Continuum effects for the shell-model calculation of the low-lying states of Oxygen drip line nuclei

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The study of the shell structure near the drip lines is one of the exciting subjects in nuclear physics. The structure of nuclei near the stability line is well described with shell-model. In the region of the neutron and proton drip lines, however, the wave functions are spread out widely which provokes the reduction of a kinetic energy and affects the effective interaction. In the case of oxygen isotopes with a large neutron excess, the $0d_{3/2}$ orbit of neutron is lying at relatively high energy. So it is reasonable to suppose that the $0d_{3/2}$ orbit is unbound. In this case, the effect from continuum states seems to be important. In order to take into account the continuum effect, we make continuum $d_{3/2}$ orbits including not only resonance sate but also the whole continuum states. We calculate some low lying states for the neutron rich oxygen isotopes and then compare to the result of ordinary shell-model calculation and to the experimental data which have become available recently.

I. INTRODUCTION

In the framework of ordinary shell-model, single particle wave functions are described as the eigenfunction of the harmonic oscillator potential. This is of course correct for the stable nuclei, but when one comes to the neutron(proton) drip line, the situation is different. Both the ground states and the distribution of the excitation strength in the neutron rich region can not be described by the ordinary shell-model. These quantities can be explained by the continuum coupling in which not only resonant state but also non-resonant continuum state are mixed through the residual interaction. There are several reliable method to take into account the continuum coupling. Among them is Gamow shell model[1] in which Gamow hartree-fock basis is generated including bound, resonant and scattering states through the analytic continuation of hartree-fock potential to the complex momentum space. Our framework is rather simple and straightforward, but can effectively take into account the whole continuum states. We focus on the Oxygen drip line nuclei, where configurations are described in the sd-shell ($0d_{5/2}, 0d_{3/2}, 1s_{1/2}$).

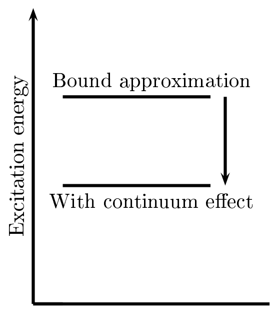

FIG. 1: schematic view of what we are considering.

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II. SHELL-MODEL CALCULATION WITH CONTINUUM EFFECT

As a starting point of the calculation, we define the single particle energy and effective interaction of the model space we are working on. We set up the model space which consists of $1s_{1/2}$ and $0d_{3/2}$. First, we modify the SDPFM interaction[2][3] so that it reproduces one neutron separation energy of ^{23}O, modifying the mono pole interaction bellow

$$\delta < 1s_1 0d_5|V|1s_1 0d_5 >_{T=1} = -0.03\text{MeV}. \quad (1)$$

This is actually less than 5% modification of the original value. Second, in order to define the single particle energy and the two body matrix elements in our model space composed of only $1s_{1/2}$ and $0d_{3/2}$ (not full sd-shell), we reproduce the energies of the low-lying states in $^{23-25}$O relative to ^{22}O, calculated with modified SDPFM, in terms of the filling configuration. The resulting single particle energies and two body matrix elements are

$$\epsilon_{s_{1/2}} = -2.752\text{MeV}, \epsilon_{d_{3/2}} = 2.217\text{MeV} \quad (2a)$$

$$< 1s_{1/2}0d_{3/2}|V|1s_{1/2}0d_{3/2} >_{J=2} = -0.516\text{MeV} \quad (2b)$$

$$< 1s_{1/2}0d_{3/2}|V|1s_{1/2}0d_{3/2} >_{J=1} = 0.740\text{MeV} \quad (2c)$$

$$< 1s_{1/2}1s_{1/2}|V|1s_{1/2}1s_{1/2} >_{J=0} = -1.319\text{MeV} \quad (2d)$$

$$< 0d_{3/2}0d_{3/2}|V|0d_{3/2}0d_{3/2} >_{J=0} = -2.3\text{MeV} \quad (2e)$$

In order to take into account the continuum effect, we generate the bound and continuum single particle states of $d_{3/2}$, solving shrödinger equation with one body hamiltonian,

$$H_0 = T + U_{WS} + V_{wall} \quad (3a)$$

$$R = 1.09A^{1/3}, a = 0.67, V_{ls} = -0.44V_0. \quad (3b)$$

V_0 is determined so that W.S. potential satisfies the relation bellow.

$$< 0d_{3/2}|H_0|0d_{3/2} >= 2.217\text{MeV} \quad (4)$$

This means $d_{3/2}$ states has the same energy as $\epsilon_{d_{3/2}}$ when we change the single particle wave function to that of harmonic oscillator. Here V_{wall} is an infinite large wall, added to discretize the spectrum obtained. The position of the wall is set at $L = 1000$[fm]. This is mainly because we would like to obtain adequately dense spectrum. One may be afraid that this is too extreme, but as we will show later the result is unchanged even when we change L up to several hundred fm.

III. LOW-LYING STATES IN $^{23-26}$O

A. Excited states of ^{24}O

For the purpose of describing the procedure of taking into account the continuum effect, we focus on the case of ^{24}O. The low-lying states of the other nuclei can be calculated almost the same manner. In the framework of shell-model, we consider the lowest configuration but $d_{3/2}$ state is expanded in terms of continuum single particle states obtained above. Therefore the excited states of ^{24}O is a superposition of these basis represented by $|iJ^+ >= |1s_{1/2} \otimes id_{3/2}; J^+ >$ $(J = 1, 2, i = 1, \cdots, n_{max})$ Here id means a continuum single particle state and i runs through the continuum basis (n_{max} corresponds to 20 MeV). With these basis, we diagonalize the hamiltonian

$$H = \sum_i \epsilon_i a_i^\dagger a_i + \frac{1}{4}\sum_{ijkl} \bar{v}_{ij;kl} a_i^\dagger a_j^\dagger a_l a_k. \quad (5)$$

For the two body interaction, we assume the two range Gaussian interaction

$$V(\vec{r}) = g_1(1 + a_1\sigma_1 \cdot \sigma_2)e^{r^2/\mu_1^2} + g_2(1 + a_2\sigma_1 \cdot \sigma_2)e^{r^2/\mu_2^2}. \quad (6)$$

We give by hand the range of the interaction $\mu_1 = 1.4, \mu_2 = 0.7$, and then we decide the remaining parameters g_i, a_i so that the matrix elements of this interaction reproduce those of SDPFM interaction shown in Eq. (2a)to(6). This means that our calculations become the same as the normal shell-model calculations when we employ the harmonic oscillator wave functions for the $0d_{3/2}$ state. In present case we obtain

$$g_1 = 91.5\text{MeV}, \qquad g_2 = -847\text{MeV} \quad (7a)$$

$$a_1 = 0.82, \qquad a_2 = 0.25 \quad (7b)$$

After the diagonalization, we get the k th $|J^+ >$ states

$$|J_k^+ >= \sum_i c_i^{(k)}|iJ^+ >, \quad (8)$$

corresponding eigenvalues are E_k $(k = 1, \cdots n_{max})$. To acquire the threshold energy, we consider the two-proton knockout reaction^{9}B(^{26}Ne,^{24} O)X. A neutron is emitted just after the reaction when the low-lying states, with energy E_k, are unbound. Since ^{26}Ne is bound nucleus, we can believe that initial state of $0d_{3/2}$ is represented by harmonic oscillator function. Hence one can consider that the emission probability is proportional to the overlap of these two state bellow

$$\begin{aligned} \text{Prob}_k &\propto | < 1s_{1/2}0d_{3/2}; J^+|J_k^+ > |^2 \\ &\propto \left|\sum_i c_i^{(k)} \langle 0d_{3/2}|id_{3/2}\rangle\right|^2 \equiv p_k \end{aligned} \quad (9)$$

We show in FIG. 2 the value of p_k, which is obtained by Eq. (9), for each spin as a function of threshold energy E_k, that is, the energy measured from the ground state of ^{23}O. We should note here that discreet data is smeared out so that the sum of p_k is equal to the integration of the curve in the figure. Vertical dashed lines mean the results of bound approximation, that is, ordinary shell-model calculation with SDPFM interaction with normal configuration. And two curves are the results where $0d_{3/2}$ orbit is considered as continuum state. Two figures represent 1^+ and 2^+ states in ^{24}O. The solid line corresponds to the result with Gauss interaction described Eq. (6), and the dotted line means one particle resonance, respectively.

One can see that the coupling with continuum states lowers the position of the peak by about 1 MeV, and still unbound. We should take head that our result is consistent with the experiment[4][5] in which no bound excited state was found in ^{24}O. As an important point, when we execute full sd-space calculation of normal shell-model, the results are not so different with that of bound approximation shown in FIG.2. We show the results of full sd-shell calculation without continuum effect in Table. I. instance

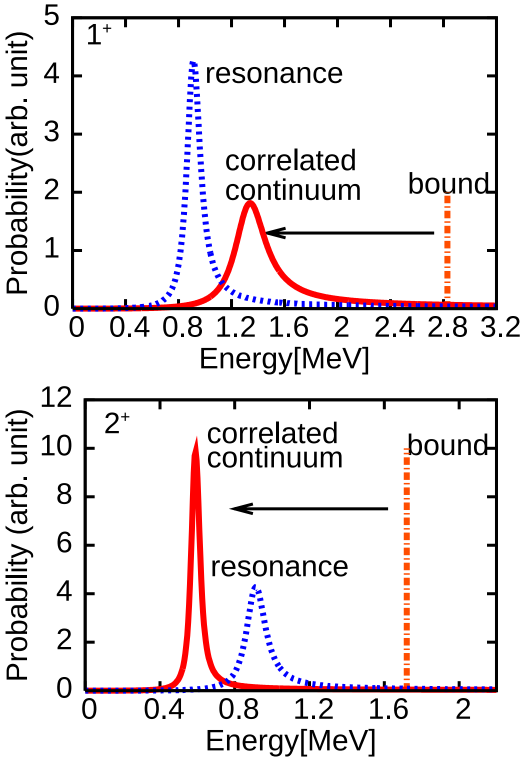

FIG. 2: Overlap value of ^{24}O. The Blue and red curves represent J^{π}=$1^+$and 2^+, respectively. Considering the continuum effect, threshold energies are lowered about 1 MeV.

	SDPFM	USD	USDA	
1^+	2.77	2.75	1.75	[MeV]
2^+	1.74	1.82	1.06	[MeV]

TABLE I: Threshold energy

B. Excited states of other oxygen isotopes

We show in FIG. 3 the results for other oxygen isotopes. Among them, "resonance." means that the result is without residual interaction. In the case of ^{23}O (see FIG. 3(a)), there is only one neutron in our model space, therefore no residual interaction acts. What one can see in this case is one particle resonance for ^{22}O core. Even with no residual interaction, the threshold energy is lowered by just the coupling with the continuum states, and rather well describes the experimental data which has been obtained recently[6].

As for ^{25}O, there are three neutron in our model space. We diagonalize the hamiltonian with the same interaction given by Eq.(6), (7a) and (7b), using the basis represented by $|(1s_{1/2})^2 \otimes id_{3/2}; 3/2^+ >$ $(i = 1, \cdots, n_{max})$. After we obtain the $|3/2^+_k >$ states, we assume the reaction of single proton striping from ^{26}F, namely, ^{9}Be(^{26}F,^{25}O)X. Here we can assume, by the same reason as for ^{24}O, that the emission probability of a neutron is proportional to the overlap value p_k written in Eq. (9). The result is shown in FIG. 3(b). Last we show the result of ^{26}O in Fig. 3(c), where red solid lines represent 0^+ states for both bound approximation and continuum coupling state, blue dotted lines for 2^+ states, and orange dashed line for the resonance.

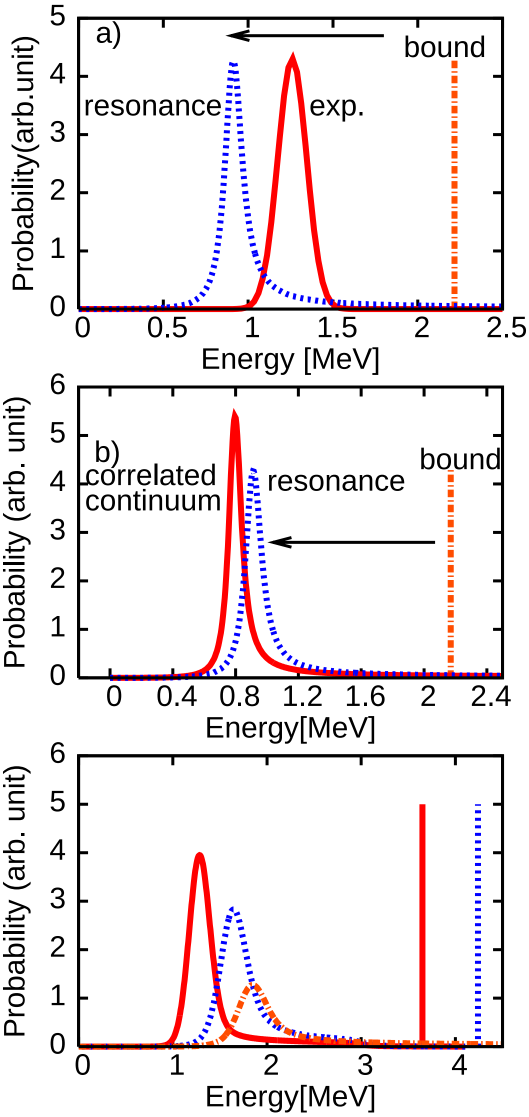

FIG. 3: Overlap values as a function of threshold energy.(a)-(c) corresponding to ^{23,25,26}O

We also show the density of a neutron in $d_{3/2}$ orbit. The density, for instance the case of ^{24}O, is defined as

$$\rho_k(r) = < J_k^+ | \hat{\rho}(r) | J_k^+ >$$
$$= |\phi_{1s}(r)|^2 + \sum_{ij}^{n_{\max}} c^{i*}_{(k)} c^{j}_{(k)} \phi^*_{id}(r) \phi_{jd}(r) \qquad (10)$$
$$\equiv \rho_{1s}(r) + \rho_{kd}(r). \qquad (11)$$

Thus, ρ_{kd} represents the density of neutron in continuum $d_{3/2}$ orbit. We show the plot of $r^2 \rho_{k_m d}(r)$ in FIG. 4,

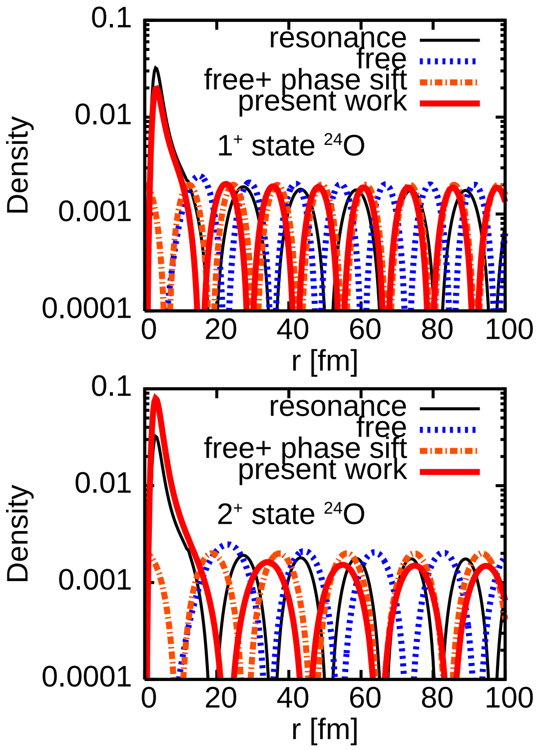

FIG. 4: Density distribution of neutron in the continuum $d_{3/2}$ orbit, 1^+ and 2^+ states in ^{24}O. Vertical axis is described in log scale

compering to that of free particle, free particle with $\pi/2$ phase shift, and the one particle resonance for the mean potential by ^{22}O core. The subscript k_m means that the energy E_{k_m} corresponds to that of the peak in FIG. 2

IV. DISCUSSION AND PERSPECTIVE

Acknowledgments

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APPENDIX A: APPENDIXES

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