Contents

Introduction

In this paper, we delve into the realm of PID (Proportional, Integral, Derivative) control, a foundational and classical approach in control systems that combines the P, I, and D components into water tank. The PID controller, renowned for its simplicity, ease of tuning, robustness, and reliability, stands as a cornerstone in industrial control technologies. Its widespread adoption in practical engineering applications underscores its effectiveness in a broad array of settings, particularly due to its strong robustness and adaptability.

This study we explore the application of both traditional tuning PID and Fuzzy controllers in the context of an interactive water level process, aiming to model a dynamic system that is both efficient and user-friendly. The use of Matlab's Simulink for modeling and testing underscores the practicality of our approach.

The findings of this research, illustrated through experimental data, highlight the dynamic process's transient and steady-state responses. Through comprehensive simulations, we demonstrate the superiority of the proposed control methodology, underscoring its potential to enhance the precision and stability of water level control systems in settings.

Water Tank System

Currently, the operational structure of a tank water level system is outlined in Figure 1. This system operates dynamically, where water enters the tank continuously and exits simultaneously. The inflow of water F_{in}, is regulated by adjusting the opening of the Control Valve (V). Users can modify the water's outflow $F_{out.}$ The variable regulated in this process is the water level L, which signifies the equilibrium between the inflow and outflow of water. With a constant valve opening V, an increase in water level L results in a higher static pressure within the tank, leading to an enhanced outflow.

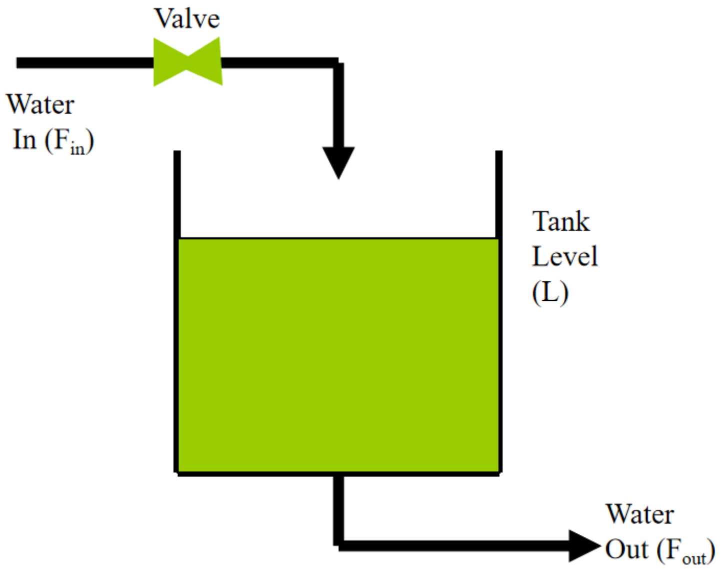

Figure 1 Water Tank System

To analyze the dynamic characteristics of the water level in response to adjustments in valve opening, the setting begins with the regulating valve fixed at a certain opening, which is then abruptly increased. Observing the water level over time allows for the derivation of a mathematical model for the water level dynamics. The variation in water level at any given moment adheres to the following material balance equation, capturing the essence of the system's behavior under such disturbances.

$$\frac{dL}{dt} = \frac{1}{F} * (F_{in} - F_{out})$$

Formula 1

$$F_{in} = Kv*V$$

Formula 2

$$F_{out} = Kv*\sqrt{L}$$

Formula 3

Where, F denotes the cross-sectional area of the water tank, and Kv is a coefficient typically treated as a constant value, reflecting the characteristics of the valve. At a specific water level, and considering a steady-state initial condition, the equation can be reformulated as follows, allowing for a focused analysis on the system's behavior under predefined conditions.

$$RC\frac{dL}{dt} + L = Kv * RV$$

Formula 4

Where, C=F is water capacity, R = $\frac{2\sqrt{L}}{Kv}$ is water resistance.

Starting from a zero initial condition, the Laplace transform of the aforementioned equation can be derived accordingly.

$$G(s) = \frac{H(s)}{Q(s)} = \frac{R}{RCs} = \frac{K}{Ts+}$$

Formula 5

Here, T = RC represents the time constant of the tank, and K = R is defined as the process gain factor, both of which play crucial roles in characterizing the dynamics of the system.

PID Controller

PID Parameter

The PID controller is a type of linear controller characterized by its proportional (P), integral (I), and derivative (D) components. The controller operates by taking the error signal e(t) which is the difference between the reference input r(t), and the actual output y(t). The controller's output u(t), is a combined function of this error signal over time, designed to adjust the control system and minimize the error.

$$u(t) = Kp \left[e(t) + \frac{T}{Ti} \int_0^t e(t)\, dt + \frac{Td}{T} \frac{de(t)}{dt} \right]$$

Formula 6

Specifically, Kp represents the proportional gain, which scales the error signal directly. A higher Kp value results in a larger response to the error, making the system react more strongly to discrepancies between the desired and actual outputs. The integral component, characterized by the integral time Ti, integrates the error over time, which helps eliminate the steady-state error and improves the system's accuracy. The derivative component, with the derivative time Td, predicts the error's future trend by taking its rate of change. This foresight allows the controller to apply corrective action in advance, thereby enhancing the system's responsiveness and reducing the time it takes to settle into the desired state. T denotes the system's sampling period, which is the interval at which the control system updates its calculations.

The PID controller's block diagram, referred to as Figure 2, would visually represent the flow of these signals through the proportional, integral, and derivative components, and show how they combine to produce the controller's output.

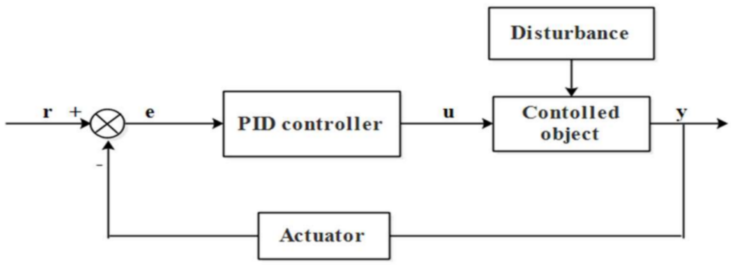

Figure 2 PID controller diagram

The PID controller functions by contrasting the measured process variable, conveyed by a transmitter, against the desired setpoint to derive an error signal. Utilizing predefined parameters, it processes this signal and dispatches the resultant command to an actuator. The actuator then autonomously adjusts the flow of materials or energy into the system in response to the controller's instructions. This action mitigates the impact of any disturbances, thereby ensuring the system meets the specified control objectives (Jiang, 2024)

PID Parameter Tuning

Tuning, the process of setting the optimal gains of the three PID parameters to achieve ideal feedback, often involves trial and error. It requires a solid understanding of how different parameters affect the system. Initially, the integral and differential parameters are set to zero, and the proportional gain is gradually increased until the loop output starts oscillating. Stability must be maintained throughout this process. Once the proportional parameter is set, overshoot. However, it also enhances the responsiveness of the system. To minimize steady-state error, both proportional and integral parameters are adjusted, followed by increasing the differential term to achieve faster response times. While increasing the differential term can reduce overshoot and enhance stability, it also makes the system more susceptible to noise. Engineers must carefully balance these characteristics when designing the control system, often making trade-offs. During debugging, the general principles involve increasing the proportional term (P), decreasing the integral term (T_i), and increasing the differential term (T_d) as long as the output remains stable and doesn't oscillate (Levis, 2021).

PID Control Water Level Simulation

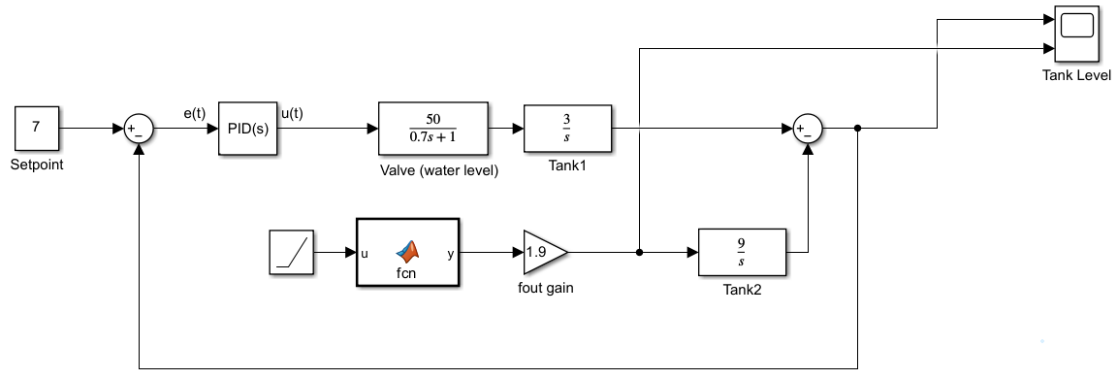

Figure 3 Simulation block diagram of classical PID control

Setpoint: The constant block labeled “7” represents that this is desired water level that the system is trying to maintain in the tanks.

PID Controller: The PID controller receives the error signal, which is the different the setpoint and the actual water level (feedback from the tank system). It then computes an output signal u(t), based on its tuning parameters (proportional gain, integral time and derivative time) to reduce the error. The proportional term K_p e(t) provides a control action proportional to the current error. The integral term $K_i \int e(t)dt$ accounts for the accmulation of past errors, aiming to eliminate steady-state errors. The derivative $K_d \frac{d}{dt}$ e(t) predicts future errors based on the rate of change, providing a damping effect to reduce overshoot. (K_p is 1.989, K_i is 6.6039, K_d is 0.104, N is 92.786)

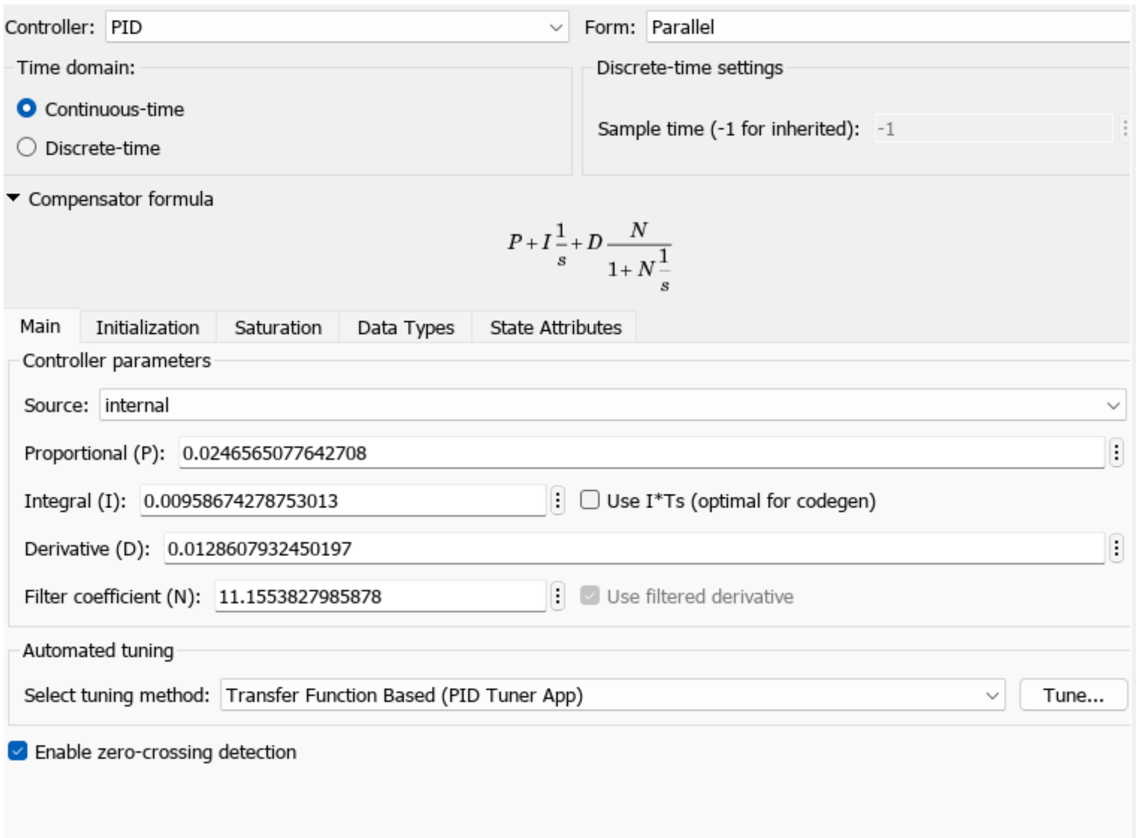

Figure 4 Setting of the PID controller in Simulink

This block has one big advantage – it has a tuning option that helps us with the setting of the P, I, D and N-parts. Part of the options window that sets the PID controller is shown in Figure 10. We can see values of P, I, D parts and also button “Tune…” that computes these values by simulation experiments on the controlled systems. We can see, that there are two branches in the scheme – the first one directly through the controlled system and the second branch goes through transfer function parameters. This branch is in this scheme just because of this tuning. If we want to tune parameters of the PID controller, we must double click on the Manual switch block which connects this “identification branch” and PID block simulates the behaviour of the system for various PID setting. In the next window (Figure 5) we can see the resulting window after the tuning.

This block possesses a significant advantage-it incorporates a tuning option facilitating the adjustment of the P, I, D and N-components. With in the options window dedicated to configuring the PID controller, depicted in Figure 4, one can observe the values assigned to the P, I and D components, alongside a “Tuning” button designed to compute these values through simulation experiments on the controlled systems. Notably, there exist two branches within the scheme: the first one traverses directly through the controlled system, while the second branch is introduced specifically for tuning purposes, involving transfer function parameters. Should one seek to fine-tune the parameters of the PID controller, a double-click on the manual switch block linking this identification branch and PID block initiates simulation experiments to evaluate various PID settings. Subsequently, the ensuing window (Figure 5) reveals the outcomes following the tuning process.

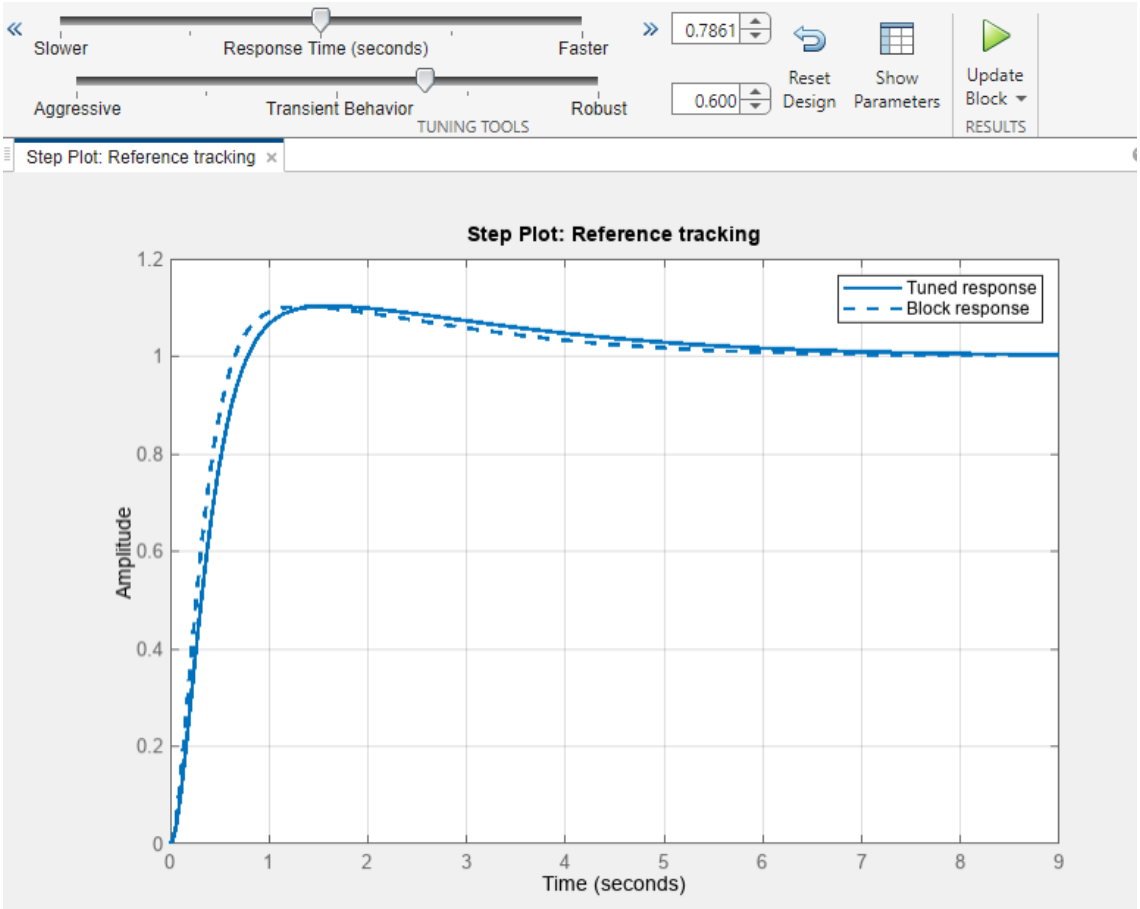

Figure 5 Tuning of the PID controller in Simulink

We can see the course of the actual output (dashed line) that is quick but with relatively big overshoot. This PID controller offers two tuning parameters – the first one affects the speed of the response and the second one aggressiveness/robustness of the controller. PID tries to set better results that is why the solid line which represents tuned response much smoother course. We can change these values to obtain our desired course. Once we are satisfied with the response, we can press button "Update block" that sets computed values of parameters P, I, D and N in the appropriate windows in Figure 6.

The depicted graph illustrates the trajectory of the actual output (represented by the dashed line), characterized by swift movement albeit accompanied by considerable overshoot. This PID controller provides two tuning parameters: the first influencing the response speed, while the second dictates the controller's aggressiveness or robustness. In its pursuit of optimal performance, the PID controller endeavors to yield superior results, as evidenced by the smoother trajectory depicted by the solid line after tuning. These parameters are adjustable to tailor the response to out preferences. Upon achieving satisfactory results, pressing the "Update" block button enables the incorporation of the computed values for parameters P, I, D and N into the corresponding windows depicted in Figure 6

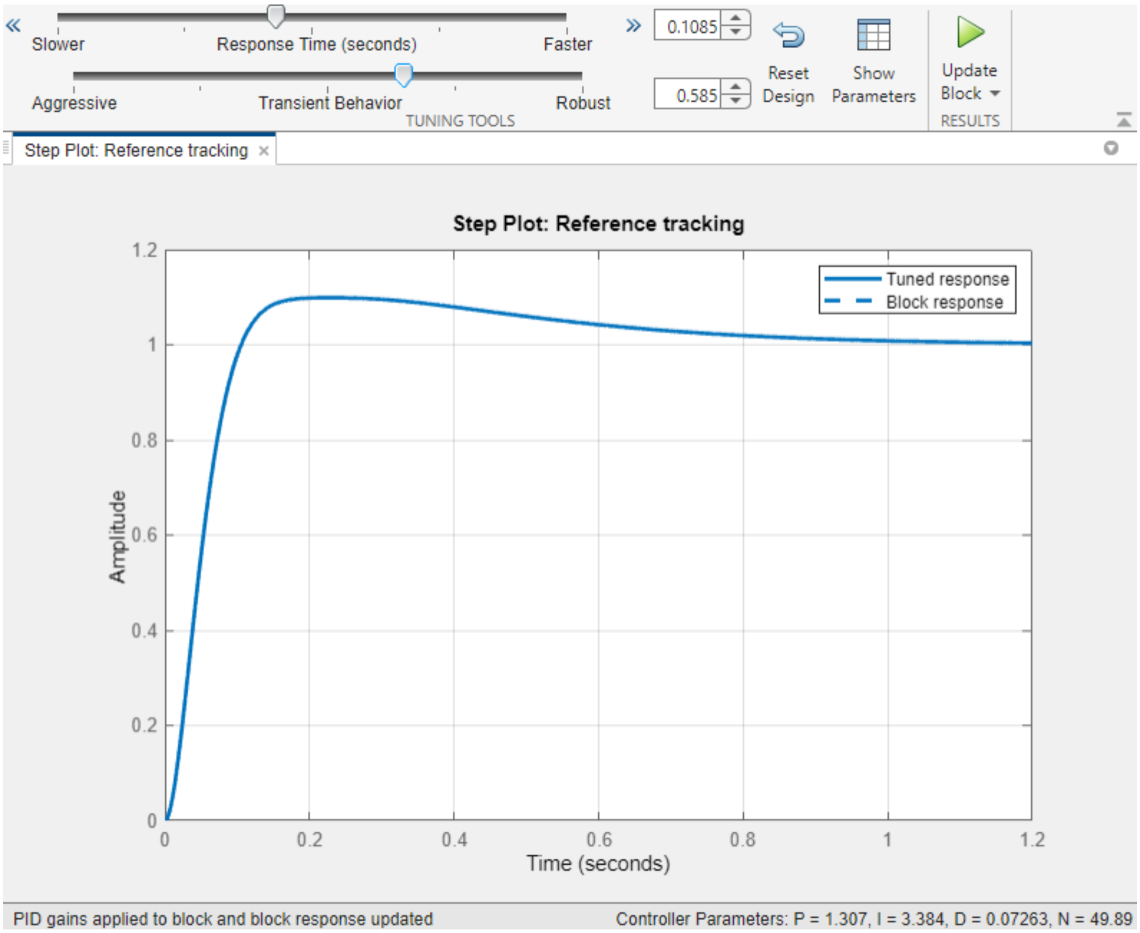

Figure 6 the output variable from the identified linearized transfer function

Valve Dynamics: The output of the PID controller u(t) is fed to a valve characterized by the transfer function, which indicates that the valve has a certain response time to changes in the signal (due to Ps+1), which means it doesn't open or close instantaneously. The valve controls the water flow into Tank1. Its dynamics are represented by a first-order lag, indicating that its response to changes is not immediate but gradually reaches the new state.

Tank Dynamics: The Tank1 has a transfer function, indicating that the water level in this tank behaves as an integrative process of the input flow. Tank 2 is connected in the feedback loop, has the same integrative behavior.

Non-linear Function Block fcn: The control signal u(t) from the output of the PID controller is modified by the non-linear function y= exp (-2*u) before it influences the system. The function exponentially reduces the control signal's effect, more so for larger signals. This exponential function decreases the effect of the control signal exponentially, it does mean a valve that has a maximum flow rate; the higher the value of u(t), the smaller the output y, implying a dampening effect on the control action.

Ramp: A ramp function typically increases or decreasae linearly over time. When used in control systems, it can represent a gradually changing setpoint, which can test the system's response to a change target or simulate a slow process change. The ramp signal changing control challenge to the system, simulating a scenario where desired water level in the tank changes over time.

System Feedback and Behavior:

The feedback loop measures the combined water level from both tanks and then processes this signal through the non-linear function block, then sends this information back to the PID controller.

The non-linear block introduces a dynamic where small errors lead to relatively large changes in the control action, but as the error increases, the control action's effectiveness is exponentially dampened. This could be a desired feature to prevent the system from overreacting to large disturbances or to stabilize the system by reducing the control action as the error grows, which could help in avoiding overshoot.

The presence of a gain block labeled "5" suggests that the feedback signal is amplified before being substracted from the setpoint, which could mean the system is desiganed to be more sensitive to errors, which can improve responsiveness and potentially affect the stability, increasing the likelihood of overshoot.

Combining a tank with a given transfer function, a non-linear function block, a gain and a ramp in a control system. The overall function is that the system is to control the water level in the tank according to a changing setpoint while considering non-linearities in the control path and ensuring that the water level changes in a smooth and controlled manner, advoiding suddenly jumps or instability that could result from a simple linear control stragtegy. The purpose of such a configuration would be to match the control system's response to the specific dynamics and constraints of the physical process, ensuring efficient and stable operation while preventing damage to the equipment or product being processed.

The model exhibits a well-tuned PID control system that stabilizes the water level at the setpoint quickly and effectively. The initial sharp decrease in outflow suggests a large initial control action is needed to quickly fill the tanks to the desired level. As the level approaches the setpoint, the control action dimishes, leading to less outflow, and stabilizes at a value that maintains the tank level at the setpoint.

The non-linear function within the feedback loop, along with the gain, may serve to prevent excessive outflow once the level is near the setpoint, hence avoiding large overshoot or oscillations.

The quick response and minimal overshoot indicate that the PID parameters are well-tuned for this specific system setip and the desired performance criteria. However, if there were any disturbances or changes in system parameyers, the PID controllrt might need to be re-tuned to maintain this level of performance.

PID Controller Results

Tank Level Graph Analysis:

The water level rapidly increases at the start, suggesting an assertive response to the system's inputs to reach the desired setpoint quickly.

Once the initial surge is complete, the water level stabilizes, indicating that the PID controller is successful in maintaining the setpoint with minimal fluctuation, demonstrating effective control without significant overshoot or prolonged oscillation.

Fout Graph Analysis:

The outflow decreases sharply initially, which correlates to a substantial initial control action to correct any disparity from the setpoint.

As the water level nears the setpoint, the outflow rate decreases and eventually stabilizes, implying that the system requires less correction to maintain the setpoint, reflecting a steady state has been reached.

These graphs shows a PID controller that rapidly brings the system to the desired water level and then effectively sustains that level with minimal correction needed, as evidenced by the reduced outflow rate. The initial strong control action is adjusted as the system reaches stability, which results in decreased energy or fuel consumption over time. This efficient response underlines the capability of PID controllers in achieving and maintaining target levels in process control applications.

PID controller successfully maintains the water tank level, with efficient energy usage.

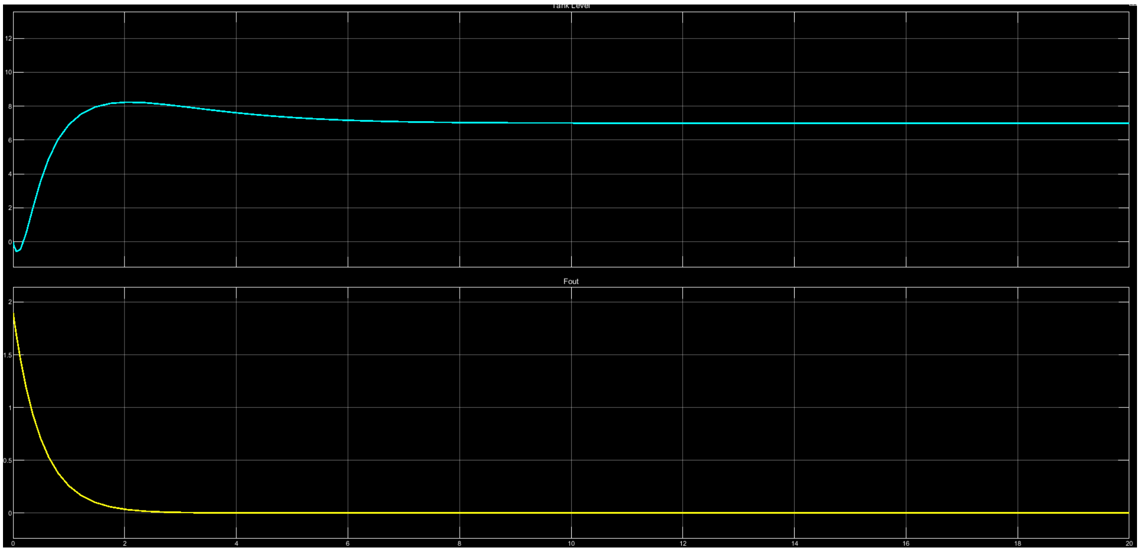

Figure 7 Simulink Results of PID control

Fuzzy Controller

Basic Fuzzy Logic Controller

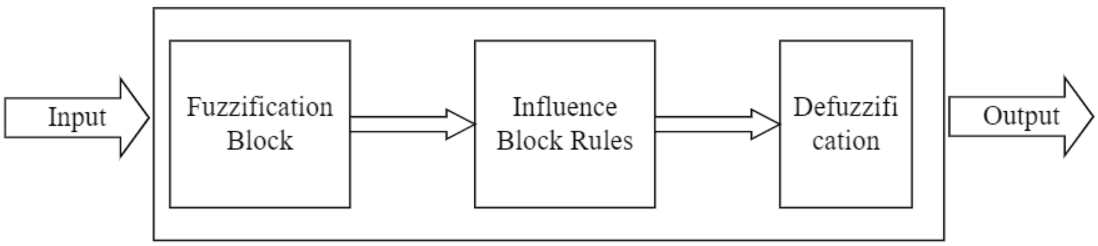

Figure 8 Basic configuration of a fuzzy logic controller

The Fuzzy Logic Controller comprises three components: the fuzzification block, the influence block rules and the defuzzification module. Figure illustrates the fundamental layout of a fuzzy logic controller. Each of these components fulfills a distinct tole in the control process and significantly influences the controller's performance and the overall system behavior. Fuzzification involves converting numerical data from the input into linguistic terms, while the influence block rules serve as the core or brain of the controller, simulating human decision-making processes. Following the inference step, the resulting output is a fuzzy value, which cannot be directly utilized for controlling the process. Therefore, the valve must undergo defuzzification to yield a precise, crisp value, thus completing the control process. The fuzzy logic controller usually works with more than two input signals (Faycal et al, 2016).

Design and Simulation

The objective of this system is to regulate the water level in a tank to a desired setpoint swiftly by adjusting the output value. The system comprises two inputs- error and rate- and one output, which controls the valve. The controller must monitor the water level and the flow rate at each sampling interval to manage the tank's inflow and outflow effectively.MATLAB software is utilized to create a simulation of this system, with the fuzzy logic controller being designed using MATLAB's Fuzzy Logic Toolbox, and the tank simulations conducted in Simulink. The controller's role is to regulate the water tank's inlet valve based on the required inflow, which is determined by the outflow. The fuzzy logic controller dictates the valve's operation by considering various potential input scenarios.

Membership Function Parameters

Open MATLAB, input Fuzzy in the pop-up window, call up fuzzy controller, set the membership function parameters, then set the fuzzy rules, and finally check the fuzzy rules.

The controller employs a rule base expressed in liguistic terms, featuring two inputs: the error in water level and the rate of change of water level, alongside one output parameter: the inlet valve control angle. Trigangular membership function are utilized to fuzzify both the input and output variables. For each of the two inputs, three fuzzy sets (Nagetive, Zero, Positive) are defined, while the output variable comprises five fuzzy sets. The ranges for the error and its time derivative, serving as inputs, are specified as follows:

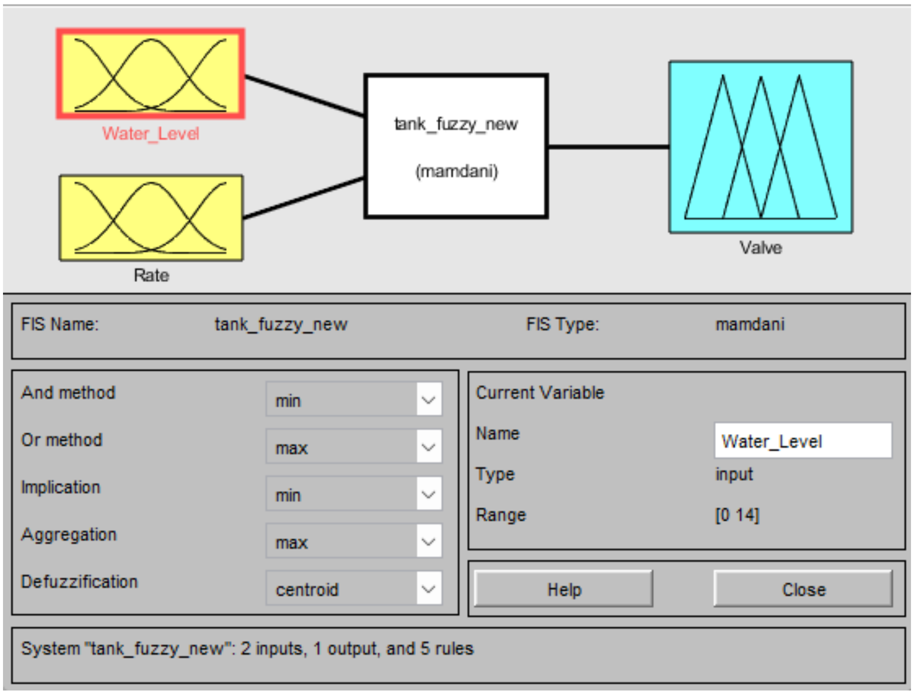

Figure 9 Graphical user interface of the fuzzy logic toolbox

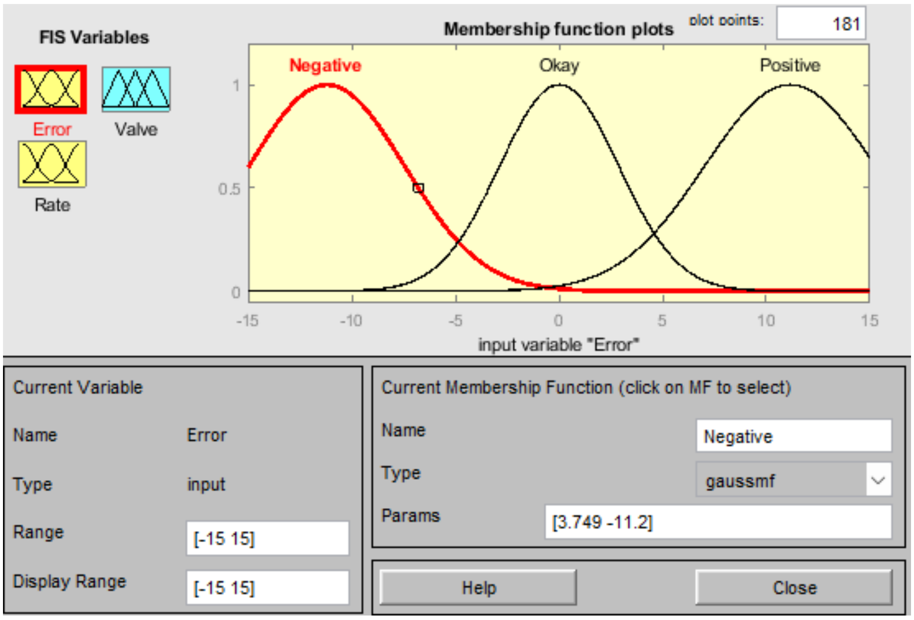

Figure 10 Water Level Membership Functions

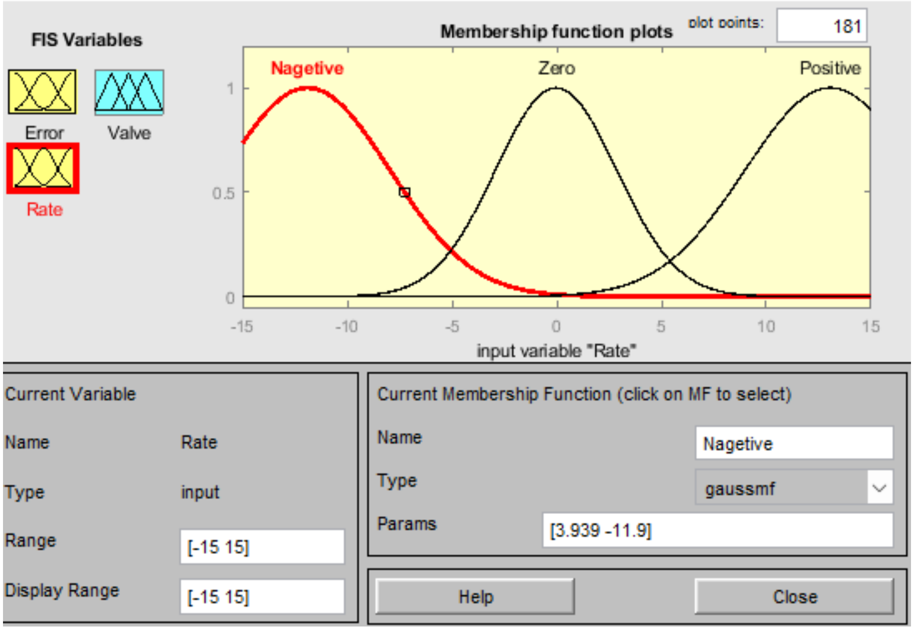

Figure 11 Rate Membership Functions

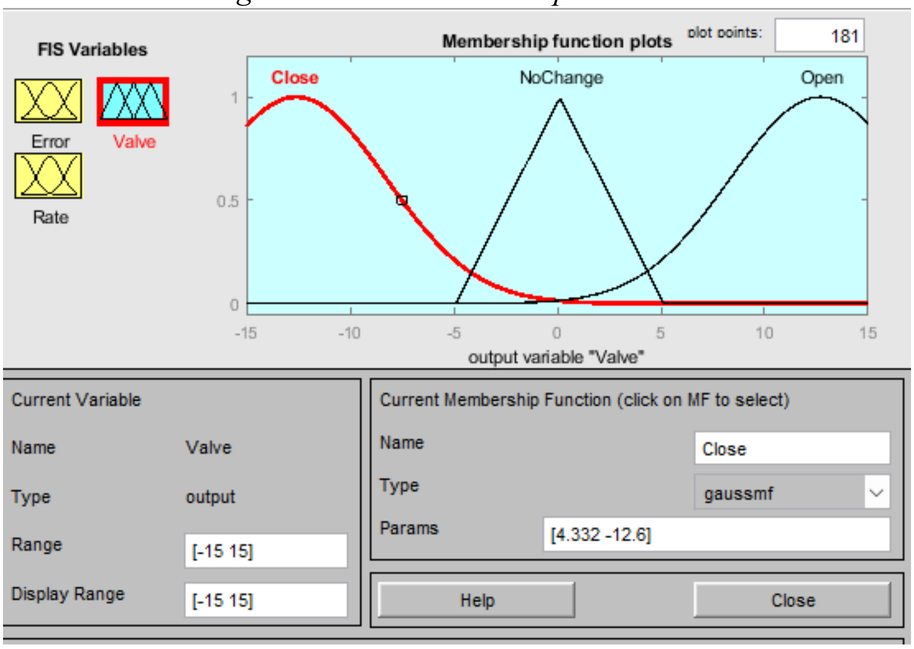

Figure 12 Valve Membership Functions

After fuzzification, as a next step a rule base will be created.

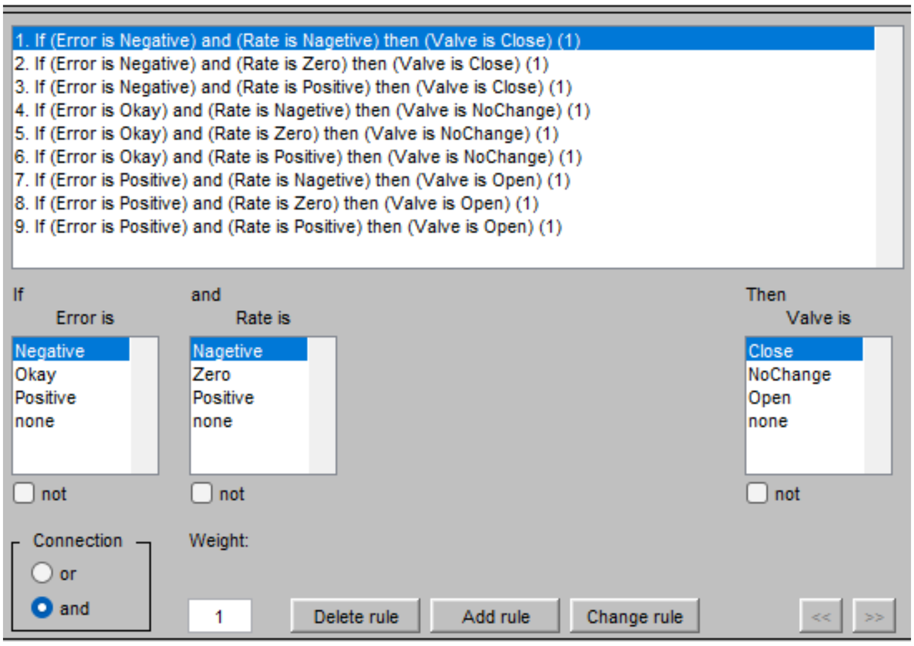

Figure 13 Rule creator editor window file

A simulation of a two-input, one-output system is conducted using the Fuzzy Logic Toolbox in MATLAB. As previously described, each of the two inputs are assigned three fuzzy levels, while the output parameter is defined with five levels. A rule base comprising five rules are employed to track the desired water level. The rule viewer tool is utilized to obtain crisp defuzzified values corresponding to the given crisp inputs, showcasing both the fuzzification and defuzzification processes. The resulting output is depicted as the red line in the output graph.

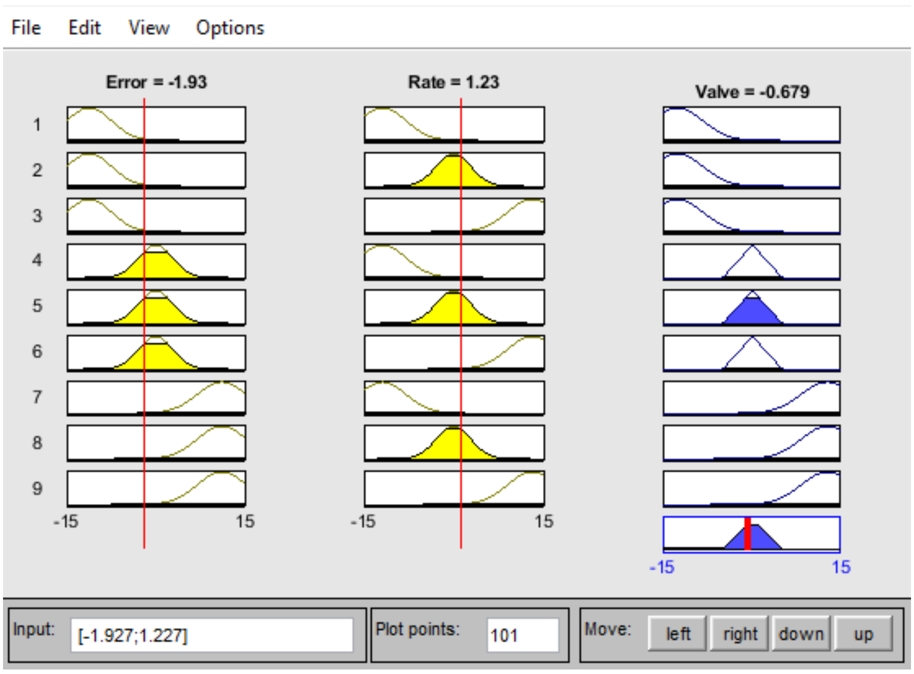

Figure 14 Five Rule Viewer window

An intriguing aspect of the water tank system is the notable disparity in the rates at which it fills and empties, attributable to the particular value of the outflow. This characteristic can be managed by fine-tuning the membership function for the close_slow valve setting, differentiatin it from the open_slow setting. The level of nunaced control is beyond the scope of a conventional PID controller. The relationship between the valve command and the rate of change in water level, as well as the relative surface of water level change, is depicted in figure 15.

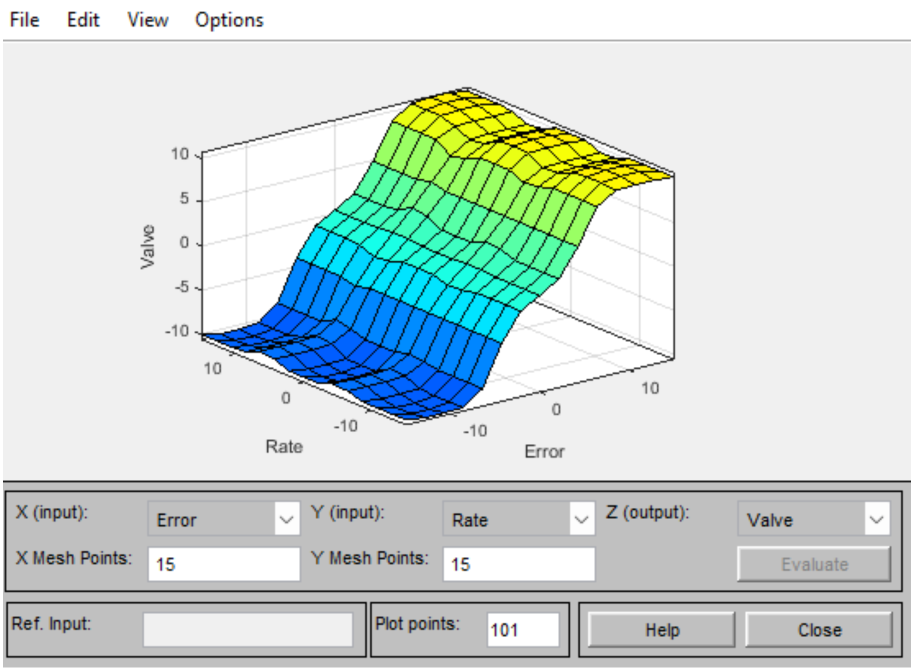

Figure 15 Surface Viewer Indicating 3D Graphical Realization of the Fuzzy Rule Base

Surface Plot Analysis:

The surface shows how the valve position is adjusted based on the combination of error and rate.

When both error and rate are negative (bottom left of the surface), the valve output is high (valve opens), likely to counteract a quickly dropping water level.

When error is negative and rate is positive (bottom right), the valve output is lower, indicating a closing valve action as the water level is below the setpoint but rising.

With a positive error and negative rate (top left), the valve output is low, indicating a closing valve action since the water level is above the setpoint and falling.

In the case of both positive error and rate (top right), the valve output is higher, suggesting the valve opens to reduce the excess water level that is still rising.

Implications:

The highest valve openings are when the error is significantly negative, regardless of the rate, showing an aggressive approach to increase the water level.

Conversely, the valve tends to close when the error is positive, showing an attempt to decrease the water level or slow its rise.

The flattest part of the surface (around the center where error and rate are near zero) indicates little to no change in the valve position, which is expected when the system is near the setpoint.

Fuzzy Control Water Level Simulation

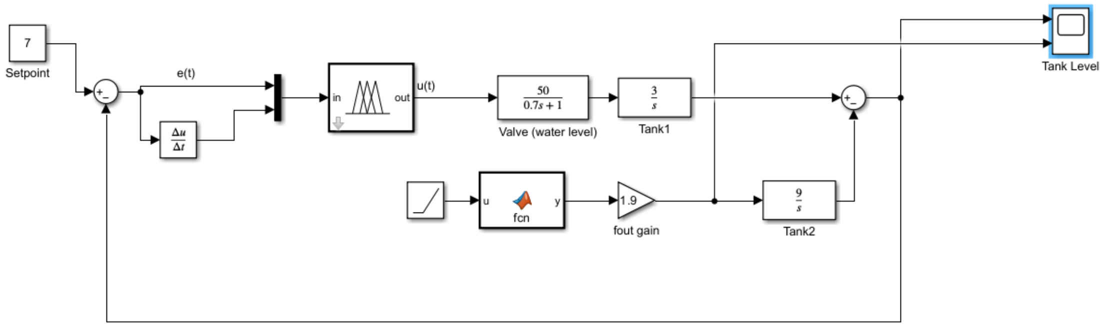

Figure 16 Fuzzy Controller Control Water Level Model

Fuzzy Control Water Level Results

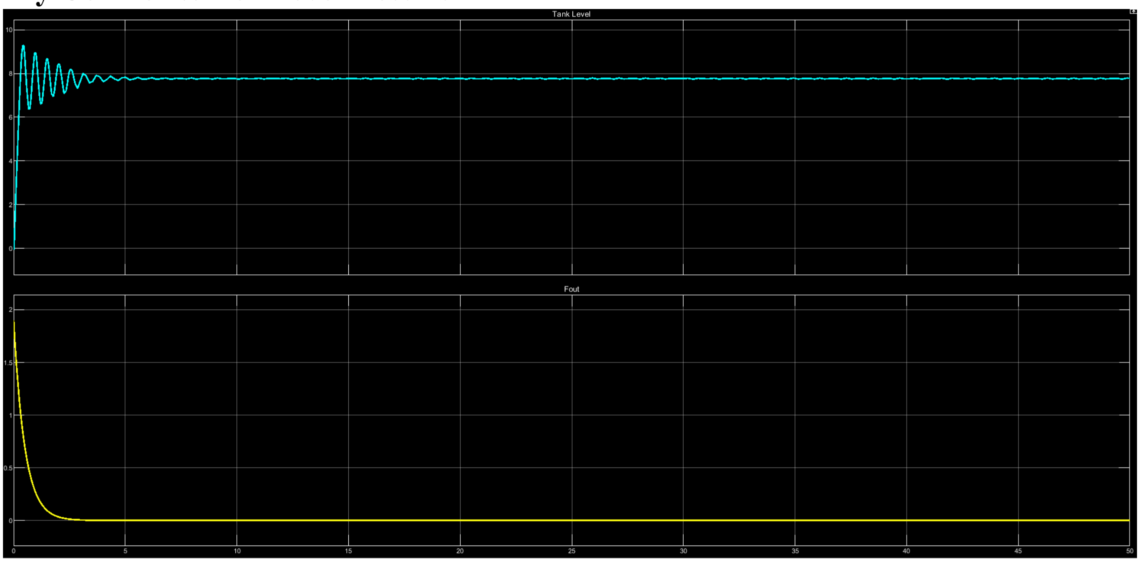

Figure 19 Fuzzy Controller Control Water Level Results

Tank Level Graph Analysis:

There's an initial rapid fluctuation in the water level, possibly due to an aggressive response from the fuzzy controller to reach the setpoint quickly. This can be a common characteristic of fuzzy logic controllers, as they initially may exert strong control action in response to being far from the setpoint.

After the initial fluctuations, the water level steadies and maintains what appears to be a constant level, very close to the setpoint. This steady state suggests that the fuzzy controller has effectively balanced the inflow and outflow to stabilize the water level.

Fout Graph Analysis:

The outflow starts high and decreases sharply, similar to an exponential decay, until it stabilizes at a low level. This is consistent with an initial large corrective action that diminishes as the water level approaches the setpoint.

The gradual decrease to a lower outflow suggests that less corrective action is needed as the system nears its target state, indicating that the controller is minimizing the actuation as the process reaches stability.

Overall Analysis:

The fuzzy logic controller appears to be effective in quickly bringing the water level to the setpoint and then maintaining that level, indicated by the eventual steady state in both the tank level and the outflow.

The rapid stabilization of outflow rate reflects the controller's ability to reduce control actions as the system reaches the desired water level, implying an efficient control process with reduced energy or resource usage once steady state is achieved.

This analysis shows the fuzzy controller's adaptability and effectiveness in managing a system with dynamic behavior, providing robust control even when faced with initial fluctuations.

Comparing PID Controller and Fuzzy Logic Controller

PID Controller (Yellow Line):

The response begins with a drastic rise, potentially an overshoot, followed by a period of oscillations that dampen over time.

The PID controller eventually brings the water level to a steady state. The overshoot and oscillations suggest that the PID parameters might be set aggressively, indicating a high proportional gain or insufficient damping from the integral or derivative components.

Fuzzy Logic Controller (Cyan Line):

The initial response is quite volatile, with large fluctuations. This might be due to the fuzzy rules being too aggressive or the membership functions not capturing the system dynamics accurately, especially when far from the setpoint.

As the system continues, the fluctuations decrease, and the response appears to stabilize, albeit with small persistent oscillations around the setpoint. This suggests the controller is somewhat effective but could be further optimized for stability.

Comparative Analysis:

Figure 17 shows the comparison of fuzzy and PID controller transient response for desired water level. It is clear from the graph that the PID controller has a large overshoot compared to the fuzzy controller and also takes a lot of time to stabilize at the desired level. Fuzzy logic on the other hand, has little overshoot and steady state error and stabilizes quickly providing accurate level control. We find that the advantages and disadvantages of PID control and fuzzy control just offset each other. As seen from the Figure 17, comparing with PID control program, the overshoot is less in fuzzy curve. Settling time reduces. And in order to get better control accuracy, PID control program used as a fine tune.

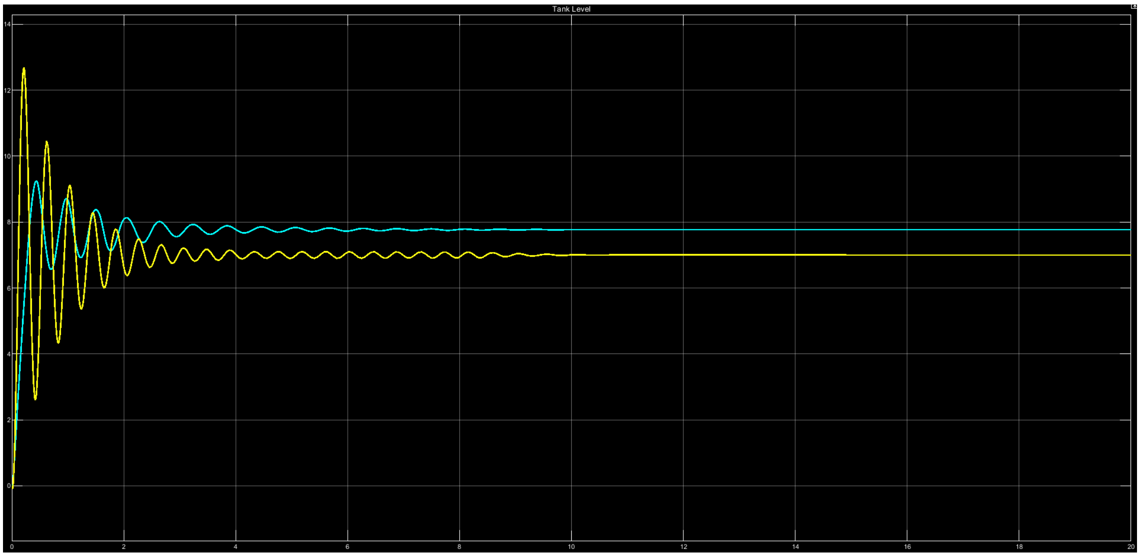

Figure 17 Comparing PID and Fuzzy Logic Controller Water Level Results

Conclusion

In this report, we explored the effectivenesss of PID and Fuzzy Logic Control (FLC) systems in maintaining a stable water level within a tank. The FLC demonstrated enhanced control performance with higher precision and a more stable response over an extended period, showcasing superior dynamic quality. This suggests that FLC is particularly suitable in situations where establishing a precise mathematical model is challenging. FLC's advantage lies in its ability to use lingustic variables to the membership functions and rules, providing a more adaptable control strategy when faced with complex, nonlinear, or uncertain system behaviors.

This report emcompasses a comprehensive process, starting with the design of a mathematical model, proceeding through simulation of the system's steady-state and dynamic behavior, system identification, and culminating in the control strategy implementation. All simulations were conducted using Matlab and Simulink, which enabled a detailed examination of the mathematical model representing the real-world system-a water tank. These simulations illustrated how simulink's PID controller could be conveniently integrated into the model and tuned to achieve the desired control response in terms of speed and robustness.

Reference

Faycal, C. Rachid, T. Abderrahmen , B. and Mohammed Amin, B. 2016 The Application of Fuzzy Control in Water Tank Level Using Arduino. Available from: https://www.researchgate.net/publication/301945883_The_Application_of_Fuzzy_Control_in_Water_Tank_Level_Using_Arduino

Jiang, P. 2024 Summary of PID Control System of Liquid Level of a Single-Capacity Tank. Available from: https://iopscience.iop.org/article/10.1088/1742-6596/1865/2/022061/pdf

Levis, M. 2021 Control Design Toolbox for the Coupled Tanks System. Available from: https://www.quanser.com/blog/mathworks-blog/leveraging-the-simulink-control-design-toolbox-for-the-quanser-coupled-tanks-system/.