Contents

Introduction

DC Motor

In our class, we studied a permanent magnet DC Motor.

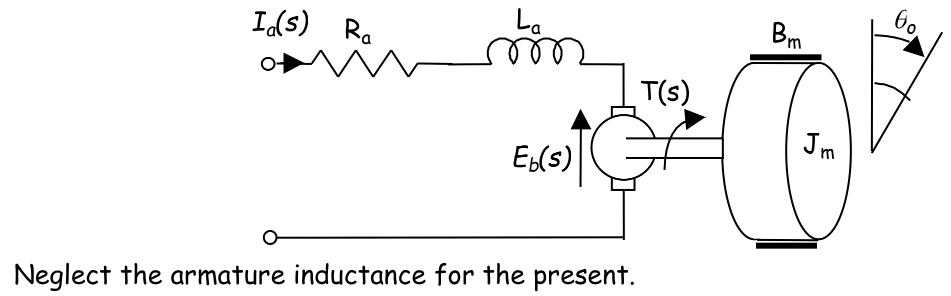

Figure1: A permanent magnet d.c. motor

According to our class, we know The torque applied to a load and the moment of inertia of the load determine the load acceleration.The load speed is equivalent to the time integral of its

acceleration.

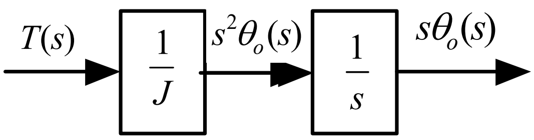

Figure2: Calculation

As the load speeds up a viscous friction torque that opposes motion builds up and reduces the amount of torque available to accelerate the load.

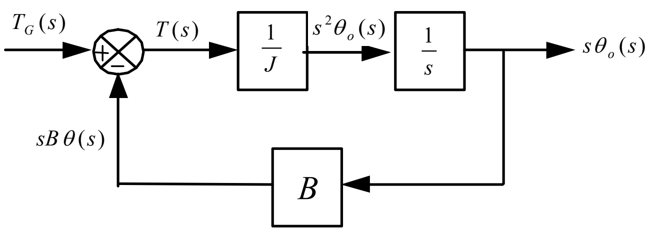

Figure3: Calculation

The current flowing in the armature circuit determines the amount of torque generated in the motor.

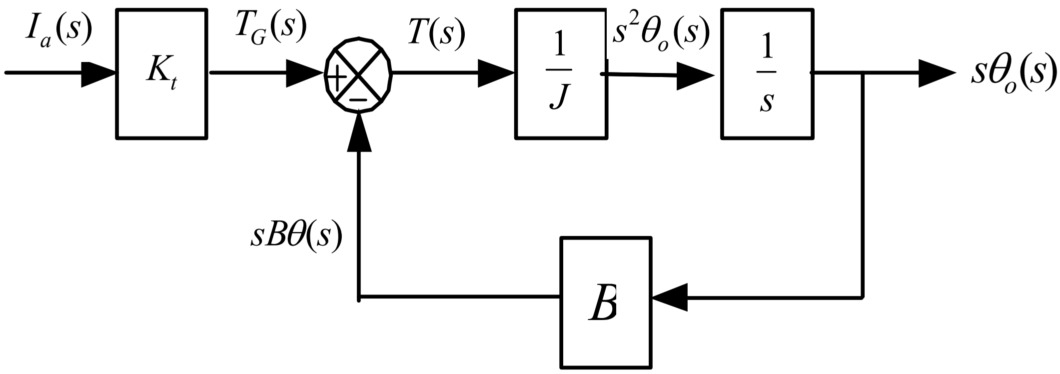

Figure4: Calculation

The armature current in turn is determined by the difference between the applied input voltage and the back emf generated in the armature windings as they move in the magnetic field set up by the motor permanent magnet. The magnitude of the back emf is proportional to the speed of rotation of the armature and provides a feedback that limits the speed of the motor for any given input voltage.

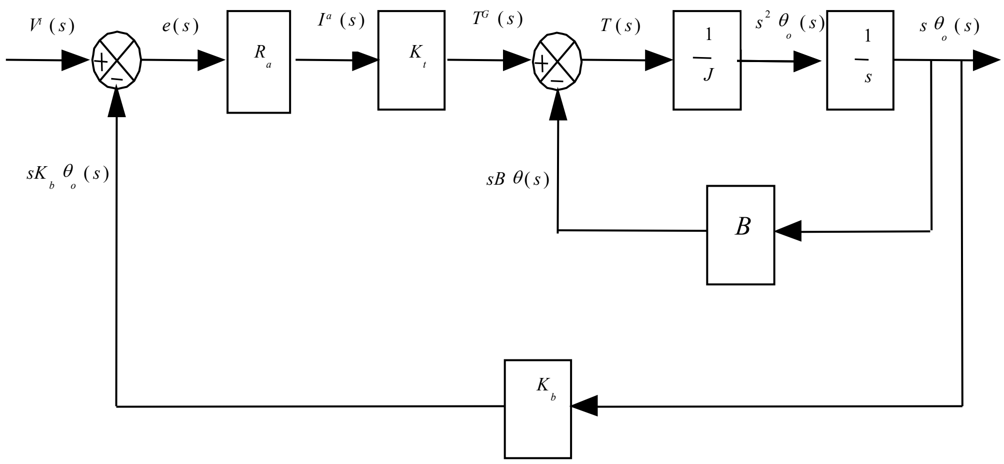

Figure5: Calculation

Today, through study the permanent dc motor, we will design a 12V DC Motor include the effect of armature inductance La. We are investigating the effective usage of a PID controller, when used to control the speed of a DC 12 V motor.

PID Controller

A PID controller is an instrument used in industrial control applications to regulate temperature, flow, pressure, speed and other process variables. PID (proportional integral derivative) controllers use a control loop feedback mechanism to control process variables and are the most accurate and stable controller.

Mathematical form

The overall control function:

$$u(t)= K_p * e(t)+K_i * \int_0^t e(t) + K_d * \frac{de(t)}{dt}$$

Where K_p, K_i and K_d all non-negative, denote the coefficients for the proportional, integral, and derivative terms respectively (sometimes denoted P, I, and D).

Design 12V DC Motor

DC Motor Formula

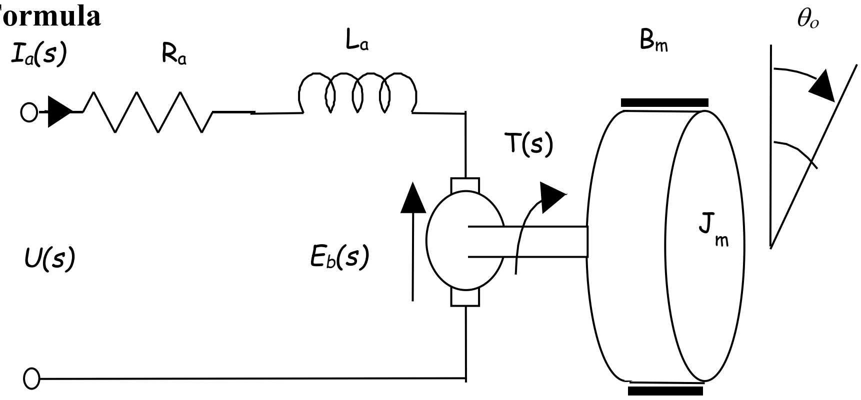

Figure6: DC Motor Parameter

T_e: Electromagnetic Torque

T_l: Load Torque

w: Angular Speed

K_t: Torque Constant

J_m: Moment of Inertia

K_b: Back EMF constant

B_m: Friction coefficient

R_a: Electrical Resistance

L_a: Electrical Inductance

$$U=E_b + I_a * R_a + L_a * \frac{di}{dt}$$

$$E_b=K_b*w$$

$$T_e - T_l = J*\frac{dw}{dt}$$

$$T_l=B_m*w$$

Formula Manipulation

$$U(s)=E_b(s) + I_a(s) * (R_a(s)+S*L_a(s))$$

$$E_b(s)=K_b*w(s)$$

$$T_e(s)-T_l(s)=S*J_m*w(s)$$

$$T_e(s)=K_t*I_a(s)$$

$$T_l=B_m*w(s)$$

DC Motor Math Function Model

Parameter	Value
Moment of inertia	$J_m = 0.000052\ Kg.$
Friction coefficient	$B_m = 0.01\ N.ms$
Back EMF constant	$K_b = 0.235\ \frac{V}{ms^{-}}$
Torque constant	$K_t = 0.235\ \frac{Nm}{A}$
Electric resistance	$R_a = 2$ ohm
Electric inductance	$L_a = 0.23\ H$
Gear ratio	gr
Load torque	$\tau_L(t)$
Angular speed	$\omega_m\ \frac{rad}{}$

DC motor parameter and values

According to the formula and parameter, we can get the model as follows:

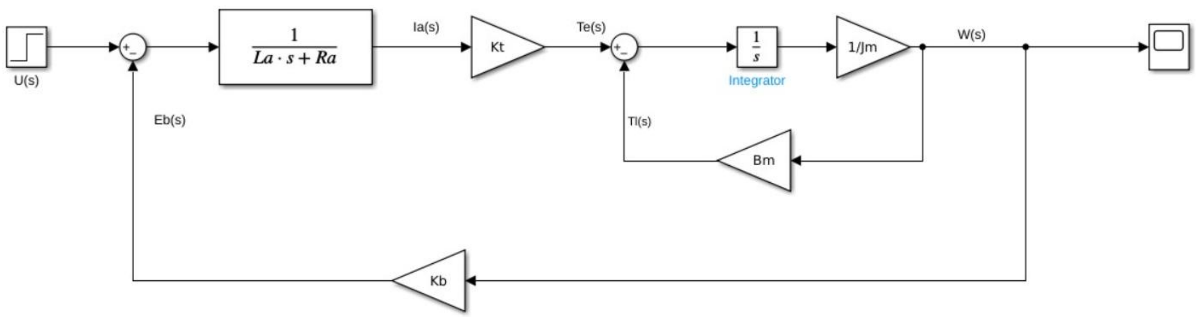

Figure6: DC Motor Model

Set Point Value	Setting Time	2
	Termination Value	5
	Stop Value	10

Chart1: DC Motor Model Set Point Value

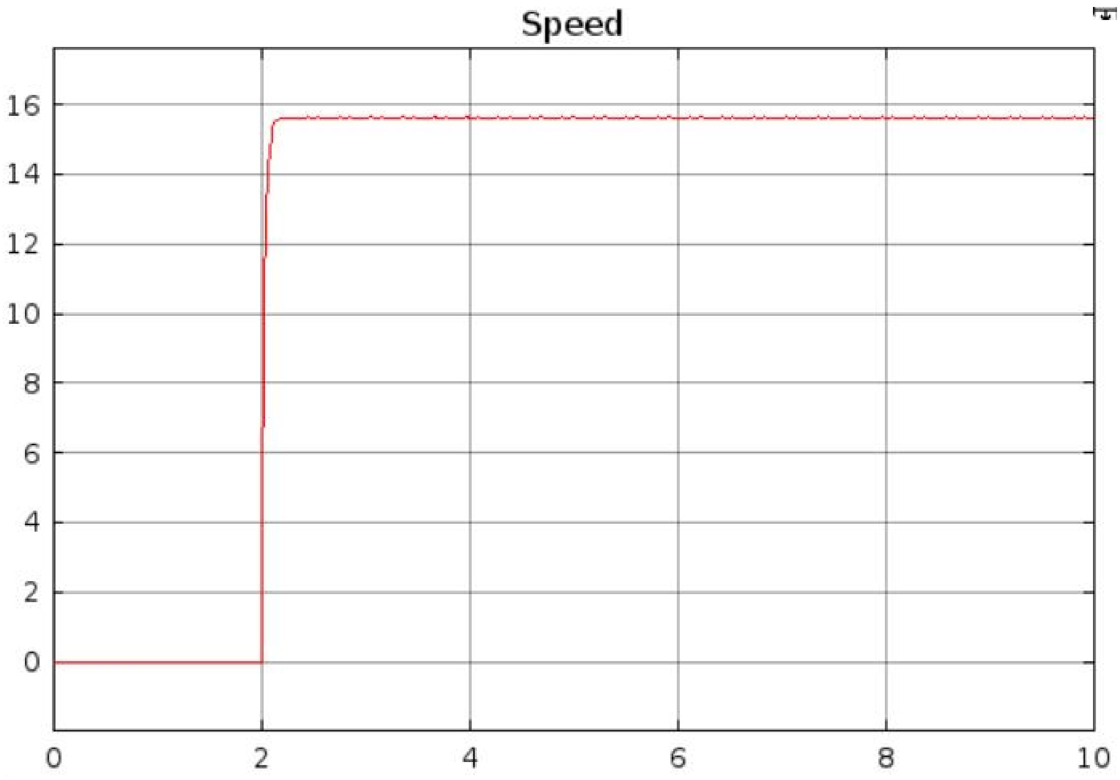

Figure7: DC Motor Model Speed Display

From this picture, we can see the rise-time equal to our setting time, when time =2.3s, the speed arrive at termination,over set point. 15.6-5=10.6, it has large Steady State Error.

$U(s)=E_b(s) + I_a(s) * (R_a(s)+S*L_a(s))$

$U(s)=K_b*w(s) + \frac{T_e(s)}{K_t} * (R_a(s)+S*L_a(s))$

$U(s)=K_b*w(s) + \frac{B_m*w(s)+S*J_m*w(s)}{K_t} * (R_a(s)+S*L_a(s))$

$U(s)=K_b*w(s) + \frac{(B_m+S*J_m)*w(s)}{K_t} * (R_a(s)+S*L_a(s))$

$U(s)= \frac{K_b*w(s)*K_t}{K_t} + \frac{(B_m+S*J_m)*w(s)}{K_t} * (R_a(s)+S*L_a(s))$

$\frac{w}{U(s)} = \frac{1}{K_b} + \frac{K_t}{(B_m+S*J_m)* (Ra(s)+S*La(s))}$

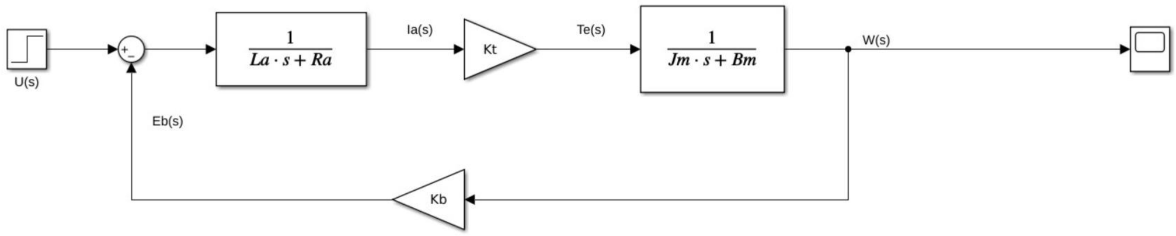

Figure8: DC Motor Transfer Function Model

Set Point Value	Setting Time	2
	Termination Value	10
	Stop Value	20

Chart2: DC Motor Transfer Function Model Set Point Value

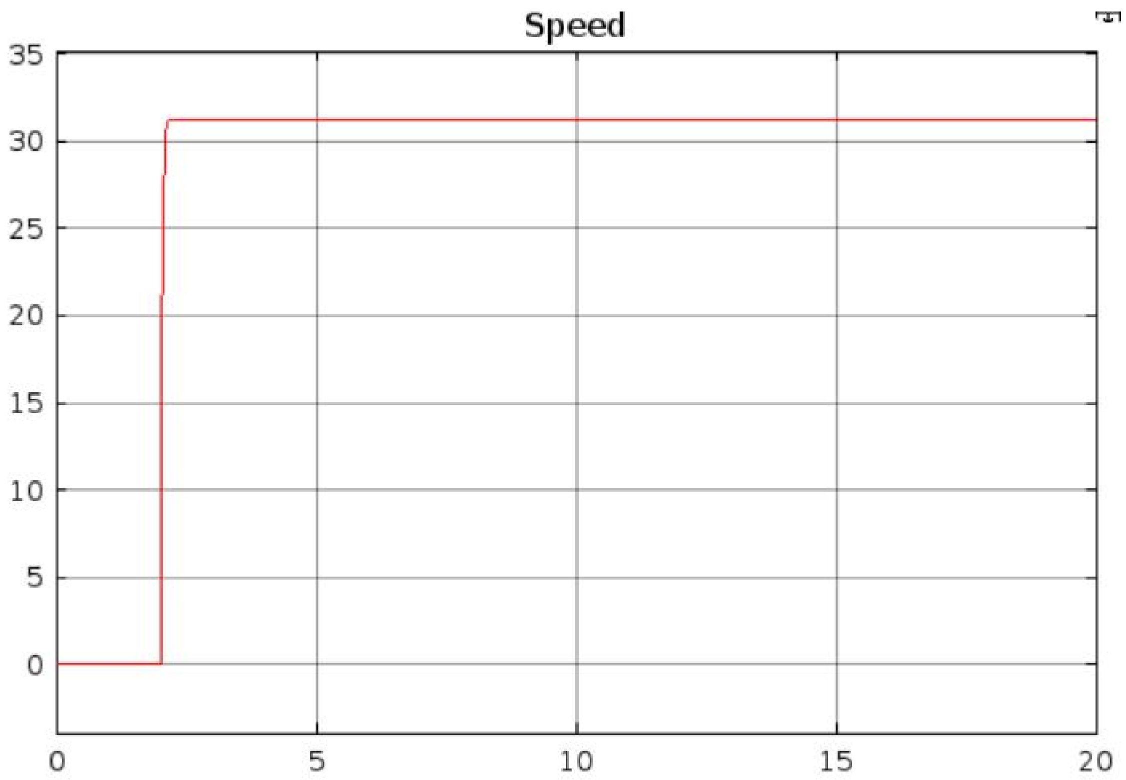

Figure9: DC Motor Transfer Function Model Speed Display

From this picture, we can see the rise-time equal to our setting time, when time =2.3s, the speed arrive at termination,over set point. 32-10=22, it has large Steady State Error.

Connect PID Controller

$$e(t)=| y(t) - r(t) |$$

$$\text{Overshoot} = \frac{Max\ value - Settling\ value}{Settling\ value} * 100\%$$

$$u(t)= K_p * e(t)+K_i * \int_0^t e(t) + K_d * \frac{de(t)}{dt}$$

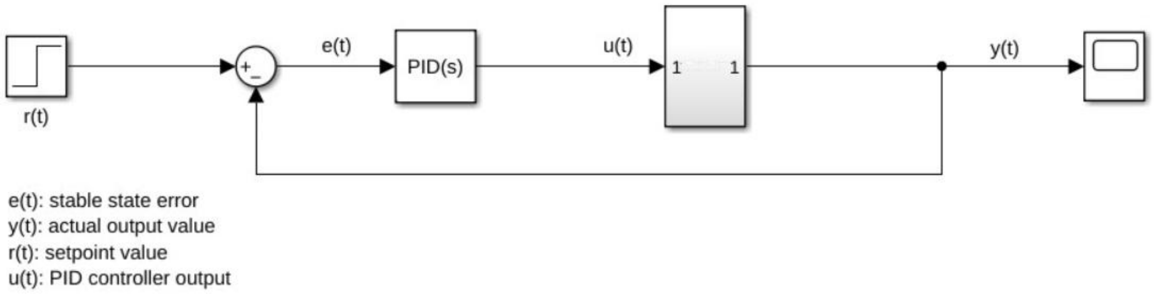

Figure10: PID Controller control DC Motor Speed

The Effective of K_p

Speed =5, K_p =1, K_i=0

Actual Value VS Setting Value	Setting Time	2	Overshoot	7.89%
	Termination Value	5	Rise-time	3.8
	Stop Value	20	Settling Time	2
	K_p	1	Steady State Error	1.1
	K_i	0		
	K_d	0		

Chart3: PID Controller control DC Motor Speed Actual Value

$$e(t)= |y(t) - r(t)|=|3.9-5|=1.1$$

$$\text{Overshoot} = \frac{Max\ value - Settling\ value}{Settling\ value} *100\%$$

$$\text{Overshoot} = \frac{4.1-3.8}{3.8} *100\%= 7.89\%$$

$$u(t)= K_p * e(t)+K_i * \int_0^t e(t) + K_d * \frac{de(t)}{dt}$$

$$u(t)= K_p * e(t)=1.1\text{(System intervention on speed)}$$

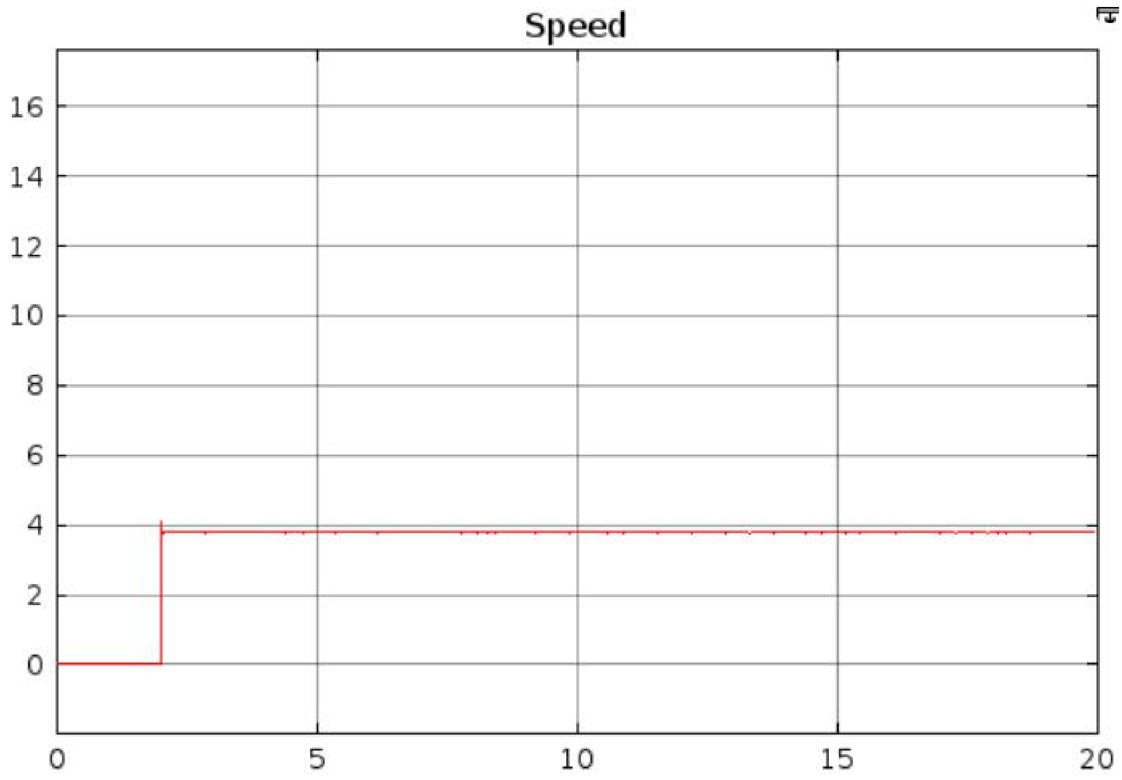

Figure11: PID Controller control DC Motor Speed =5 Display

Speed =5, K_p =2, K_i=0

Actual Value VS Setting Value	Setting Time	2	Overshoot	18.6%
	Termination Value	5	Rise-time	4.3
	Stop Value	20	Settling Time	2
	K_p	2	Steady State Error	0.7
	K_i	0		
	K_d	0		

Chart4: PID Controller control DC Motor Speed Actual Value

e(t)= |y(t) - r(t) |=|4.3-5|=0.7

$$\text{Overshoot} = \frac{Max\ value - Settling\ value}{Settling\ value} * 100\%$$

$$\text{Overshoot} = \frac{5.1 - 4.3}{4.3} * 100\% = 18.6\%$$

$$u(t) = K_p * e(t) + K_i * \int_0^t e(t) + K_d * \frac{de(t)}{dt}$$

$u(t) = K_p * e(t)$=1.4(System intervention on speed)

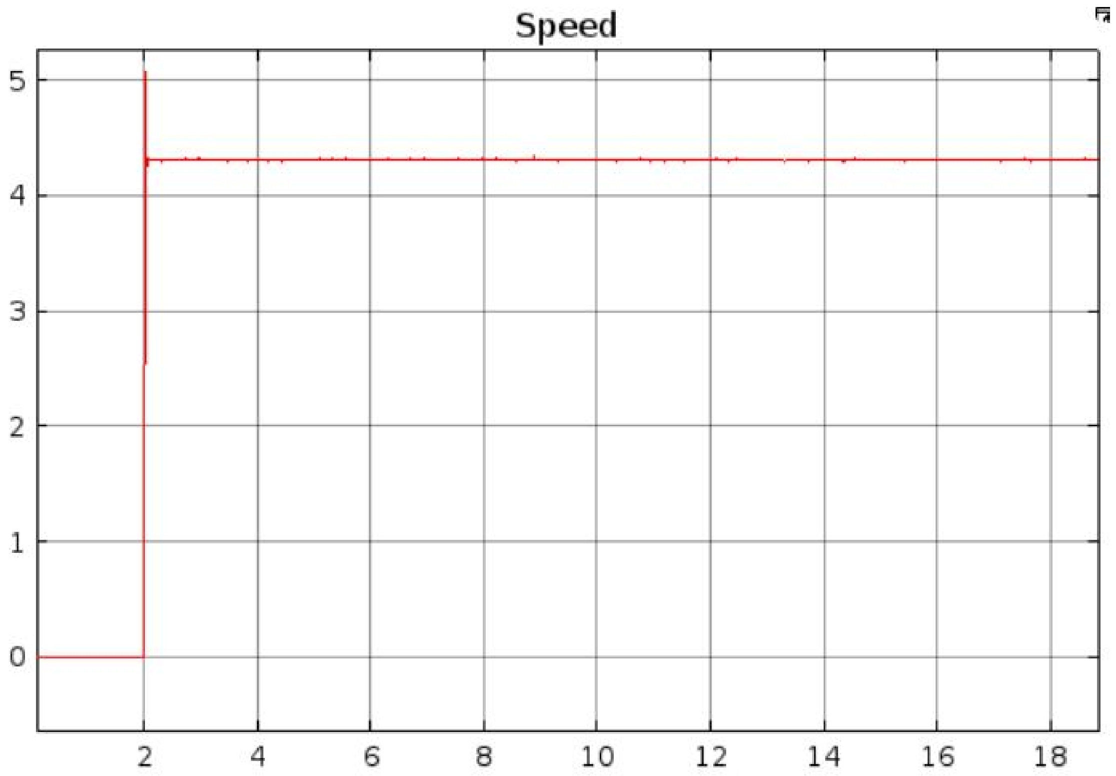

Figure12: PID Controller control DC Motor Speed =5 Display

Analysis Figure 11 and Figure 12, The proportional part speeds up the response speed of the system and reduces the steady state error of the system. The rise-time is increased. However, if the proportional coefficient is too large, the overshoot will increase, the number of oscillations will increase, the adjustment time will lengthen, and the dynamic performance will deteriorate. If the proportional coefficient is too large, the closed-loop system will become unstable.

Speed =5, K_p =1, K_i=1

Actual Value VS Setting Value	Setting Time	2	Overshoot	0%
	Termination Value	5	Rise-time	3.8
	Stop Value	20	Settling Time	12.5
	K_p	1	Steady State Error	0
	K_i	1		
	K_d	0		

Chart5: PID Controller control DC Motor Speed Actual Value

Speed =5, K_p =1, K_i=4

Actual Value VS Setting Value	Setting Time	2	Overshoot	0%
	Termination Value	5	Rise-time	3.8
	Stop Value	20	Settling Time	4
	K_p	1	Steady State Error	0
	K_i	4		
	K_d	0		

Chart6: PID Controller control DC Motor Speed Actual Value

e(t)= |y(t) - r(t) |=|5-5|=0

$$\text{Overshoot} = \frac{Max\ value - Settling\ value}{Settling\ value} * 100\%$$

$$\text{Overshoot} = \frac{5-5}{5} * 100\% = 0\%$$

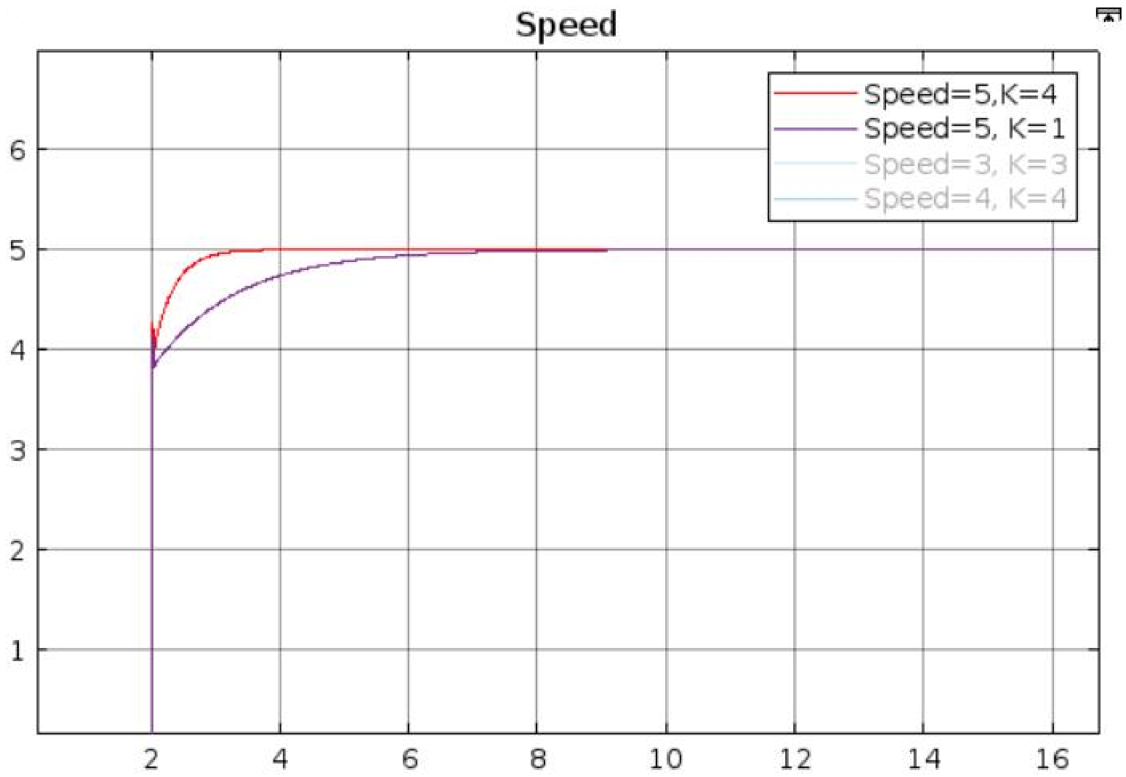

Figure13: PID Controller control DC Motor Speed =5 Display

Analysis Figure13 and Figure 11. When adding K_i, the steady state error is reduced and the system is close the setpoint value. If K_i is too large, it maybe reduce the system stability. At the same time, the overshoot is reduced, but the settling time is increased. The rise-time is same. If K_i is increased, K_p is constant, the settling time is reduced.

Speed =5, K_p =0, K_i=1

Actual Value VS Setting Value	Setting Time	2	Overshoot	0%
	Termination Value	5	Rise-time	2
	Stop Value	20	Settling Time	3.8
	K_p	0	Steady State Error	0
	K_i	1		
	K_d	0		

Chart7: PID Controller control DC Motor Speed Actual Value

Speed =5, K_p =0, K_i=4

<table>
<tr><td rowspan="6">Actual Value VS Setting Value</td><td>Setting Time</td><td>2</td><td>Overshoot</td><td>0%</td></tr>
<tr><td>Termination Value</td><td>5</td><td>Rise-time</td><td>2</td></tr>
<tr><td>Stop Value</td><td>20</td><td>Settling Time</td><td>2.1</td></tr>
<tr><td>K_p</td><td>0</td><td rowspan="3">Steady State Error</td><td rowspan="3">0</td></tr>
<tr><td>K_i</td><td>4</td></tr>
<tr><td>K_d</td><td>0</td></tr>
</table>

Chart8: PID Controller control DC Motor Speed Actual Value

$$e(t)= |y(t) - r(t)| = |5-5| = 0$$

$$\text{Overshoot} = \frac{Max\ value - Settling\ value}{Settling\ value} * 100\%$$

$$\text{Overshoot} = \frac{5-5}{5} * 100\% = 0\%$$

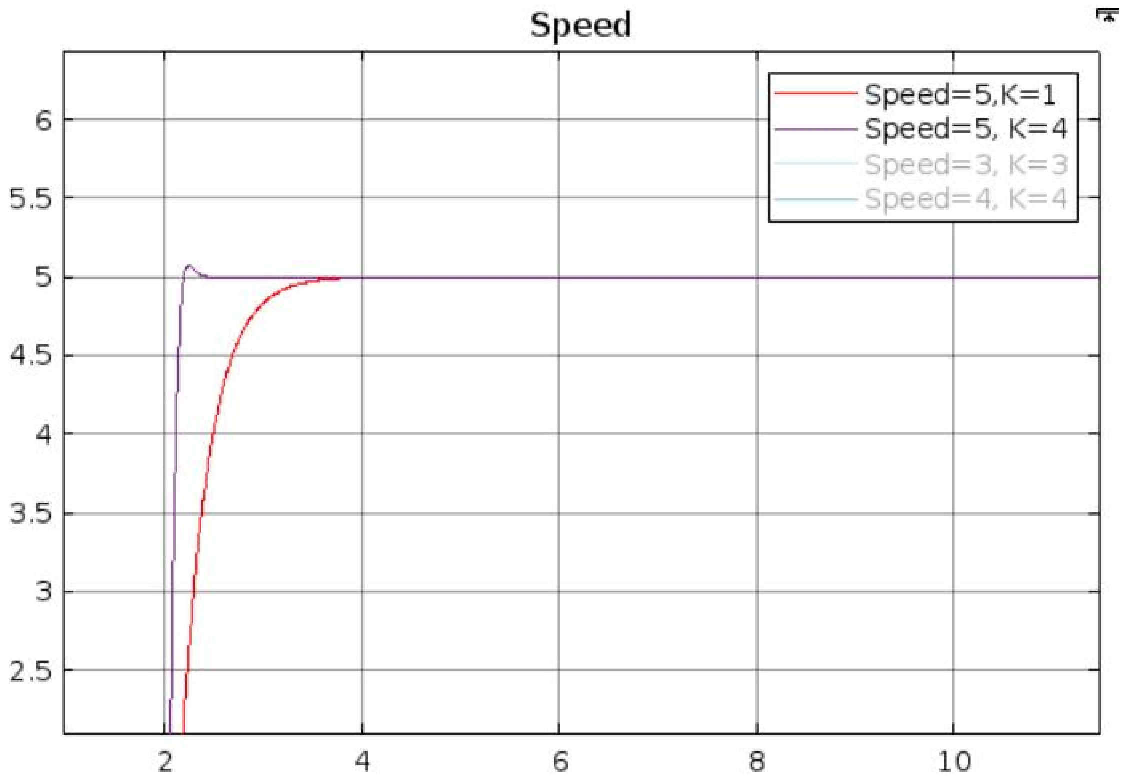

Figure14: PID Controller control DC Motor Speed =5 Display

Analysis Figure14. When adding K_i, the steady state error is reduced and the system is close the setpoint value. If K_i is too large, it maybe reduce the system stability. At the same time, the overshoot is reduced, If K_i is increased, K_p is constant, the settling time is reduced.

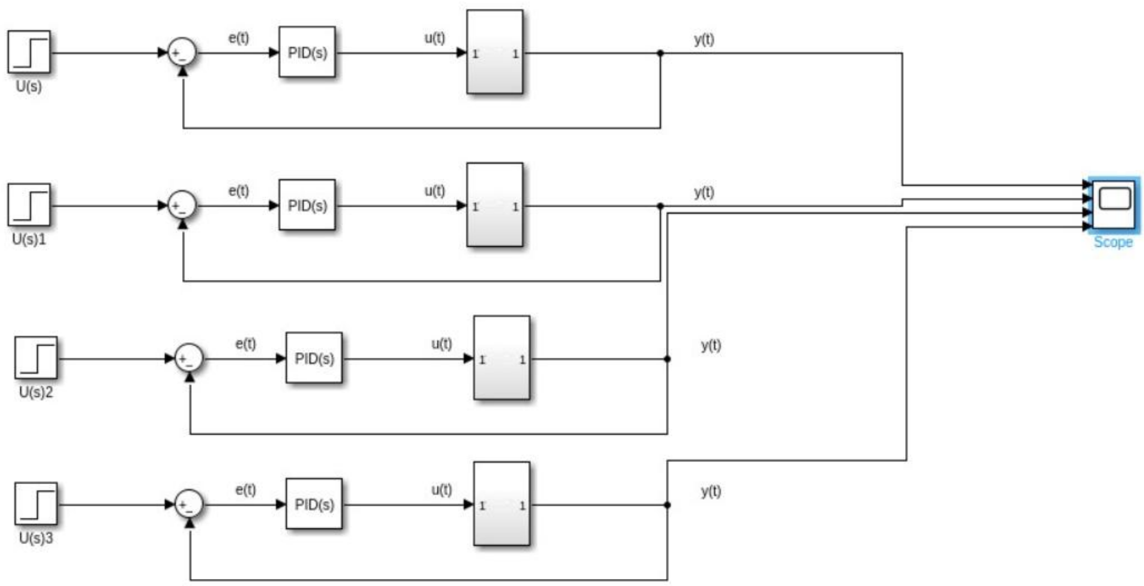

Figure15: PID Controller control DC Motor Speed=5

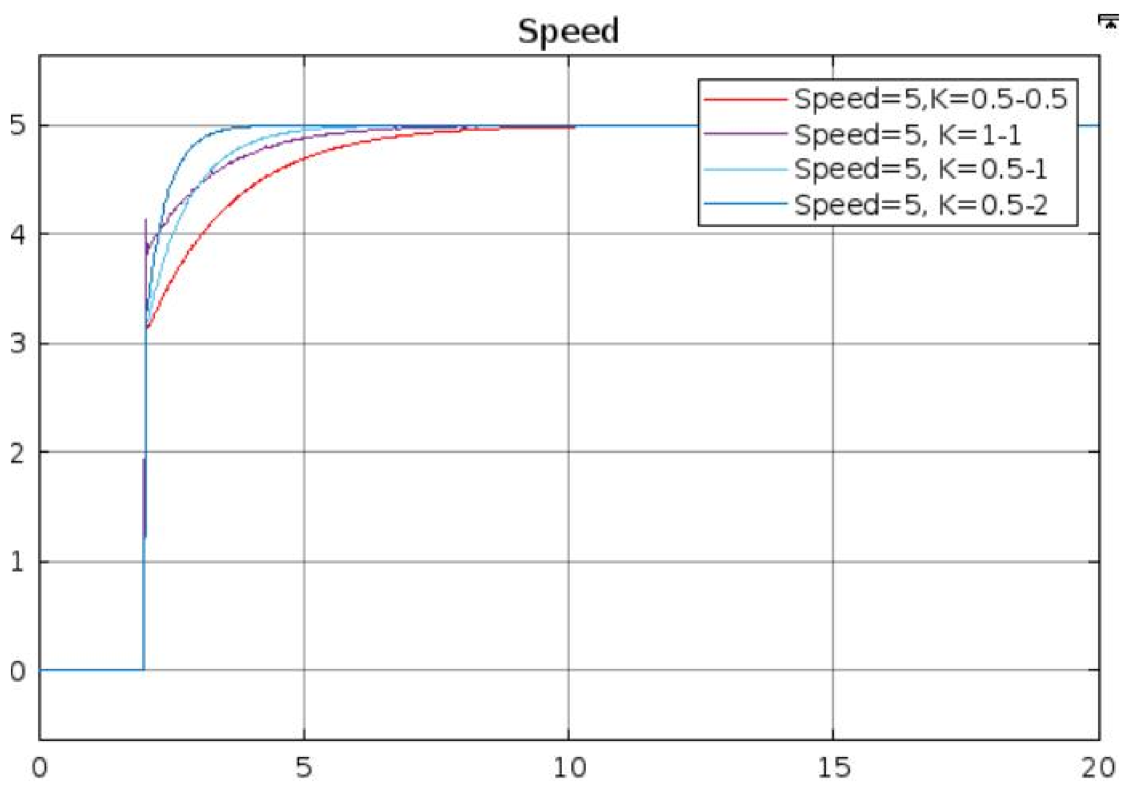

Figure16: PID Controller control DC Motor Speed =5

Analysis Figure11-16, we investigate different K_i and K_p, then we contrast the overshoot, rise-time, steady state error and settling time, we observe when $K_i = 0.5$ and $K_p = 2$, the speed is close to the setpoint desired value. This is because the response speed is fastest, the overshoot is minimum, the steady stable error is minimum.

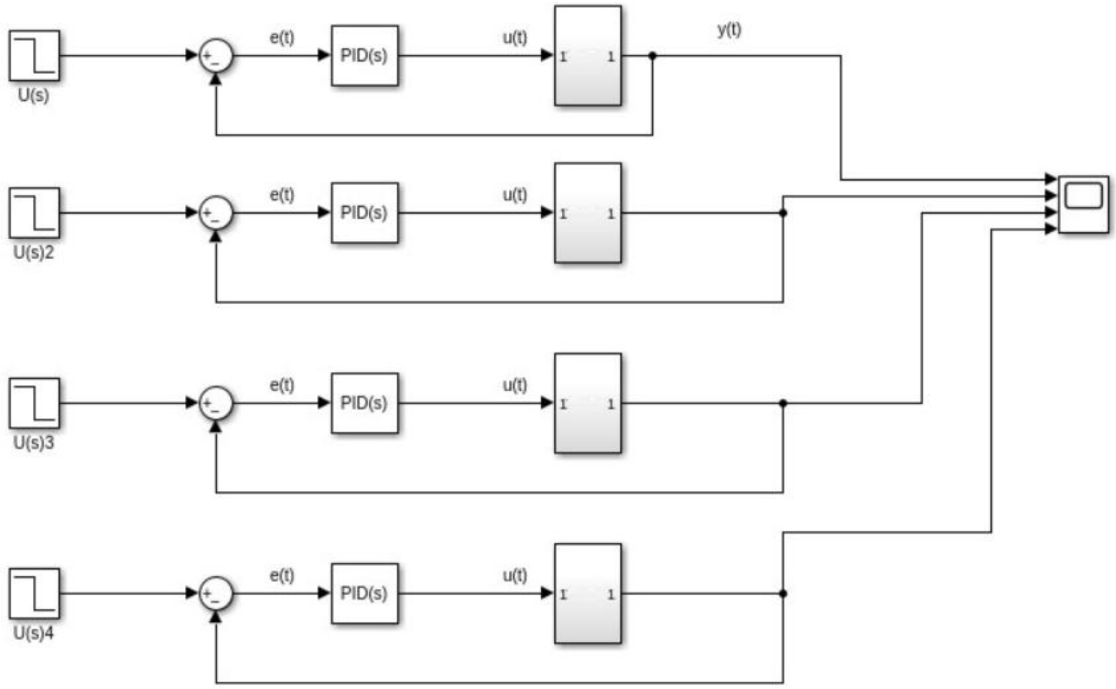

Figure17: PID Controller control DC Motor Speed=-3

Speed = -3, K_p =2, K_i=0

Actual Value VS Setting Value	Setting Time	2	Overshoot	18.2%
	Termination Value	-3	Rise-time	2.58
	Stop Value	20	Settling Time	2
	K_p	2	Steady State Error	0.42
	K_i	0		
	K_d	0		

Chart9: PID Controller control DC Motor Speed Actual Value

e(t)= |y(t) - r(t) |=|2.58-3|=0.42

$$\text{Overshoot} = \frac{Max\ value - Settling\ value}{Settling\ value} * 100\%$$

$$\text{Overshoot} = \frac{-3.05 - (-2.58)}{-2.58} * 100\% = 18.2\%$$

Speed = -3, K_p =3, K_i=0

Actual Value VS Setting Value	Setting Time	2	Overshoot	25.5%
	Termination Value	-3	Rise-time	2.71
	Stop Value	20	Settling Time	2
	K_p	3	Steady State Error	0
	K_i	0		
	K_d	0		

Chart10: PID Controller control DC Motor Speed Actual Value

e(t)= |y(t) - r(t) |=|2.58-3|=0.42

$$\text{Overshoot} = \frac{Max\ value - Settling\ value}{Settling\ value} * 100\%$$

$$\text{Overshoot} = \frac{-3.4-(-2.71)}{-2.71} * 100\% = 25.5\%$$

Speed = -3, K_p =0.5, K_i=1

Actual Value VS Setting Value	Setting Time	2	Overshoot	0%
	Termination Value	-3	Rise-time	1.8
	Stop Value	20	Settling Time	6.3
	K_p	0.5	Steady State Error	0
	K_i	1		
	K_d	0		

Chart11: PID Controller control DC Motor Speed Actual Value

e(t)= |y(t) - r(t) |=|3-3|=0

$$\text{Overshoot} = \frac{Max\ value - Settling\ value}{Settling\ value} * 100\%$$

$$\text{Overshoot} = \frac{-3-(-3)}{-3} * 100\% = 0\%$$

Speed = -3, K_p =0.5, K_i=2

Actual Value VS Setting Value	Setting Time	2	Overshoot	0%
	Termination Value	-3	Rise-time	1.9
	Stop Value	20	Settling Time	4
	K_p	0.5	Steady State Error	0
	K_i	2		
	K_d	0		

Chart12: PID Controller control DC Motor Speed Actual Value

e(t)= |y(t) - r(t) |=|3-3|=0

$$\text{Overshoot} = \frac{Max\ value - Settling\ value}{Settling\ value} * 100\%$$

$$\text{Overshoot} = \frac{3-3}{3} * 100\% = 0\%$$

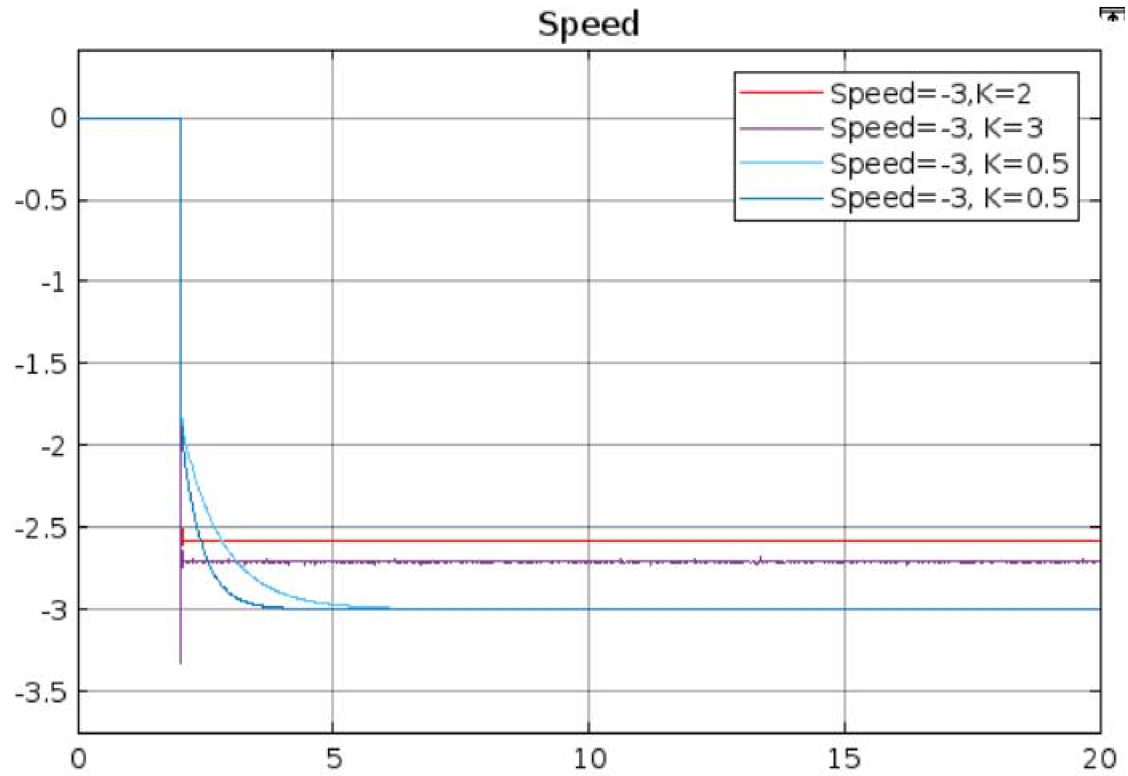

Figure18: PID Controller control DC Motor Speed =-3 Display

Analysis Figure18, we investigate negative speed, we have 4 table to explain these situation. When speed is constant, we change K_p, when K_p is increased, it increased the response of system and close to setpoint value, the overshoot is increased, it maybe cause system instability, the steady stable error is reduced. From this picture, when K_p is increased, K_p=3 it caused oscillate.

When speed and K_p are constant, we change K_t, when K_t is increased, the overshoot is reduced, the steady stable error is reduced. From this picture, when K_t is increased, the settling time is reduced and we can see that when K_t =2, K_p =0.5, the speed is close to setpoint desired value.

Speed =0, K_p =1, K_i =1

<table>
<tr><td rowspan="6">Actual Value VS Setting Value</td><td>Setting Time</td><td>2</td><td>Overshoot</td><td>0</td></tr>
<tr><td>Termination Value</td><td>0</td><td>Rise-time</td><td>0</td></tr>
<tr><td>Stop Value</td><td>20</td><td>Settling Time</td><td>0</td></tr>
<tr><td>K_p</td><td>1</td><td rowspan="3">Steady State Error</td><td rowspan="3">0</td></tr>
<tr><td>K_i</td><td>1</td></tr>
<tr><td>K_d</td><td>0</td></tr>
</table>

Chart13: PID Controller control DC Motor Speed Actual Value

e(t)= |y(t) - r(t) |=|0-0|=0

$$\text{Overshoot} = \frac{Max\ value - Settling\ value}{Settling\ value} * 100\%$$

Overshoot = 0

$$u(t)= K_p * e(t)+K_i * \int_0^t e(t) + K_d * \frac{de(t)}{dt}$$

$$u(t)= K_p * e(t)= 0$$

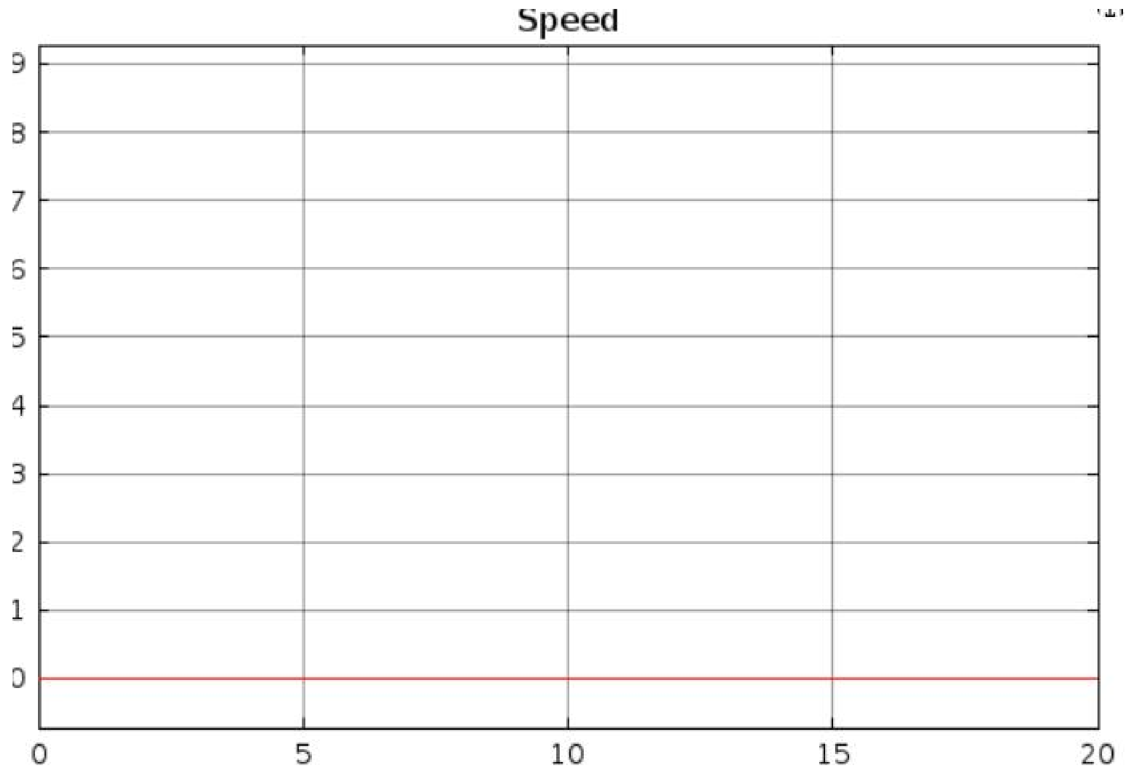

Figure19: PID Controller control DC Motor Speed=0 Display

When speed setpoint current equal 0, all value change 0.

Speed =1, K_p =1, K_i =1

Actual Value VS Setting Value	Setting Time	2	Overshoot	0%
	Termination Value	1	Rise-time	0.75
	Stop Value	20	Settling Time	10
	K_p	1	Steady State Error	0
	K_i	1		
	K_d	0		

Chart14: PID Controller control DC Motor Speed Actual Value

Speed =2, K_p =1, K_i =1

Actual Value VS Setting	Setting Time	2	Overshoot	0%
	Termination Value	2	Rise-time	1.6
	Stop Value	20	Settling Time	10
	K_p	1	Steady State Error	0

Value	K_i	1		
	K_d	0		

Chart15: PID Controller control DC Motor Speed Actual Value

Speed =3, K_p =1, K_i =1

Actual Value VS Setting Value	Setting Time	2	Overshoot	0%
	Termination Value	3	Rise-time	2.4
	Stop Value	20	Settling Time	10
	K_p	1	Steady State Error	0
	K_i	1		
	K_d	0		

Chart16: PID Controller control DC Motor Speed Actual Value

Speed =4, K_p =1, K_i =1

Actual Value VS Setting Value	Setting Time	2	Overshoot	0%
	Termination Value	4	Rise-time	3.4
	Stop Value	20	Settling Time	10
	K_p	1	Steady State Error	0
	K_i	1		
	K_d	0		

Chart17: PID Controller control DC Motor Speed Actual Value

$$e(t)= |y(t) - r(t)| = |1-1| = 0$$

$$\text{Overshoot} = \frac{Max\ value - Settling\ value}{Settling\ value} * 100\%$$

$$\text{Overshoot} = \frac{1-1}{1} * 100\% = 0\%$$

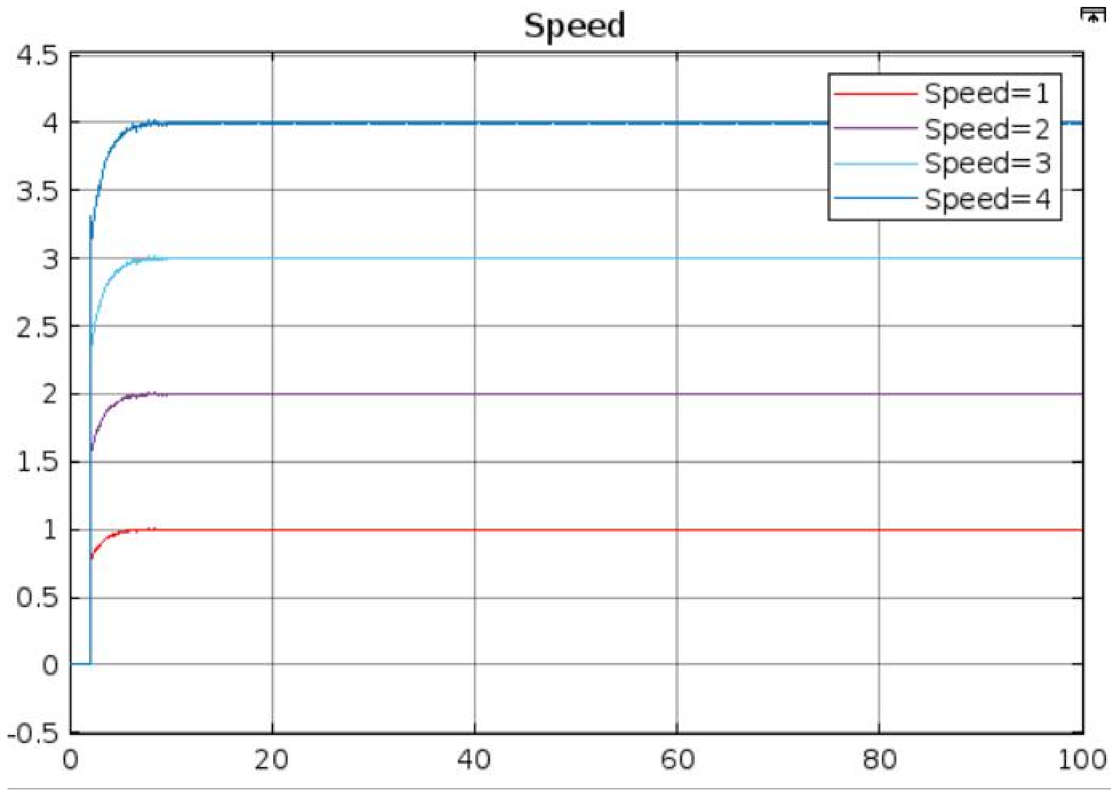

Figure20: PID Controller control DC Motor Positive Speed Display

Analysis Figure20, when K_p and K_i are constant, we change the speed and observe that except rise-time, other factors have no influence. The rise-time is increased, this is because rise-time and speed are related, the speed is greater, the rise-time is longer and the response speed is lower.

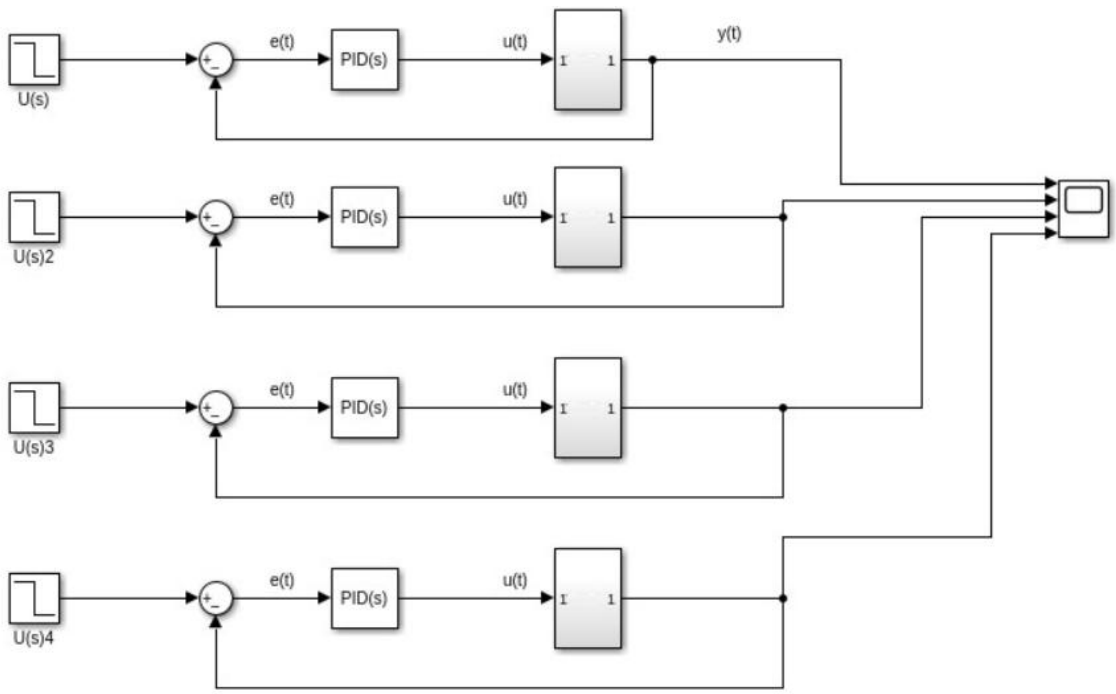

Figure21: PID Controller control DC Motor Negative Speed

Speed =-2, K_p =0.5, K_i =1

Actual Value VS Setting Value	Setting Time	2	Overshoot	0%
	Termination Value	-2	Rise-time	1.28
	Stop Value	20	Settling Time	10
	K_p	0.5	Steady State Error	0
	K_i	1		
	K_d	0		

Chart18: PID Controller control DC Motor Speed Actual Value

Speed =-4, K_p =0.5, K_i =1

Actual Value VS Setting Value	Setting Time	2	Overshoot	0%
	Termination Value	-4	Rise-time	2.6
	Stop Value	20	Settling Time	10
	K_p	0.5	Steady State Error	0
	K_i	1		
	K_d	0		

Chart19: PID Controller control DC Motor Speed Actual Value

Speed =-6, K_p =0.5, K_i =1

Actual Value VS Setting Value	Setting Time	2	Overshoot	0%
	Termination Value	-6	Rise-time	3.85
	Stop Value	20	Settling Time	10
	K_p	0.5	Steady State Error	0
	K_i	1		
	K_d	0		

Chart20: PID Controller control DC Motor Speed Actual Value

Speed =-8, K_p =0.5, K_i =1

Actual Value VS Setting Value	Setting Time	2	Overshoot	0%
	Termination Value	-8	Rise-time	4
	Stop Value	20	Settling Time	10
	K_p	0.5	Steady State Error	0
	K_i	1		
	K_d	0		

Chart21: PID Controller control DC Motor Speed Actual Value

$$e(t)= |y(t) - r(t)| =|2-2|=0$$

$$\text{Overshoot} = \frac{Max\ value - Settling\ value}{Settling\ value} * 100\%$$

$$\text{Overshoot} = \frac{-2-(-2)}{-2} * 100\% = 0\%$$

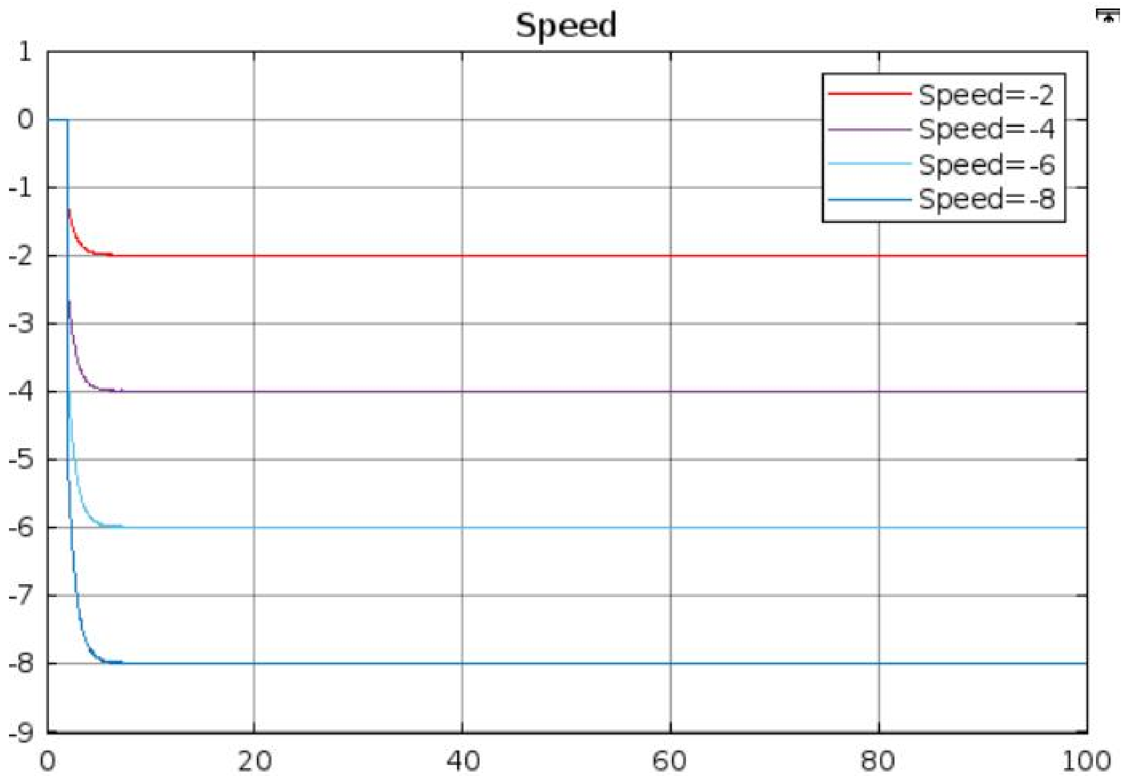

Figure22: PID Controller control DC Motor Negative Speed Display

Analysis Figure22, when K_p and K_i are constant, we change the negative speed and observe that except rise-time, other factors have no influence. The rise-time is increased, this is because rise-time and speed are related, the speed is greater, the rise-time is longer and the response speed is lower.

Conclusion

This report we are using Matlab Simulink design a DC motor, and connect the PID controller, we change the speed, K_p and K_i ,then get the desired speed of the motor.

We investigate, demonstrate and explain in detail the observed effects of changes in controller gain, such as: Overshoot, Rise-time, Settling Time and Steady State Error.

Parameter	Oscillate	Overshoot	Rise-time	Settling Time	Steady State Error
Speed	/	/	Increase	/	/
K_p	Too large	Depend on situation	Depend on situation	Depend on situation	Reduce
K_i	Too large	Reduce	Reduce	Reduce	Reduce

Chart22: PID Controller Parameter Influence

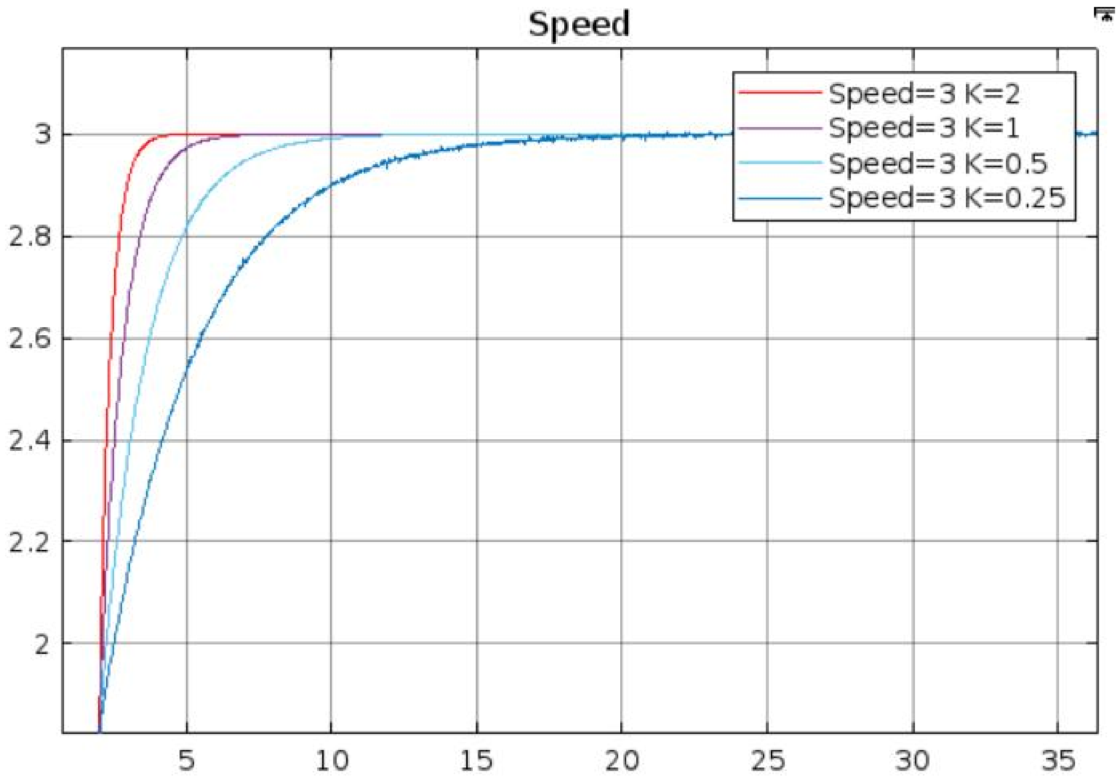

Figure23: PID Controller control DC Motor Speed =3 Display

Through this report, we can find that when K_p and K_i are too large, the system will have oscillate, it will effect the stability of the system, so we change different K_p and K_i, finally when K_p =0.5 and K_i =2, this close to setpoint value.

Reference

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