Abstract

Oceans cover nearly 70 percent of the Earth's surface, and waves, tides and currents make up different forms of motion: waves are generated by the wind and ocean currents are caused by differences in water temperature and Earth's rotation. These ocean movements not only maintain the balance of ocean ecosystems, but also play an important role in human activities[1]. This report will focus on exploring different devices that use wave energy to generate electricity and their applicability in different climates. By describing several wave energy devices, such as Wave Profile Devices, Oscillating Water Columns and Wave Capture Devices, and comparing their performance under different wave conditions and water depths, the development potential of wave energy as a clean, renewable energy source is demonstrated.

Keywords: Ocean Waves, Wave Profile Devices, Oscillating Water Columns, Wave Capture Devices

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Section1

1 Introduction

With the increasing global energy problems and the impact of the greenhouse effect, developing ocean renewable energy is becoming more important. Among the different types of ocean energy, wave energy is very helpful because it has large reserves [9]. Wave energy comes from the movement of waves on the ocean surface. This movement is caused by wind, and it contains both kinetic and potential energy[10]. Because ocean waves are periodic and oscillate, we can use different devices to capture the energy they produce.
In Section1, we will focus on wave energy devices and their potential. Wave energy conversion involves three main steps [11]: 1. capturing the wave, 2. converting wave energy into mechanical energy, and 3. changing it into electricity. We will describe several devices that are already in use or under study, such as Wave Profile Devices, Oscillating Water Columns, and Wave Capture Devices. Wave Profile Devices convert the up-and-down movement of waves into mechanical energy. Oscillating Water Columns use wave energy to create air pressure. Wave Capture Devices collect wave energy as potential energy. The report will compare the use of these devices in different wave conditions, depths, and places.
In Section 2, we will study the behavior of ocean waves in different water depths using MATLAB. We will calculate key wave properties, like wavelength, speed, particle velocity, and pressure, to understand how these change when water becomes deeper or shallower. Understanding waves is important for practical applications, such as protecting coastlines from erosion and helping engineers design structures like oil rigs, ports, and seawalls [12]. It also helps us understand the effects of water movement and wave pressure on the marine environment [13]. Linear wave theory will be used for this analysis because it helps us understand small waves more easily. Waves behave differently in shallow and deep waters. In shallow water, waves interact with the seabed, while in deep water, waves are affected by their period and wavelength [14]. In this section, we will 1. calculate wavelength and speed at different depths and periods, 2. analyze particle velocity in both shallow and deep wa-

ter, and 3. find the pressure and energy in a wave. 4 calculates the wavelength and celerity of an ocean wave at a depth of 150 meters. These calculations will help us understand how waves behave near the shore and in deeper waters.

2 Ocean Wave Devices

2.1 Wave Profile Devices

2.1.1 Principles

[2] provides that a wave profile device is a wave energy device that moves with the shape and motion of the wave by floating on or near the surface of the sea, or moves up and down under water affected by changes in wave pressure. If the size of the device is small and the period length of the relative wave is short, it is called a "point absorber", such as Fig.1. If the device is larger or longer, it is called a "linear absorber" or "wave attenuator", such as Fig.2, the medium device. The main difference between the two is how they convert wave energy between the absorber and the reaction point, which can be done by oscillating the water body inside the float, the oscillating part, or the buoy[15].

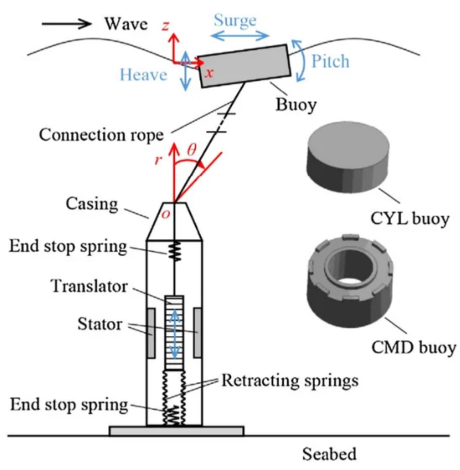

Figure 1: Point Absorber with One Generator.(Source: [2])

2.1.2 Kinds of Wave Profile Devices

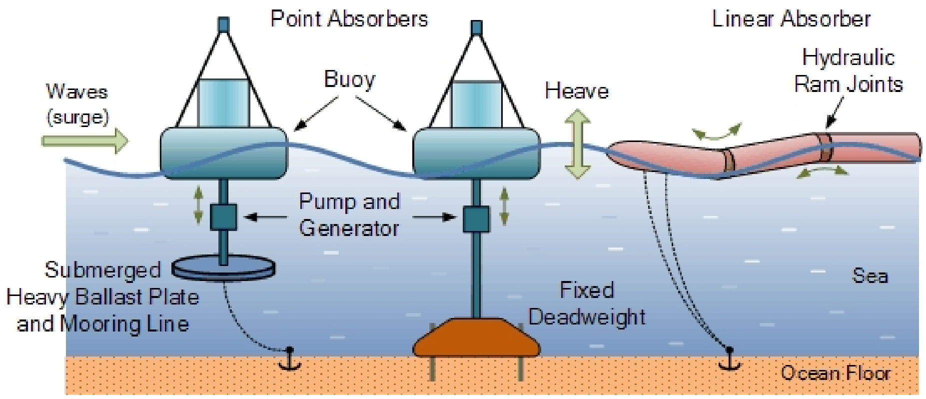

Figure 2: Wave Profile Devices(Source: [2]).

According to[2]. The oscillation of the wave creates a relative motion between the absorber and the reaction point. From Fig.2, these devices on the left uses a heavy ballast plate suspended below the buoy and secured by mooring lines on the seafloor to prevent the buoy from drifting away, allowing the point absorber to operate in deep water areas away from shore.
The right device is a linear absorber (wave attenuator), combine[2] we know, this device aligns itself with the direction of the wave and moves with it. As the wave passes, it takes energy from the motion between the two arms. The device is held in place by a mooring line and moves like a snake, bending downward in the troughs of the wave and arching up at the crests to generate energy.

2.1.3 Example-Powerbuoy

Figure 3: OPT-Powerbuoy1 (source:[3]).

Figure 4: OPT-Powerbuoy2 (source:[3]).

Such devices already have small-scale applications, such as powering navigational buoys. From Fig.3 and Fig.4. According to[3],the PB3 PowerBuoy® developed by Ocean Power Technologies (OPT) is a typical point absorber application. The PB3 PowerBuoy® is self-charging, continuously harvesting energy from waves to act as an undisrupted power supply (UPS), and has been proven in the U.S. Navy's coastal safety programs, including testing under harsh conditions such as hurricanes. It is designed to operate at depths of 20 meters to 3,000 meters and is able to float above the point of use and maintain position via mooring. Its system architecture includes buoys, masts and moorings with self-monitoring, data collection, processing and transmission capabilities to support proactive maintenance strategies to improve equipment reliability and operational efficiency. It payload can be shipped via ISO containers, allowing for quick and easy installation with minimal on-site preparation prior to deployment.

2.2 Oscillating Water Column

2.2.1 Principles

An oscillating water column (OWC) is a common coastal wave energy device, usually mounted on rocks or cliffs near deep water. It consists of partially submerged chambers that convert wave energy into air pressure.

The OWC device drives a turbine at the orifice to generate airflow in a single direction by oscillating the water surface caused by waves,such as Fig.5 and Fig.7. From Fig.6, we know the process of OWC working. Its working principle is divided into two stages: Firstly, the wave energy compresses the air in the cavity, converting the fluid energy into mechanical energy; The generator then converts the mechanical energy into electricity. The OWC device can also be used as a pump, using the movement of the water surface to capture wave energy like a piston. In order to maintain a single direction of rotation when the direction of the air flow is constantly changing, the device needs to have a self-correcting turbine[5].

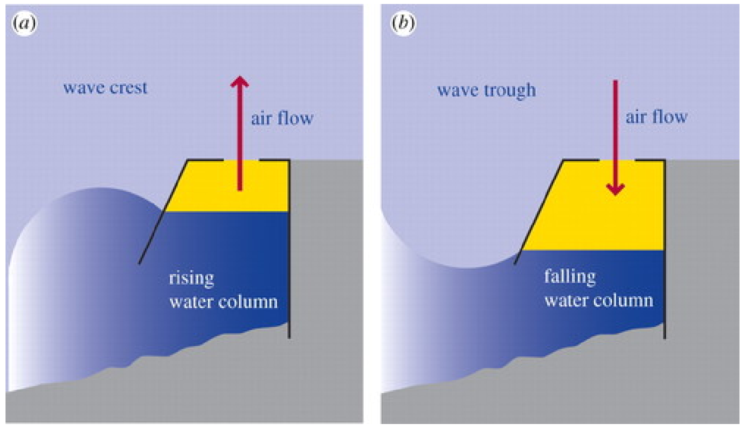

Figure 5: Oscillating the Water Surface(source: [4])

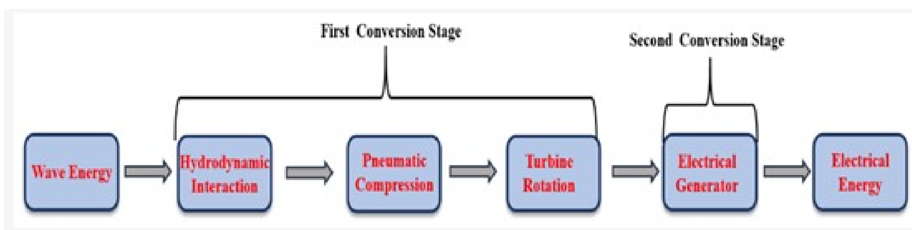

Figure 6: The Process of OWC Working(source:[5]).

2.2.2 Example-LIMPET

From Fig.7 Lslay Limpet Wave Power Station. According to[6] and [4]. A typical example of OWC is LIMPET (Land Installed Marine Powered Energy Transformer), deployed off the coast of the Scottish island of Islay. Use the waves to generate electricity, use the air that the waves displace. It consists of a cement box, which is located on the shoreline, and waves can rush into it from below. Air is pushed out of the box from the top, where a turbine is turned, and when the water back, the air is pulled back into the box, the world's first commercial wave energy device, connected to the UK national grid in 2000. Wavegen reports that "the unit's performance has been optimized for an annual average wave intensity of 15 to 25kW/m." The water column feeds a pair of counter-rotating turbines. LIMPET had an initial power of 500 kW, later adjusted to 250 kW, and was retired in 2012. This device not only validates the feasibility of OWC technology, but also provides valuable data support for coastal wave energy applications. LIMPET has the advantage of being relatively inconspicuous compared to other power generation facilities. Unless you walk on it, you won't know it's there. Like Fig.8.

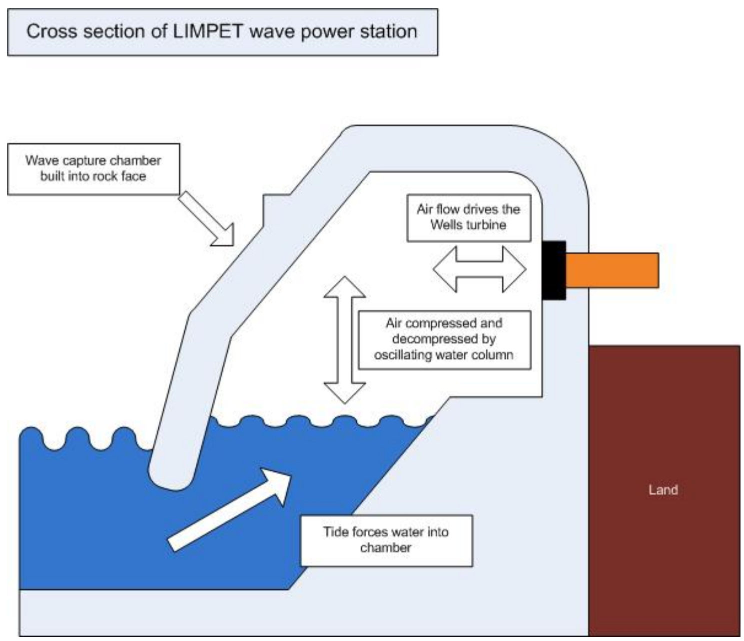

Figure 7: Lslay Limpet Wave Power Station (source: [4]).

Figure 8: Lslay Limpet Wave Power Plant (source: [6]).

2.3 Wave Capture Device

2.3.1 Principles

A wave capture device (or overtopping wave energy device) is a coastal or near-shore wave energy device that converts the kinetic energy of waves and tides into potential energy by raising seawater into a reservoir above sea level.
The wave capture device has simple structure, no complex moving parts, and is easy to maintain. However, since it rely on low head, it has a relatively low power output. Floating versions have been developed to improve this, but they still require waves strong enough to fill the reservoir[2]. For modelling, a linear system approach can be considered, where the water up and down the ramp is considered as the main oscillating part and the ramp wave crests act as the power output system. However, this approach is difficult to apply accurately due to its complex non-linear behaviour, so a more practical approach that relies on real-world data to model overtopping flows over time is used. Figure 9 shows a flow diagram for the Wave Dragon device[7].

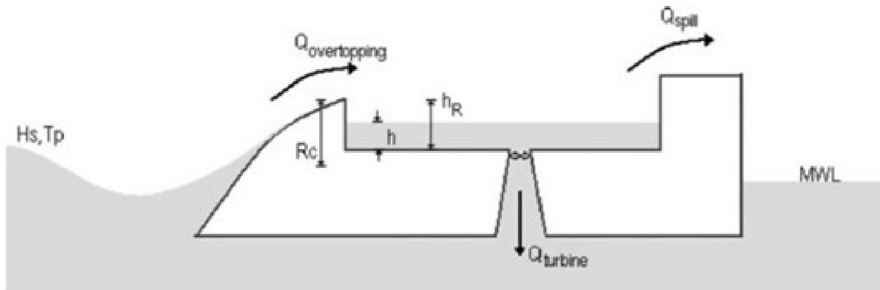

Figure 9: A Flow Diagram for The Wave Dragon Device(Source: [7]).

2.3.2 Example-Wave Dragon Device

From Fig.9. Wave Dragon, developed in Denmark, is a typical example of a overtopping converter. The Wave Dragon consists of two wave reflectors that direct the waves to the reservoir behind the slope. The reservoir sits above sea level and collects and temporarily stores water from wave overflows, which flow through hydroelectric turbines to generate electricity. To maintain platform stability, Wave Dragon is designed to be large and heavy, using additional water weight to balance the platform. In addition, its design simplifies maintenance, and the more expensive parts of the maintenance can be done on land, making it a cost-effective option for offshore power generation[8] and [16].

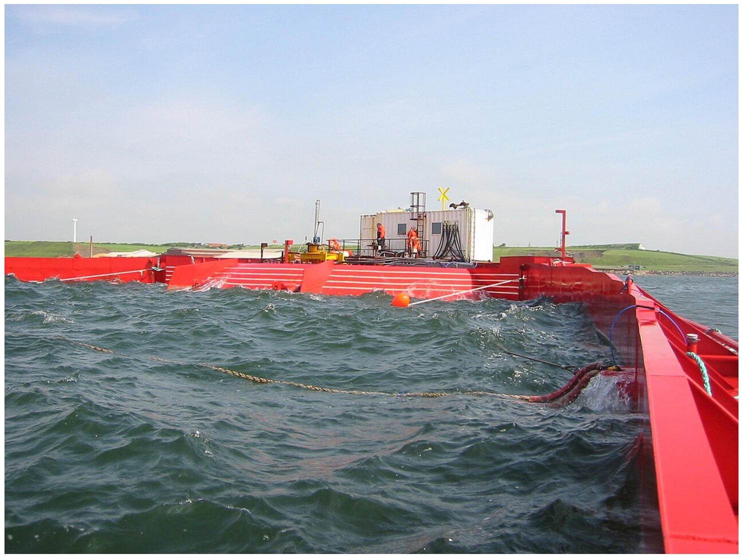

Figure 10: Wave Dragon(Source: [8]).

Here is a table comparing the features of WECs. It shows their main characteristics in a clear way. Table1 helps us understand how each type of WEC works, where it is best used, and provides examples of real devices.

General	**Type**	**Water Depth**	**Climate**	**Location**	**Typical Device**	**Reference**
Wave Profile Devices	Point Absorber	In any ocean depth over 20 m and up to 3 km	High energy	Offshore	PowerBuoy	[7]
Oscillating Water Columns	Oscillating Water Column	Shallow water	Lower wave power (250 kW)	Shoreline	Limpet	[7]
Wave Capture Devices	Overtopping	25 metres (82 ft)	More energetic wave climate	Coastal or near-shore	Wave Dragon	[8]

Table 1: Wave Energy Devices Comparison

Section2

3 Theory

3.1 Linear wave theory

Linear wave theory, also called Airy wave theory, explains how small waves move on the surface of water. This theory assumes that the water depth is the same everywhere and that the water flow is smooth, not compressible, and without rotation. It works well for studying small waves because it makes the equations simpler while still giving accurate results for most real-life uses[14]. We will introduce below the formula from [17] Equation II-1-11 and II-1-7.

1.Wavelength

$$L = \frac{gT^2}{2\pi}\sqrt{\tanh\left(\frac{4\pi^2 d}{T^2 g}\right)} \tag{1}$$

Where:

L: Wavelength
g: Gravitational acceleration (9.81m/s$^{\mathbf{2}}$)
T: Wave period
d: Water depth

2.Wave Celerity

$$C = \frac{L}{T} \tag{2}$$

Where:

C: Wave Celerity
L: Wavelength
T: Wave period

3.Wave Behavior with Depth
In shallow water (d less than L/20), waves are influenced by the seabed, and both L and C increase with depth.
In deep water (d greater than L/2), waves are unaffected by depth, and both L and C stabilize.

3.2 Horizontal and Vertical Velocities of Water Particles

In ocean waves, water particles move in different ways depending on the water depth. In deep water, where the depth is more than half the wavelength, the particles move in almost circular paths. In shallow water, the paths become more oval-shaped because of the seabed. This change from circular to oval motion is important for understanding how waves move, carry energy, and shift sand or sediments [18].

We will introduce below formula to calculate horizontal and vetical velocity from [17] Equation II-1-22 and II-1-23. **1. Horizontal Velocity(u) with Time**

$$u = \frac{H}{2} \cdot \frac{gT}{L} \cdot \frac{\cosh\left[\frac{2\pi(z+d)}{L}\right]}{\cosh\left(\frac{2\pi d}{L}\right)} \cos\theta \tag{3}$$

2. Vertical Velocity(w) with Time

$$w = \frac{H}{2} \cdot \frac{gT}{L} \cdot \frac{\sinh\left[\frac{2\pi(z+d)}{L}\right]}{\cosh\left(\frac{2\pi d}{L}\right)} \sin\theta \tag{4}$$

$$\theta = kx - \omega t \tag{5}$$

3. Horizontal Velocity(u) with Depth

$$u = \frac{H}{2} \cdot \frac{gT}{L} \cdot \frac{\cosh\big(k(z+d)\big)}{\cosh(kd)} \tag{6}$$

4. Vertical Velocity(w) with Depth

$$w = \frac{H}{2} \cdot \frac{gT}{L} \cdot \frac{\sinh\big(k(z+d)\big)}{\cosh(kd)} \tag{7}$$

Where:

H: Wave Hight
k: Wave Number
T: Wave period
z: Depth below the Surface (z=0 at surface)
w: Angular Frequency

Note:

$$k = \frac{2\pi}{L} \tag{8}$$

$$\omega = \frac{2\pi}{T} \tag{9}$$

5. Shallow vs. Deep Water Conclude[17] **In shallow water**, velocities decay slowly with depth.

In deep water, velocities decay exponentially with depth, reducing seabed interaction.

6.Variation with Time
Particle velocities vary sinusoidally with time, following the wave phase (kx - wt)

3.3 Wave Pressure and Energy Flux

Wave Pressure

In ocean waves, the total pressure experienced at a point beneath the surface is the sum of static and dynamic pressures. The static pressure results from the weight of the water column above the point, increasing linearly with depth. The dynamic pressure arises from the motion of the water particles induced by the wave and varies with both depth and time. According to [17] Equation II-1-39 and II-1-58), this combined pressure distribution is crucial for understanding wave forces on submerged structures and the behavior of the water column under wave action.

1. Static Pressure

$$P_{\text{stat}} = -\rho g z \tag{10}$$

2. Dynamic Pressure (Wave-Induced)

$$P_{\text{dyn}} = \rho g A \frac{\cosh\big(k(d+z)\big)}{\cosh(kd)} \tag{11}$$

Where:

A = H/2: Wave amplitude.

3. Total Absolute Pressure

$$P_{\text{abs}} = P_{\text{gauge}} + P_{\text{atmos}} \tag{12}$$

4. Gauge Pressure

$$P_{\text{gauge}} = P_{\text{stat}} + P_{\text{dyn}} \tag{13}$$

Energy Flux

The energy flux per unit width of wave crest is:

$$F = \frac{\rho g H^2 C}{8} \tag{14}$$

This represents the energy transported by the wave per second per meter of crest. Additionally, dynamic pressure decreases with depth, becoming negligible near the seabed.

3.4 Iterative Solution for Wavelength

In water depths, the wavelength must satisfy a special equation. According to [17] Equation II-1-10 andII-1-17 This equation is complex and needs to be solved using iterative numerical methods. The process starts with a guess and is updated using techniques like the Newton-Raphson method or fixed-point iteration. The steps are repeated until the solution is accurate, with the difference between steps becoming very small. This method gives a precise result for depths where shall ow or deep-water formulas do not work[17].

1. Initial Wavelength

$$L_{\text{old}} = \frac{gT^2}{2\pi} \tag{15}$$

This equation is transcendental and requires an iterative numerical solution.

2. Iterative Techniques

$$L_{\text{new}} = \frac{gT^2}{2\pi} \cdot \tanh\left(\frac{2\pi d}{L_{\text{old}}}\right) \tag{16}$$

4 Methodology

4.1 Question 1

Question 1 calculates how wavelength (L) and celerity (C) change for linear ocean waves in different water depths. The first part uses a fixed wave period of 10 seconds with depths from 100 meters to 10 meters. The second part uses wave periods from 3 to 10 seconds with depths from 1 to 200 meters. The calculations follow Equation II-1-11 from the Coastal Protection Manual and are done using MATLAB. The code is simple: 1.Defined parameters; 2.Calculates wavelength and celerity for a wave period of 10 seconds and depths from 1 to 200 meters; 3.Created plots to show these changes separately and together; 4.Updated to use wave periods from 3 to 10

seconds. 5.Calculated wavelength and celerity for each period and depth; 6.Separate plots are made to compare shorter and longer wave periods.
Define Parameters and Constants: The wave period (T) and depth range (d) are defined:

```
T = 10;              % Wave period in seconds (fixed for the first part)
g = 9.81;            % Gravitational acceleration (m/s^2)
start = 1;           % Starting depth in meters
finish = 200;        % Final depth in meters
d = start:1:finish;  % Depths incremented by 1 meter
```

We use Equation(1) and (2). MATLAB loop implementation:

```
for cnt = 1:length(d)
    L(cnt) = ((g*T^2)/(2*pi)) * sqrt(tanh((4*pi^2*d(cnt))/(T^2*g)));
    C(cnt) = L(cnt) / T;
end
```

For wave periods ranging from 3 to 10 seconds, the same equations are applied across a depth range from 1 to 200 meters:

```
T = 3:1:10;  % Wave periods from 3 to 10 seconds
for cnt2 = 1:length(T)
    for cnt = 1:length(d)
        L(cnt,cnt2) = ((g * T(cnt2)^2) / (2 * pi)) * sqrt(tanh((4 * pi^2* d(cnt)) / (T(cnt2)^2 * g))
        C(cnt,cnt2) = L(cnt,cnt2) / T(cnt2);
    end
end
```

The results for multiple wave periods are plotted to show their dependency on depth:

```
figure()
plot(d, L(:,1), 'b', 'LineWidth', 2); hold on;
plot(d, L(:,2), 'c', 'LineWidth', 2);
plot(d, L(:,3), 'y', 'LineWidth', 2);
plot(d, L(:,4), 'k', 'LineWidth', 2);
xlabel('Water depth in meters');
ylabel('Wavelength in meters');
legend('T=3s', 'T=4s', 'T=5s', 'T=6s');
title('Variation in L with d for different wave periods');
grid on;
```

4.2 Question 2

Question2 aims to is to find the maximum horizontal and vertical velocities of water particles in both shallow and deep-water waves. The wave period is 10 seconds. The steps include: 1.Checking if the wave is shallow, deep, or transitional by comparing the depth to the wavelength; 2,Calculating how horizontal and vertical velocities change with depth; 3.Writing a script to calculate how horizontal velocity changes with time at a specific depth for shallow and deep-water waves. We use Equation(6)(7)(8)(9).The velocities are computed across a range of depths:

```
z = -d:d/100:0;  % Depth vector from seabed to surface
for cnt = 1:length(z)
    u(cnt) = (H/2) * (g * T / L) * ((cosh(2 * pi * (z(cnt) + d) / L)) / (cosh(2 * pi * d / L)));
    w(cnt) = (H/2) * (g * T / L) * ((sinh(2 * pi * (z(cnt) + d) / L)) / (cosh(2 * pi * d / L)));
end
```

For a fixed depth, the horizontal velocity is calculated over time (t) using Equation(3)

```
omega = 2 * pi / T;  % Angular frequency
k = 2 * pi / L;      % Wave number
t = 0:0.01:3*T;      % Time vector (3 wave periods)
z = -2;              % Depth below surface (in meters)
x = 0;               % Horizontal location
for cnt = 1:length(t)
    u(cnt) = (H/2) * (g * T / L) * ((cosh(2 * pi * (z + d) / L)) / (cosh(2 * pi * d / L))) * cos(k *
end
```

4.3 Question 3

Question3 aims to find the pressure distribution and energy flux in a wave with a period of 10 seconds and an amplitude of 2.5 meters. The wave moves in water that is 100 meters deep. The steps are: 1.Calculate the absolute, gauge, and dynamic pressures at different depths. 2.Plot how these pressure values change with depth. 3.Compute the energy flux in kW/m for the wave. The wave properties, fluid density, and other constants are initialized

```
Amp = 2.5;          % Amplitude of the wave in meters
H = 2 * Amp;        % Wave height (twice the amplitude)
g = 9.81;           % Gravitational acceleration in m/s^2
```

```
rho = 1025;          % Density of water in kg/m^3
T = 10;              % Wave period in seconds
d = 100;             % Wave depth in meters
z = -d:0.1:0;        % Depth range from seabed to surface in 0.1 m increments
patmos = 101325;     % Atmospheric pressure in Pa
```

The following pressure components are computed for each depth (z), we use Equation(10)(11)(12)(13).

```
for cnt = 1:length(z)
    pdym(cnt) = (rho * g * H / 2) * cosh(2 * pi * (z(cnt) + d) / L) / cosh(2 * pi * d / L);
    pstat(cnt) = -rho * g * z(cnt);
    pgauge(cnt) = pdym(cnt) + pstat(cnt);
    pabs(cnt) = pgauge(cnt) + patmos;
end
```

The energy flux (F) per meter of wave crest is calculated using Eqution(2)and(14)

```
Pow = (rho * g^2 * H^2 * T) / (32 * pi);  % Energy flux in W/m
disp('This energy flux in this wave per meter wave crest is');
disp(Pow);
```

4.4 Question 4

Question 4 calculates the wavelength and celerity of an ocean wave at a depth of 150 meters. The steps are simple. 1.Defined the wave period, gravity, and water depth; 2.An initial guess for the wavelength using the deep-water formula; 3.Updated the wavelength step by step until the change is very small or a set number of steps is reached. Once the wavelength is found, the celerity is calculated by dividing the wavelength by the wave period; 4.The results are shown in the command window and as a bar chart. The wave period , water depth, and other constants are initialized

```
T = 10; % Wave period in seconds
g = 9.81; % Gravitational acceleration in m/s^2
d = 150; % Water depth in meters
```

Initial guess for wavelength, we using Equation(15)

```
L_old = (g * T^2) / (2 * pi); % Deep water approximation
tolerance = 1e-6; % Convergence tolerance
max_iter = 100; % Maximum iterations
iteration = 0; % Initialize iteration counter
```

Iterative Calculation, we using Equation(16) to calculate,if the loop runs 100 times, it will stop to avoid running forever. The method is fast because the tanh function becomes stable as the depth gets larger.

```
while true
    % Update wavelength using the iterative equation
    L_new = (g * T^2) / (2 * pi) * tanh((2 * pi * d) / L_old);
    % Increment iteration counter
    iteration = iteration + 1;
    % Check convergence
    if abs(L_new - L_old) < tolerance
        break;
    end
    % Update L_old for the next iteration
    L_old = L_new;

    % Check if maximum iterations reached
    if iteration >= max_iter
        break;
    end
end
```

We used the Eqution(2) to calculate celerity, providing a complete description of the wave properties.

```
C = L_new / T;
fprintf('Celerity = %.4f m/s\n', C);
```

The results are presented visually to highlight the relative values of wavelength (L) and celerity (C).

```
figure()
bar(1, L_new, 'FaceColor', 'b'); % Visualize Wavelength
hold on;
bar(2, C, 'FaceColor', 'r'); % Visualize Celerity
xticks([1 2])
xticklabels({'Wavelength (L)', 'Celerity (C)'})
ylabel('Value (meters or meters/second)')
title('Wave Properties at Depth = 150 m')
grid on;
```

5 Results and Discuss

5.1 Question 1

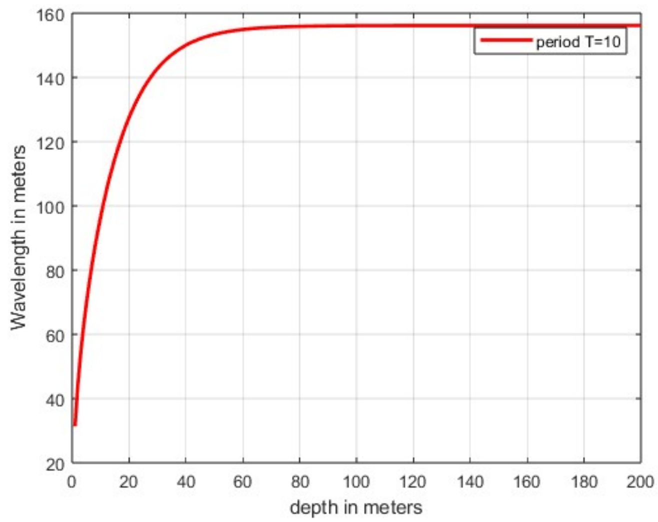

Figure 11: Wavelength vs. Depth.

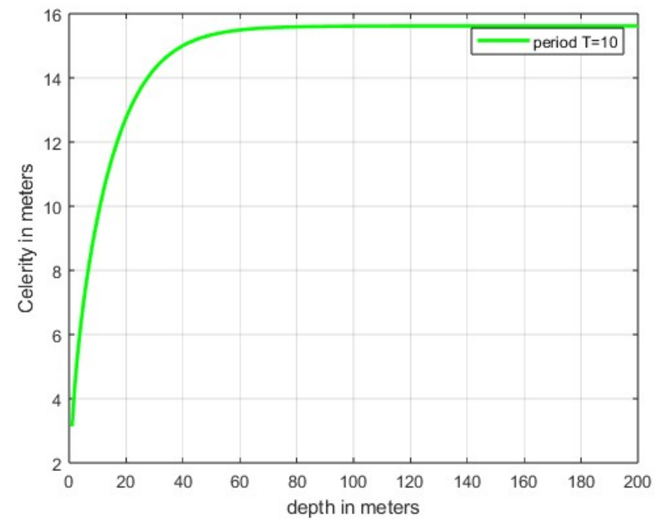

Figure 12: Celerity vs. Depth

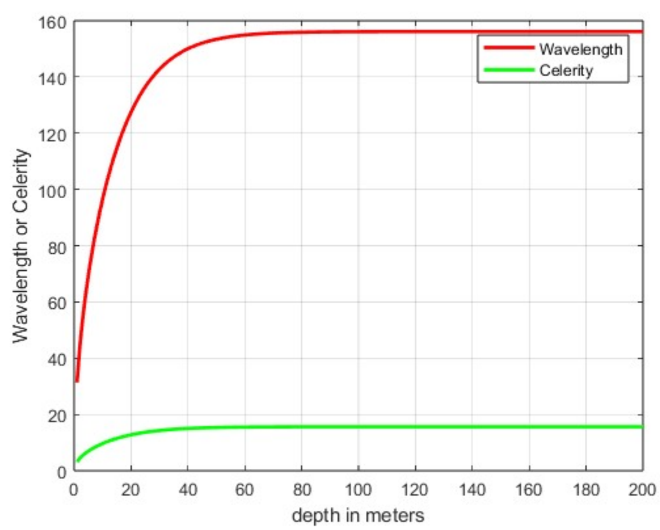

Figure 13: Combined Plot

We will analysis the influence of celerity and wavelength for depth. Figure12 graph shows that the celerity (wave speed) also increases quickly in shallow water and stabilizes at around 16 m/s in deep water. In shallow water, the wave speed depends on the depth of the water. As the depth increases, the wave speed becomes faster. In deep water, the wave speed is only affected by the wave period and the wavelength, so it stays constant. Figure11 shows that the wavelength increases quickly in shallow water (near the surface) and becomes stable in deeper water. The wavelength around 160 meters in deep water. This is because, in shallow water, the wave interacts with the seabed, making the wavelength shorter. In deep water, the wavelength depends only on the wave period and no longer changes with depth. Figure13 combines the wavelength and celerity curves. Both curves increase quickly in shallow water and then level off in deeper water. This is because wave speed is directly related to the wavelength. When the wavelength becomes stable, the wave speed also stays constant. This graph helps us compare how the wavelength and celerity behave as the water gets deeper.

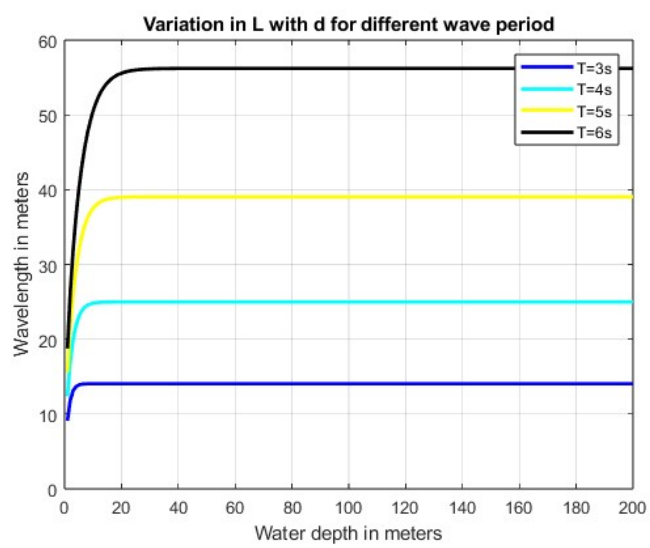

Figure 14: Wavelength for Shorter Periods (T = 3s to 6s)

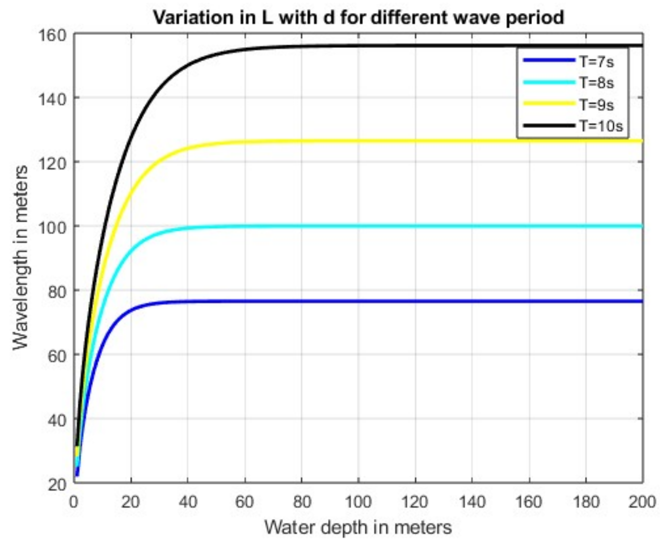

Figure 15: Wavelength for Longer Periods (T = 7s to 10s)

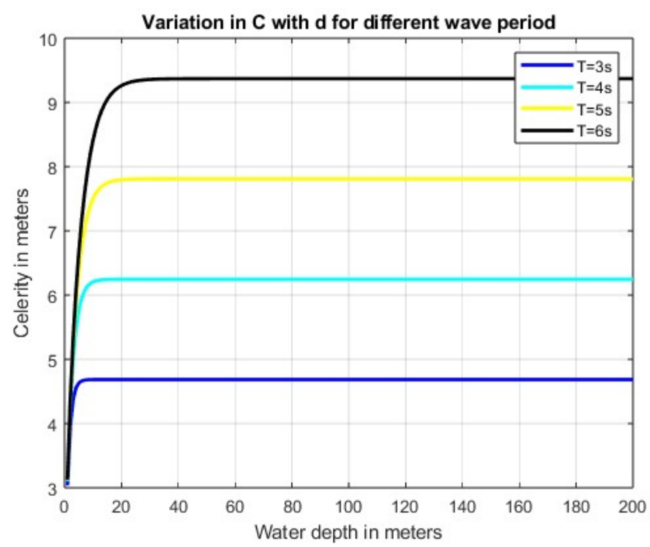

Figure 16: Celerity for Shorter Periods (T = 3s to 6s)

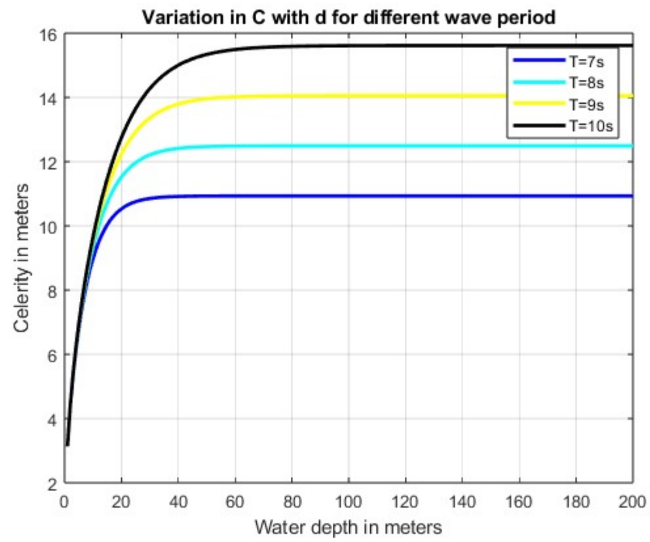

Figure 17: Celerity for Longer Periods (T = 7s to 10s)

We will analysis celerity and wavelength for time period. Figure14 shows that for shorter wave periods, the wavelength (L) increases rapidly with depth in shallow water and stabilizes in deep water. The stabilization occurs at smaller wavelengths compared to longer periods. For example, T=3s results in the shortest wavelength, while T=6s produces a longer wavelength. This is because shorter periods correspond to waves with lower energy and shorter propagation distances. Figure15 shows that for longer wave periods, the wavelength increases with depth in shallow water but stabilizes at much larger values in deep water. For instance, T=10s results in the longest wavelength, reaching over 150 meters. This is because waves with longer periods have more energy and can propagate over longer distances with larger wavelengths.Figure16 shows that the wave celerity (C) increases with depth in shallow water and stabilizes in deep water. For shorter periods, such as T=3s, the wave celerity stabilizes at lower speeds, around 5 m/s. The celerity is directly proportional to the wavelength, so shorter-period waves travel more slowly. Figure17 shows that for longer periods, the wave celerity increases with depth and stabilizes at higher speeds. For T=10s, the celerity stabilizes at about 15 m/s in deep water. This is because longer-period waves have longer wavelengths, which result in faster propagation speeds. Both the wavelength and celerity depend on the wave period. Longer-period waves result in greater wavelengths and higher speeds, especially in deep water, while shorter-period waves have shorter wavelengths and slower speeds. These trends match the predictions of linear wave theory.

5.2 Question 2

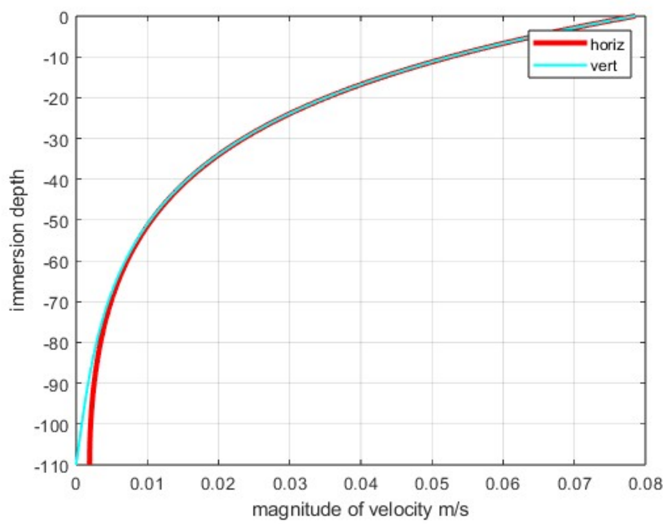

Figure 18: Magnitude of Velocity vs. Depth.

Figure18 based on the given depth ($d = 110$ m) and the calculated wavelength (L), the wave was classified as a deep-water wave. This is because the ratio of depth to wavelength (d/L) is greater than 1/2. In deep water, the wave motion does not interact with the seabed, and the particle velocities decrease exponentially with depth. The horizontal velocity (u) decreases as the depth increases. At the surface, the horizontal velocity is highest because the wave energy is concentrated near the surface. Deeper below the surface, the velocity becomes very small due to the diminishing influence of the wave motion. The vertical velocity (w) also decreases with depth. Near the surface, the water particles move up and down with higher speed, but this motion reduces as the depth increases. At greater depths, the vertical velocity becomes negligible, similar to the horizontal velocity. The graph shows that the horizontal velocity (u) is slightly higher than the vertical velocity (w) at all depths. This is because the horizontal motion of water particles dominates the wave-induced motion, while the vertical motion is smaller. Both velocities decrease exponentially with depth, reflecting typical behavior in deep-water waves.

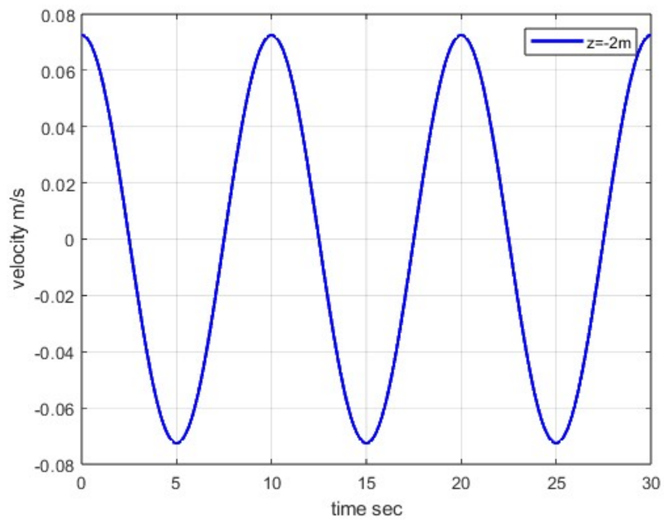

Figure 19: Time vs. Velocity.

Figure19 shows the variation of horizontal velocity (u) over time for a water particle located at $z = -2$m below the surface. The velocity oscillates between a maximum of about 0.075m/s and a minimum of about -0.075m/s, forming a sinusoidal pattern. This indicates the back-and-forth motion of water particles caused by the wave. The periodic nature of the velocity matches the wave period ($T =$ 10s). The velocity completes one full oscillation every 10 seconds, as expected for a wave with this period. This regular pattern shows how water particles move in a predictable way under wave action. At $z = -2$m, the velocity is smaller compared to particles closer to the surface. This decrease in velocity is consistent with wave theory, where water particle motion decreases as the depth increases. The influence of the wave becomes negligible as you move further below the surface.

5.3 Question 3

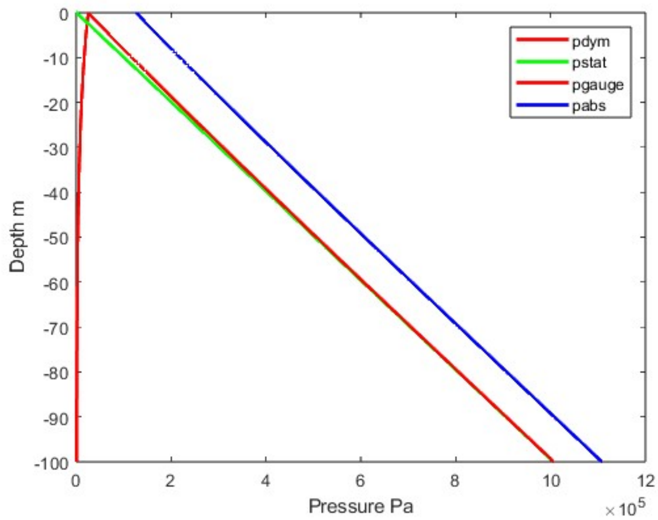

Figure 20: Pressure vs. Depth.

Figure20 shows that the dynamic pressure (p_{dyn}) decreases as the depth increases. Near the surface, the dynamic pressure is highest because the wave motion is strongest. As the depth increases, the effect of the wave diminishes, and the dynamic pressure approaches zero near the seabed. The static pressure (p_{stat}) increases linearly with depth. This happens because static pressure is caused by the weight of the water column above the point of measurement. Deeper points experience higher static pressure.The gauge pressure (p_{gauge}) is the sum of the dynamic and static pressures. It is highest at the surface because both components are significant. As the depth increases, the gauge pressure becomes dominated by the static pressure, which increases with depth. The absolute pressure (p_{abs}) includes the atmospheric pressure in addition to the gauge pressure. It is highest at the seabed due to the sum of atmospheric, static, and dynamic pressures. The atmospheric pressure ensures that the absolute pressure is always higher than the gauge pressure. The energy flux of the wave is calculated to be a specific value (output as Pow in the code). This energy flux represents the power carried by the wave per meter of wave crest. Higher wave height and period result in greater energy flux, showing the wave's potential to generate energy or cause coastal impact.

5.4 Question 4

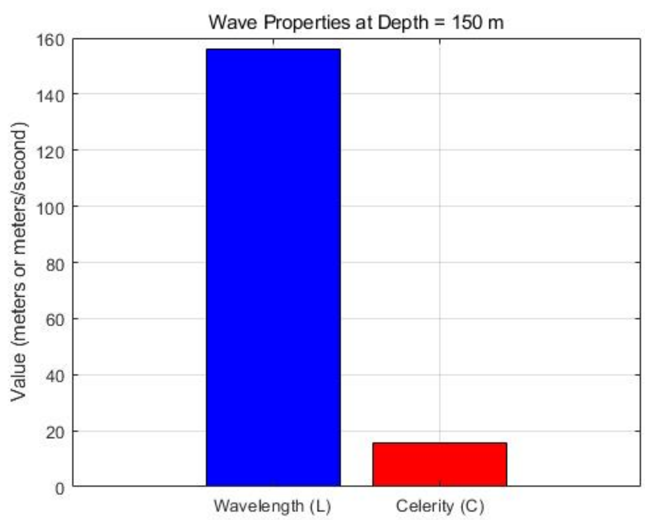

Figure 21: Wave Properties at Depth = 150 m.

Figure21 shows the wave properties at a depth of 150meters. The blue bar represents the wavelength, and the red bar represents the celerity. The wavelength is much larger than the celerity, with a value of about 150meters, while the celerity is around 15meters per second. This means that the wave travels a long distance in each cycle, but the speed of the wave is relatively lower compared to the wavelength. The difference in values is expected, as wavelength depends heavily on depth and wave period.

6 Conclusion

This report discusses three important types of wave energy devices. Wave Profile Devices, like the PB3 PowerBuoy®, are placed in deep water. They use buoys to move with the waves and turn that movement into electricity. These devices can work in very deep water, up to 3,000 meters, and are built to survive storms. Oscillating Water Columns, like the LIMPET in Scotland, use waves to push air through turbines, which creates energy. These devices are best for places near the shore or on islands with steady waves. Wave Capture Devices, like the Wave Dragon, use the overflow of waves to create energy. These work well in shallow water or nearshore areas. Wave energy is a clean and renewable energy source, but it has problems like low efficiency, weather damage, and high repair costs.

The analysis used MATLAB to study how waves behave. It calculated wavelength, speed, pressure, and velocity for different depths. The study found that wavelength becomes stable in deep water, and wave speed increases as the wavelength grows. The pressure in waves changes with depth. Dynamic pressure, which comes from wave motion, gets weaker as you go deeper. Static and absolute pressures increase because of the weight of water above. The way water particles move also changes. In deep water, particles move in circles. In shallow water, the movement becomes oval because of the seabed. The velocity of the particles also becomes much slower as the depth increases. The study also calculated the energy in waves, showing that waves have high energy. The velocity of the waves over time followed a wave-like pattern, matching the movement of the ocean surface.

The report shows that wave energy can be an important renewable energy source. The findings help in making devices that can work better in different ocean conditions. However, there are challenges. Devices can break in storms, and fixing them costs a lot of money. Also, the current devices only use about 30 percent of the wave's energy. Finally, using wave energy on a large scale might hurt the marine environment, so more research is needed. To solve these problems, scientists need to make devices stronger and more efficient. They also need to study how to combine wave energy with other clean energy sources. This study shows that understanding wave behavior can help create better devices and make wave energy more useful in the future.

7 Including Matlab code

Question1:

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Name: Jinyan Yang                    %
%Student Number: D00264564            %
%Date: 18/10/2024                     %
%this script find the wave length for %
%waves of a period of 10 seconds and  %
%water depths varying from 10 to 100  %
%meters                               %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%do the housekeeping

clear                 %clear the workspace
clc                   %clear the command window
clear all             %clear any open figures

%now declare everything that we know
T = 10;               % wave period in minutes
g = 9.81;             % acceration due to gravity in m/s^2
inc = 1;              %this is the depth increment in m
start = 1;            %start depth
finish = 200;         %finish depth
d = start:inc:finish; %these are the depth in m

for cnt = 1:length(d)
    L(cnt)=((g*T^2)/(2*pi))*sqrt(tanh((4*pi^2*d(cnt))/(T^2*g)));
    C(cnt)= L(cnt)/T;
end

figure()
plot(d,L,'r','LineWidth', 2)
xlabel('depth in meters')
ylabel('Wavelength in meters')
grid on
legend ('period T=10')

figure()
plot(d,C,'g','LineWidth', 2)
xlabel('depth in meters')
ylabel('Celerity in meters')
grid on
legend ('period T=10')

figure()
plot(d,L,'r','LineWidth', 2)
hold on
plot(d,C,'g','LineWidth', 2)
xlabel('depth in meters')
ylabel('Wavelength or Celerity')
grid on
legend ('Wavelength','Celerity')
```

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Name: Jinyan Yang                    %
%Student Number: D00264564           %
%Date: 18/10/2024                    %
%this script find the wave length for %
%waves of a period of 10 seconds and  %
%water depths varying from 10 to 100  %
%meters and periods 3 seconds to 10 s %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%do the housekeeping

clear                 %clear the workspace
clc                   %clear the command window
clear all             %clear any open figures

%now declare everything that we know
T = 3:1:10;               % wave period in minutes
g = 9.81;             % acceration due to gravity in m/s^2
inc = 1;              %this is the depth increment in m
start = 1;            %start depth
finish = 200;         %finish depth
d = start:inc:finish; %these are the depth in m

for cnt2 = 1:length(T)
    for cnt = 1:length(d)
        L(cnt,cnt2)=((g*T(cnt2)^2)/(2*pi))*sqrt(tanh((4*pi^2*d(cnt))/(T(cnt2)^2*g)));
        C(cnt,cnt2)= L(cnt,cnt2)/T(cnt2);
    end
end

%now plot the results
figure()
plot(d,L(:,1),'b','LineWidth', 2)
hold on
plot(d,L(:,2),'c','LineWidth', 2)
plot(d,L(:,3),'y','LineWidth', 2)
plot(d,L(:,4),'k','LineWidth', 2)
grid on
xlabel('Water depth in meters')
ylabel('Wavelength in meters')
legend ('T=3s','T=4s','T=5s','T=6s')
title('Variation in L with d for different wave period')

figure()
plot(d,L(:,5),'b','LineWidth', 2)
hold on
plot(d,L(:,6),'c','LineWidth', 2)
plot(d,L(:,7),'y','LineWidth', 2)
plot(d,L(:,8),'k','LineWidth', 2)
grid on
xlabel('Water depth in meters')
ylabel('Wavelength in meters')
legend ('T=7s','T=8s','T=9s','T=10s')
title('Variation in L with d for different wave period')

figure()
```

```
plot(d,C(:,1),'b','LineWidth', 2)
hold on
plot(d,C(:,2),'c','LineWidth', 2)
plot(d,C(:,3),'y','LineWidth', 2)
plot(d,C(:,4),'k','LineWidth', 2)
grid on
xlabel('Water depth in meters')
ylabel('Celerity in meters')
legend ('T=3s','T=4s','T=5s','T=6s')
title('Variation in C with d for different wave period')

figure()
plot(d,C(:,5),'b','LineWidth', 2)
hold on
plot(d,C(:,6),'c','LineWidth', 2)
plot(d,C(:,7),'y','LineWidth', 2)
plot(d,C(:,8),'k','LineWidth', 2)
grid on
xlabel('Water depth in meters')
ylabel('Celerity in meters')
legend ('T=7s','T=8s','T=9s','T=10s')
title('Variation in C with d for different wave period')
% figure()
% plot(d,L,'r','LineWidth', 2)
% xlabel('depth in meters')
% ylabel('Wavelength in meters')
% grid on
% legend ('period T=10')
%
% figure()
% plot(d,C,'g','LineWidth', 2)
% xlabel('depth in meters')
% ylabel('Celerity in meters')
% grid on
% legend ('period T=10')
%
% figure()
% plot(d,L,'r','LineWidth', 2)
% hold on
% plot(d,C,'g','LineWidth', 2)
% xlabel('depth in meters')
% ylabel('Wavelength or Celerity')
% grid on
% legend ('Wavelength','Celerity')
```

Question2:

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Name: Jinyan Yang                    %
%Student Number: D00264564            %
%Date: 25/10/2024                     %
%this script find the wave length for %
%a 10 sec wave. it checks to see if   %
%the wave is a deep or shallow water  %
%wave and tells the user. it then     %
%finds the variation in water particle%
%horizontal and vertical velocities   %
%with depth and plots the results     %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%do the housekeeping

clear                 %clear the workspace
clc                   %clear the command window
clear all             %clear any open figures

%define waht we know
T = 10;               % wave period in minutes
g = 9.81;             % acceration due to gravity in m/s^2

%pick a water depth to sart with
d = 110; %my first guess in m
H = 0.25; % half meter wave height until we settle on a depth

%find the wavelength for this wave

L=((g*T^2)/(2*pi))*sqrt(tanh((4*pi^2*d)/(T^2*g)));

% now check if the wave is deep or shallow

if d/L < 1/20
    disp('This is a shallow water wave')
elseif d/L > 1/2
    disp('This is a deep water wave')
elseif d/L >= 1/20 && d/L <= 1/2
    disp('This is a transitional water wave')
else
    disp('Something went wrong')
end

% define z

z= -d:d/100:0;


%now use a for loop to find the max horizontal and vertical velocities at
%each deoth. To do this we let cos theta and sin theta be equal to one
%respectivey

for cnt = 1:length(z)
    u(cnt) = (H/2)*(g*T/L)*((cosh(2*pi*(z(cnt)+d)/L))/(cosh(2*pi*d/L)));
    w(cnt) = (H/2)*(g*T/L)*((sinh(2*pi*(z(cnt)+d)/L))/(cosh(2*pi*d/L)));
end
```

```
figure()
plot(u,z,'r','LineWidth',3)
hold on
plot(w,z,'c','LineWidth',1.5)
xlabel('magnitude of velocity m/s')
ylabel('immersion depth')
legend('horiz','vert')
grid on
ylim([-d 0])
```

Question3:

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Name: Jinyan Yang                    %
%Student Number: D00264564            %
%Date: 25/10/2024                     %
%this script find the wave length for %
%a 10 sec wave. It then plot the      %
%variation in horizontal particle     %
%velocity as it varies with thime for %
%some chosen depth and plts the rsults%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%do the housekeeping

clear                 %clear the workspace
clc                   %clear the command window
close all             %clear any open figures

%define water know
Amp =2.5;  %Amplitude of the wave in m
H = 2*Amp; % waveheight is twice the amplitude
g = 9.81;  % acceration due to gravity in m/s^2
rho = 1025; %density of water kg/m3
T = 10;    % wave period in minutes
d = 100; %wave depth in m
z= -d:0.1:0; %immersion vector
patmos = 101325;%atmosperic pressure

%Now find wave length, L

L=((g*T^2)/(2*pi))*sqrt(tanh((4*pi^2*d)/(T^2*g)));


for cnt = 1:length(z)
    pdym(cnt)= rho*g*H*cosh(2*pi*(z(cnt)+d)/L)/(2*cosh(2*pi*d/L));
    pstat(cnt)= -rho*g*z(cnt);
    pgauge(cnt)= pdym(cnt)+ pstat(cnt);
    pabs(cnt)= pgauge(cnt)+patmos;
end

figure()
plot(pdym,z,'r','linewidth',2)
hold on
plot(pstat,z,'g','linewidth',2)
plot(pgauge,z,'r','linewidth',2)
plot(pabs,z,'b','linewidth',2)
xlabel('Pressure Pa')
ylabel('Depth m')
legend('pdym','pstat','pgauge','pabs')

figure()
plot(pdym,z,'r','linewidth',2)
xlabel('Pressure Pa')
ylabel('Depth m')
legend('pdym','pstat','pgauge','pabs')

Pow = (rho*g^2*H^2*T)/(32*pi); % power in kW/m
disp('This energy flux in this wave per metre wave crest is')
```

```
Pow
```

Question4:

```matlab
% Housekeeping
clear;
clc;

% Given constants
T = 10; % Wave period in seconds
g = 9.81; % Gravitational acceleration in m/s^2
d = 150; % Water depth in meters

% Initial guess for wavelength
L_old = (g * T^2) / (2 * pi); % Initial approximation for deep water
tolerance = 1e-6; % Convergence tolerance
max_iter = 100; % Maximum iterations
iteration = 0; % Initialize iteration counter

% Iterative calculation using while loop
while true
    % Update wavelength using the iterative equation
    L_new = (g * T^2) / (2 * pi) * tanh((2 * pi * d) / L_old);

    % Increment iteration counter
    iteration = iteration + 1;

    % Check convergence
    if abs(L_new - L_old) < tolerance
        fprintf('Converged after %d iterations: Wavelength = %.4f m\n', iteration, L_new);
        break;
    end

    % Update L_old for the next iteration
    L_old = L_new;

    % Check if maximum iterations reached
    if iteration >= max_iter
        fprintf('Solution did not converge after %d iterations.\n', max_iter);
        break;
    end
end

% Calculate Celerity (wave speed)
C = L_new / T;
fprintf('Celerity = %.4f m/s\n', C);

% Visualization
figure()
bar(1, L_new, 'FaceColor', 'b'); % Visualize Wavelength
hold on;
bar(2, C, 'FaceColor', 'r'); % Visualize Celerity
xticks([1 2])
xticklabels({'Wavelength (L)', 'Celerity (C)'})
ylabel('Value (meters or meters/second)')
title('Wave Properties at Depth = 150 m')
grid on;
```

References

[1] M. M.L., S. R., and G. J., "How can we use ocean energy to generate electricity?" online, n.d., https://kids.frontiersin.org/articles/10.3389/frym.2021.609510.

[2] E. S. Team, "Wave energy — earthsci.org," https://earthsci.org/mineral/energy/wavpwr/wavepwr.html, 2024, [Accessed 01-12-2024].

[3] O. P. Technologies, "Pb3 powerbuoy — oceanpowertechnologies.com," https://oceanpowertechnologies.com/platform/opt-pb3-powerbuoy/, n.d., [Accessed 01-12-2024].

[4] W. Contributors, "Islay limpet," https://en.wikipedia.org/w/index.php?title=IslayLIMPET, n.d., [Accessed 02-12-2024].

[5] R. Gayathri, J.-Y. Chang, C.-C. Tsai, and T.-W. Hsu, "Wave energy conversion through oscillating water columns: A review," *Journal of Marine Science and Engineering*, vol. 12, no. 2, p. 342, 2024.

[6] C. C. News, "Limpet: Land installed marine powered energy transformer," https://news.climate.columbia.edu/2010/05/19/limpet-land-installed-marine-powered-energy-transformer/, n.d., [Accessed 01-12-2024].

[7] G. Bevilacqua and B. Zanuttigh, "Overtopping wave energy converters: General aspects and stage of development," 2016, 2015 study published in 2016.

[8] W. Dragon, "World class offshore wave and wind energy for a renewable future," https://www.wavedragon.com/, n.d., [Accessed 01-12-2024].

[9] H. Zhang and G. Aggidis, "Nature rules hidden in the biomimetic wave energy converters," *Renewable and Sustainable Energy Reviews*, vol. 97, pp. 28–37, 2018.

[10] M. Golušin, S. Dodić, and S. Popov, "Strategic priorities of sustainable energy development," *U: Sustainable Energy Managment*, pp. 243–333, 2013.

[11] "Wave Energy — Alternative Energy Tutorials — alternative-energy-tutorials.com," https://www.alternative-energy-tutorials.com/wave-energy/wave-energy., [Accessed 01-12-2024].

[12] "Scientists quantify how wave power drives coastal erosion — news.mit.edu," https://news.mit.edu/2020/wave-power-coastal-erosion-0316, [Accessed 01-12-2024].

[13] T. Pang, X. Wang, R. A. Nawaz, G. Keefe, and T. Adekanmbi, "Coastal erosion and climate change: A review on coastal-change process and modeling," *Ambio*, vol. 52, no. 12, pp. 2034–2052, 2023.

[14] Wikipedia, "Airy wave theory," https://en.wikipedia.org/w/index.php?title=Airy_wave_theory_oldid=1233487520, 2024, [Accessed 02-12-2024].

[15] ScienceDirect, "Tidal power - an overview — sciencedirect topics," https://www.sciencedirect.com/topics/engineering/tidal-power, 2024, [Accessed 01-12-2024].

[16] W. Contributors, "Wave dragon - wikipedia," https://en.wikipedia.org/wiki/WaveDragon, n.d., [Accessed 01-12-2024].

[17] R. M. Sorensen, *Basic Coastal Engineering.* Springer, 2006.

[18] C. Wiki, "Shallow-water wave theory," https://www.coastalwiki.org/wiki/Shallow-water_wave_theory#The_influence_of_water_depth_on_wave_characteristics, n.d., [Accessed 02-12-2024].