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Fairness in Rankings and Recommendations: An Overview

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Abstract We increasingly depend on a variety of data-driven algorithmic systems to assist us in many aspects of life. Search engines and recommender systems amongst others are used as sources of information and to help us in making all sort of decisions from selecting restaurants and books, to choosing friends and careers. This has given rise to important concerns regarding the fairness of such systems. In this work, we aim at presenting a toolkit of definitions, models and methods used for ensuring fairness in rankings and recommendations. Our objectives are three-fold: (a) to provide a solid framework on a novel, quickly evolving, and impactful domain, (b) to present related methods and put them into perspective, and (c) to highlight open challenges and research paths for future work.

Keywords Fairness · Rankings · Recommendations

1 Introduction

Nowadays, algorithmic systems driven by large amounts of data are increasingly being used in all aspects of society. Such systems offer enormous opportunities. They accelerate scientific discovery in all domains, including

personalized medicine and smart weather forecasting, they automate tasks, they help in improving our life through personal assistants and recommendations, they have the potential of transforming society through open government, to name just a few of their benefits.

Often, such systems are used to assist, or, even replace human decision making in diverse domains. Examples include software systems used in school admissions, housing, pricing of goods, credit score estimation, job applicant selection, and sentencing decisions in courts and surveillance. A prominent case is the COMPAS software used in courts in the US to assist bail and sentencing decisions through a risk assessment algorithm that predicts future crime.

The ubiquitous use of such systems may create possible threats of economic loss, social stigmatization, or even loss of liberty. For instance, a known study by ProPublica found that in COMPAS, the false positive rate for African American defendants, namely people labelled “high-risk” who did not re-offend, was nearly twice as high as that for white defendants [2]. Another well-known study shows that names used predominantly by men and women of colour are much more likely to generate ads related to arrest records [65].

Data-driven systems are also being employed by search and recommendation engines, social media tools, and news outlets, among others. Recent studies report that social media has become the main source of online news with more than 2.4 billion internet users, of which nearly 64.5% receive breaking news from social media instead of traditional sources [49]. Thus, to a great extent, such systems play a central role in shaping our experiences and influencing our perception of the world. There are many reports questioning the output of such systems. For instance, a known study on search results showed

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evidence for stereotype exaggeration in images returned when people search for professional careers [39].

In this article, we pay special attention to the concept of fairness in rankings and recommender systems. By fairness, we typically mean lack of discrimination (bias). Bias may come from the algorithm, reflecting, for example, commercial or other preferences of its designers, or even from the actual data, for example, if a survey contains biased questions, or, if some specific population is misrepresented in the input data.

As fairness is an elusive concept, an abundance of definitions and models of fairness have been proposed as well as several algorithmic approaches for fair rankings and recommendations making the landscape very convoluted. In order to make real progress in building fair-aware systems, we need to de-mystify what has been done, understand how and when each model and approach can be used, and, finally, distinguish the research challenges ahead of us.

Therefore, we follow a systematic and structured approach to explain the various sides of and approaches to fairness. We start by presenting general fairness definitions. Then, we focus on models and definitions for rankings and recommendations. We organize them in a taxonomy and highlight their differences and commonalities. Specifically, we distinguish between *individual* and *group* fairness, *consumer* and *producer* fairness, and fairness for *single* and *multiple* outputs. We continue with describing solutions for fair rankings and recommendations. We organize them into *pre-processing approaches*, that aim at transforming the data to remove any underlying bias or discrimination, *in-processing approaches*, that aim at modifying existing or introducing new algorithms that result in fair rankings and recommendations, and *post-processing approaches*, that modify the output of the algorithm. Within each category, we further classify approaches along several dimensions. We discuss other cases where a system needs to make decisions and where fairness is also important, and present open research challenges pertaining to fairness in the broader context of data management.

To the best of our knowledge, this is the first survey that provides a toolkit of definitions, models and methods used for ensuring fairness in rankings and recommendations. Recent tutorials, with a stricter focus than ours, focusing on concepts and metrics of fairness and the challenges in applying these to recommendations and information retrieval, as well as to scoring methods, are surveyed, respectively, in [22] and [6,51]. On the other hand, this article has a much wider coverage and depth, presenting a structured survey and comparison of methods and models for ensuring fairness in rankings and recommendations.

The remaining of this survey is organized as follows. Section 2 presents the core definitions of fairness, and Section 3 reviews definitions of fairness that are applicable specifically to rankings and recommenders. Section 4 discusses a distinction of the methods for achieving fairness, while Sections 5, 6 and 7 organize and present in detail the pre-, in- and post-processing methods. Section 8 offers a comparison between the in- and post-processing methods. Section 9 studies how we can verify whether a program is fair. Finally, Section 10 elaborates on critical open issues and challenges for future work, and Section 11 summarizes the status of the current research on fairness in ranking and recommender systems.

2 Fairness Models

2.1 General Fairness Definitions

Most approaches to algorithmic fairness interpret fairness as *lack of discrimination* asking that the results of an algorithm should not be influenced by attributes that are not relevant to the task at hand. Such attributes are called *protected*, or *sensitive*, and often include among others gender, religion, age, sexual orientation, and race.

According to the *level* of fairness, fairness formulations are distinguished into [21]:

- *Individual fairness* definitions are grounded on the premise that similar individuals should be treated similarly.
- *Group fairness* definitions partition individuals into groups based on the value of one, or more of their protected attributes and ask that all groups are treated similarly.

To operationalize fairness, we need to define similarity for the input and the output of an algorithm. For *input similarity*, we need a means of quantifying similarity of individuals in the case of individual fairness, and, a way of partitioning individuals into groups, in the case of group fairness. For *output similarity*, for both individual and group fairness, we need a formal definition of equal treatment.

Input similarity. Let V be the set of individuals. For individual fairness, a first approach is to define a distance metric $d : V \times V \rightarrow R$ between each pair of individuals [21]. Such metric should be task-specific, that is two individuals may be similar for one task and dissimilar for another. For example, two individuals may be consider similar (e.g., have similar qualifications) when it comes to being admitted to college but different when

it comes to receiving a loan. The metric may be externally imposed, e.g., by a regulatory body, or externally proposed, e.g., by a civil rights organization. Ideally, the metric should express the ground truth, or, the best available approximation of it. Finally, this metric should be made public, and open to discussion and refinement.

In the case of group fairness, the challenge lies in determining how to partition individuals into groups. Often looking only at the protected attributes does not suffice, since there may be other *proxy* attributes correlated with the protected ones, a case also known as *redundant encoding*.

Output similarity. Specifying what it means for individuals, or groups of individuals to be treated similarly is an intricate problem, from both a social and a technical perspective. From a social perspective, a fundamental distinction is made between equity and equality. Simply put, *equality* refers to treating people equally, while *equity* refers to treating people according to their needs, so that they all finally receive the same output, even when some individuals are disadvantaged. Yet another differentiation is between disparate treatment and disparate impact. *Disparate treatment* is the often illegal practice of treating an entity differently based on the protected attribute. *Disparate impact* refers to cases where the output depends on the protected attribute, even if all entities are treated the same way. The disparate impact doctrine was solidified in the US after [Griggs v. Duke Power Co. 1971] where a high school diploma was required for unskilled work, excluding applicants of color.

From a technical perspective, how these notions are translated into quantifiable measures depends clearly on the specific task. In the next section, we discuss well-known definitions of output similarity for the classification task.

2.2 Fairness in Classification

Most work so far has focused on classification algorithms used in decision making. Although this survey considers ranking and recommendations, we will present a short survey of this work for completeness and also because many approaches to the fair ranking and recommendation tasks build on definitions of fairness in classification.

Note also, that the various fairness formulas in this and the following sections can be seen both as: (a) definitions of fairness, i.e., conditions that a system must satisfy for being fair and (b) measures of fairness. For instance, we can measure how much a fairness condition

is violated, or, define a fairness condition by setting a threshold on a measure of fairness.

Individual Fairness. In the case of individual fairness in classification, intuitively the output of the classifier should be similar for similar individuals. A way to express this requirement in the case of probabilistic classifiers is the following [21]. A classifier M maps individuals V to outcomes A . In the case of probabilistic classifiers, these are randomized mappings from individuals to probability distributions over outcomes. Specifically, to classify $v \in V$, we choose an outcome $a \in A$ according to the distribution $M(x)$. The classifier is individually fair if the mapping $M : V \rightarrow \Delta(A)$ satisfies the (D, d) -Lipschitz property, that is, $\forall v, u \in V$, $D(M(v), M(u)) \leq d(v, u)$, where D is a distance measure between probability distributions, and d a distance metric between individuals. In words, the distance between probability distributions assigned by the classifier M should be no greater than the actual similarity between individuals.

Another form of fairness is *counterfactual fairness* [44]. The intuition in this case, is that an output is fair towards an individual if it is the same in both the actual world and a counterfactual world where the individual belonged to a different group. Causal inference is used to formalize this notion of fairness.

Group Fairness. For defining similarity of treatment in classification, a variety of statistical approaches have been proposed. For simplicity, let us assume two groups, namely, the protected group G^+ and the non-protected (or, privileged) group G^- . Assume that Y is the actual and \hat{Y} the predicted output of a binary classifier, and 1 is the positive class that leads to a favorable decision, e.g., someone getting a loan, or being admitted at a competitive school. Let S be the predicted probability for a certain classification.

Statistical approaches to group fairness can be divided as follows [25, 66]:

- *base rates approaches*: that use only the predicted output,
- *accuracy approaches*: that use the predicted and the actual output, and
- *calibration approaches*: that use the predicted probability and the actual output.

Base rate fairness compares the probability $P(\hat{Y} = 1|v \in G^+)$ that an individual v receives the favorable outcome when v belongs to the protected group G^+ with the corresponding probability $P(\hat{Y} = 1|v \in G^-)$ that v receives the favorable outcome when v belongs to the non-protected group G_1 . To compare the two, we may take their ratio [73], [23]: $\frac{P(\hat{Y}=1|v \in G^+)}{P(\hat{Y}=1|v \in G^-)}$ or, their

difference [14] $1 - (P(\hat{Y} = 1|v \in G^+) - P(\hat{Y} = 1|v \in G^-))$.

When these probabilities are equal, we have a special type of fairness termed *demographic*, or *statistical parity*. Statistical parity preserves the input ratio, that is, the demographics of the individuals receiving a favorable outcome are the same as the demographics of the underlying population. Statistical parity is also a natural way to model equity: members of each group have the same chance of getting the favorable output.

However, base rate fairness ignores the actual output, i.e., the classification may be fair, but it may not reflect the ground truth. Accuracy and calibration look at traditional classification evaluation measures and require that the classifier works equally well in terms of prediction errors for both groups.

Accuracy-based fairness warrants that various types of classification errors (e.g., true positives, false positives) are equal across groups. Depending on the type of classification errors considered, the achieved type of fairness takes different names [32]. For example, the case in which, we ask that $P(\hat{Y} = 1|Y = 1, v \in G^+) = P(\hat{Y} = 1|Y = 1, v \in G^-)$ is called *equal opportunity*. Comparing equal opportunity with statistical parity, again the members of the two groups have the same chance of getting the favorable outcome, but only when these members qualify. Thus, equal opportunity is more close to an equality interpretation of fairness.

Calibration-based fairness considers probabilistic classifiers that predict a probability for each class [1,41]. In general, a classification algorithm is considered to be well-calibrated if: when the algorithm predicts a set of individuals as having probability p of belonging to the positive class, then approximately a p fraction of this set are actual members of the positive class. In terms of fairness, intuitively, we would like the classifier to be equally well-calibrated for both groups. An example calibration-based fairness is asking that for any predicted probability score p in $[0, 1]$, the probability of actually getting a favorable outcome is equal for both groups, i.e., $P(Y = 1|S = p, v \in G^+) = P(Y = 1|S = p, v \in G^-)$.

Group-based measures in general tend to ignore the merits of each individual in the group. Some individuals in a group may be better for a given task than other individuals in the group, which is not captured by some group-based fairness definitions. This issue may lead to two problematic behaviors, namely, (a) the *self-fulfilling prophecy* where by deliberately choosing the less qualified members of the protected group we aim at building a bad track record for the group and (b) *reverse tokenism* where by not choosing a well qualified member of the non-protected group we aim at creating convinc-

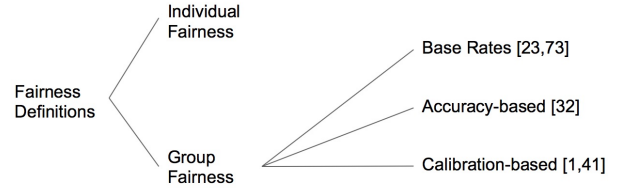


Fig. 1 Fairness Definitions in Classification.

ing refutations for the members of the protected group that are also not selected.

Figure 1 presents a summary of the fairness definitions in classification.

2.3 Multi-Dimensional Fairness

In the previous section, we distinguished fairness models based on the *Level* of fairness into individual and group fairness. When it comes to ranking and recommender systems, we can further classify fairness models based on: the *Side* dimension, that considers consumer and producer fairness, and the *Graduality* dimension, that considers single and multiple outputs (Figure 2). Next, we discuss these two dimensions in details.

Side of fairness. We distinguish between consumer and producer fairness. *Consumer fairness* focuses on the individuals that receive a ranked output, e.g., a search result, or a recommendation. In abstract terms, we would like similar users, or groups of users to receive similar rankings or recommendations. For instance, we may ask that race does not influence the job recommendations that different applicants receive. On the other hand, *producer fairness* focuses on the individuals that are being ranked, or recommended. In this case, we would like similar individuals or groups to be ranked, or, be recommended in a similar way, e.g., to appear in similar positions in a ranking. For instance, we may ask that the political orientation of an article does not affect its ranking in a search result.

When comparing consumer and producer fairness, we note that the latter is passive, since typically producers do not seek suggestion opportunities that cover their information needs, but instead just wait for the users to ask for suggestions. There are cases in which a system may require fairness for both consumers and providers, when for instance both of them belong to protected groups. For example, assume a rental property business that may wish to treat minority applicants as a protected class and ensure that they can have access to properties similar to other renters, while at the

same time, wish to treat minority landlords as a protected class and ensure that highly qualified tenants are referred to them at the same rate as to other landlords.

Different types of recommendation systems may call for specializations of consumer and producer fairness. For example, group recommendation systems recommend items to groups of users as opposed to a single user, for example a movie to friends, an event to an online community, or a excursion to a group of tourists [5, 35]. In this case, we may have different types of consumer fairness, since now the consumer is not just a single user. Similarly, there are bundle and package recommendation systems that recommend complex items, or sets of items, instead of just a single item, for example a set of places to visit, or courses to attend [68]. In this case, we may have different types of producer fairness, since now the recommended items are composite. We discuss these special types of fairness in Section 3.3.

Graduality of fairness. We distinguish between single output and multiple output. In the case of a *single output* of the system, i.e., a ranking or recommendation, we seek to define fairness in this context. On the other hand, in the case of *multiple output*, we consider a sequence of rankings, or recommendations. For example, in the case where the same items appear in results of multiple search queries, or the same users receive more than one recommendation. In this case, we may ask for *eventual*, or *amortized* consumer, or producer fairness, i.e., we ask that the consumers or producers are treated fairly in the sequence of rankings of recommendations as a whole, although they may be treated unfairly in a single ranking or recommendation.

A special case is *sequential recommenders* that suggest items of interest by modeling the sequential dependencies over the user-item interactions in a sequence. This means that the recommender treats the user-item interactions as a dynamic sequence and takes the sequential dependencies into account to capture the current and previous user preferences for increasing the quality of recommendations [64]. The system recommends different items at each interaction, while retaining knowledge from past interactions. Interestingly, due to the multiple user-item interactions in sequential recommender systems, fairness correction can be performed, while moving from one interaction to the next.

Other Refinements. We can refine consumer and producer fairness dimensions further by considering the additional stakeholders that may be involved in a recommendation system besides the consumers and the producers. For example, in a recommendation system, the items recommended may belong to various providers. In this case, we may ask for producer fairness with respect to the *providers* of the items, instead of the

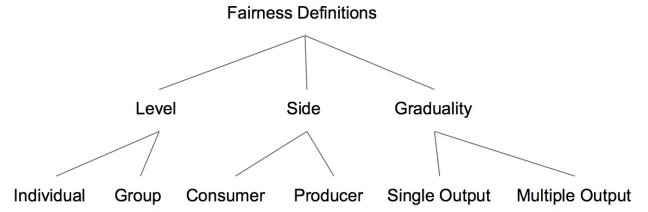


Fig. 2 Fairness Definitions Taxonomy.

single items. For example, in an online craft marketplace, we may want to ensure market diversity and avoid monopoly domination, where the system wishes to ensure that new entrants to the market get a reasonable share of recommendations even though they have fewer shoppers than established vendors. This may also be interpreted as group fairness with the protected attribute being the provider. Other forms of fairness include: (a) fairness for the owners of the recommendation system, especially when the owners are different than the producers, and (b) fairness for system regulators and auditors, for example, data scientists, machine learning researchers, policymakers and governmental auditors that are using the system for decision making.

3 Fairness in Rankings and Recommenders

In this section, we review a number of definitions of fairness that are applicable specifically to rankings and recommenders.

3.1 Fairness in Rankings

The output of many algorithms is a list of items where the items are ranked based on some measure of their relative quality for the application at hand. For example, the items in the output of a search query are ranked mainly based on their relevance to the query. In the following, we will refer to the measure of quality, as *utility*.

Formally, given a set of items, $\{i_1, i_2, \dots, i_N\}$, a *ranking* is an assignment (mapping) of items to ranking positions. Most approaches to fairness in ranking consider producer fairness, that is, their goal is to ensure that the items being ranked are treated fairly.

A central issue in ranking is the manifestation of *position bias*, i.e., the fact that people tend to consider only the items that appear in the top few positions of the ranking. Even more the attention that items receive is highly skewed with regards to their position in the list. At a high level, output similarity for the ranking

Rank	ID	Group	Score	Rank	ID	Group	Score	Rank	ID	Group	Score
1	x299	Blue	0.56	1	x78	Red	0.55	1	x78	Red	0.55
2	x78	Red	0.55	2	x23	Red	0.44	2	x299	Blue	0.56
3	x45	Blue	0.45	3	x87	Red	0.16	3	x23	Red	0.44
4	x329	Blue	0.44	4	x232	Red	0.15	4	x45	Blue	0.45
5	x23	Red	0.44	5	x299	Blue	0.56	5	x87	Red	0.16
6	x981	Blue	0.25	6	x45	Blue	0.45	6	x329	Blue	0.44
7	x665	Blue	0.23	7	x329	Blue	0.44	7	x232	Red	0.15
8	x724	Blue	0.18	8	x981	Blue	0.25	8	x981	Blue	0.25
9	x87	Red	0.16	9	x665	Blue	0.23	9	x665	Blue	0.23
10	x232	Red	0.15	10	x724	Blue	0.18	10	x724	Blue	0.18

Fig. 3 (left) An example ranking (center) reranking with $p = 1$, (right) reranking with $p \geq 0.5$, Red is the protected value.

task refers to offering similar visibility to similar items or group of items, that is, placing them at similar positions in the ranking, especially for top positions.

Fairness constraints. One type of group fairness is achieved by **constraining the number of items from the different groups that can appear in the top positions in the ranking**. Specifically, in the *fairness constraints* approach [18], given a number of protected attributes, or, properties, an upper bound $U_{l,k}$ and a lower bound $L_{l,k}$ is specified on the number of items with property l that are allowed to appear in the top- k positions of the ranking. For example, given that color is the protected attribute, and red the protected group, the middle ranking in Figure 3 satisfies the constraint $L_{red,4} = 1$, since there is at least 1 item with property red in the top-4 positions.

Discounted cumulative fairness. The Discounted Cumulative Gain (DCG) is a standard way of measuring ranking quality. DCG gives emphasis on having higher utility scores at first positions:

$$DCG@k(r) = \sum_{i=1}^k \frac{2^{util(i)} - 1}{\log_2(i+1)}, \quad (1)$$

where k stands for the size of the ranking analyzed. The DCG value is then normalized by the DCG of the perfect ranking order, for obtaining NDCG.

One can focus on the representation (i.e., number of items) of the protected group in the top- p ranking positions for various values of p [69]. Let G^+ be the protected and G^- be the other group. The idea of *discounted cumulative fairness* is to accumulate the number of items belonging to the protected group G^+ at discrete positions in the ranking (e.g., at positions $p = 5, 10, \dots$) and discount these numbers accordingly. i.e., decrease the values as we move to lower positions in the list. Three different definitions based on this general idea are given.

The first one, the *normalized discounted difference* (rND) of a ranking r , is defined as follows:

$$rND(r) = \frac{1}{opt_rND} \sum_{p=5,10}^N \frac{1}{\log_2(p)} \left| \frac{|G_{1..p}^+|}{p} - \frac{|G^+|}{N} \right| \quad (2)$$

where N is the size of the list, opt_rND is the optimal value for the given N and protected group size $|G^+|$. For example, for the ranking in Figure 3, we have $opt_rND = \frac{1}{\log_2(5)} \left| \frac{4}{5} - \frac{4}{10} \right| = 0.93$ (which corresponds to the optimal ranking in the middle), while for the random ranking in the left, we have $rND(r) = \frac{1}{0.93} \left(\left(\frac{1}{\log_2(5)} \left| \frac{2}{5} - \frac{4}{10} \right| + \frac{1}{\log_2(10)} \left| \frac{4}{10} - \frac{4}{10} \right| \right) \right) = 0$ and for the ranking in the right $rND(r) = \frac{1}{0.93} \left(\left(\frac{1}{\log_2(5)} \left| \frac{3}{5} - \frac{4}{10} \right| + \frac{1}{\log_2(10)} \left| \frac{4}{10} - \frac{4}{10} \right| \right) \right) = 0.5$.

A variation called *normalized discounted ratio* is also proposed where in Equation 2 instead of dividing with p , i.e., the total number of items up to position p , we divide with $|G_{1..p}^-|$ and instead of dividing with N , we divide by $|G^-|$.

Finally, the *normalized KL divergence* (rKL) definition of fairness uses KL-divergence to compute the expectation of the difference between the membership probability distribution of the protected group at the top- p positions (for $p = 5, 10, \dots$) and in the overall population.

Fairness of exposure. A problem with the discounted cumulative approach is the fact that it does not account for skewness in visibility. Counting items at discrete positions does not fully capture the fact that minimal differences in relevance scores may translate into large differences in visibility for different groups because of position bias that results in a large skew in the distribution of exposure.

To address this, in the *fairness of exposure* approach [61], each position j in the ranking is assigned a specific value that represents the importance of the position, i.e., the fraction of users that examine an item at position j . This is modeled by using a position discount vector v , where v_j represents the importance of position j . Rankings are probabilistic. In particular, a ranking of

N items in N positions is modeled as a doubly stochastic $N \times N$ matrix P , where $P_{i,j}$ is the probability that item i is ranked at position j .

Given vector v , the exposure of item i in ranking P is defined as:

$$Exposure(i|P) = \sum_{j=1}^N P_{i,j} v_j \quad (3)$$

The exposure of a group G is defined as the average exposure of the items in the group:

$$Exposure(G|P) = \frac{1}{|G|} \sum_{i \in G} Exposure(i|P) \quad (4)$$

In analogy to base rate statistical parity in classification, we get a *demographic parity definition* of ranking fairness by asking that the two groups get the same exposure:

$$\frac{Exposure(G^+|P)}{Exposure(G^-|P)} = 1 \quad (5)$$

As with classification, we can also get additional statistical fairness definitions by taking into account the actual output, in this case, the utility of the items (e.g., their relevance to a search query q). This is called *disparate treatment constraint* in [61], and it is expressed by asking that the exposure that the two groups receive is proportional to their average utility:

$$\frac{Exposure(G^+|P)}{Utility(G^+|q)} = \frac{Exposure(G^-|P)}{Utility(G^-|q)} \quad (6)$$

Yet another definition, termed *disparate impact*, considers instead of just the exposure, the impact of a ranking. Impact is modeled by the click-through rate (CTR) which is estimated as a function of both exposure and relevance. This definition asks that the impact of the ranking of the two groups is proportional to their average utility:

$$\frac{CTR(G^+|P)}{Utility(G^+|q)} = \frac{CTR(G^-|P)}{Utility(G^-|q)} \quad (7)$$

A fairness of exposure approach has also been taken to define individual fairness in rankings. Specifically, *equity of attention* [10] asks that each item i receives attention a_i (i.e., views, clicks) that is proportional to its utility $util_i$ (i.e., relevance to a given query):

$$\frac{a_1}{util_1} = \frac{a_2}{util_2}, \forall i_1, i_2 \quad (8)$$

Amortized fairness has also been proposed. In general, it is unlikely that equity of attention can be satisfied in any single ranking. For example, multiple items

may be similarly relevant for a given query, yet they obviously cannot all occupy the same ranking position. A sequence $\rho^1 \dots \rho^m$ of rankings offers *amortized equity of attention* [10], if each item receives cumulative attention proportional to its cumulative relevance, i.e.:

$$\frac{\sum_{l=1}^m a_1^l}{\sum_{l=1}^m util_1^l} = \frac{\sum_{l=1}^m a_2^l}{\sum_{l=1}^m util_2^l}, \forall i_1, i_2. \quad (9)$$

In this case, unfairness is defined as the distance between the attention and utility distributions.

$$unfairness(\rho^1, \dots, \rho^m) = \sum_{i=1}^n \left\| \sum_{j=1}^m a_i^j - \sum_{j=1}^m util_i^j \right\| \quad (10)$$

A normalized version of this unfairness definition that considers the number N of items to be ranked and the number m of rankings in the sequence, is proposed in [12]. Formally:

$$unfairness(\rho^1, \dots, \rho^m) = \frac{1}{N} \frac{1}{m} \sum_{i=1}^N \left\| \sum_{j=1}^m a_i^j - \sum_{j=1}^m r_i^j \right\| \quad (11)$$

3.2 Fairness in Recommenders

Recommendation systems retrieve interesting items for users based on their profiles and their history. Depending on the application and the recommendation system, history may include for example explicit user ratings of items, or, selection of items (e.g., views, clicks). In general, recommenders estimate a score, or, rating, $\hat{s}(u, i)$ for a user u and an item i that reflects the preference of user u for item i , or, in other words, the relevance of item i for user u . Then, a recommendation list I is formed for user u that includes the items having the highest estimated score for u . These scores can be seen as the utility scores in the case of recommenders.

A simple approach to defining producer-side (that is, item-side) fairness for recommendations is to consider the recommendation problem as a classification problem where the positive class is the recommendation list. Then, any of the definitions in Section 2.1 are readily applicable to defining producer-side fairness. Yet, another approach to defining producer-side fairness is to consider the recommendation list I as a ranked list r and apply the various definitions described in Section 3.1.

Next, we present a number of approaches proposed specifically for recommenders and discuss their relationship with approaches presented for classification and ranking.

Unfairness in predictions. A measure of consumer-side unfairness that looks into the discrepancy between the prediction behavior for *protected* and *non-protected* users is introduced in [70]. Specifically, accuracy-based fairness metrics are proposed that measure the difference between the predicted and actual scores (i.e., the errors in prediction) of the items recommended to users in the protected group G^+ and the items recommended to users in the non-protected group G^- .

Let N be the size of the recommendation list, $E_{G^+}[\hat{s}]_j$ and $E_{G^-}[\hat{s}]_j$ be the average predicted score (\hat{s}) that an item j receives for the protected users and non-protected users respectively, and $E_{G^+}[s]_j$ and $E_{G^-}[s]_j$ be the corresponding average actual score (s) of item j . Alternatives for defining unfairness can be summarized as follows:

Value unfairness (U_{val}) counts inconsistencies in estimation errors across groups, i.e., when one group is given higher or lower predictions than their true preferences. That is:

$$U_{val} = \frac{1}{N} \sum_{j=1}^n |(E_{G^+}[\hat{s}]_j - E_{G^+}[s]_j) - (E_{G^-}[\hat{s}]_j - E_{G^-}[s]_j)|. \quad (12)$$

Value unfairness occurs when one group of users is consistently given higher or lower predictions than their actual preferences. *Absolute unfairness* (U_{abs}) counts inconsistencies in absolute estimation errors across user groups. That is:

$$U_{val} = \frac{1}{n} \sum_{j=1}^N |E_{G^+}[\hat{s}]_j - E_{G^+}[s]_j| - |E_{G^-}[\hat{s}]_j - E_{G^-}[s]_j|. \quad (13)$$

Absolute unfairness is unsigned, so it captures a single statistic representing the quality of prediction for each group. *Underestimation unfairness* (U_{under}) counts inconsistencies in how much the predictions underestimate the true ratings. That is:

$$U_{under} = \frac{1}{n} \sum_{j=1}^N |\max\{0, E_{G^+}[s]_j - E_{G^+}[\hat{s}]_j\} - \max\{0, E_{G^-}[s]_j - E_{G^-}[\hat{s}]_j\}|. \quad (14)$$

Underestimation unfairness is important when missing recommendations are more critical than extra recommendations. *Overestimation unfairness* (U_{over}) counts inconsistencies in how much the predictions overestimate the true ratings and is important when users may

be overwhelmed by recommendations. That is:

$$U_{over} = \frac{1}{n} \sum_{j=1}^N |\max\{0, E_{G^+}[\hat{s}]_j - E_{G^+}[s]_j\} - \max\{0, E_{G^-}[\hat{s}]_j - E_{G^-}[s]_j\}|. \quad (15)$$

Finally, *non-parity unfairness* (U_{par}) counts the absolute difference between the overall average ratings of protected users and non-protected users. That is:

$$U_{par} = |E_{G^+}[\hat{s}] - E_{G^-}[\hat{s}]|. \quad (16)$$

Calibrated recommendations. A calibration-based approach to producer-side fairness is proposed in [62]. A classification algorithm is considered to be well-calibrated if the predicted proportions of the groups in the various classes agree with their actual proportions in the input data. In analogy, the goal of a *calibrated recommendation algorithm* is to reflect the interests of a user in the recommendations, and with their appropriate proportions. Intuitively, the proportion of the different groups of items in a recommendation list should be similar with their corresponding proportions in the history of the user. As an example, consider movies as the items to be recommended and genre as the protected attribute.

For quantifying the degree of calibration of a list of recommended movies, with respect to the user's history of played movies, this approach considers two distribution of the genre z for each movie i , $p(z|i)$. Specifically, $p(z|u)$ is the distribution over genres z of the set of movies in the history of the user u :

$$p(z|u) = \frac{\sum_{i \in H} w_{u,i} \cdot p(z|i)}{\sum_{i \in H} w_{u,i}}, \quad (17)$$

where H is the set of movies played by user u in the past and $w_{u,i}$ is the weight of movie i reflecting how recently it was played by u .

In turn, $q(z|u)$ is the distribution over genres z of the list of movies recommended to u :

$$q(z|u) = \frac{\sum_{i \in I} w_{r(i)} \cdot p(z|i)}{\sum_{i \in I} w_{r(i)}} \quad (18)$$

where I is the set of recommended movies, and $w_r(i)$ is the weight of movie i due to its rank $r(i)$ in the recommendation list.

To compare these distributions, several methods can be used, like for example, the Kullback-Leibler (KL) divergence that is employed as a calibration metric.

$$C_{KL}(p, q) = \sum_z p(z|u) \log(p(z|u)/\tilde{q}(z|u)), \quad (19)$$

where $p(z|u)$ is the target distribution and $\tilde{q}(z|u) = (1 - \alpha)q(z|u) - \alpha p(z|u) \approx q(z|u)$ with small $\alpha > 0$ is used to

handle the fact that KL-divergence diverge for $q(z|u) = 0$ and $p(z|u) > 0$. KL-divergence ensures that the genres that the user rarely played will also be reflected in the recommended list with their corresponding proportions; namely, it is sensitive to small discrepancies between distributions, it favors more uniform and less extreme distributions, and in the case of perfect calibration, its value is 0.

Pairwise Fairness. Instead of looking at the scores that the items receive, *pairwise fairness* looks at the relative position of pairs of items in a recommendation list. The pairwise approach proposed in [9] is an accuracy-based one where the positive class includes the items that receive positive feedback from the user, such as clicks, high ratings, or increased user engagement (e.g., dwell-time). For simplicity, in the following we will assume only click-based feedback.

Let $r(u, j)$ be 1 if user u clicked on item j and 0 otherwise. Assume that $\hat{r}(u, j)$ is the predicted probability that u clicks on j and g a monotonic ranking function on $\hat{r}(u, j)$. Let I be the set of items, and G^+ and G^- the group of protected and non-protected items, respectively. *Pairwise accuracy* is based on the probability that a clicked item is ranked above another unclicked item, for the same user:

$$P(g(\hat{r}(u, j)) > g(\hat{r}(u, j')) | r(u, j) > r(u, j'), j, j' \in I) \quad (19)$$

For succinctness, let $c_u(j, j') = \mathbb{1}[g(\hat{r}(u, j)) > g(\hat{r}(u, j'))]$. The main idea is to ask that the two groups G^+ and G^- have similar pairwise accuracy. Specifically, we achieve pairwise fairness if:

$$P(c_u(j, j') | r(u, j) > r(u, j'), j \in G^+, j' \in I = P(c_u(j, j') | r(u, j) > r(u, j'), j \in G^-, j' \in I) \quad (20)$$

The authors also consider actual engagement by conditioning that the items have been engaged with the same amount.

We can also distinguish between intra- and inter-group fairness. *Intra-group pairwise fairness* is achieved if the likelihood of a clicked item being ranked above another relevant unclicked item from the same group is the same independent of the group, i.e., when both j and j' in Eq. 20 belong to the same group, *Inter-group pairwise fairness* is achieved if the likelihood of a clicked item being ranked above another relevant unclicked item from the opposite group is the same independent of the group, i.e., j and j' in Eq. 20 belong to opposite groups.

3.3 Fairness in Group-Recommenders

With the expansion of social media, another form of recommendations has emerged; namely group recommendations [50, 56]. Instead of an individual user requesting recommendations from the system, a group can make a request as well. A standard example of group recommendations is the following: a group of friends wants to watch a movie and each one has their own likes and dislikes. The system needs to properly balance them, and offer to the group a list of items that has a degree of relevance to each member. The typical way for doing so, is to apply a recommendation method to each member individually, and then aggregate the separate lists into one for the group. For the aggregation phase, intuitively, for each item, we can calculate the average score across all users in the group preference scores for the item (*average approach*). As an alternative, we can use the minimum function rather than the average one (*least misery approach*). Several approaches recently focus on how to ensure fairness on group recommendations by attempting to minimize the feeling of dissatisfaction within group members.

Dissatisfaction fairness. For counting fairness, a measure of quantifying the satisfaction, or *utility*, of each user in a group given a list of recommendations for this group, can be used, namely by checking how relevant the recommended items are to each user [46]. Formally, given a user u in a group G and a set I of N items recommended to G , the individual utility $U(u, I)$ of the items I for u is defined with respect to their relevance for u , $rel(u, i)$, as:

$$U(u, I) = \frac{\sum_{i \in I} rel(u, i)}{N \times rel_{max}}$$

or

$$U(u, I) = \frac{\sum_{i \in I} rel(u, i)}{\sum_{i \in I(u, N)} rel(u, i)}$$

where $I(u, N)$ denotes the set of items, which are among the top- N favourite items of u .

In turn, the overall satisfaction of users about the group recommendation quality, or *group utility*, is estimated via aggregating all the individual utilities. This is called *social welfare*, $SW(G, I)$, and is defined as:

$$SW(G, I) = \frac{\sum_{u \in G} U(u, I)}{|G|}. \quad (21)$$

Then, for estimating fairness, we need to compare the utilities of the users in the group. Intuitively, for example, a list that minimizes the dissatisfaction of any user in the group can be considered as the most fair. In this sense, fairness enforces the least misery principle among users utilities, emphasising the gap between

the least and highest utilities of the group members. Following this concept, fairness can be defined as:

$$F(G, I) = \min\{U(u, I), \forall u \in G\}. \quad (22)$$

Similarly, fairness can encourage the group members to achieve close utilities between each other using variance:

$$F(G, I) = 1 - \text{Variance}(\{U(u, I), \forall u \in G\}). \quad (23)$$

Pareto optimal fairness. Instead of computing users' individual utility for a list of recommendations by summing up the relevance scores of all items in the list [46], the item positions in the recommendation list can be considered [57]. Specifically, the solution for making fair group recommendations is based on the notion of Pareto optimality, which means that an item i is *Pareto optimal* for a group if there exists no other item j that ranks higher according to all users in the group, i.e., there is no item j that dominates item i . *N-level Pareto optimal*, in turn, is a direct extension that contains items dominated by at most $N - 1$ other items, and is used for identifying the N best items to recommend. Such a set of items is fair by definition, since it contains the top choices for each user in the group.

Fairness in package-to-group recommendations.

Given a group G , an approach to fair package-to-group recommendations is to recommend to G a package of items P , by requiring that for each user u in G , at least one item high in u 's preferences is included in P [60]. Even if such a resulting package is not the best overall, it is fair, since there exists at least one item in P that satisfies each user in G .

Specifically, two different aspects of fairness are examined [60]: (a) fairness proportionality, ensuring that each user finds a sufficient number of items in the package that he/she likes compared to items not in the package, and (b) fairness envy-freeness, ensuring that for each user there is a sufficient number of items in the package that he/she likes more than other users do. Formally:

m-proportionality. For a group G of users and a package P , the m -proportionality of P for G is defined as:

$$F_{prop}(G, P) = \frac{|G_P|}{|G|}, \quad (24)$$

where $G_P \subseteq G$ is the set of users in G for which P is m -proportional. In turn, P is m -proportional to a user u , if there exist at least m items in P , such that, each one is ranked in the top- $\delta\%$ of the preferences of u over all items in I , for an input parameter δ .

m-envy-freeness. For a group of users G and a package P , the m -envy-freeness of P for G is defined as:

$$F_{ef}(G, P) = \frac{|G_{ef}|}{|G|}, \quad (25)$$

where $G_{ef} \subseteq G$ is the set of users in G for which P is m -envy-free. In turn, P is m -envy-free for a user u , if u is envy-free for at least m items in P , i.e., each item's $rel(u, i)$ is in the top- $\delta\%$ of the preferences in the set $\{rel(v, i) : v \in G\}$.

Sequential hybrid aggregation. An approach for fair sequential group recommenders targets two independent objectives [64]. The first one considers the group as an entity and aims at offering the best possible results, by maximizing the overall group satisfaction over a sequence of recommendations. The satisfaction of each user u_i in a group G for the group recommendation Gr_j received at the j round of recommendations, is computed by comparing the quality of recommendations that u_i receives as a member of the group over the quality of recommendations u_i would have received as an individual. Given the list $A_{u_i, j}$ with the top- k items for u_i , the user's satisfaction is calculated based on the group recommendation list, i.e., for every item in Gr_j , we sum the score as they appear in each user's $A_{u_i, j}$, over the ideal case for the user, by sum the scores of the top- k items in $A_{u_i, j}$. Formally:

$$sat(u_i, Gr_j) = \frac{\sum_{d_z \in Gr_j} p_j(u_i, d_z)}{\sum_{d_z \in A_{u_i, j}} p_j(u_i, d_z)} \quad (26)$$

The second objective considers the group members independently and aims to behave as fairly as possible towards all members, by minimizing the variance between the user satisfaction scores. Intuitively, this variance represents the potential disagreement between the users in the group. Formally, the disagreement is defined as:

$$\begin{aligned} groupDis(G, \mathcal{GR}) = \\ \max_{u_i \in G} satO(u_i, \mathcal{GR}) - \min_{u_i \in G} satO(u_i, \mathcal{GR}) \end{aligned} \quad (27)$$

where $satO(u_i, \mathcal{GR})$ is the overall satisfaction of u_i for a sequence \mathcal{GR} of recommendations defined as the average of the satisfaction scores after each round. That is, group disagreement is the difference in the overall satisfaction scores between the most satisfied and the least satisfied user in the group. When this measure takes low values, the group members are all satisfied to the same degree.

Table 1 Fairness definitions taxonomy in Rankings, Recommenders and Group-Recommenders.

	Individual	Group	Consumer	Producer	Single	Multiple	Criterion
Rankings							
Fairness constraints [18]		✓		✓	✓		position
Discounted cumulative fairness [69]		✓		✓	✓		position
Fairness of exposure [61]		✓		✓	✓		position/utility
Equity of attention [10]	✓			✓		✓	position/utility
Recommenders							
Calibrated recommendations [62]		✓		✓	✓		number of items
Value/Absolute unfairness [70]		✓	✓		✓		error on predictions
Under/Overestimation unfairness [70]		✓	✓		✓		error in ratings
Non-parity unfairness [70]		✓	✓		✓		ratings
Group-Recommenders							
Dissatisfaction fairness [46]	✓		✓		✓		user satisfaction
Pareto optimal fairness [57]	✓		✓		✓		position
Proportionality fairness [60]	✓		✓		✓		number of items
Envy-freeness fairness [60]	✓		✓		✓		number of items
Sequential hybrid aggregation [64]	✓		✓			✓	user satisfaction

3.4 Summary

Table 1 presents a summary of the various definitions of fairness used in rankings and recommenders. We make several observations.

We observe that these fairness definitions are based on one of the following criteria: (a) position of item in the ranking or recommendation list (b) item utility (c) prediction error (d) rating, and (e) number of items. User satisfaction is defined through item utility.

In rankings, fairness is typically defined for the items to be ranked, matching all existing definitions to producer fairness. In recommenders, the distinction between consumer fairness and producer fairness makes more sense, given that they focus either on the individuals that receive a recommendation or the individuals that are recommended. Nevertheless, most existing works target consumer fairness.

In group recommenders, typically, fairness is considered as a way to evaluate if the system takes into consideration the individual preferences of each single user in the group, making all approaches in the research literature to target at individual and consumer fairness. Only recently, there are few approaches that focus on another form of fairness that is applicable when we consider a sequence of rankings, or recommendations, instead of just a single one.

In what follows, we will study how these models and definitions of fairness are applied to algorithms.

4 Methods for Achieving Fairness

Methods for fair ranked outputs and recommendations can be distinguished into the following categories:

- *Pre-processing methods* aim at transforming the data to remove any underlying bias or discrimination.
- *In-processing methods* aim at modifying existing or introducing new algorithms that result in fair rankings and recommendations, e.g., by removing bias.
- *Post-processing methods* modify the output of the algorithm.

We will use this taxonomy to organize and present the related works that we describe in the next sections.

5 Pre-processing Methods

Often, bias can exist in the underlying data on which systems are trained [15], and it can take two forms. *Bias in the rows* of the data exists when there are not enough representative individuals from minority (sub)groups. For example, according to a Reuters article [20], Amazon’s experimental automated system to review job applicants’ resumes showed a significant gender bias towards male candidates over females that was due to historical discrimination in the training data.

Bias in the columns is when features are biased (correlated) with sensitive attributes. For example, zip code tends to predict race due to a history of segregation [34]. Direct discrimination occurs when protected attributes are used explicitly in making decisions (i.e., *disparate treatment*). More pervasive nowadays is indirect discrimination, in which protected attributes are not used but reliance on variables correlated with them leads to significantly different outcomes for different groups, also known as *disparate impact* (Section 2.1).

To address bias and avoid discrimination, several methods have been proposed for pre-processing data. Many of these methods are studied in the context of

classification, while a few have been proposed in the context of recommender systems.

Suppression. To tackle bias in the data, a naïve approach sometimes used in practice is to simply *omit the protected attribute* (say, race or gender) when training the classifier [37]. Of course, simply excluding a protected variable is insufficient to avoid discriminatory predictions, as any included variables that are correlated with the protected variables still contain information about the protected characteristic, and the classifier still learns the discrimination reflected in the training data. For example, answers to personality tests identify people with disabilities [67]. Word embeddings trained on Google News articles exhibit female/male gender stereotypes [11]. To tackle such dependencies, one can further find the attributes that correlate most with the sensitive attribute and remove these as well.

Class Relabeling. This approach, also known as massaging [37], changes the labels of some objects in the dataset in order to remove the discrimination from the input data. A good selection of which labels to change is essential. The idea is to consider a subset of data from the minority group as promotion candidates, and change their class label. Similarly, a subset of the majority group is chosen as demotion candidates. To select the best candidates for relabeling, a ranker is used that ranks the objects based on their probability of having positive labels. For example, a Naïve Bayesian classifier can be used for both ranking and learning [36, 37]. Then, the top- k minority, for promotion, objects and the bottom- k majority, for demotion, objects are chosen. The number k of pairs needed to be modified to make a dataset D discrimination-free can be calculated as follows.

Let us assume as before two groups, namely, the protected group G^+ and the non-protected (or, privileged) group G^- . If we modify k objects from each group, the resulting discrimination will be:

$$\frac{p^- - k}{|G^-|} - \frac{p^+ + k}{|G^+|} = \text{disc}(D) - k\left(\frac{1}{|G^+|} + \frac{1}{|G^-|}\right) = \text{disc}(D) - \left(k \frac{|D|}{|G^-||G^+|}\right) \quad (28)$$

To reach zero discrimination, the number of modifications needed is:

$$k = \frac{\text{disc}(D) \times |G^-| \times |G^+|}{|D|} \quad (29)$$

where p^+ (p^-) are the number of positive objects that belong to the minority group (majority group). Discrimination $\text{disc}(D)$ in D is the probability of being in the positive class between the objects in the minority group versus those in the majority group.

Sex	Ethnicity	Highest degree	Job type	Class
M	Native	H. school	Board	+
M	Native	Univ.	Board	+
M	Native	H. school	Board	+
M	Non-nat.	H. school	Healthcare	+
M	Non-nat.	Univ.	Healthcare	−
F	Non-nat.	Univ.	Education	−
F	Native	H. school	Education	−
F	Native	None	Healthcare	+
F	Non-nat.	Univ.	Education	−
F	Native	H. school	Board	+

Fig. 4 Job application example [37].

The problem of classification without discrimination w.r.t. a sensitive attribute is a multi-objective optimization problem. Lowering the discrimination will result in lowering the accuracy and vice versa.

Reweighting. The previous approach is rather intrusive as it changes the labels of the objects. Instead of that, weights can be assigned to the objects to compensate for the bias [37]. The idea is to assign lower weights to objects that have been deprived or favored. Then, the weights can be used directly in any method based on frequency counts.

A frequently used family of analytical methods are grouped under *propensity score matching* [33]. Such methods model the probability of each object or group receiving the treatment and use these predicted probabilities or “propensities” to make up for the confounding of the treatment with the other variables of interest and balance the data.

A simple *probability-based reweighting* method is the following [37]. Let us consider the sensitive attribute S . Then, every object x will be assigned a weight:

$$w(x) = \frac{P_{exp}(S = x(S) \wedge class = x(class))}{P_{obs}(S = x(S) \wedge class = x(class))} \quad (30)$$

i.e., the weight of an object will be the expected probability to see an instance with its sensitive attribute value and class given independence, divided by its observed probability.

For example, consider the dataset in Figure 4. If the dataset is unbiased, then the sensitive attribute S (i.e., sex in our example) and the class are statistically independent. Then, the expected probability for females to be promoted would be: $P_{exp}(sex = F \wedge class = +) = 0.5 * 0.6 = 0.3$. In reality, however, the observed probability based on the dataset is $P(sex = F \wedge class = +) = 0.2$. Hence, one can use a re-weighting factor $w(x) = 0.3/0.2 = 1.5$ to balance the bias in the dataset.

Entropy balancing aims at covariate balance in data for binary classification [31]. It relies on a maximum entropy reweighting scheme that calibrates individual

weights so that the reweighed groups satisfy a set of balance constraints that are imposed on the sample moments of the covariate distributions. The balance constraints ensure that the reweighed groups match exactly on the specified moments adjusting in this way inequalities in representation. The generated weights can be passed to any standard classifier.

Adaptive Sensitive Reweighting uses a convex model to estimate distributions of underlying labels with which to adapt weights [42]. It assumes that there exists an (unobservable) underlying set of class labels corresponding to training samples that, if predicted, would yield unbiased classification with respect to a fairness objective. It searches for sample weights that make weighted training on the original dataset also train towards those labels, without explicitly observing them.

More specifically, consider a binary probabilistic classifier, which produces probability estimates $\hat{P}(Y = y_i) = 1 - \hat{P}(Y \neq y_i)$. For training samples i with features x_i and class labels y_i , there exists an underlying (i.e. unobservable) class labels \tilde{y}_i that yield estimated labels \hat{y}_i which conform to designated fairness and accuracy trade-offs. The training goal is to minimize both weighted error on observed labels as well as the distance between weighted observed labels and unweighed underlying labels:

$$\min \sum_i w_i \hat{P}(\hat{y}_i \neq y_i) \quad (31)$$

$$\min \sum_i (w_i \hat{P}(\hat{y}_i \neq y_i) - \hat{P}(\hat{y}_i \neq \tilde{y}_i))^2 \quad (32)$$

To simultaneously adjust training weights alongside classifier training, a classifier-agnostic iterative approach is proposed: first, a classifier is fully trained based on uniform weights, and then, the method appropriately readjusts those weights. This process is repeated until convergence.

Data transformation. A common theme is the importance of balancing discrimination control against utility of the processed data. This can be formulated as an optimization problem for producing preprocessing transformations that trade off discrimination control, data utility, and individual distortion [52]. Assuming S is the one or more protected (sensitive) variables, X denotes other non-protected variables, and Y is an outcome random variable. The goal is to determine a randomized mapping $P_{\hat{X}, \hat{Y} | X, Y, S}$ that transforms both the training data and the test data. The mapping should satisfy three properties.

– *Discrimination Control.* The first objective is to limit the dependence of the transformed outcome \hat{Y}

on the protected variables S , which requires the conditional distribution $P_{\hat{Y} | S}$ to be close to a target distribution P_{Y_T} for all values of S .

– *Distortion Control.* The mapping $P_{\hat{X}, \hat{Y} | X, Y, S}$ should satisfy distortion constraints to reduce or avoid certain large changes (e.g. a very low credit score being mapped to a very high credit score).

– *Utility Preservation.* The distribution of (\hat{X}, \hat{Y}) should be statistically close to the distribution of (X, Y) . This is to ensure that a model learned from the transformed data (when averaged over the protected variables S) is not too different from one learned from the original data. For example, a bank’s existing policy for approving loans does not change much when learnt over the transformed data.

Database repair. Handling bias in the data can be considered a *database repair problem*. One approach is to *remove information about the protected variables from the set of covariates* to be used in predictive models [23, 48]. A test for disparate impact based on how well the protected class can be predicted from the other attributes and a data repair algorithm for numerical attributes have been proposed [23]. The algorithm “strongly preserves rank”, which means it changes the data in such a way that predicting the class is still possible. A chain of conditional models can be used for both protecting and adjusting variables of arbitrary type [48]. This framework allows for an arbitrary number of variables to be adjusted and for each of these variables and the protected variables to be continuous or discrete.

Another data repair approach is based on measuring the discriminatory causal influence of the protected attribute on the outcome of an algorithm. This approach removes discrimination by repairing the training data in order to *remove the effect of any discriminatory causal relationship between the protected attribute and classifier predictions* [58]. This work introduced the notion of *interventional fairness*, which ensures that the protected attribute does not affect the output of the algorithm in any configuration of the system obtained by fixing other variables at some arbitrary values. The system repairs the input data by inserting or removing tuples, changing the empirical probability distribution to remove the influence of the protected attribute on the outcome through any causal pathway that includes inadmissible attributes, i.e. attributes that should not influence the protected attribute.

Data augmentation. While the aforementioned approaches modify the training data, either explicitly (e.g., by suppressing attributes or changing class labels) or implicitly, e.g., by adding weights, a different approach is to leave the training data as is and just augment it with additional data [55]. This framework starts

Table 2 Pre-processing Methods.

	bias in rows	bias in columns	fairness	algorithm
Suppression [37]		✓	group	any
Class Relabeling [36, 37]	✓		group	ranker
Reweighting [31, 37, 42]		✓	group/ individual	ranker
Data transformation [52]		✓	group/ individual	ranker
Data repair [23, 48, 58]	✓	✓	group	ranker
Data augmentation [55]	✓	✓	group/ individual	matrix factorization

from an existing matrix factorization recommender system that has already been trained with some input (ratings) data, and adds new users who provide ratings of existing items. The new users’ ratings, called antidote data, are chosen so as to improve a socially relevant property of the recommendations that are provided to the original users. The proposed framework includes measures of both individual and group unfairness.

5.1 Summary of Pre-processing Methods

Table 2 summarizes pre-processing approaches to fairness based on whether they focus on bias in rows or columns, the level of fairness (individual or group) and the algorithm that will use the pre-processed data.

Suppression as a brute-force approach does not depend on the algorithm. Many of the pre-processing methods are studied in the context of classification/ranking. The class relabeling approach works with different rankers (e.g., a Naive Bayes classifier, or nearest neighbor classifier). The reweighting methods are parameter-free as they do not rely on a ranker. Hence, they can work with any ranking algorithm. There is also a large literature on reweighting methods in economics and statistics that could be worth looking at in the context of ranking and recommendation methods.

One pre-processing method has been proposed in the context of recommender system, and in particular for matrix factorization. This approach has studied both individual and group fairness [55]. In general, group fairness is easier to track and handle.

There is an abundance of machine learning algorithms used in practice for search and recommendations (and in general) dictating a clear need for a future systematic study of the relationship between dataset features, algorithms, and pre-processing performance.

6 In-processing Methods

In-processing methods for achieving fairness in rankings and recommendations focus on modifying existing or introducing new models or algorithms. In this section, we survey in a unified way these methods, by distinguishing between learning approaches and approaches using preference functions.

6.1 Learning Approaches

For both rankings and recommenders, learning approaches typically use machine learning to construct ranking models, most often using a set of labeled training data as input. In general, the ranking model ranks unseen lists in a way similar to the ranking of the training data. The overall goal is to learn a model that minimizes a *loss function* that captures the distance between the learned and the input ranking.

Various approaches exist, varying on the form of training data and the type of loss function [47]. In the *point-wise approach* (e.g., [19]), the training data are (item, relevance-score) pairs for each query. In this case, learning can be seen as a regression problem where given an item and a query, the goal is to predict the score of the item. In the *pair-wise approach*, the training data are pairs of items where the first item is more relevant than the second item for a given query [13, 24]. In this case, learning can be seen as a binary classification problem where given two items, the classifier decides whether the first item is better than the second one. Finally, in the *list-wise approach* (e.g., [16]), the input consists of a query and a list of items ordered by their relevance to the query. Note that the loss function takes many different forms depending on the approach. For example, in the pair-wise approach, loss may be computed as the average number of inversions in a ranked output.

In the following, we present a number of approaches towards making the ranking models fair. Note that the proposed approaches can be adopted to work for different types of input data and loss functions.

6.1.1 Adding Regularization Terms

A general in-processing approach to achieving fairness is by adding *regularization terms* to the loss function of the learning model. These regularization terms express measures of unfairness that the model must minimize in addition to the minimization of the original loss function. Depending on the form of the training data, the loss function and the measure of fairness, different instantiations of this general approach are possible.

The DELTR approach [72] extends the ListNet [16] learning to rank framework. ListNet is a list-wise framework where the training set consists of a query q and a list of items ordered by their relevance to q . ListNet learns a ranking function f that minimizes a loss function L_{LN} that measures the extent to which the ordering \hat{r} of items induced by f for a query differs from the ordering r that the items appear in the training set for this query. The loss function L_{DELTR} of DELTR is:

$$L_{DELTR}(r(q), \hat{r}(q)) = L_{LN}(r(q), \hat{r}(q)) + \lambda Unfairness(\hat{r}(q)) \quad (33)$$

L_{DELTR} extends the original loss function L_{LN} of ListNet with a term that imposes a fairness constraint. Parameter λ controls the trade-off between ranking utility (i.e., distance from input ranking r captured by the original loss function) and fairness.

Exposure is used as a measure of unfairness in the produced output. Specifically:

$$Unfairness(r(q)) = \max\{0, Exposure(G^+|r(q)) - Exposure(G^-|r(q))^2\} \quad (34)$$

Using the squared hinge loss makes the loss function differentiable. Also, the model prefers rankings in which the exposure of the protected group is not less than the exposure of the non-protected group but not vice versa.

A regularization approach is also taken for recommender systems in [38]. Let U and I denote random variables for the users and items respectively and R denote a random variable for the recommendation output. Let also S be the sensitive attribute, that is, information to be ignored in the recommendation process, like, for example, the gender of a user or the popularity of an item. The goal in this case, is to achieve recommendation, or statistical, independence. This means to include no information about the sensitive feature that influences the outcome, as well as recommendations should satisfy a recommendation independence constraint.

The core of this regularization approach is included in Equation 35 that adopts a regularizer imposing a constraint of independence, while training the recommendation model.

$$\sum_D \text{loss}(r_i, r(x_i, y_i, s_i)) - \eta \cdot \text{ind}(R, S) + \lambda \cdot \text{reg}(\theta) \quad (35)$$

where η is the independence parameter that controls the balance between independence and accuracy and ind is the independence term, i.e., the regularizer to constrain independence; the larger value indicates that recommendations and sensitive values are more independent. Loss is the empirical loss, while λ is the regularization

parameter and θ is the L2 regularizer. Several alternatives can be used for the independence term, like, for example, the mutual information with histogram models or normal distributions, or by exploiting distance measures as in the case of distribution matching using the Bhattacharyya distance.

6.1.2 Learning via Variational Autoencoders

Variational Autoencoders (VAE) are proposed as the state-of-the-art for the collaborative filtering task in recommenders. With a multinomial likelihood generative model and a controlled regularization parameter, it is possible to estimate normal distribution parameters in the middle layer of an MLP, that enriches the rating data representation and outperforms previous neural network based approaches [45]. The situation requires drawing samples from the inferred distributions in order to propagate values to the decoder, but it is not a trivial task to take gradients when having a sampling step. The *re-parametrization trick* [40] is to re-parameterize the sampled values by incorporating a normal distributed noise, so the gradient can back-propagate through the sampled variable during the training.

Instead of using only the re-parametrization trick during the training phase, the noise variable can be incorporated in the test phase of VAE as well [12], in order to enhance fairness in the ranking order of recommendations (using as definition of fairness, Equation 10). The motivation here is that different noise distributions directly affect the rankings, depending on how frequently the latent values vary around the mean inside the interval defined by the variance. Specifically, it is experimentally shown that the noisy effect of the Gaussian and uniform distributions vary the output scores when having the same data as input, while unfairness is reduced despite of a small decrease in the quality of the ranking. The higher the variance of the new component, the greater the effect in the predicted scores, and consequently in the ranking order.

6.1.3 Learning Fair Representations

The main idea in this approach is to learn a fair representation of the input data and use it for the task at hand. Previous work in fair classification used this idea to achieve fairness by introducing an intermediate level Z between the input space X that represents individuals and the output space Y that represents classification outcomes [73]. Z should be a fair representation of X that best encodes X and obfuscates any information about membership in the protected group. Specifically, Z is modeled as a multinomial random variable of size

k where each of the k values represents a prototype (cluster) in the space of X .

The goal is to learn Z such that to minimize a loss function L :

$$L = \lambda_x L_x + \lambda_z L_z + \lambda_y L_y \quad (36)$$

where the first term, L_x , refers to the quality of the encoding, i.e., expresses the requirement that the distance from points in X to their representation in Z should be small, the second term, L_z , refers to fairness, and the last term, L_y , refers to accuracy, i.e., the prediction based on the representation should be accurate. Parameters λ_x , λ_y and λ_z are hyper-parameters that control the trade-off among these three objectives.

One can enforce different forms of fairness by appropriately defining the L_z objective. Statistical parity is used in [73] captured by the following objective:

$$Pr(z = k|x \in G^+) = Pr(z = k|x \in G^-), \forall k \quad (37)$$

that is, the probability that a random element that belongs to the protected group of X maps to a particular prototype of Z is equal to the probability that a random element that belongs to the non-protected group of X maps to the same prototype.

The fair representation approach has also been used for fair ranking instead of classification. This was achieved by modifying the last objective L_y in Equation 36 to represent accuracy in the case of ranking as opposed to accuracy in classification [69]. The modified objective asks that the distance between the ground truth ranking and the estimated ranking is small.

6.2 Linear Preference Functions

In some applications, items are ranked based on a score that is a weighted linear combination of the values of their attributes. Specifically, let i be an item with d scoring attributes, $i[1] \dots i[d]$. A linear ranking function f uses a weight vector $w = (w_1, \dots, w_d)$, to compute a utility (goodness) score for each item i , $f(i) = \sum_{j=1}^d w_j i[j]$.

In this case, fairness is formulated as the following problem. Given function f with weight vector $w = (w_1, \dots, w_d)$, find a function f^* with weight vector $w^* = (w_1^*, \dots, w_d^*)$, such that, f^* produces a fair ranking and its weights are as close to the weights of the original f as possible, i.e., $\text{cosine}(w, w^*)$ is minimized [7].

7 Post-processing Methods

Post-processing approaches are agnostic to the ranking or the recommendation algorithm. Typically, they take

as input a ranking r and a specification of the required form of fairness and produce a new ranking \hat{r} that satisfies the fairness requirements and respects the initial ranking, to the extent possible.

7.1 Fairness as a Generative Process

Let us consider a simple case of group parity, where we ask that a specific proportion p , $0 \leq p \leq 1$, of the items at the top k positions in the ranking belong to the protected group.

Given p and a ranking r , the generative process introduced in [69] creates a ranking \hat{r} by: (a) initializing \hat{r} to the empty list, and (b) incrementally adding items to \hat{r} . Specifically, for each position j in r , a Bernoulli trial is performed with p . If the trial succeeds, we select the best available, i.e., most highly ranked in r , item from the protected group. Otherwise, we select the best available item from the non-protected group. An example is shown in Figure 3.

The produced ranking \hat{r} satisfies the in-group monotonicity constraints. This means that, within each group, the items are ordered with decreasing qualifications. It is also shown that under some assumptions the ranking also maximizes the utility expressed as the average score of the items in the top- k positions [71].

A statistical test for this generative model is proposed in [71]. Specifically, given that at a specific position we have seen a specific number of items from each group, an one-tailed Binomial test is used to compare the null hypotheses that the ranking was generated using the model with parameter $p^* = p$, or with $p^* < p$, which would mean that the protected group is represented less than desired.

7.2 Fair Ranking as a Constraint Optimization Problem

Another post-processing approach formulates the problem of producing a fair ranking \hat{r} as an optimization problem. Let F be a fairness measure for rankings and let U be a measure of the utility of a ranking for a particular task, for example, let $U(r|q)$ be the relevance of ranking r for a given query q . There are two general ways of formulating an optimization problem involving fairness $F(r)$ and utility $U(r|q)$, namely:

- (MAXFCOU) maximizing fairness subject to a constraint in utility,
- (MAXUCOF) maximizing utility subject to a constraint in fairness.

In the MAXFCOU formulation, the underlying idea is to produce a ranking that is as fair as possible while remaining relevant to q (e.g., [10]). For example, we ask for the most fair ranking \hat{r} among all rankings such that \hat{r} also satisfies a utility constraint, e.g., the loss in utility with regards to the original ranking r remains below a given threshold θ :

$$\begin{aligned} \hat{r} &= \operatorname{argmax}_{\hat{r}} F(\hat{r}) \\ \text{s. t., } &\text{distance}(U(r|q), U(\hat{r}|q)) \leq \theta. \end{aligned}$$

Alternatively, in the MAXUCOF formulation, we look for the ranking that has the the maximum possible utility among all rankings whose fairness is sufficient (e.g., [10], [61]). For example, the approach proposed in [61] produces a ranking r such that:

$$r = \operatorname{argmax}_r U(r|Q) \quad \text{s.t. } r \text{ is fair.}$$

We characterize such MAXUCOF approaches as post-processing, since they assume that the utility of each item is known, or can be estimated. Thus, implicitly there is an original, non-fair ranking in which the items are ordered solely by their utility. Then, given these individual utilities, a new ranking is produced that also satisfies a fairness constraint.

In general, the complexity of both the MAXFCOU and MAXUCOF optimization problems depends on the type of the utility and fairness functions, and the form of the constraints. In some cases, the optimization problems can be solved using linear integer programming (ILP), e.g., [10], or, in some special cases using a dynamic programming algorithm e.g., [18].

Constraints can be also used for producing fair recommendation packages of items for groups of users [60]. The intuition of the method is to greedily construct a package P , by adding in rounds to P the item that satisfies the largest number of non-satisfied users. Specifically, given that $\text{sat}_G(P)$ denotes the users in G that are satisfied by P , at each round, the goal is to maximize:

$$f_G(P, i) = |\text{sat}_G(P \cup i) \setminus \text{sat}_G(P)|. \quad (38)$$

When considering fairness proportionality (Equation 25), $\text{sat}_G(i)$ contains the users for which item i belongs in their top- $\delta\%$ most preferable items. For envy-freeness fairness (Equation 24), $\text{sat}_G(i)$ contains the users that are envy-free for the item i . That is, P is fair for a user u (m-proportional, or m-envy-free), if there are at least m items i in P , such that, $u \in \text{sat}_G(i)$.

This method is generalized to include constraints that restrict the set of candidate packages that can be recommended to a group of users. Two types of constraints are discussed, namely *category* and *distance* constraints. In simple words, with category constraints, when selecting an item from a specific category, we remove the items of this category from the candidate set. With distance constraints, we consider as candidate

items only the items that when added to the existing solution satisfy the specific input distance constraints.

7.3 Fairness via Calibration Methods

Calibration methods suggest to re-rank a list of items as a post-processing step. Equation 18 (in Section 3.2) quantifies the degree of calibration for recommender's outputs based on specific metrics [62].

To determine the optimal set I^* of N recommended items – movies in the suggested scenario – the maximum marginal relevance function [17] is used:

$$I^* = \operatorname{argmax}_{I, |I|=N} (1 - \lambda) \cdot s(I) - \lambda \cdot C_{KL}(p, q(I)) \quad (39)$$

where $\lambda \in [0, 1]$ determines the trade-off between the prediction scores $s(i)$ of the movies $i \in I$, with $s(I) = \sum_{i \in I} s(i)$, and the calibration metric (Equation 18). Namely, the trade-off between accuracy and calibration is controlled by λ . Greedily, the method starts with an empty set, and iteratively adds one movie at a time, namely the movie that maximizes Equation 39.

In a similar way, for group recommendations, the best suggestions I for a group G should maximize the social welfare $SW(G, I)$ (Equation 21) and fairness $F(G, I)$ (Equation 22 and 23), using the scheme [46]:

$$\lambda \cdot SW(G, I) + (1 - \lambda) \cdot F(G, I). \quad (40)$$

A greedy solution is to select an item that, when added to the current recommendation list, achieves the highest fairness $F(G, I)$. More time-efficient alternatives are offered via integer programming techniques.

When considering the notion of Pareto optimality for group recommendations, a simple heuristic can be used to compile and approximately identify the list of the top- N recommendations for a group [57]: Specifically, given N' , where $N' > N$ is the largest number of items the system can recommend for an individual user, the method proceeds as follows: (a) it requests the top- N' recommendations for each user in the group, (b) it takes their union, and (c) it identifies the N -level Pareto optimal items among the items in the union.

7.4 Fairness in Multiple Outputs

When providing rankings or recommendations, users typically pay more attention to the first positions, and attention wears off for items in lower positions in the ranking. In a situation where the greatest estimated probabilities are quite close or equal to each other, the algorithm needs to arrange them in a proper order and necessarily present high scores in high positions. This

promotes an unfair result that can be mitigated in the long term, by changing the position of items in sequential rounds of rankings or recommendations.

In the case of rankings, one approach [10] is to require that ranked items receive attention that is proportional to their utility in a sequence of rankings (Equation 9). This way, the unfair position one item appears in a single ranking can be compensated in the next rankings when it changes position, and the whole session contemplates long-term fairness. However, the act of reducing unfairness implies reduction in the ranking quality, due to the perturbation of the utility-based rankings. The trade-off between ranking quality and fairness is formulated as a constrained optimization problem [10] targeting at minimizing unfairness subject to constraints on quality (i.e., lower-bound the minimum acceptable quality).

Specifically, for a sequence $\rho^1 \dots \rho^m$ of rankings where the items are ordered by the utility score, inducing zero quality loss, the aim is at reordering them into $\rho^{1*} \dots \rho^{m*}$, to minimize the distance between the attention (A) and utility (U) distributions with constraints on the NDCG-quality loss in each ranking. Formally:

$$\min \sum_i |A_i - U_i|, \quad (41)$$

subject to $\text{NDCG-quality@k}(\rho^j, \rho^{j*}) \geq \theta, j = 1, \dots, m$, where A_i and U_i denote the cumulative attention and utility scores that the item i gained across all rankings.

Consider a different scenario, where a group of users, interacts with a recommender multiple times [64]. When following traditional methods for group recommendations, like the average aggregation method and the least misery one, the degree of *satisfaction* for each user in the group (e.g., Equation 26), can not be good enough for all users in the group, leading to cases in which almost none of the reported items are of interest to some users in the group. That is, the recommender system is unfair to these users, and unfairness continues throughout a number of recommendations rounds. To overcome the drawbacks of the *average* and the *least misery* aggregation methods, and capitalize on their advantages, an aggregation method, called *sequential hybrid aggregation method*, offers a weighted combination of them [64]. Specifically:

$$\text{score}(G, d_z, j) = (1 - \alpha_j) * \text{avgScore}(G, d_z, j) + \alpha_j * \text{leastScore}(G, d_z, j). \quad (42)$$

For a group G , $\text{avgScore}(G, d_z, j)$ returns the score of the item d_z as it is computed by the average aggregation method during round j , and $\text{leastScore}(G, d_z, j)$ returns the least satisfied user's score of d_z at round j .

To self-regulate the value of α between 0 and 1, so as to more effectively describe the consensus of the group, α is set dynamically in each iteration by subtracting the minimum satisfaction score of the group members in the previous iteration, from the maximum score:

$$\alpha_j = \max_{u \in G} \text{sat}(u, Gr_{j-1}) - \min_{u \in G} \text{sat}(u, Gr_{j-1}), \quad (43)$$

where $\text{sat}(u, Gr_{j-1})$ defines the satisfaction of user u for the group recommendations Gr_{j-1} at round $j - 1$.

The dynamic calculations of α counteracts the individual drawbacks of the average and least misery method. Intuitively, if the group members are equally satisfied at the last round, then α takes low values, and the aggregation will closely follow that of average, where everyone is treated as an equal. On the other hand, if one group member is extremely unsatisfied in a specific round, then α takes a high value and promotes that member's preferences on the next round.

8 Summary of In- and Post-processing Methods

In this section, we summarize the in-processing and post-processing approaches for achieving fairness. Overall, there exist methods that have been proposed both in the context of rankings and recommender systems. Table 3 organizes the methods based on (a) the level of fairness, namely, individual or group, (b) the side of fairness, namely, consumer or producer, and (c) the graduality of fairness, namely, if a method focuses on a single or multiple outputs.

In general, all existing *learning* and *linear preference functions* in-processing approaches target group and producer fairness. Most approaches consider a single output with the recent exception of [12] using VAEs that considers multiple outputs. Regarding the post-processing approaches, we observe that there exist works focusing on all different options of fairness definitions. As in the pre-processing case, post-processing methods treat the algorithms for producing rankings and recommendations as black boxes, without changing their inner workings. As a consequence, they can lead to unpredictable losses in accuracy (which is especially true for pre-processing methods).

Recently, [73] combines both pre-processing and in-processing strategies by jointly learning a fair representation of the data and the classifier parameters. This approach has two main limitations: (a) it leads to a non-convex optimization problem and does not guarantee optimality, and (b) the accuracy of the classifier depends on the dimension of the fair representation, which needs to be chosen rather arbitrarily.

Table 3 In- and Post-processing Methods.

	Individual	Group	Consumer	Producer	Single	Multiple
In-processing methods						
Adding regularization terms		[72, 38]		[72, 38]	[72, 38]	
Learning fair representations		[73]		[73]	[73]	
Learning with VAEs		[12]		[12]		[12]
Linear preference functions		[7]		[7]	[7]	
Post-processing methods						
Fairness as a generative process		[69, 71]		[69, 71]	[69, 71]	
Fairness as a constraint optimization problem	[60, 10]	[61, 18]	[60]	[61, 10, 18]	[60, 61, 18]	[10]
Fairness with calibration methods	[57, 46]	[62]	[57, 46]	[62]	[57, 62, 46]	
Fairness in multiple rounds	[64, 10]		[64]	[10]		[64, 10]

9 Verifying Fairness

In the previous sections, we studied methods for achieving fairness. In this section, we ask *how we can verify whether a program is fair*. In *program fairness verification*, our goal is to analyze a given decision-making program and construct a proof of its fairness or unfairness — just as a traditional static program verifier would prove correctness of a program with respect to, for example, lack of divisions by zero.

Building a fairness verifier, one has to answer: (a) what class of decision-making programs the program model will capture, (b) what the set of possible inputs to the program is, (c) how to describe what it means for the program to be fair, and (d) how to fully automate the verification process.

A *fairness verifier* takes a decision-making program P and a dataset or a population model M as input. While using a concrete dataset simplifies the verification problem [26], it also raises questions of whether the dataset is representative for the population for which we are trying to prove fairness. A population model M can be a probabilistic model that defines a joint probability distribution on the inputs of P [3].

There are many ways to define when and why a program is fair or unfair. For example, if we want to prove group fairness — for example, that the algorithm is just as likely to hire a minority applicant (m) as it is for other, non-minority applicants we could define a post-condition like the following:

$$\frac{Pr[P(v)=true|v_s=m]}{Pr[P(v)=true|v_s \neq m]} > 1 - \epsilon$$

The verifier then will prove or disprove that P is fair for the given population. For group fairness, the verification problem reduces to computing the probability of a number of events with respect to the program and the population model. For individual fairness, proving fairness requires more complex reasoning involving multiple runs of the program, a notoriously hard problem.

In the case of a negative result, the verifier should provide the users with a proof of unfairness. Depending on the fairness definition, producing a human readable proof might be challenging as the argument might involve multiple and potentially infinite inputs. For example, in the case of group fairness, it might be challenging to explain why the program outputs true on 40% of the minority inputs and on 70% of the majority inputs.

In *fairness-aware programming* [4], developers can state fairness expectations natively in their code, and have a run-time system monitor decision-making and report violations of fairness. This approach is analogous to the notion of assertions in modern programming languages. For instance, the developer might assert that $x > 0$, indicating that they expect the value of x to be positive at a certain point in the code. The difficulty, however, is that fairness definitions are typically probabilistic, and therefore detecting their violation cannot be done through a single execution as in traditional assertions. Instead, we have to monitor the decisions made by the procedure, and then, using statistical tools, infer that a fairness property does not hold with reasonably high confidence.

For example, consider a movie recommendation system, where user data has been used to train a recommender that, given a user profile, recommends a single movie [4]. Suppose that the recommender was constructed with the goal of ensuring that male users are not isolated from movies with a strong female lead. Then, the developer may add the following specification to their recommender code:

$$@spec(pr(femaleLead(r)|s = male) > 0.2)$$

The above specification ensures that for male users, the procedure recommends a movie with a female lead at least 20% of the time.

To determine that a procedure f satisfies a fairness specification ϕ , we need to maintain statistics over the inputs and outputs of the procedure f as it is being applied. Specifically, we compile the specification ϕ into run-time monitoring code that executes every time f is

applied, storing aggregate results of every probability event appearing in ϕ . In the earlier example with movie recommendation, the monitoring code would maintain the number of times the procedure returned true for a movie with a female lead. Again, a big challenge is checking individual fairness. In this case, the run-time system has to remember all decisions made explicitly, so as to compare new decisions with past ones.

10 Open Challenges

As we have just begun to realize the need for fairness in rankings and recommender systems, we next highlight few critical open issues and challenges for future work, which aim to support advanced services for making accountable complex rankings and recommender systems.

A codification of definitions. As described in Sections 2 and 3, there is not a universal definition for expressing fairness in rankings and recommenders. Instead, the list of potential definitions is very long.

Specifically, just for fairness alone, there exist a plethora of different definitions. Some of them are not even compatible, in the sense that there is no method that can satisfy all of them simultaneously, except in highly constrained special cases [41]. Furthermore, there is also a need to make explicit the correspondence between these definitions and the interpretation of bias or diversity, that each of them materializes. The limitations of each definition, the compatibility among them, the incurred trade-offs, the domain of applicability, assumptions and parameters are not well understood yet.

Moreover, it is interesting to see how the general public views fairness in decision making. By testing people's perception of different fairness definitions, we can understand definitions of fairness that are appropriate for particular contexts [29],[53]. One such attempt investigates which definitions people perceive to be the fairest in the context of loan decisions [59], and whether fairness perceptions change with the addition of sensitive information (i.e., race of the loan applicants).

Lack of data. A major challenge is that the available data is often limited [8]. This way, any analysis is done with data that has been acquired independently, through a process on which the data scientist has limited control. Collecting more data for analysis is challenging, and will help to discover more types of biases on it. In the long run, one could envision benchmarks for measuring the societal impact of an algorithm along the lines of the TPC benchmark¹ for database performance.

A unified approach for the data pipeline. One limitation of the current work is that fairness has been studied for specific tasks in isolation, with most current work in fairness focusing on the classification task with the goal of non-discrimination. However, there is a need to consider fairness along the whole data pipeline [63]. This includes pre-processing steps, such as data selection, acquisition, cleaning, filtering and integration. Pre-processing for removing bias, can be viewed as the action of repairing, e.g., by replacing, modifying, or deleting data that cause bias. This pipeline also includes post-processing steps, such as data representation, data visualization and user interfaces. For example, how results are presented can introduce bias, and this is why we need to understand the implications for fairness.

Lack of evaluation tools. Besides coming up with the correct way of defining fairness in rankings and recommenders, there is also a need for tools for investigating bias, and evaluating the quality of a dataset, an algorithm, or a system. There are some first attempts, such as IBM's AI Fairness 360² and Tensor Flow's Fairness Indicators³. However, both of them focus mainly on statistical group measures of fairness in classification. Concepts, like context and provenance, are important in designing such tools.

In this direction, we also need efficient ways for measuring fairness and monitoring its evolution over time. Previous research in stream processing and incrementally maintaining statistics may be relevant here.

Perhaps the most pending question is how to quantify the long-term impact of enforcing methods that target at ensuring fairness. Would they work in favor of the social good, or would they backfire in ways that we cannot predict?

Lack of real applications of fairness. While a lot of work is done in a research setting, we still do not see many actual applications and their results. There are many challenges in making algorithms and systems fairer in the real-world. A company, for example, needs to consider its business metrics (e.g., click-through rate, purchases) and make sure that these are not affected. For example, how to design algorithms that take into account all these different objectives is challenging.

An example of a real application of fairness is found in LinkedIn [27], where they developed a fair framework and they applied it to LinkedIn Talent Search. Online A/B tests showed considerable improvement in the fairness metrics without a significant impact on the

¹ <http://www.tpc.org/information/benchmarks.asp>

² <https://aif360.mybluemix.net/>

³ <https://aif360.mybluemix.net/>

business metrics, which paved the way for deployment to LinkedIn users worldwide.

A multi-level architecture of value systems and algorithms. A problem intrinsic to the definition of all fairness definitions in rankings and recommenders is the fact that they attempt to quantify philosophical, legal, often elusive, and even controversial notions of justice and social good. Complexity is aggregated when notions that reflect value systems and beliefs interact with the mechanisms for implementing them. In the very least, there should be a clear distinction between what constitutes a belief and what is the mechanism, or measure, for codifying this belief. From a technical point of view, we should then be able to focus on assessing whether a proposed measure is an appropriate codification of a given belief as opposed to assessing the belief itself.

This calls for developing different levels of abstractions and mappings between them. This is somehow reminiscent of how data independence is supported in database management systems by the three-level architecture with the physical, conceptual, and external level and the mappings between these levels [54]. At the lower level, we could have beliefs and value systems and at the higher level fairness definitions. Intermediate levels could be used to support transformations for getting from the lower to the higher level.

Fairness in systems and algorithms. While the focus of this survey is on about fairness in decision-making processes, and in particular on rankings and recommenders systems, there are several other cases where a system needs to make decisions, and where fairness is also important. There are several examples more familiar to the database community, where fairness has been studied, such as cache allocation [43], multiple resource allocation [28] and scheduling [30]. How to embed models and algorithms for fairness into any system that involves any type of decision making is an open challenge and opportunity. Of course, these may need to evolve and new models and algorithms may be needed in different contexts, where the principles of fairness may be different. What works well for a recommendation problem may not work well for a query optimizer or resource sharing in cloud computing.

11 Conclusions

Ranking and recommender systems have several applications, such as hiring, lending, and college admissions, where the notion of fairness is very important since decision making is involved. We have only begun to understand the nature, representation and variety of the sev-

eral definitions of fairness, and the appropriate methods for ensuring it.

In this article, we follow a systematic and structured approach to explain the various sides of and approaches to fairness. First, we lay the ground by presenting general fairness definitions. Then, we zoom in on models and definitions for rankings and recommendations. We organize them in a taxonomy and highlight their differences and commonalities. This analysis naturally leads to a number of open questions, like: (a) How do fairness definitions fare? (b) Which definition is suitable for which context? (c) How do people perceive fairness in different contexts? (d) What does it mean to be fair after all? Is there a unified way to be able to judge whether an outcome or an algorithm is fair?

We move on to describing solutions for fair rankings and recommendations. We organize the approaches to tackle unfairness or ensure a fairer outcome into pre-, in- and post-processing approaches. Within each category, we further classify them along several dimensions. It is still very early to say which one works best for which context. There is no evaluation that puts them all under the same lens and there are generally not conclusive results as to which fare better. It may be the case that a combination of methods should be applied, e.g., combining pre-processing and in-processing steps. This is also an open question as different efforts have adapted a single angle in the problem.

While the focus of this survey is on about fairness in rankings and recommender systems, we discuss several other cases where a system needs to make decisions and fairness is also important, and how we can verify whether a program is fair. Finally, we discuss open research challenges pertaining to fairness in the broader context of data management and on designing, building, managing, and evaluating fair data systems and applications.

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