Washington State University
School of Electrical Engineering and Computer Science
Fall 2021

CptS 440/540 Artificial Intelligence
Homework 10 – Solution
Due: November 18, 2021 (11:59pm)

General Instructions: Put your answers to the following problems into a PDF document and upload the document as your submission for Homework 10 for the course CptS 440 Pullman (all sections of CptS 440 and 540 are merged under the CptS 440 Pullman section) on the Canvas system by the above deadline. Note that you may submit multiple times, but we will only grade the most recent entry submitted before the deadline.

1. Consider the table of data below, which contains 11 examples of the class value BuyJersey based on the features Weather, Uniform and Win. Suppose we want to classify the new instance <Weather=cloudy, Uniform=gray, Win=yes> using the Naïve Bayes learning method. Compute the following. Show your work. Use at least three-digit precision in all answers. Note: use the "add 1 / |values|" method only if the original P(feature | class) = 0.

Weather	Uniform	Win	BuyJersey
clear	crimson	yes	yes
clear	crimson	no	yes
clear	gray	yes	yes
clear	gray	no	no
cloudy	crimson	yes	yes
cloudy	crimson	no	yes
cloudy	gray	no	no
rainy	crimson	yes	yes
rainy	crimson	no	yes
rainy	gray	yes	no
rainy	gray	no	no

   a. Compute the prior probabilities P(BuyJersey=yes) and P(BuyJersey=no).
   b. Compute P(Weather | BuyJersey) for all values of Weather ∈ {clear, cloudy, rainy} and BuyJersey ∈ {yes, no}.
   c. Compute P(Uniform | BuyJersey) for all values of Uniform ∈ {crimson, gray} and BuyJersey ∈ {yes, no}.
   d. Compute P(Win | BuyJersey) for all values of Win ∈ {yes, no} and BuyJersey ∈ {yes, no}.
   e. Compute P(BuyJersey=yes | Weather=cloudy, Uniform=gray, Win=yes) and P(BuyJersey =no | Weather=cloudy, Uniform=gray, Win=yes).
   f. Which class would Naïve Bayes choose for the new instance?

Solution:

a. P(BuyJersey=yes) = 7/11 = 0.636
   P(BuyJersey=no) = 4/11 = 0.364

b. P(Weather=clear | BuyJersey=yes) = 3/7 = 0.429
   P(Weather=cloudy | BuyJersey=yes) = 2/7 = 0.286
   P(Weather=rainy | BuyJersey=yes) = 2/7 = 0. 286
   P(Weather=clear | BuyJersey=no) = 1/4 = 0.25
   P(Weather=cloudy | BuyJersey=no) = 1/4 = 0.25
   P(Weather=rainy | BuyJersey=no) = 2/4 = 0.5

c. P(Uniform=crimson | BuyJersey=yes) = 6/7 = 0.857
   P(Uniform=gray | BuyJersey=yes) = 1/7 = 0.143
   P(Uniform= crimson | BuyJersey=no) = 0/4 = 0
      Since values(Uniform)=2, recompute as (0+1)/(4+2) = 1/6 = 0.167
   P(Uniform=gray | BuyJersey=no) = 4/4 = 1

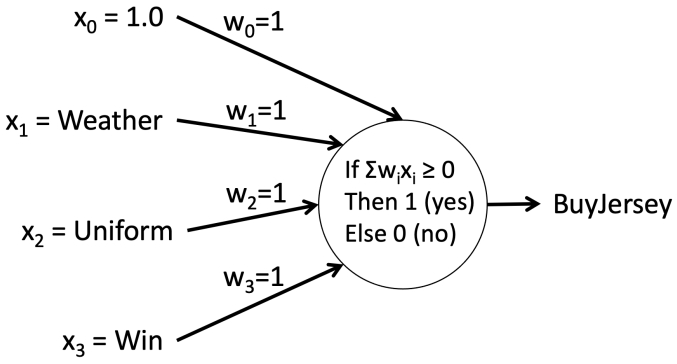
d. P(Win=yes | BuyJersey=yes) = 4/7 = 0.571
   P(Win=no | BuyJersey=yes) = 3/7 = 0.429
   P(Win=yes | BuyJersey=no) = 1/4 = 0.25
   P(Win=no | BuyJersey=no) = 3/4 = 0.75

e. P(BuyJersey=yes | Weather=cloudy, Uniform=gray, Win=yes)
   = α * P(Weather=cloudy, Uniform=gray, Win=yes | BuyJersey=yes) * P(BuyJersey=yes)
   = α * P(Weather=cloudy | BuyJersey=yes) * P(Uniform=gray | BuyJersey=yes) * P(Win=yes | BuyJersey=yes) * P(BuyJersey=yes)
   = α * (2/7) * (1/7) * (4/7) * (7/11)
   = α * 0.015 = <u>0.395</u>
   P(BuyJersey=no | Weather=cloudy, Uniform=gray, Win=yes)
   = α * P(Weather=cloudy, Uniform=gray, Win=yes | BuyJersey=no) * P(BuyJersey=no)
   = α * P(Weather=cloudy | BuyJersey=no) * P(Uniform=gray | BuyJersey=no) * P(Win=yes | BuyJersey=no) * P(BuyJersey=no)
   = α * (1/4) * (1) * (1/4) * (4/11)
   = α * 0.023 = <u>0.605</u>
   α = 1 / (0.015 + 0.023) = 26.316

f. Naïve Bayes would classify the new instance as BuyJersey=no.

2. *Perceptron*. Train a perceptron on the 11 examples in the above table and then use the perceptron to classify the new instance. Specifically,
   a. Translate the examples (including the BuyJersey class value) according to the mapping: clear → 0, cloudy → 1, rainy → 2, crimson → 0, gray → 1, no→ 0, yes → 1. Show a new table of examples using this mapping.

b. Translate the new instance according to the mapping in part (a). Show the resulting instance. How would the perceptron below (before any training) classify the new instance? Show your work.
c. Train the perceptron below using one pass through the 11 examples by updating the weights according to the perceptron learning rule (slide 34 of Learning lecture). Assume the initial weights are all equal to 1.0, and $\eta = 0.5$. Consider each example in the order presented in the table above and show the weight updates for each incorrectly-classified example. Show your work. *Hint*: There should be three weight updates.
d. How would the trained perceptron from part (c) classify the new instance <Weather=cloudy, Uniform=gray, Win=yes>? Show your work.



Solution:

a. Table.

Weather	Uniform	Win	BuyJersey
0	0	1	1
0	0	0	1
0	1	1	1
0	1	0	0
1	0	1	1
1	0	0	1
1	1	0	0
2	0	1	1
2	0	0	1
2	1	1	0
2	1	0	0

b. New instance: $<x_1=1, x_2=1, x_3=1>$
$w_0x_0 + w_1x_1 + x_2x_2 + w_3x_3 = (1)(1) + (1)(1) + (1)(1) + (1)(1) = 4$
Since $4 \geq 0$, the output of the network is 1 (BuyJersey=yes)

c. One pass.

```
Example 1 correct
Example 2 correct
Example 3 correct
Example 4 incorrect
  w0 = w0 + n(y – o)x0 = 1 + (0.5)(0 – 1)(1) = 0.5
  w1 = w1 + n(y – o)x1 = 1 + (0.5)(0 – 1)(0) = 1.0
  w2 = w2 + n(y – o)x2 = 1 + (0.5)(0 – 1)(1) = 0.5
  w3 = w3 + n(y – o)x3 = 1 + (0.5)(0 – 1)(0) = 1.0
  New weights: w0=0.5 w1=1.0 w2=0.5 w3=1.0
Example 5 correct
Example 6 correct
Example 7 incorrect
  w0 = w0 + n(y – o)x0 = 0.5 + (0.5)(0 – 1)(1) = 0.0
  w1 = w1 + n(y – o)x1 = 1.0 + (0.5)(0 – 1)(1) = 0.5
  w2 = w2 + n(y – o)x2 = 0.5 + (0.5)(0 – 1)(1) = 0.0
  w3 = w3 + n(y – o)x3 = 1.0 + (0.5)(0 – 1)(0) = 1.0
  New weights: w0=0.0 w1=0.5 w2=0.0 w3=1
Example 8 correct
Example 9 correct
Example 10 incorrect
  w0 = w0 + n(y – o)x0 = 0.0 + (0.5)(0 – 1)(1) = -0.5
  w1 = w1 + n(y – o)x1 = 0.5 + (0.5)(0 – 1)(2) = -0.5
  w2 = w2 + n(y – o)x2 = 0.0 + (0.5)(0 – 1)(1) = -0.5
  w3 = w3 + n(y – o)x3 = 1.0 + (0.5)(0 – 1)(1) = 0.5
  New weights: w0=-0.5 w1=-0.5 w2=-0.5 w3=0.5
Example 11 correct

Final Weights: w0=-0.5 w1=-0.5 w2=-0.5 w3=0.5
```

d. New instance (-0.5)(1) + (-0.5)(1) + (-0.5)(1) + (0.5)(1) = -1.0
Since -1.0 < 0, the output of the network is 0 (BuyJersey=no)

3. *CptS 540 Students Only*: Put the 11 training examples from the initial table above (used in Problem 1) into an ARFF file suitable for input to WEKA. Follow the procedure below to run the Naive Bayes classifier.
   a. Download and install WEKA from www.cs.waikato.ac.nz/ml/weka/downloading.html.
   b. Start WEKA and choose the Explorer mode.
   c. Under the Preprocess tab, choose "Open file..." and load your ARFF file.
   d. Under the Classify tab, choose the "bayes→NaiveBayes" classifier.
   e. Under Test options, choose "Use training set".
   f. Click Start to run the classifier on your data.
   g. Include your ARFF file and WEKA's output in your submission.

Solution:

ARFF file:

```
@relation buyjersey

@attribute weather {clear, cloudy, rainy}
@attribute uniform {crimson, gray}
@attribute win {yes, no}
@attribute buyjersey {yes, no}

@data
clear,crimson,yes,yes
clear,crimson,no,yes
clear,gray,yes,yes
clear,gray,no,no
cloudy,crimson,yes,yes
cloudy,crimson,no,yes
cloudy,gray,no,no
rainy,crimson,yes,yes
rainy,crimson,no,yes
rainy,gray,yes,no
rainy,gray,no,no
```

WEKA Output:

```
=== Run information ===

Scheme:       weka.classifiers.bayes.NaiveBayes
Relation:     buyjersey
Instances:    11
Attributes:   4
              weather
              uniform
              win
              buyjersey
Test mode:    evaluate on training data

=== Classifier model (full training set) ===

Naive Bayes Classifier

                Class
Attribute         yes     no
               (0.62) (0.38)
============================
weather
  clear            4.0    2.0
  cloudy           3.0    2.0
  rainy            3.0    3.0
  [total]         10.0    7.0

uniform
  crimson          7.0    1.0
  gray             2.0    5.0
  [total]          9.0    6.0

win
  yes              5.0    2.0
  no               4.0    4.0
  [total]          9.0    6.0
```

```
Time taken to build model: 0 seconds

=== Evaluation on training set ===

Time taken to test model on training data: 0 seconds

=== Summary ===

Correctly Classified Instances          10               90.9091 %
Incorrectly Classified Instances         1                9.0909 %
Kappa statistic                          0.8136
Mean absolute error                      0.2047
Root mean squared error                  0.2399
Relative absolute error                 43.6901 %
Root relative squared error             49.8209 %
Total Number of Instances               11

=== Detailed Accuracy By Class ===

                 TP Rate   FP Rate   Precision   Recall    F-Measure   MCC       ROC Area   PRC Area
Class
                 0.857    0.000    1.000      0.857    0.923      0.828    1.000     1.000     yes
                 1.000    0.143    0.800      1.000    0.889      0.828    1.000     1.000     no
Weighted Avg.    0.909    0.052    0.927      0.909    0.911      0.828    1.000     1.000

=== Confusion Matrix ===

 a b   <-- classified as
 6 1 | a = yes
 0 4 | b = no
```

4. *Extra Credit*. Continue training the perceptron in 2(c) until it correctly classifies all 11 examples. Show the final perceptron weights. *Hint*: The perceptron should correctly classify all 11 examples on the 6th pass through the examples (where 2(c) is the 1st pass).

Solution:

```
Pass #1 (same as 2c)
  Example 1 correct
  Example 2 correct
  Example 3 correct
  Example 4 incorrect, new weights: w0=0.5 w1=1 w2=0.5 w3=1
  Example 5 correct
  Example 6 correct
  Example 7 incorrect, new weights: w0=0 w1=0.5 w2=0 w3=1
  Example 8 correct
  Example 9 correct
  Example 10 incorrect, new weights: w0=-0.5 w1=-0.5 w2=-0.5 w3=0.5
  Example 11 correct
Pass #2
  Example 1 correct
  Example 2 incorrect, new weights: w0=0 w1=-0.5 w2=-0.5 w3=0.5
```

```
  Example 3 correct
  Example 4 correct
  Example 5 correct
  Example 6 incorrect, new weights: w0=0.5 w1=0 w2=-0.5 w3=0.5
  Example 7 incorrect, new weights: w0=0 w1=-0.5 w2=-1 w3=0.5
  Example 8 incorrect, new weights: w0=0.5 w1=0.5 w2=-1 w3=1
  Example 9 correct
  Example 10 incorrect, new weights: w0=0 w1=-0.5 w2=-1.5 w3=0.5
  Example 11 correct
Pass #3
  Example 1 correct
  Example 2 correct
  Example 3 incorrect, new weights: w0=0.5 w1=-0.5 w2=-1 w3=1
  Example 4 correct
  Example 5 correct
  Example 6 correct
  Example 7 correct
  Example 8 correct
  Example 9 incorrect, new weights: w0=1 w1=0.5 w2=-1 w3=1
  Example 10 incorrect, new weights: w0=0.5 w1=-0.5 w2=-1.5 w3=0.5
  Example 11 correct
Pass #4
  Example 1 correct
  Example 2 correct
  Example 3 incorrect, new weights: w0=1 w1=-0.5 w2=-1 w3=1
  Example 4 incorrect, new weights: w0=0.5 w1=-0.5 w2=-1.5 w3=1
  Example 5 correct
  Example 6 correct
  Example 7 correct
  Example 8 correct
  Example 9 incorrect, new weights: w0=1 w1=0.5 w2=-1.5 w3=1
  Example 10 incorrect, new weights: w0=0.5 w1=-0.5 w2=-2 w3=0.5
  Example 11 correct
Pass #5
  Example 1 correct
  Example 2 correct
  Example 3 incorrect, new weights: w0=1 w1=-0.5 w2=-1.5 w3=1
  Example 4 correct
  Example 5 correct
  Example 6 correct
  Example 7 correct
  Example 8 correct
  Example 9 correct
  Example 10 correct
  Example 11 correct
Pass #6
  Example 1 correct
  Example 2 correct
  Example 3 correct
  Example 4 correct
```

```
  Example 5 correct
  Example 6 correct
  Example 7 correct
  Example 8 correct
  Example 9 correct
  Example 10 correct
  Example 11 correct
Final Weights:
 w0=1 w1=-0.5 w2=-1.5 w3=1
```