30,000$ to $70,000$ times smaller than the Sun ($R_\odot = 6.96 \times 10^5$ km).
   - Neutron stars have densities of $8 \times 10^{13} - 2 \times 10^{15}$ g/cm^3, about the nuclear matter density ($\rho_0 = 2.8 \times 10^{14}$ g/cm^3).
   - There are large variations in predicted radii and maximum masses because of the uncertainties in the equations of state near and above ρ_0. The present theoretical limit is on the order of $M_{\max} \sim 2.5\, M_\odot$.

Outline

1. Introduction
   - Basic Concepts of Neutron Stars
   - Energy Loss from a Neutron Star
   - Overall Motivation and Objectives
2. Equations of State
3. Neutrino Emissivities
   - Direct Urca Process
   - Neutrino Pair Bremsstrahlung
4. Summary

Energy Loss by the Neutrino Emission

1. The characteristic way of energy loss from a neutron star is expected to proceed via the emission of neutrinos.
2. One of the most remarkable aspects is that neutrinos become temporarily trapped within the star during collapse.
3. The typical neutrino matter cross section is $\sigma \approx 10^{-40}\,\mathrm{cm}^2$, resulting in a mean free path $\lambda \approx (\sigma n)^{-1} \approx 10\,\mathrm{cm}$, with the baryon number density $n \simeq 2 - 3\, n_0$ ($n_0 \simeq 0.16\,\mathrm{fm}^{-3}$).
4. This length is much less than the proto-neutron star radius $R > 20\,\mathrm{km}$.

Energy Loss by the Neutrino Emission

1. Very shortly after formation the escape of neutrinos from the interior occurs on a diffusion time $\tau \simeq 3R^2/\lambda c \approx 10\,\mathrm{s}$.
2. The loss of neutrinos initially warms the stellar interior. The core temperature more than doubles, reaching $\sim 50\,\mathrm{MeV}$ $(6 \times 10^{11}\,\mathrm{K})$.
3. After $10 - 20\,\mathrm{s}$, however, the steady emission of neutrinos begins to cool the interior. Because the cross section $\sigma \propto \lambda^{-1}$ scales as the square of the mean neutrino energy, the condition $\lambda > R$ is achieved in about $50\,\mathrm{s}$. The star becomes transparent to neutrinos, and its cooling rate accelerates.
4. This stage lasts up to about 10^5 years of age, and the photon emission overtakes neutrinos only when the internal temperature falls to $\sim 10^8\,\mathrm{K}$.

Energy Loss by the Neutrino Emission

1. The important neutrino-emitting process leading to the cooling of neutron stars is the so-called direct Urca process:

$$n \longrightarrow p + e^- + \bar{\nu}_e, \qquad\qquad p + e^- \longrightarrow n + \nu_e.$$

These are the well known β-decay of the neutron and electron capture on protons. But these processes are not usually allowed on the surface of a neutron star because the densities are too low to ensure momentum conservation.

Energy Loss by the Neutrino Emission

1. For a long time, the dominant mode of neutrino emission is thought to be the so-called modified Urca process:

$$n + n \longrightarrow n + p + e^- + \bar{\nu}_e, \qquad n + p + e^- \longrightarrow n + n + \nu_e.$$

These processes are basically the direct Urca process, with the addition of a bystander particle whose sole purpose is to enhance the phase space.

Energy Loss by the Neutrino Emission

1. Besides the processes stated above, there exist several other neutrino-emitting processes, like neutrino pair bremsstrahlung:

$$n + n \longrightarrow n + n + \nu + \bar{\nu}, \qquad n + p \longrightarrow n + p + \nu + \bar{\nu}.$$

   Like the modified Urca process, the neutrino pair bremsstrahlung plays a crucial role in neutron star cooling.
2. All these processes together make the so-called standard cooling scenario for a neutron star.

Outline

Aims and Motivation

1. Work out a general formalism and a calculational scheme that may be carried out to determine the neutrino emissivities of any process, with a given effective interaction.
2. First, I focused on the direct Urca process and studied the β-decay of the neutron. I aimed at calculating the neutrino emissivities due to the direct Urca process.
3. For this purpose I constructed a numerical program in order to carry out a five-dimensional integration over the phase space.

Aims and Motivation

1. I went on to study the neutrino pair bremsstrahlung in detail and evaluated the corresponding Feynman amplitudes analytically.
2. As a principal stage, I developed a generic code for calculating the Feynman amplitude of any process.
3. At last, I calculated the rate of energy loss associated with the neutrino emission from neutrino pair bremsstrahlung.

Aims and Motivation

1. An approximate treatment of the neutrino emission:
   - Carried out by Friman and Maxwell in 1979.
   - A nucleon-nucleon interaction that consists of a long-range, one-pion-exchange tensor part and a short-range part parametrized with nuclear Fermi liquid (Landau) parameters.
2. A proper approach of the neutrino emission:
   - Introduce the many-body contributions from the equation of state in order to generate a medium modified interaction.
   - To describe the nucleon-nucleon interaction I have used the meson propagators, and the appropriate coupling constants.
   - The only medium effects I have included, are the effective masses for the various baryons, and their corresponding densities and chemical potentials.

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Equations of State (EoS)

1. The properties of neutron stars are determined by the EoS for dense matter. Properties like the mass range, the mass-radius relationship, the crust thickness and the cooling rate can all be extracted from the EoS. The same EoS is also crucial in calculating the energy released in a supernova explosion.
2. The EoS of neutron star matter below neutron drip, which occurs at densities around $4 \times 10^{11}\ \mathrm{g/cm^3}$, and at densities above neutron drip but below the saturation density ($\rho_0 = 2.8 \times 10^{14}\ \mathrm{g/cm^3}$) of nuclear matter is relatively well known.
3. In the high-density range above ρ_0, the physical properties of matter are still uncertain so that the associated EoS is only very poorly known.

Equations of State (EoS)

1. In calculating the rate of neutrino emission, I have applied various EoS and composition of matter in order to include the many-body correlations in an effective way.
2. There are primarily three EoS I have applied for the sake of many-body correlations. Here I give the description of these three EoS:
   - EoS I: pure nuclear matter with two-body interactions,
   - EoS II: pure nuclear matter with three-body interactions,
   - EoS III: the presence of hyperons in nuclear matter with two-body interactions.

Outline

Direct Urca Process

1. The star as a whole is in thermal, β and charge equilibrium, and the interior matter occupies its ground state. Because of the extremely high densities and relatively low temperatures involved (in comparison with the relevant Fermi energies), the electrons, protons, and neutrons in the interior are all degenerate.
2. The direct Urca process can take place only near the Fermi energies of participating particles and simultaneous conservation of energy and momentum require the inequality, $p_F(e) + p_F(p) \geqslant p_F(n)$, to be satisfied. This leads to the well known threshold for the proton fraction $x_p = n_p/n_B \geqslant 1/9 \approx 11\%$, where $n_B = n_n + n_p$ is the total baryon density.

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Direct Urca Process

The rate of energy loss associated with the neutrino emission from the direct Urca process, is obtained by integrating the total phase space available to the participating particles, multiplied by the corresponding Feynman amplitude and total neutrino energy,

$$\varepsilon_{\text{Urca}} = \frac{2^4}{(4\pi)^6}\cos^2\theta_C \int |\mathbf{p}_n| dE_n \int |\mathbf{p}_p| dE_p \int \sin\theta_p d\theta_p \int d\phi_p \times \\ \times \int \sin\theta_{\bar{\nu}_e} d\theta_{\bar{\nu}_e} \frac{E_{\bar{\nu}_e}}{E_e} |\mathbf{p}_{\bar{\nu}_e}| |\mathcal{M}|^2 f(E_n)(1 - f(E_p))(1 - f(E_e)).$$

To perform the remaining integrations, I developed a numerical program, in which I implemented the above expression for emissivity.

Direct Urca Process

I depicted the produced rates of energy loss as a function of the number density of particles. In this context, I picked out two temperatures: 10^8 K and 10^9 K, which are of relevance for the study of neutrino emission. In the case of the direct Urca process, I compared the results with the approximate rates obtained by the expression of Prakash *et al.* (1991):

$$\varepsilon_{\mathrm{Urca}} = \frac{457\pi}{10080} G_F^2 \cos^2\theta_C (1 + 3g_A^2) \frac{m_n^* m_p^* \mu_e}{\hbar^{10} c^5} (k_B T)^6 \Theta_t .$$

Here Θ_t is the threshold factor $\Theta(p_F^e + p_F^p - p_F^n)$, which is $+1$ if the argument exceeds 0, and is 0 otherwise. However, it ought to be stressed that the above result was derived by assuming $|\mathbf{p}|d|\mathbf{p}| \simeq mdE$.

Direct Urca Process

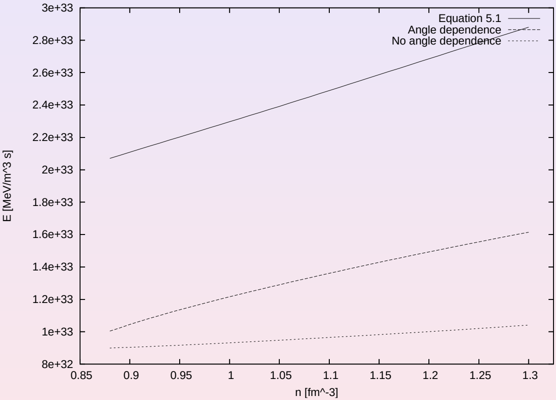
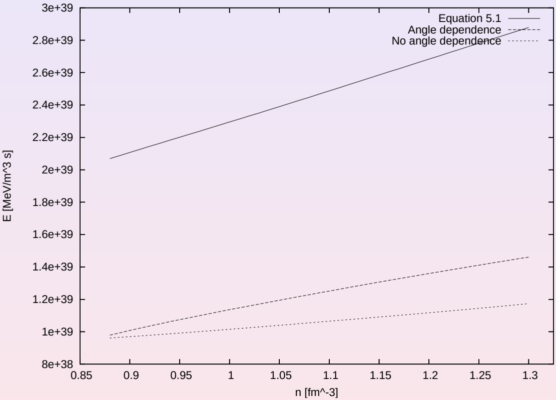


Figure: Emissivities for the direct Urca process at $T = 10^8$ K for EoS I. Angle dependence is included.

Direct Urca Process



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Direct Urca Process

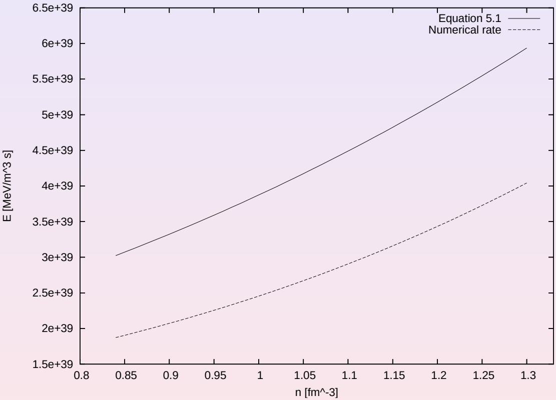


Figure: Emissivities for the direct Urca process at $T = 10^9$ K for EoS II. Angle dependence is excluded.

Direct Urca Process

1. Although the densities range from $0.02\,\mathrm{fm}^{-3}$ to $1.3\,\mathrm{fm}^{-3}$, the direct Urca process starts only at a density $n = 5.5\, n_0$ $= 0.88\,\mathrm{fm}^{-3}$.
   This is due to the requirement of the minimum proton fraction, being $x_p = n_p/n \geqslant 14.08\%$.
   This means in turn that the triangle inequality, $p_F^e + p_F^p \geqslant p_F^n$, is to be fulfilled so as to ensure simultaneous conservation of energy and momentum for the process.
2. When including the angle dependence the phase space of the interacting particles is expected to increase, resulting in larger energy loss rates.

Direct Urca Process

1. The rates with angle dependence get closer to the approximate rates, indicating that the expression of Prakash is a reliable estimate for the emissivities. To my knowledge, this is the first time this has been checked.
2. As expected, the three-body interactions tend to boost the energy loss by neutrino emission, yielding a rapid emission rate with increasing density.
3. In addition, the EoS with three-body interactions reduces the threshold proton fraction to $x_p \geqslant 14.05\%$, causing the direct Urca process to start at $n = 5.25\, n_0 = 0.84\,\mathrm{fm}^{-3}$.

Direct Urca Process

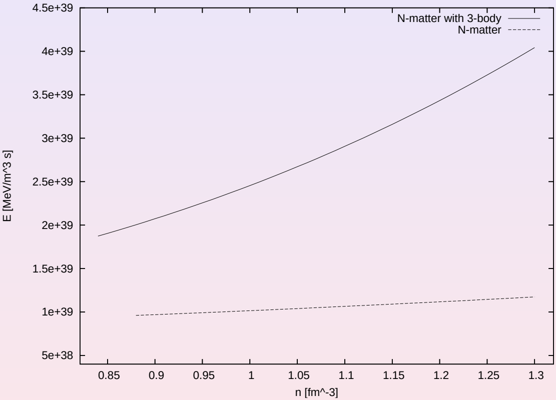


Figure: Emissivities for the direct Urca process at $T = 10^9$ K for EoS I and II. Angle dependence is excluded.

Direct Urca Process

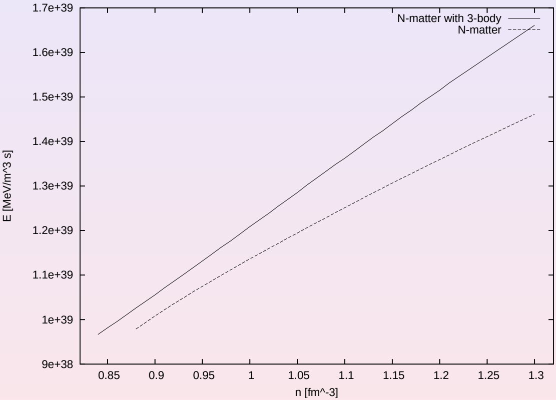


Figure: Emissivities for the direct Urca process at $T = 10^9$ K for EoS I and II. Angle dependence is included.

Direct Urca Process

1. From the results for the direct Urca process we deduce that the three-body interactions (EoS II) enhance the proton fraction considerably and thereby the increase of energy loss.
2. As the proton fraction gets higher, the chemical potentials of the interacting particles become also larger.
3. We also note that the EoS II allows the direct Urca process to start at $n = 5.25\, n_0 = 0.84\,\mathrm{fm}^{-3}$, while the EoS I ignite the process at $n = 5.5\, n_0 = 0.88\,\mathrm{fm}^{-3}$.
4. The angle dependence plays a crucial role in the energy loss by neutrinos.

Outline

Neutrino Pair Bremsstrahlung

Concerning the neutrino emission from neutrino pair bremsstrahlung, the rate of energy loss is obtained by integrating the total phase space available to the participating particles, multiplied by the corresponding Feynman amplitude and total neutrino energy,

$$\varepsilon_{\nu\bar{\nu}} = \frac{2^6}{(4\pi)^{10}} \int |\mathbf{p}_1| dE_1 \int |\mathbf{p}_2| dE_2 \int \frac{|\mathbf{p}_3|}{E_4} dE_3 \int \sin\theta_3 d\theta_3 \int d\phi_3 \times$$
$$\times \int |\mathbf{q}_\nu||\mathbf{q}_{\bar{\nu}}| dE_\nu \int \sin\theta_\nu d\theta_\nu \int \sin\theta_{\bar{\nu}} d\theta_{\bar{\nu}} |\mathcal{M}|^2 (E_\nu + E_{\bar{\nu}}) \times$$
$$\times f(E_1) f(E_2)(1 - f(E_3))(1 - f(E_4)) \cos^2\theta_C .$$

The remaining eight-dimensional integration was calculated numerically.

The $nn\nu\bar{\nu}$ Process

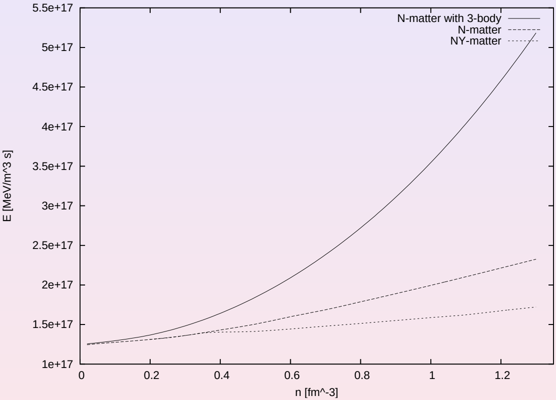
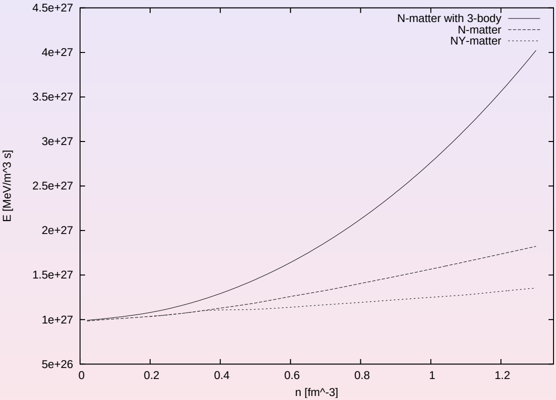


Figure: Emissivities for the $nn\nu\bar{\nu}$ process at $T = 10^8$ K for EoS $\mathrm{I} - \mathrm{III}$. Angle dependence is excluded.

The $nn\nu\bar{\nu}$ Process



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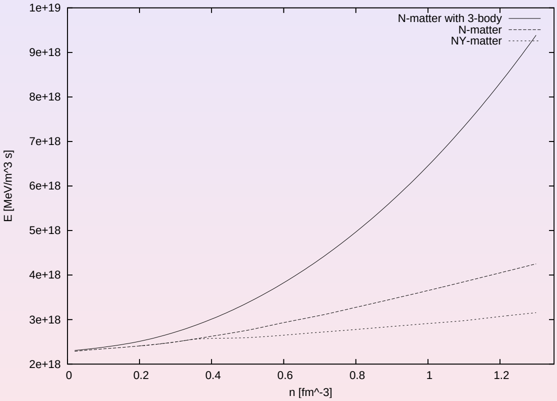

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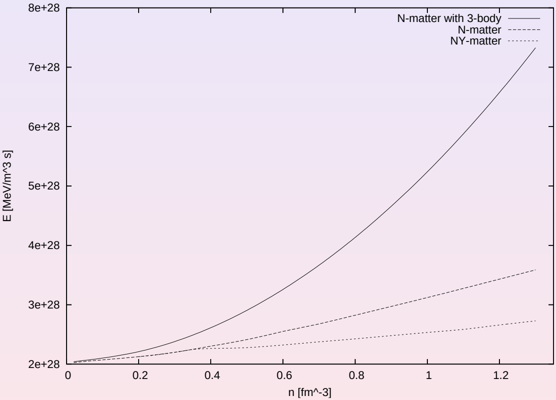

Figure: Emissivities for the $nn\nu\bar{\nu}$ process at $T = 10^9$ K for EoS $\mathrm{I} - \mathrm{III}$. Angle dependence is included.

The $nn\nu\bar{\nu}$ Process

1. It is interesting to notice that the $nn\nu\bar{\nu}$ process will take place no matter what the number density is.
   This is due to the presence of the additional neutron which increases the phase space of the interacting particles.
   This in turn implies that the conservation of energy and momentum is possible for the entire density range $n = 0.02 - 1.3\,\mathrm{fm}^{-3}$.

The $nn\nu\bar{\nu}$ Process

1. From the energy loss rates obtained for the $nn\nu\bar{\nu}$ process, we infer that the three-body interactions associated with the EoS II cause significant energy-loss rates when compared with EoS I and III. This is a consequence of the relatively large proton fraction caused by EoS II.
2. The hyperons with two-body interactions tend to give a rather constant energy loss through the entire density range.
3. The angle dependence gives relatively larger energy-loss rates as compared to the results obtained by ignoring angles. It seems that the relative difference here is by a factor of $18 - 20$.

Future Work

The one-boson interactions we have considered here, may be defin-ed as follows

$$V = \bar{u}(\mathbf{p}_1{}')\bar{u}(\mathbf{p}_2{}')\left(\sum_{j=1}^{5} v_j(\mathbf{k})F_j\right)u(\mathbf{p}_1)u(\mathbf{p}_2),$$

where F_j ($j = 1, ..., 5$) denote the Fermi invariant quantities, being

$$S = 1^{(1)}1^{(2)}, \quad V = \gamma_\mu^{(1)}\gamma_\mu^{(2)}, \quad T = \frac{1}{2}\sigma_{\mu\nu}^{(1)}\sigma_{\mu\nu}^{(2)},$$

$$A = i\gamma_5^{(1)}\gamma_\mu^{(1)}i\gamma_5^{(2)}\gamma_\mu^{(2)}, \quad P = \gamma_5^{(1)}\gamma_5^{(2)}.$$

Future Work

1. In our treatment, the $v_j(\mathbf{k})$ was given by the meson propagators only.
2. The only medium effects we have included, are the effective masses for the various baryons, and their corresponding densities and chemical potentials.
3. The next progress is to use the EoS to parametrize $v_j(\mathbf{k})$, in order to obtain an effective medium dependent interaction consistent with the associated EoS.

Concluding Remarks

1. From the results we obtained for the energy loss rates, we conclude that the direct Urca process would lead to more rapid cooling than the bremsstrahlung process.
2. The modified Urca process discussed in Chapter 1, is expected to give energy losses similar to that of the bremsstrahlung process. The $np\nu\bar{\nu}$ results will be presented in a forthcoming publication together with modified Urca rates.
3. While an exact treatment of neutron star cooling lies still in the future, it is clear already what the important physical principles are and how the calculation will be done.