MASTER THESIS PROJECT FOR AUGUST GEELMUYDEN: LATTICE QCD AND CHIRAL CONDENSATES

The Standard Model of Particle Physics (SM) has been widely successful in describing the measured particle spectrum and composition of matter ranging from quarks and gluons to multi-hadron systems. Such systems constitute approximately 5% of the observable matter-energy within the Universe. Yet the theory alone cannot explain the origins of the remaining 95% of matter and energy, dubbed 'Dark Matter' and 'Dark Energy', respectively. Further, the SM does not provide the requisite amount of charge conjugation and parity (CP) symmetry violation to account for the observed matter/anti-matter asymmetry. Therefore any physical description of such phenomena requires a theory that goes *beyond* the Standard Model (BSM), while at the same time encompassing the Standard Model and its predictions related to ordinary matter.

There are numerous candidate BSM theories and a description of these theories is beyond the scope of this project. However, these theories all share certain operator traits that can be utilized in a formal study involving numerical methods of quantum chromodynamics (QCD), the gauge theory describing the strong interactions. When viewed as a low-energy effective theory of some larger (still unknown) fundamental theory, the SM, consisting of renormalizable operators of dimension 4 or less, is augmented with BSM operators of dimension $>$ 4. The structures of these higher dimensional operators can be determined generically assuming the larger fundamental theory respects certain symmetries (e.g. the combined charge conjugation, parity and time reversal (CPT) invariance). Different candidate BSM theories will give different couplings for these higher-dimension operators. When BSM theories are expressed in this manner, all the benefits inherent to effective field theories (EFTs) follow through, such as a hierarchy in operator complexity and a systematic power counting of terms.

Because of the generality of these higher-dimension operators, calculations of these operators can be performed with their unknown coefficients as free parameters using lattice QCD (LQCD), to be later fixed by candidate BSM theories. Conversely, a phase-space investigation of these parameters can be studied to rule-out different BSM theories. The study of these higher-dimension operators with a lattice regulator is plagued by hard renormalization ambiguities. The lattice discretized version of these higher-dimension operators can cause mixing of their coefficients with lower-dimension operators, making the separation and extraction of BSM observables from standard QCD observables very difficult. To address this issue, we have proposed a new method [1, 2] based on the use of the gradient flow [3, 4] to circumvent such mixing, as well as to address other issues related to renormalization.

The overall goal of this research program is to calculate nuclear observables induced by these general BSM operators using lattice QCD. Such observables include the electric dipole moment (EDM) of nucleon and few-body nuclear systems. In this thesis project we focus on the part of the calculations related to the CP violation within the Standard Model.

CP VIOLATION WITHIN THE STANDARD MODEL AND ELECTRIC DIPOLE MOMENTS

The EDMs of the neutron and proton are very sensitive probes of CP-violating sources beyond those contained in the SM. In fact, the current bound on the neutron EDM strongly constrains many models of BSM physics. At current experimental accuracies, a nonzero nucleon EDM cannot be accounted for by the phase in the quark-mass matrix. This implies that such a signal is either caused by a nonzero QCD θ term or by genuine BSM physics which, at low energies, can be parametrized in terms of higher-dimensional CP-violating quark-gluon operators. Irrespective of the origin, the signal for the nucleon EDM will be small and largely masked by strong-interaction physics, which presents a formidable challenge to the interpretation of such a signal. To disentangle the origin of a nonzero EDM measurement (e.g. θ term or BSM), a quantitative understanding of the underlying hadronic physics is required.

In the presence of N_f fermions, the general QCD Lagrangian in Euclidean space with a θ term reads

$$\mathcal{L}_\theta = \frac{1}{4} F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) + \bar{\psi}\, D\psi + \bar{\psi}_L\, M\, \psi_R + \bar{\psi}_R\, M^\dagger\, \psi_L - i\theta_q q(x), \tag{1}$$

where ψ is an N_f component vector in flavor space, M is the quark mass matrix, and $q(x)$ is the topological charge density. The parameter θ_q and the imaginary part of the quark mass are related, so that the relevant θ parameter is given by

$$\theta = \theta_q + \arg\det M. \tag{2}$$

This term is CP-violating and induces, for example, an electric dipole moment within the neutron. One of the most stringent constraints on possible violations of parity and time reversal (equivalently CP symmetry assuming CPT invariance) symmetries is inferred from measurements of the neutron EDM.

A. The Gradient flow

In this section we give a short introduction to the gradient flow that is at the base of the method we propose to use for this project. The gradient flow (GF), for gauge [3] and fermion [4] fields, used in combination with a lattice regulator, can probe the non-perturbative dynamics of QCD in advantageous manners. It is defined by a differential equation that gauge and fermion fields satisfy as a function of the space-time coordinates x and of a new scale, the flow time t.

The works by Lüscher and Weisz [3–5] give us a complete understanding of the continuum limit of observables at non-vanishing flow time. This allows us to use the GF to define observables that otherwise would be difficult to compute with standard methods. For example the GF has been used to give operational definitions to the fundamental parameters of QCD as the strong coupling [3, 6, 7] or other quantities, as the chiral condensate [4] and the energy-momentum tensor [8, 9]. The GF can also be used to define new relative ways to set the scale in lattice QCD calculations [3, 10].

In particular the topological susceptibility [3] and the chiral condensate [4] can be computed with rather high precision because with the gradient flow mixing with lower-dimensional operators can be avoided. Local operators at non-vanishing flow time have a very simple renormalization pattern [3–5], but to relate them with the operators at vanishing flow time one has to rely to Ward identities (WI) [4, 9, 11] or to the small flow-time expansion.

The goal of this project is to use the gradient flow to define and compute all the possible CP-violating contributions to the EDM of nucleons and few-body nuclei. This is possible because the gradient flow allows a direct calculation of the topological charge density at non-vanishing flow time and, in combination with a small flow-time expansion, the simplification of the renormalization pattern of higher-dimensional operators.

The current experimental limit on the neutron EDM is $|d_N| < 2.9 \cdot 10^{-13}\, e\,\mathrm{fm}$ [12] and experiments are underway to improve this bound by one to two orders of magnitude. In fig. 1 we show the history of neutron EDM limits from experimental measurements. The bound on the proton EDM is induced from the ^{199}Hg EDM limit [13] and is $|d_P| < 7.9 \cdot 10^{-12}\, e\,\mathrm{fm}$. Recently the bound on the ^{225}Ra EDM has also improved dramatically [14], $\left|d(^{225}\mathrm{Ra})\right| < 1.4 \cdot 10^{-10} e$ fm. While this specific measurement is not competitive at the moment with the ^{199}Hg one it has the potential to achieve rapid improvements by many orders of magnitude through a series of experimental upgrades. Plans exist to probe the EDM of the proton directly (and other light nuclei) in storage rings [15] with a proposed sensitivity of $10^{-16}\, e \cdot \mathrm{fm}$, thus improving the current bounds by several orders of magnitudes and covering a wide range where BSM physics can show its footprint. Test of fundamental symmetries and the measurement of atomic EDMs represent one of the main research thrusts of FRIB. Specific rare isotopes produced at FRIB, such as ^{229}Pa and ^{225}Ra are expected to have a much larger EDM than most atoms providing an ideal environment to improve dramatically the current upper limits. There is a huge effort underway to improve limits or find EDMs like the experimental proposals (p,d)EDM experiment at BNL, the neutron EDM experiment at ORNL, ILL, FRM-2, FNAL,PSI/KEK/TRIUMF, the charged particle (d, p)EDM at COSY and the lepton EDM at J-PARC, FNAL aiming at a sensitivity of $10^{-16}\, e$ fm. In order to translate the above experimental bound into a constraint on θ and CP-violating BSM operators, one needs to compute their contribution to the nucleon or deuteron EDMs. Nucleon EDMs arising from the QCD θ term or BSM physics have been calculated both in models [17] and in chiral perturbation theory [18, 19]. In the latter approach, the nucleon EDMs are calculated in terms of effective CP-odd hadronic interactions that have the same symmetry properties as the underlying CP-odd sources at the quark level (for a review, see [20]). The calculated EDMs depend on several low-energy constants (LECs) whose sizes are in most cases unknown and need to be estimated or calculated with lattice QCD.

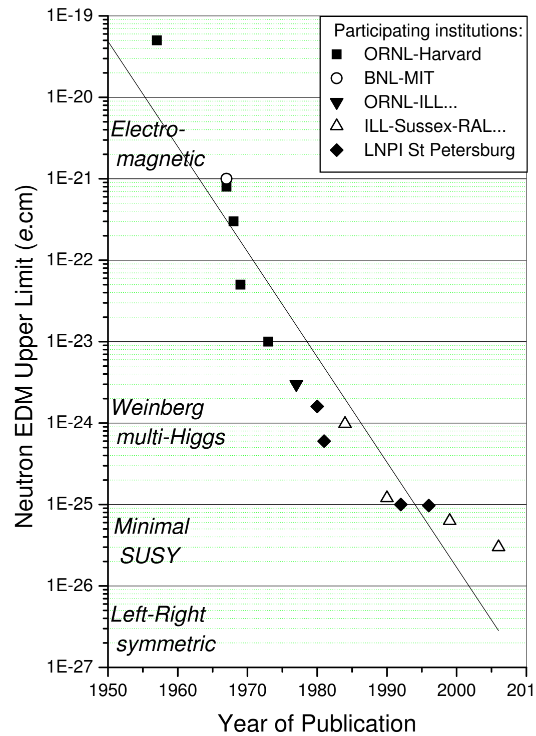

FIG. 1. History of neutron EDM limits. Figure from ref. [16].

Lattice QCD can thus be used to perform an *ab initio* calculation of the nucleon and light nuclei EDMs. For the θ term, this has already been shown in the pioneering works in refs. [21, 22] and later in ref. [23, 24] (for BSM sources only the nucleon EDMs arising from the quark EDMs have been calculated with lattice QCD [25]). The calculation of the EDM within a lattice (discretized) formulation of QCD is very non-trivial, and presents, together with the standard LQCD systematics, large difficulties for two main reasons. The renormalization of the CP-odd operators and the degradation of the signal-to-noise ratio towards the chiral limit. Additionally, the θ term itself introduces an imaginary term in the real Euclidean action, which produces a sign problem and precludes the use of standard stochastic methods employed by lattice QCD.

In this project we make use of a new method [1, 2], that we have recently developed, based on the gradient flow for the gauge fields [3] and fermion fields [4] that has no renormalization ambiguities and, to our knowledge, is the only method that allows a theoretical sound continuum limit for all the physical quantity of interest.

The results will be published in scientific journals, if possible.

PROGRESS PLAN AND MILESTONES

The aims and progress plan of this thesis are as follows

- Fall 2017: study the gradient flow for gauge fields [3] and fermions [4].
- Fall 2017: implement the code that reads in the gauge configurations and then computes the gradient flow for gauge fields as well as the energy perturbatively.
- Spring 2018: Calculate the perturbative fermion energies for quarks and their current densities.
- Spring 2018: Study then the chiral condensate and the strange content of the nucleon. to extract the θ-term contribution to the nucleon EDM.
- Spring 2018: The last part deals with a proper write-up of the thesis.

The thesis is expected to be handed in May/June 2018.

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