Realistic Three-Nucleon Effective Interaction from the Folded-Diagram Theory

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The three-body forces

1. **Necessary** for reproducing binding energies of light nuclei.
2. Importance for medium and heavy nuclei is still not fully clarified.

Medium and heavy nuclei

Three-body contributions originate from:

1. The **'true'** three-body forces:
   (CD Bonn + TM99 3NF, AV18 + Illinois 3NF, CFPT (3NLO))
2. Three-body terms of the **effective interaction**

Effective Interaction

The many-body Schrödinger equation

$$H\Psi = E\Psi \qquad \text{with} \qquad H = H_1 + H_0, \qquad H_0 = T + U, \qquad H_1 = V - U$$

is practically impossible to solve in the complete Hilbert space and is usually being solved in a truncated space. Defining projection operators P (on the model space) and Q (on the excluded space), so that

$$P + Q = 1 \qquad \text{and} \qquad PQ = 0$$

the complete Hilbert-space eigenvalue problem can be replaced by the model space eigenvalue problem:

$$PH_{eff}P\Psi = EP\Psi \qquad \text{with} \qquad H_{eff} = H_0 + V_{eff}$$

Here V_{eff} is the effective interaction, acting solely within the chosen model space.

The Folded-Diagram Theory

To construct the effective interaction, one starts with introducing the $\hat{Q}$-box[a]:

$$\hat{Q}(\omega) = PH_1P + PH_1Q\frac{1}{\omega - QH_0Q}Q\hat{Q}(\omega)P$$

If the model space is assumed to be **degenerate**, the effective interaction can be expressed in terms of $\hat{Q}$-box and its energy derivatives as follows:

$$V_{eff} = \lim_{n\to\infty} V_{eff}^{(n)} \quad \text{and} \quad V_{eff}^{(n)} = \hat{Q}(\omega_0) + \sum_{m=1}^{\infty}\frac{1}{m!}\frac{d^m\hat{Q}(\omega_0)}{d\omega^m}\left\{V_{eff}^{(n-1)}\right\}^m$$

Here ω_0 is the unperturbed energy of the initial many-body state (the starting energy) and $V_{eff}^{(0)} = \hat{Q}(\omega_0)$.

To perform calculations with N-particles in the model space, one in principle needs to construct the N-body effective interaction.

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[a] T. T. S. Kuo and E. Osnes, Folded-Diagram Theory of the Effective Interaction in Atomic Nuclei, Springer Lecture Noters in Physics (Springer, Berlin, 1990) Vol. 364

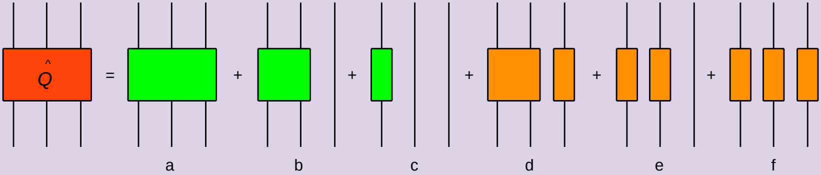
Diagrammatic representation of the $\hat{Q}$-box

The $\hat{Q}$-box is a sum of all possible topologically distinct diagrams which are:

1. **Irreducible:** the intermediate many-particle states between each pair of vertices belong to the excluded space Q.
2. **Valence linked:** all the interaction vertices are linked (via fermion lines) to at least one valence (model space) line.

These diagrams can be either connected (consisting of a single piece), or disconnected.

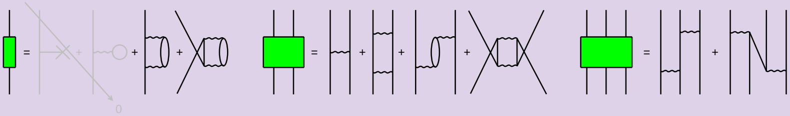
The three-body $\hat{Q}$-box



The three-body contributions originate from connected diagrams of type **a**, the disconnected diagrams **d**, **f** and from **three-body folded-diagrams**. Constructing the **two-body** effective interaction, one usually neglects contributions from disconnected terms. To account for the small three-body correlations accurately, **all of the three-body $\hat{Q}$-box terms must be included**.

Connected terms of the three-body $\hat{Q}$-box

The connected terms are evaluated up to second order in perturbarion theory and containg the following diagrams:



Two-body potential used to calculate the diagrams is the G-matrix obtained with CD-Bonn 2000 NN potential and h.o. s.p. basis.

Disconnected terms of the three-body $\hat{Q}$-box

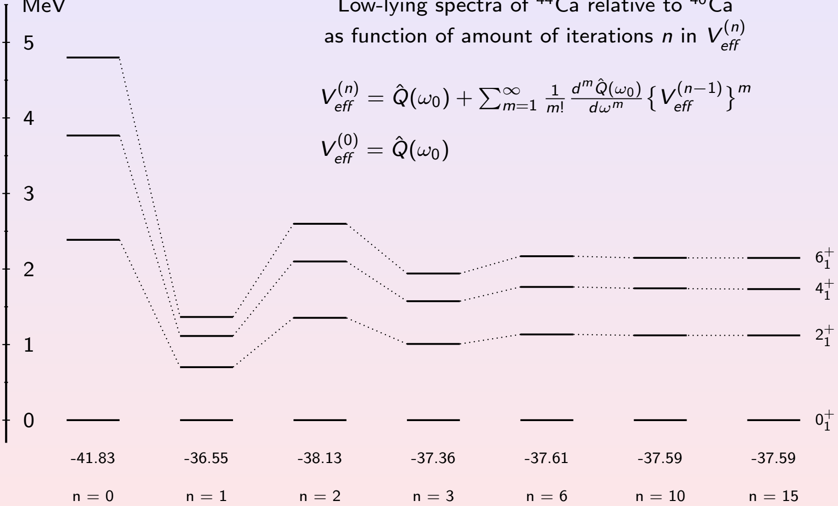
The disconnected diagrams from different folds and with the same amount of interaction vertices cancel out **exactly**. Hence, once the connected terms has been calculated, the disconnected terms can be obtained by evaluating disconnected folded-diagrams of corresponding connected terms.

Iterations Convergence

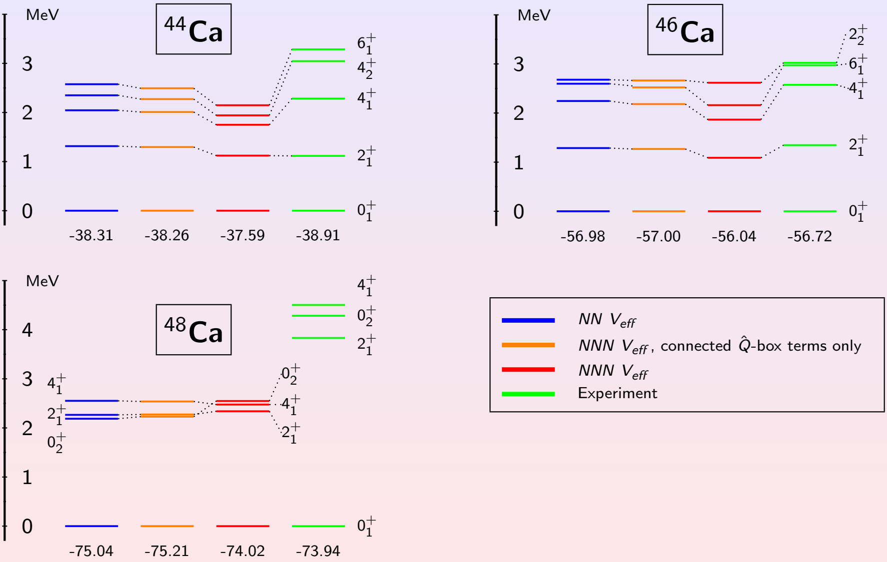
Low-lying spectra of ^{44}Ca relative to ^{40}Ca
as function of amount of iterations n in $V_{eff}^{(n)}$

$$V_{eff}^{(n)} = \hat{Q}(\omega_0) + \sum_{m=1}^{\infty} \frac{1}{m!} \frac{d^m \hat{Q}(\omega_0)}{d\omega^m} \{ V_{eff}^{(n-1)} \}^m$$

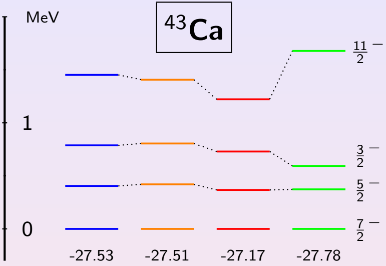
$$V_{eff}^{(0)} = \hat{Q}(\omega_0)$$

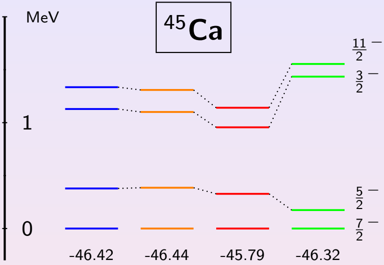


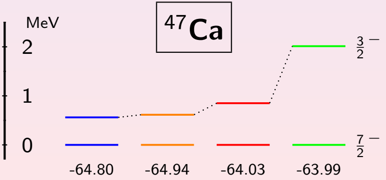
Shell-model calculations of even ^{40}Ca isotopes



Shell-model calculations of odd ^{40}Ca isotopes







NN V_{eff}
NNN V_{eff}, connected $\hat{Q}$-box terms only
NNN V_{eff}
Experiment

Conclusions

1. Three-body terms of the effective interactions constructed with two-body force are **repulsive**.
2. Contributions from the three-body terms of the effective interaction cannot be neglected.
3. The disconnected terms of the three-body $\hat{Q}$-box must be taken into account.

Work in progress

1. Inclusion of the **'true'** three-body forces: (CD Bonn + TM99 3NF, AV18 + Illinois 3NF, CFPT (3NLO))
2. Implementation of the Folded-Diagram Theory for the case of **non-degenerate model space** - allows for the use of self-consistent basis.
3. Comparison with other schemes for constructing effective interactions: LS resummation scheme, LS similarity transformation.