

**MASTER THESIS PROJECT FOR AUGUST  
GELMUYDEN: ASPECTS AND APPLICATIONS  
OF THE GRADIENT FLOW IN LATTICE QCD:  
STRONG COUPLING, ELECTRIC DIPOLE  
MOMENTS AND STRANGE CONTENT OF THE  
NUCLEONS**

The Standard Model of Particle Physics (SM) has been widely successful in describing the measured particle spectrum and composition of matter ranging from quarks and gluons to multi-hadron systems. Such systems constitute approximately 5% of the observable matter-energy within the Universe. Yet the theory alone cannot explain the origins of the remaining 95% of matter and energy, dubbed ‘Dark Matter’ and ‘Dark Energy’, respectively. Further, the SM does not provide the requisite amount of charge conjugation and parity (CP) symmetry violation to account for the observed matter/anti-matter asymmetry. Therefore any physical description of such phenomena requires a theory that goes *beyond* the Standard Model (BSM), while at the same time encompassing the Standard Model and its predictions related to ordinary matter.

There are numerous candidate BSM theories and a description of these theories is beyond the scope of this project. However, these theories all share certain operator traits that can be utilized in a formal study involving numerical methods of quantum chromodynamics (QCD), the gauge theory describing the strong interactions. When viewed as a low-energy effective theory of some larger (still unknown) fundamental theory, the SM, consisting of renormalizable operators of dimension 4 or less, is augmented with BSM operators of dimension  $> 4$ . The structures of these higher dimensional operators can be determined generically assuming the larger fundamental theory respects certain symmetries (e.g. the combined charge conjugation, parity and time reversal (CPT) invariance). Different candidate BSM theories will give different couplings for these higher-dimension operators. When BSM theories are expressed in this manner, all the benefits inherent to effective field theories (EFTs) follow through, such as a hierarchy in operator complexity and a systematic power counting of terms.

Because of the generality of these higher-dimension operators, calculations of these operators can be performed with their unknown coefficients as free parameters using lattice QCD (LQCD), to be later fixed by candidate BSM theories. Conversely, a phase-space investigation of these parameters can be studied to rule-out different BSM theories. The study of these higher-dimension operators with a lattice regulator is plagued by hard renormalization ambiguities. The lattice discretized version of these higher-dimension operators can cause mixing of their coefficients with lower-dimension operators, making the separation and extraction of BSM observables from standard QCD observables very difficult. To address this issue, we have proposed a new method [1, 2] based on the use of the gradient flow [3, 4] to circumvent such mixing,

as well as to address other issues related to renormalization.

The overall goal of this research program is to calculate nuclear observables induced by these general BSM operators using lattice QCD. Such observables include the electric dipole moment of nucleon and few-body nuclear systems and the strange content of nucleons. In this thesis project we focus on the perturbative calculations needed to match the results obtained and non-zero flow time with the physical results.

## STRANGE CONTENT OF A NUCLEON

A very popular example of dark matter (DM) candidate is a weakly interacting massive particle (WIMP) like the ones that are predicted in a large class of models [5]. Definite evidences for a direct detection of WIMPs have only been recently suggested, with various ongoing experiments providing rather severe constraints for the parameters of many DM models.

A possible scenario for the detection of a WIMP type of DM particles relies on the idea that the WIMP, due to its assumed large mass, produces a Higgs boson that couples to the various quark flavor scalar density operators taken between nucleon states (see fig. 1). At zero momentum transfer, the cross section for spin independent elastic WIMP–nucleon ( $\chi N$ ) scattering reads [6]

$$\sigma_{\chi N} \sim \left| \sum_f G_f(m_\chi^2) f_{T_f} \right|^2 \quad \text{with} \quad f_{T_f} = \frac{m_f}{m_N} \langle N | \bar{q}_f q_f | N \rangle. \quad (1)$$

The functions  $G_f$  depend on several parameters of the particular BSM theory used for the calculation of  $\sigma_{\chi N}$ , which include the WIMP mass,  $m_\chi$ . The dimensionless and renormalization group invariant (RGI) coupling  $f_{T_f}$  depends on the mass  $m_f$  of the quark of flavor  $f$  and the nucleon mass,  $m_N$ . As evident from Eq. (1), the cross section,  $\sigma_{\chi N}$  depends quadratically on  $f_{T_f}$ , and is therefore very sensitive to the size of the scalar content contributions of different flavours. A O(10%) variation of  $f_{T_f}$ , for example, can lead to significant changes in  $\sigma_{\chi N}$ . It is therefore necessary to compute accurately and with controlled error the hadronic matrix elements  $\langle N | \bar{q}_f q_f | N \rangle$ .

In principle lattice QCD can provide a determination of these nucleon matrix elements from first principles. In this regard, two approaches have been followed for the calculation of the light and strange quark content of the nucleons. The first is based on the Feynman-Hellman theorem [7] that relates the nucleon scalar matrix element to the dependence of the nucleon mass on the quark masses, while the second relies on so-called “direct method” that involves evaluating the hadronic matrix elements directly.

Both approaches are numerically very challenging. In the first method, a number of simulations at different values of the strange quark mass are needed in order to

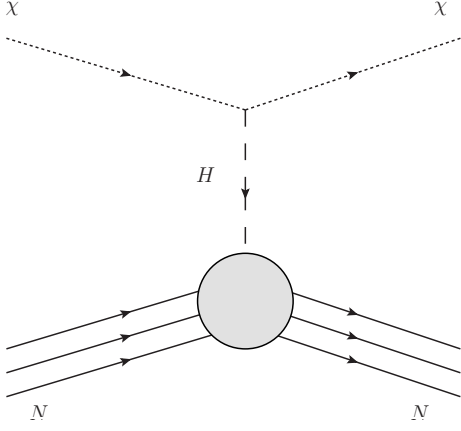


FIG. 1. The Higgs-boson exchange contribution to the WIMP-Nucleon low energy scattering process.

evaluate the derivative. In the direct method, the computation of the disconnected diagrams is required, which is a highly demanding computational task. In addition, as pointed out in ref. [8], when lattice discretizations that break chiral symmetry are used, a mixing between the bare light and strange scalar quark density matrix ele-

ments occurs under renormalization.

Despite these challenges, there exist a number of lattice QCD computations of the strange quark content of the nucleon (see the review [9]). Although these calculations indicate that the strange quark content of the nucleon is smaller than suggested from  $\chi$ PT, their quoted results are affected by large statistical errors for the reasons mentioned above. This makes it impossible to make any definitive conclusions about the hadronic component of matrix elements that couple to DM. To make any radical progress in this area, a novel approach to this problem is required.

### CALCULATION OF HADRONIC MATRIX ELEMENTS THAT COUPLE TO DM PROBES

This calculation is based on the possibility to relate the matrix element of a scalar density  $\bar{\chi}_r(t, x)\chi_s(t, x)$  at non-vanishing flow-time between nucleon states with the one at vanishing flow-time. This can be done considering the small flow-time expansion or finding the proper WI as for the case of the chiral condensate. The small flow-time expansion of the scalar density

$$S^{rs}(t, x) = \bar{\chi}_r(t, x)\chi_s(t, x), \quad (2)$$

reads

$$S^{rs}(t, x) = c_0(t)M^{rs} + c_1(t)M^{rs}\text{Tr}[M^2] + c_2(t)(M^3)^{rs} + c_3(t)S^{rs}(0, x) + O(t) \quad (3)$$

where  $c_0 \sim 1/t$  and  $c_{1,2,3} \sim \log t$ . If one considers the subtracted matrix element

$$\mathcal{C}^{\text{sub}}(t, x) = \langle \mathcal{N}\mathcal{O}(t)\mathcal{N}^\dagger \rangle - \langle \mathcal{O}(t) \rangle \langle \mathcal{N}\mathcal{N}^\dagger \rangle \quad (4)$$

the small flow-time expansion contains only the term proportional to  $c_3$

$$\mathcal{C}^{\text{sub}}(t, x) = c_3(t)\mathcal{C}^{\text{sub}}(0, x). \quad (5)$$

At finite flow-time no mixing is present. To compute the physical matrix element at  $t = 0$  one needs to compute  $c_3(t)$ . Using the chiral WIs one can show that

$$c_3(t) = \frac{G_{\pi,t}}{G_\pi} + O(t). \quad (6)$$

This implies that to compute the matrix element we are interested in we need to compute

$$\frac{G_\pi}{G_{\pi,t}} \cdot [\langle \mathcal{N}S^{rs}(t)\mathcal{N}^\dagger \rangle - \langle S^{rs}(t) \rangle \langle \mathcal{N}\mathcal{N}^\dagger \rangle]. \quad (7)$$

We observe that the renormalization factor of  $S^{rs}(t)$  simplifies with the one of  $G_{\pi,t}$ , thus the only renormalization

factor needed is the one for  $G_\pi$ , i.e.  $Z_P$ . The matrix element (7) should be computed in a window of  $t$ -values, not too small to avoid big discretization errors and not too large to avoid contributions from higher dimensional operators. Once the continuum limit is performed at values of  $t$  fixed in physical units, the matrix element (7) should be independent of  $t$  if  $t$  is not too large.

### CP VIOLATION WITHIN THE STANDARD MODEL AND ELECTRIC DIPOLE MOMENTS

The EDMs of the neutron and proton are very sensitive probes of CP-violating sources beyond those contained in the SM. In fact, the current bound on the neutron EDM strongly constrains many models of BSM physics. At current experimental accuracies, a nonzero nucleon EDM cannot be accounted for by the phase in the quark-mass matrix. This implies that such a signal is either caused by a nonzero QCD  $\theta$  term or by genuine BSM physics which, at low energies, can be parametrized in terms of higher-dimensional CP-violating quark-gluon operators. Irrespective of the origin, the signal for the nucleon EDM will be small and largely masked by strong-interaction

physics, which presents a formidable challenge to the interpretation of such a signal. To disentangle the origin of a nonzero EDM measurement (e.g.  $\theta$  term or BSM), a quantitative understanding of the underlying hadronic physics is required.

In the presence of  $N_f$  fermions, the general QCD Lagrangian in Euclidean space with a  $\theta$  term reads

$$\mathcal{L}_\theta = \frac{1}{4} F_{\mu\nu}^a(x) F_{\mu\nu}^a(x) + \bar{\psi} D \psi + \bar{\psi}_L M \psi_R + \bar{\psi}_R M^\dagger \psi_L - i \theta_q q(x), \quad (8)$$

where  $\psi$  is an  $N_f$  component vector in flavor space,  $M$  is the quark mass matrix, and  $q(x)$  is the topological charge density. The parameter  $\theta_q$  and the imaginary part of the quark mass are related, so that the relevant  $\theta$  parameter is given by

$$\theta = \theta_q + \arg \det M. \quad (9)$$

This term is CP-violating and induces, for example, an electric dipole moment within the neutron. One of the most stringent constraints on possible violations of parity and time reversal (equivalently CP symmetry assuming CPT invariance) symmetries is inferred from measurements of the neutron EDM.

## THE GRADIENT FLOW

In this section we give a short introduction to the gradient flow that is at the base of the method we propose to use for this project. The gradient flow (GF), for gauge [3] and fermion [4] fields, used in combination with a lattice regulator, can probe the non-perturbative dynamics of QCD in advantageous manners. It is defined by a differential equation that gauge and fermion fields satisfy as a function of the space-time coordinates  $x$  and of a new scale, the flow time  $t$ .

The works by Lüscher and Weisz [3, 4, 10] give us a complete understanding of the continuum limit of observables at non-vanishing flow time. This allows us to use the GF to define observables that otherwise would be difficult to compute with standard methods. For example the GF has been used to give operational definitions to the fundamental parameters of QCD as the strong cou-

pling [3, 11, 12] or other quantities, as the chiral condensate [4] and the energy-momentum tensor [13, 14]. The GF can also be used to define new relative ways to set the scale in lattice QCD calculations [3, 15].

In particular the topological susceptibility [3] and the chiral condensate [4] can be computed with rather high precision because with the gradient flow mixing with lower-dimensional operators can be avoided. Local operators at non-vanishing flow time have a very simple renormalization pattern [3, 4, 10], but to relate them with the operators at vanishing flow time one has to rely to Ward identities (WI) [4, 14, 16] or to the small flow-time expansion.

The goal of this project to perform the matching of the correlation functions computed at non-zero flow time with the physical quantities such as the EDM, the strange content of the nucleon or, as a test ground, the energy.

The results will be published in scientific journals, if possible.

## PROGRESS PLAN AND MILESTONES

The aims and progress plan of this thesis are as follows

- Fall 2017: study the gradient flow for gauge fields [3] and fermions [4].
- Fall 2017: development of gauge field Feynman rules with the gradient flow.
- Fall 2017 and Spring 2018: perturbative calculation of the energy defined with the gradient flow (this can be matched with the non-perturbative calculation to extract the  $\Lambda_{\text{QCD}}$  parameter).
- Spring 2018: development of Feynman rules for fermion fields at non-zero flow time.
- Spring 2018: sample perturbative calculation with fermions (quark propagator, fermion bilinears)
- Spring 2018: The last part deals with a proper write-up of the thesis.

The thesis is expected to be handed in May/June 2018.

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[1] A. Shindler, J. de Vries, and T. Luu, PoS **LAT-TICE2014**, 251 (2014), 1409.2735.  
[2] A. Shindler, T. Luu, and J. de Vries, Phys. Rev. **D92**, 094518 (2015), 1507.02343.  
[3] M. Lüscher, JHEP **1008**, 071 (2010), 1006.4518.  
[4] M. Lüscher, JHEP **1304**, 123 (2013), 1302.5246.  
[5] G. Bertone, D. Hooper, and J. Silk, Phys.Rept. **405**, 279 (2005), hep-ph/0404175.  
[6] J. R. Ellis, K. A. Olive, and C. Savage, Phys.Rev. **D77**, 065026 (2008), 0801.3656.  
[7] R. Feynman (1939).

[8] C. Michael, C. McNeile, and D. Hepburn (UKQCD Collaboration), Nucl.Phys.Proc.Suppl. **106**, 293 (2002), hep-lat/0109028.  
[9] R. D. Young and A. W. Thomas, Nucl.Phys. **A844**, 266C (2010), 0911.1757.  
[10] M. Lüscher and P. Weisz, JHEP **1102**, 051 (2011), 1101.0963.  
[11] Z. Fodor, K. Holland, J. Kuti, D. Negradi, and C. H. Wong, JHEP **1211**, 007 (2012), 1208.1051.  
[12] P. Fritzsch and A. Ramos (2013), 1301.4388.  
[13] H. Suzuki, PTEP **2013**, 083B03 (2013), 1304.0533.

- [14] L. Del Debbio, A. Patella, and A. Rago, JHEP **11**, 212 (2013), 1306.1173.
- [15] S. Borsanyi, S. Dür, Z. Fodor, C. Hoelbling, S. D. Katz, et al., JHEP **1209**, 010 (2012), 1203.4469.
- [16] A. Shindler, Nucl. Phys. **B881**, 71 (2014), 1312.4908.