Washington State University
School of Electrical Engineering and Computer Science
Fall 2021

CptS 440/540 Artificial Intelligence
Homework 6 – Solution
Due: October 14, 2021 (11:59pm pacific time)

General Instructions: Put your answers to the following problems into a PDF document and upload the document as your submission for Homework 6 for the course CptS 440 Pullman (all sections of CptS 440 and 540 are merged under the CptS 440 Pullman section) on the Canvas system by the above deadline. Note that you may submit multiple times, but we will only grade the most recent entry submitted before the deadline.

1. Convert the following first-order logic sentence into conjunctive normal form (CNF). There is no need to show intermediate steps.

   $\forall x$ Stench(x) $\Rightarrow \exists y$ Adjacent(x,y) $\wedge$ At(Wumpus,y)

   Solution:
   Step 1: $\forall x$ $\neg$Stench(x) $\vee$ $\exists y$ Adjacent(x,y) $\wedge$ At(Wumpus,y)
   Step 2: Negations already inside
   Step 3: Variables already standardized
   Step 4: $\forall x$ $\neg$Stench(x) $\vee$ (Adjacent(x,SK1(x)) $\wedge$ At(Wumpus,SK1(x)))
   Step 5: $\neg$Stench(x) $\vee$ (Adjacent(x,SK1(x)) $\wedge$ At(Wumpus,SK1(x)))
   Step 6: ($\neg$Stench(x) $\vee$ (Adjacent(x,SK1(x))) $\wedge$ ($\neg$Stench(x) $\vee$ At(Wumpus,SK1(x)))

2. Convert each of the following English statements into a single first-order logic sentence using the following constants and predicates.
   - Constants: Apples, Oranges, Chess, Go, John, Mary.
   - Predicates:
     - Likes(x,y): person x likes food y
     - Plays(x,y): person x plays game y

   a. If a person likes Apples, then they play Chess.
   b. If a person likes Oranges, then they play Go.
   c. A person likes Apples or Oranges, but not both.
   d. John likes Apples.
   e. Mary does not like anything that John likes.

   Solution:
   a. $\forall x$ Likes(x,Apples) $\Rightarrow$ Plays(x,Chess)
   b. $\forall x$ Likes(x,Oranges) $\Rightarrow$ Plays(x,Go)
   c. $\forall x$ ($\neg$Likes(x,Apples) $\wedge$ Likes(x,Oranges)) $\vee$ (Likes(x,Apples) $\wedge$ $\neg$Likes(x,Oranges))

d. Likes(John,Apples)
e. $\forall x$ Likes(John,x) $\Rightarrow$ ¬Likes(Mary,x)

3. Convert each of the first-order logic sentences in Problem 2 to conjunctive normal form (CNF). There is no need to show intermediate steps. Assign each clause a number: C1, C2, etc.

   Solution:
   a. C1: ¬Likes(x,Apples) ∨ Plays(x,Chess)
   b. C2: ¬Likes(x,Oranges) ∨ Plays(x,Go)
   c. C3: Likes(x,Oranges) ∨ Likes(x,Apples)
      C4: ¬Likes(x,Oranges) ∨ ¬Likes(x,Apples)
   d. C5: Likes(John,Apples)
   e. C6: ¬Likes(John,x) ∨ ¬Likes(Mary,x)

4. Using the clauses from Problem 3, perform a resolution proof by refutation to prove the query "Plays(Mary,Go)". Specifically,
   a. Show the clause corresponding to the negation of the query and assign it a number.
   b. Show each resolution step by indicated the two clauses being resolved (be sure to standardize the variables), the resulting clause (assign it a new number), and any necessary variable substitutions. Conclude your proof with a statement of what was proven.

   Solution:
   a. C7: ¬Plays(Mary,Go)

   b. Resolve C7 and C2:
      C2: ¬Likes(x_1,Oranges) ∨ Plays(x_1,Go)
      C7: ¬Plays(Mary,Go)
      Substitution: {x_1 / Mary}
      C8: ¬Likes(Mary,Oranges)

      Resolve C8 and C3:
      C3: Likes($x2$,Oranges) ∨ Likes(x_2,Apples)
      C8: ¬Likes(Mary,Oranges)
      Substitution: {x_2 / Mary}
      C9: Likes(Mary,Apples)

      Resolve C9 and C6:
      C6: ¬Likes(John,x_3) ∨ ¬Likes(Mary,x_3)
      C9: Likes(Mary,Apples)
      Substitution: {x_3 / Apples}
      C10: ¬Likes(John,Apples)

Resolve C10 and C5:
C5: Likes(John,Apples)
C10: ¬Likes(John,Apples)
C11: empty clause

Thus, ¬Plays(Mary,Go) must be false; therefore, Plays(Mary,Go) is true.

5. *CptS 540 Students Only*. Create an input file for the Vampire theorem prover that can be used to solve Problem 4. Include your input file and the corresponding Vampire output in the PDF document for your Homework 6 solution.

   Solution:

   Input:

```
fof(a1, axiom,
    ! [X] : (likes(X,apples) => plays(X,chess))).

fof(a2, axiom,
    ! [X] : (likes(X,oranges) => plays(X,go))).

fof(a3, axiom,
    ! [X] : ((~likes(X,apples) & likes(X,oranges)) | (likes(X,apples) &
~likes(X,oranges)))).

fof(a4, axiom, likes(john,apples)).

fof(a5, axiom,
    ! [X] : (likes(john,X) => ~likes(mary,X))).

fof(c1, conjecture, plays(mary,go)).
```

Output:

```
% Refutation found. Thanks to Tanya!
% SZS status Theorem for hw6
% SZS output start Proof for hw6
2. ! [X0] : (likes(X0,oranges) => plays(X0,go)) [input]
3. ! [X0] : ((~likes(X0,oranges) & likes(X0,apples)) | (likes(X0,oranges)
& ~likes(X0,apples))) [input]
4. likes(john,apples) [input]
5. ! [X0] : (likes(john,X0) => ~likes(mary,X0)) [input]
6. plays(mary,go) [input]
7. ~plays(mary,go) [negated conjecture 6]
8. ~plays(mary,go) [flattening 7]
10. ! [X0] : (plays(X0,go) | ~likes(X0,oranges)) [ennf transformation 2]
11. ! [X0] : (~likes(mary,X0) | ~likes(john,X0)) [ennf transformation 5]
13. ~likes(X0,oranges) | plays(X0,go) [cnf transformation 10]
15. likes(X0,oranges) | likes(X0,apples) [cnf transformation 3]
18. likes(john,apples) [cnf transformation 4]
19. ~likes(mary,X0) | ~likes(john,X0) [cnf transformation 11]
20. ~plays(mary,go) [cnf transformation 8]
22. plays(X0,go) | likes(X0,apples) [resolution 15,13]
26. 1 <=> likes(mary,apples) [avatar definition]
28. likes(mary,apples) <- (1) [avatar component clause 26]
35. likes(mary,apples) [resolution 22,20]
36. 1 [avatar split clause 35,26]
37. ~likes(john,apples) <- (1) [resolution 28,19]
39. $false <- (1) [subsumption resolution 37,18]
40. ~1 [avatar contradiction clause 39]
41. $false [avatar sat refutation 36,40]
% SZS output end Proof for hw6
% ------------------------------
% Version: Vampire 4.5.1 (commit unknown)
% Termination reason: Refutation

% Memory used [KB]: 4861
% Time elapsed: 0.001 s
% ------------------------------
% ------------------------------
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