Cpts 515. 9/14/2020

App of SCC Alg.

A looping SCC is one with

① size > 1, or

② size = 1 but there is a self-loop on the node.

A looping SCC contains a loop. (a cycle).

Most important app of SCC Alg:

① Identifying loops on a graph.

② Identifying inf. long run of a software system defined by a graph.

③ Arguing "liveness" in a software system.

barrier is high.

properties held on infinite runs.

Other applications: social network & grouping.

security on a group. (Access table type authentication defined on a network of parties)

Why entry barrier for a topic is important:

① Low barrier = high school kids can do "research" in the area.

good

it's open for everyone

bad

the area is simple ← wasting time

OR Nobody understands it

takes decades.

② high barrier = only a few truly understand it. there is depth inside the area.

Reasoning on inf. paths of a graph.

inf. paths = ω-paths.

Exaple.

Given: a directed graph G
three designated nodes v_1, v_2, v_3.

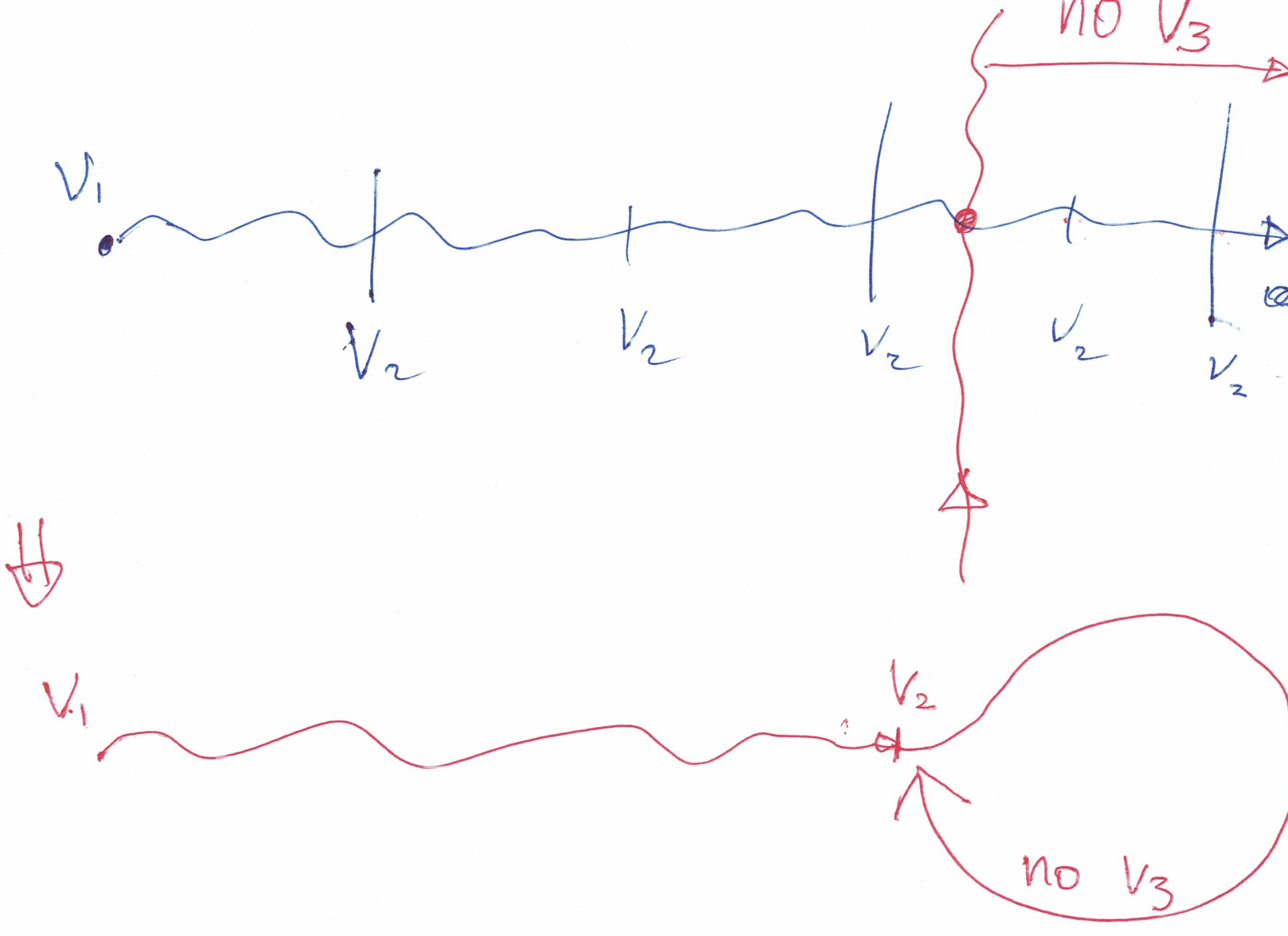
Question: ∃ an ω-path α s.t.

① α starts from v_1

② α passes v_2 for inf. # of times,

③ α passes v_3 for only finitely many times. ?

using SCC.



no V_3
V_1
V_2
V_2
V_2
V_2
V_2
∞
V_1
V_2
no V_3

Alg: (ideas) we need check:

① $v_1 \rightsquigarrow v_2$ ⟸ easy by DFS.

② There is a loop on v_2 where there is no v_3.

A. Drop v_3 from G

B. Run SCC on the resulting graph

C. Make sure there is a looping SCC containing v_2.

Generalization:

① Define a formalism/logic on properties defined on infinite paths.

② Each point has a color; e.g.

③. Formal Examples:

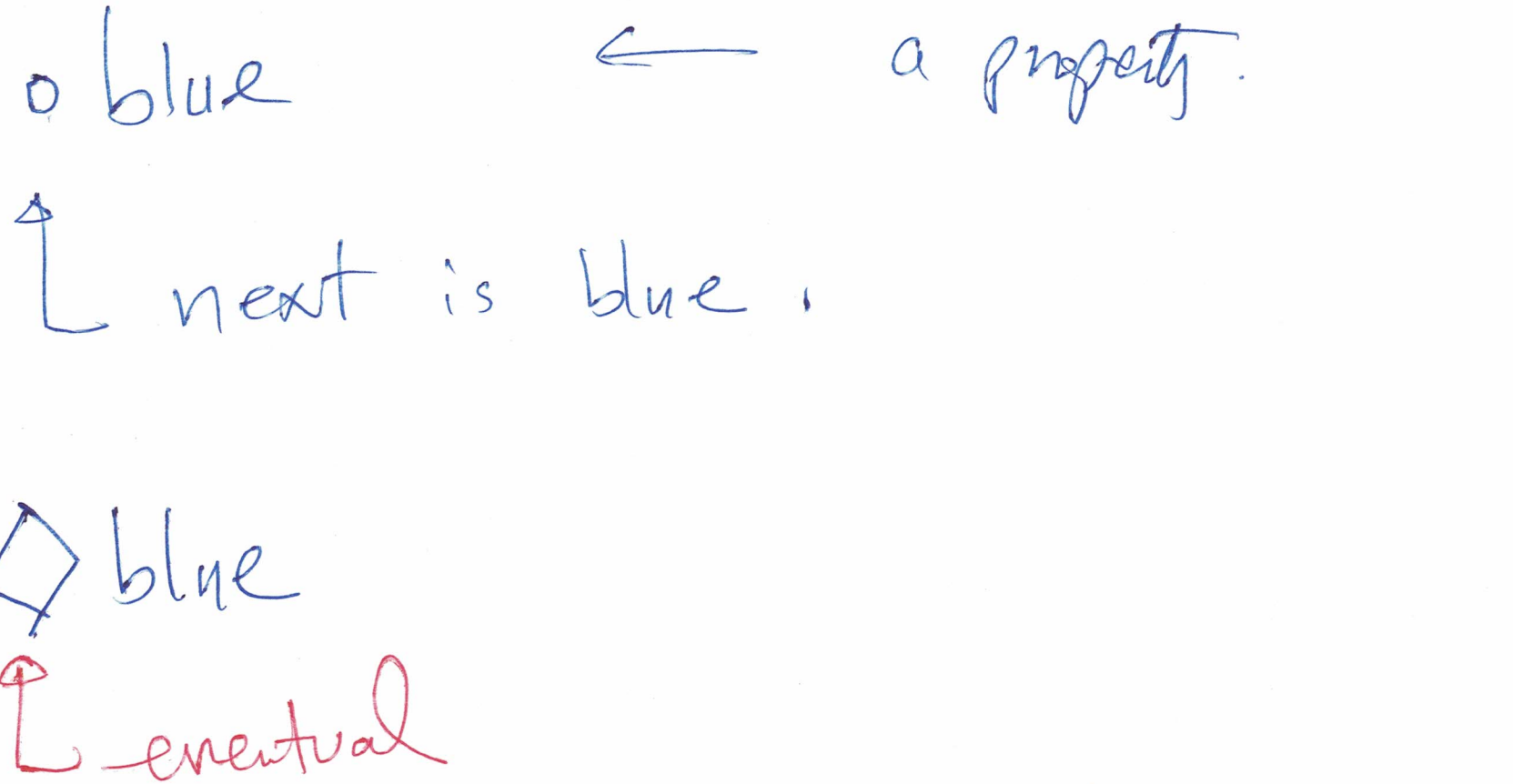
$\Box$ blue. is a property.

↑ forever

$\alpha \models \Box$ blue

("α sat. the property $\Box$ blue")

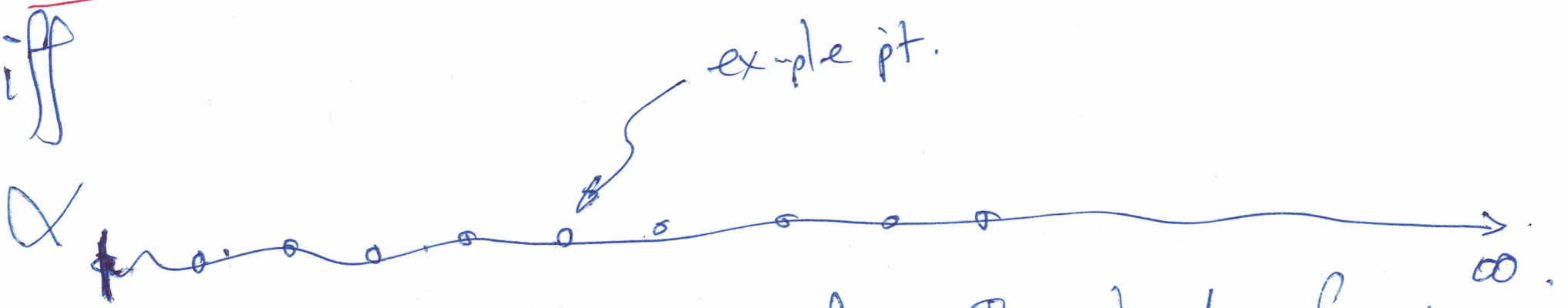
if you look from the start of α,
- any point afterwards (including the start)
is blue.



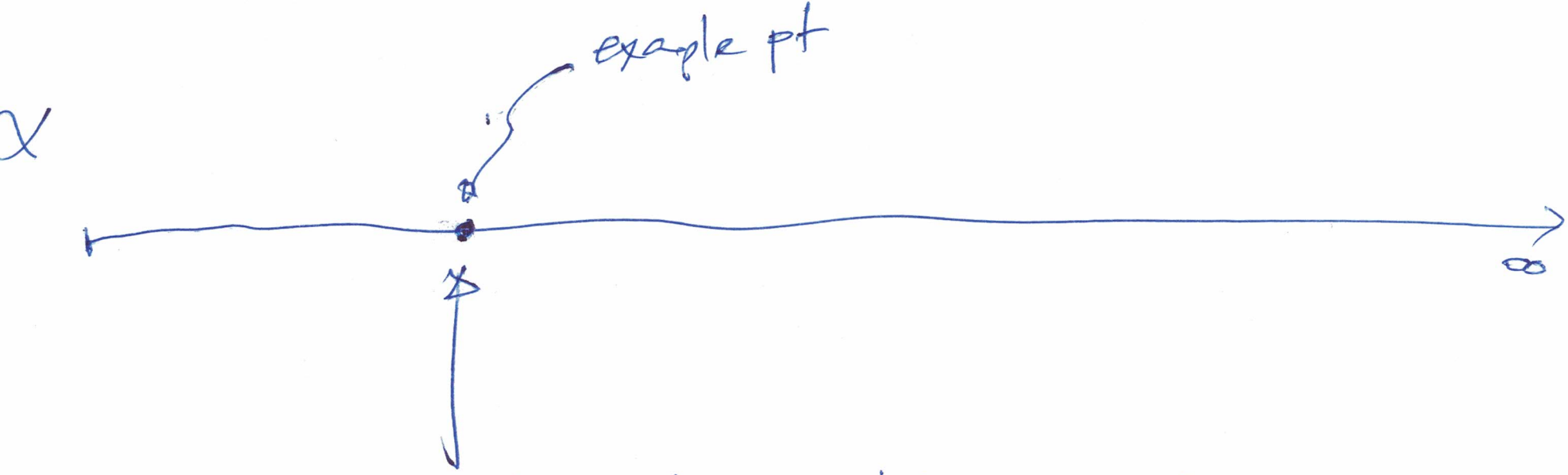
$\alpha \models \Diamond$ blue iff when you look forward from the start of α, there is a future point (which could be the start) on which you see blue.

more cuplex Exaple:

$\alpha \models \Box(\text{blue} \longrightarrow \Diamond \text{green})$



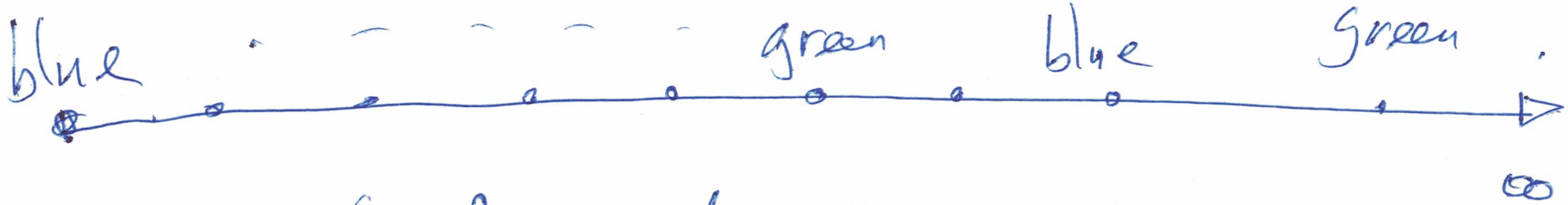
when I look forward from the start of α, "blue $\longrightarrow \Diamond$ green" is true on <u>every</u> time point.



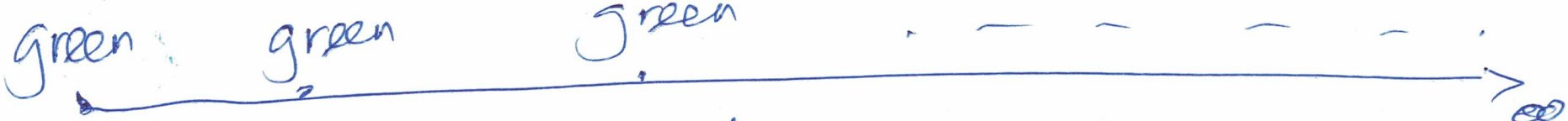
On this pt, blue ⟶ ◇ green is true.

Can you give an exple of sat.

$\Box(\text{blue} \rightarrow \Diamond \text{green})$?



(inf. # of blue's and green's). ✓



(inf # of green's). ✓

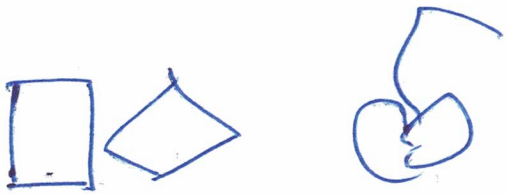
red red red green red red green green ....

works!

red red yellow - — — — —

(no blue, no green)

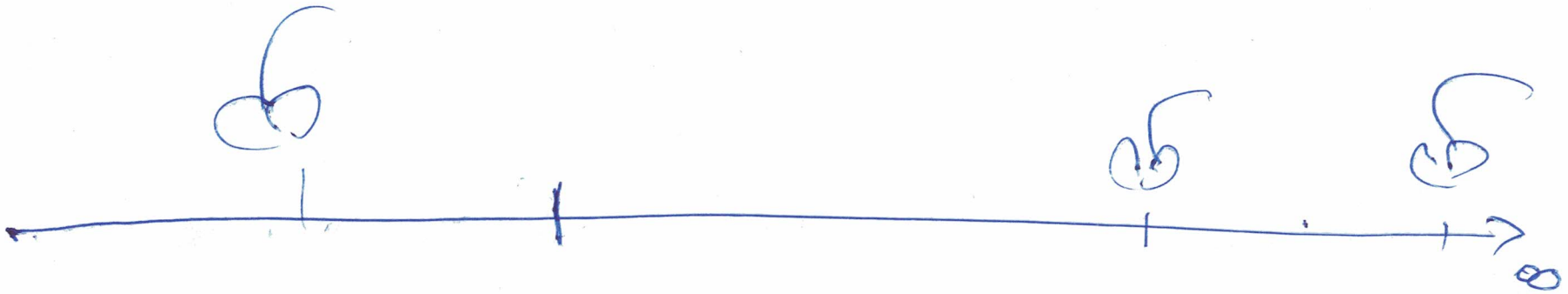
Real world:



means

some property

The prpety is inf. often,



Summary: (Thomy Francez'05)

all such properties (formed by $\Box$, $\Diamond$, $\circ$)

can be automatically checked using SCC:

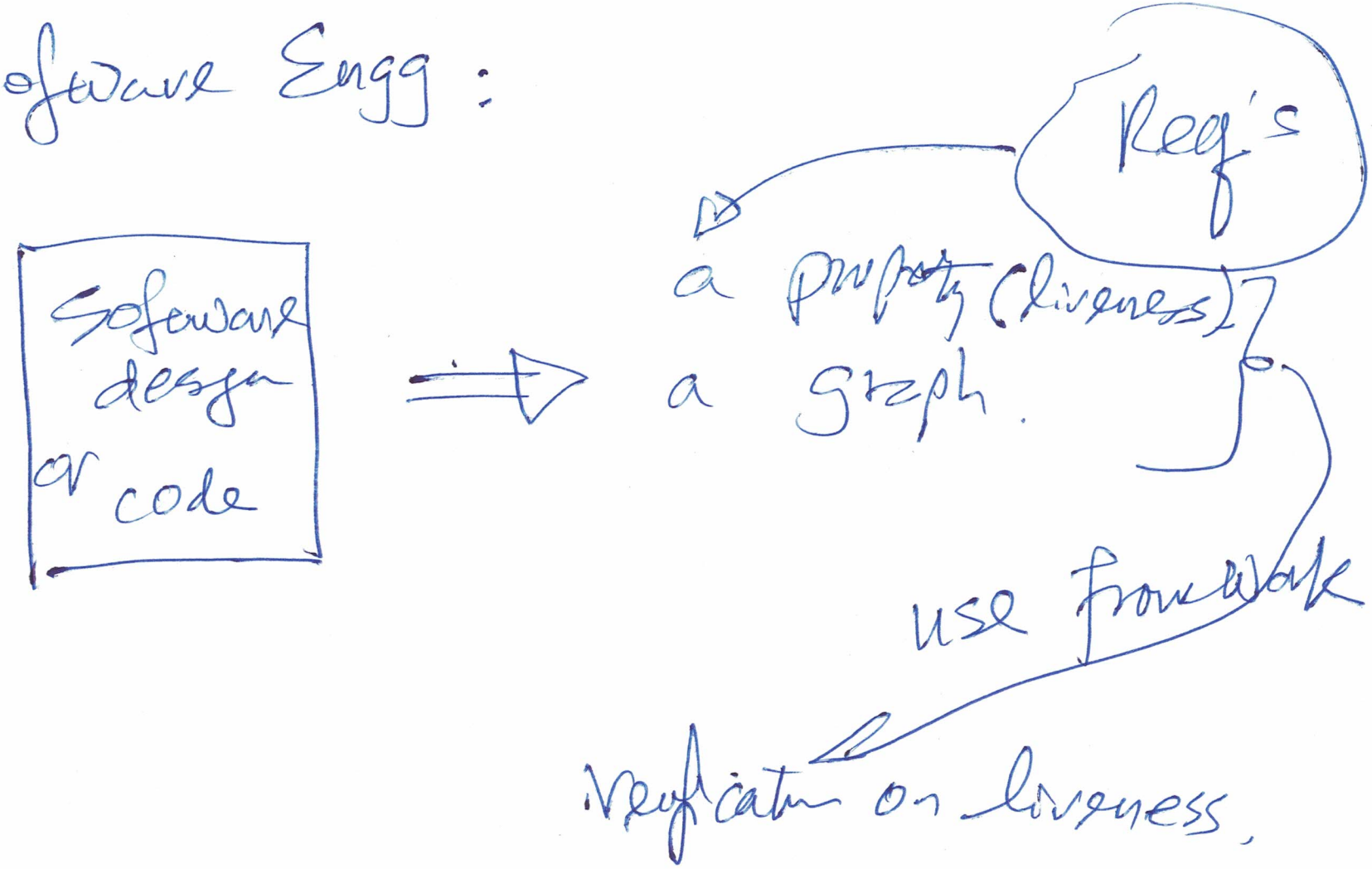
Given: a graph G with start v_0;

a property $\mathcal{G}$.

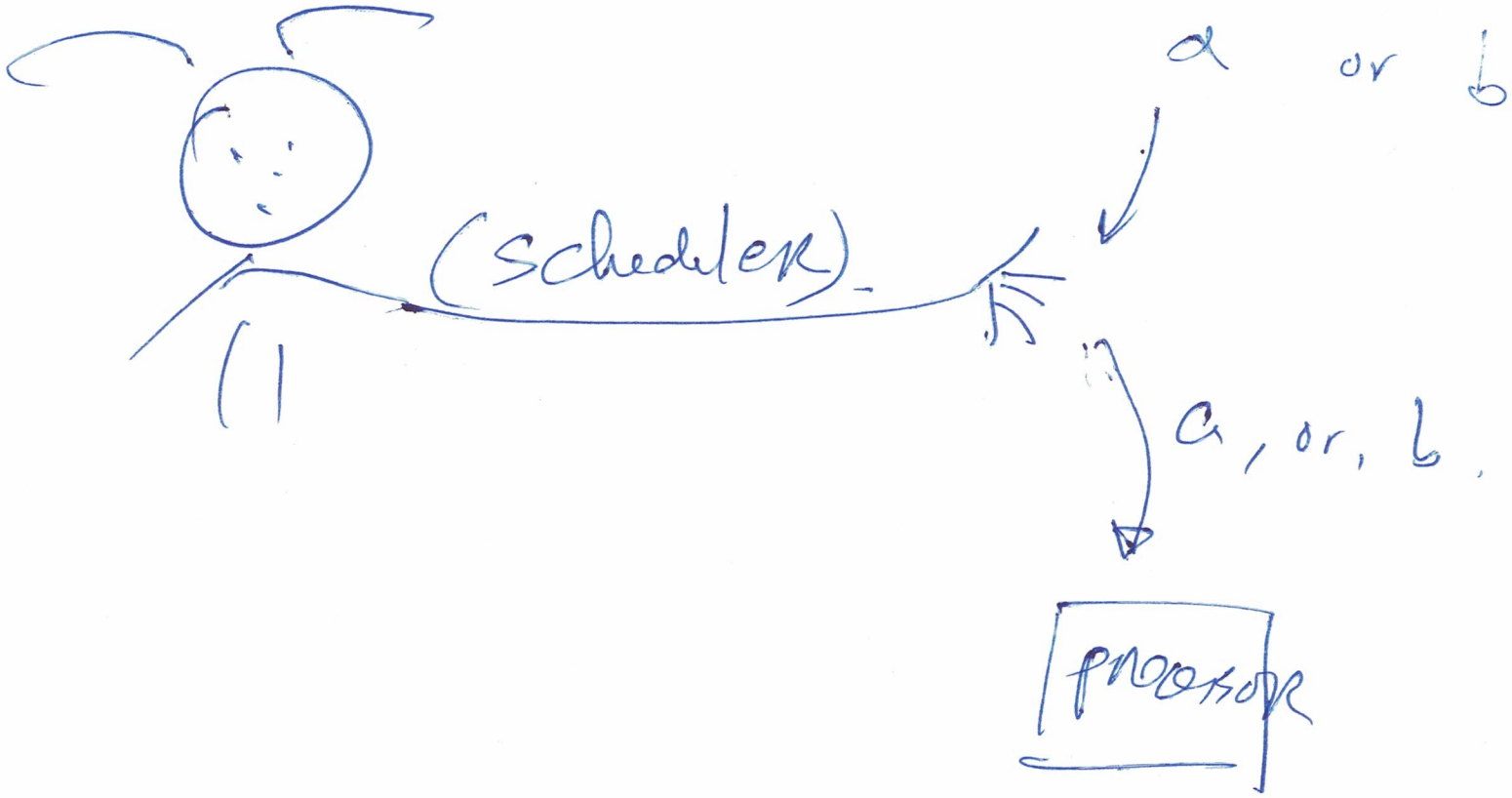
Question: Is there an inf. walk starting from v_0 s.t. the walk sat. the property $\mathcal{G}$?

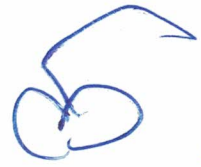
(you can pre-assign a color to each node in G; "color" can be interpreted differently in reality; e.g. red ⇒ rainy, blue ⇒ leaf, ...)

In Software Engg :



I have a design for a job scheduler;
Suppose that we have two kinds of
jobs: a, b.



"scheduler is fair" =  .

you need "prove" ← software testing is useless here.

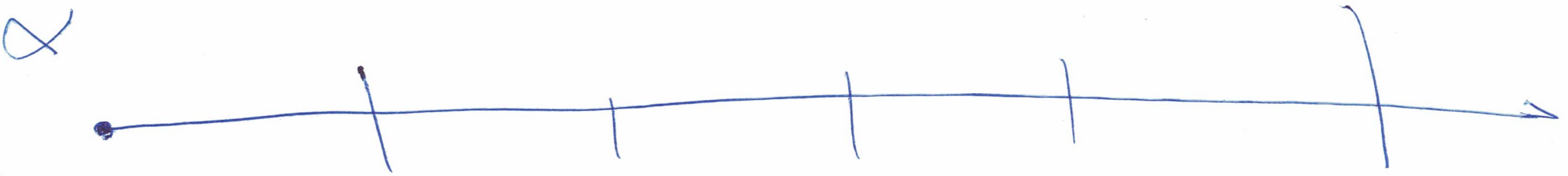
YOUR scheduler ⊨  .

① Formalize  into the logic mentioned earlier

② translate scheduler into a graph

③ use the framework.

What's fairness? You def.



there are ∞f. many positrons/tones s.t.

$\#_a \approx \#_b$ scheduled so far.