540 Artificial Intelligence
Homework 7
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1.
   a. P (Win = true, Uniform = crimson, Weather = clear) = 0.18
   b. P (Weather = clear) = 0.18 + 0.08 + 0.06 + 0.08 = 0.40
   c. P (Uniform = crimson) = 0.18 + 0.08 + 0.05 + 0.06 +0.07 + 0.08 = 0.52
   d. P (Win = true | Weather = clear) = P (Win = true, Weather = clear) / P (Weather = clear) = (0.18 + 0.08) / (0.4) = 0.65
   e. P (Win = true | Weather = cloudy ∨ Weather = rainy)
      = P (Win = true ∧ (Weather = cloudy ∨ Weather = rainy)) / P (Weather = cloudy ∨ Weather = rainy)
      = (0.18 + 0.14) / (0.34 + 0.26)
      = 0.32/0.60 ≈ 0.53

2. Bayes rule and normalization:
   P (Win | Practice = true ∧ Healthy = true)
   = **P** (Practice = true ∧ Healthy = true | Win) * **P** (Win) / P (Practice = true ∧ Healthy = true)
   = α * **P** (Practice = true ∧ Healthy = true | Win) * **P** (Win)
   = α < P (Practice = true ∧ Healthy = true | Win = true) * P (Win = true), P (Practice = true ∧ Healthy = true | Win = false) * P (Win = false) >
   = α <0.8*0.7, 0.4*0.3>
   = α <0.56, 0.12>
   ∵ α = 1 / (0.56+0.12) ≈ 1.47
   ∴ **P** (Win | Practice = true ∧ Healthy = true) = <0.82, 0.18>

3.
   a. $breeze = \{\neg b_{1,1}, b_{2,1}, b_{1,2}\}$
      $known = \{\neg p_{1,1}, \neg p_{2,1}, \neg p_{1,2}, p_{3,1}\}$
      $frontier = \{Pit_{1,3}, Pit_{3,2}\}$
      $other = \{Pit_{2,3}, Pit_{3,3}\}$

   b. $\mathbf{P}(Pit_{2,2} \mid breeze, known)$
      = $\mathbf{P}(Pit_{2,2}, breeze, known)$ / $\mathbf{P}(breeze, known)$
      = α $\mathbf{P}$ $(Pit_{2,2}, breeze, known)$
      = $\alpha \sum_{frontier} \sum_{other} \mathbf{P}\big(Pit_{2,2}, breeze, known, frontier, other\big)$
      = $\alpha \sum_{frontier} \sum_{other} \mathbf{P}\big(breeze|Pit_{2,2}, known, frontier, other\big) *$
      $\mathbf{P}\big(Pit_{2,2}, known, frontier, other\big)$
      Since *breeze* is independent of *other* given $Pit_{2,2}$, *known*, and *frontier*:
      = $\alpha \sum_{f} \sum_{o} \mathbf{P}\big(breeze \mid Pit_{2,2}, known, frontier\big) \mathbf{P}\big(Pit_{2,2}, known, frontier, other\big)$
      = $\alpha \sum_{f} \mathbf{P}\big(breeze \mid Pit_{2,2}, known, frontier\big) \sum_{o} \mathbf{P}\big(Pit_{2,2}, known, frontier, other\big)$

Since $Pit_{2,2}$, *known*, *frontier*, *other* are independent of each other:
$= \alpha \sum_f \mathbf{P}\big(breeze \mid Pit_{2,2}, known, frontier\big) \sum_o \mathbf{P}\big(Pit_{2,2}\big)\mathrm{P}(known)\mathrm{P}(frontier)\mathrm{P}(other\big)$
$= \alpha\ \mathbf{P}(Pit_{2,2})\ \mathrm{P}(known)\ \sum_f \mathbf{P}\big(breeze \mid Pit_{2,2}, known, frontier\big)\ \mathrm{P}(frontier)\ \sum_o \mathrm{P}(other)$
Letting $\alpha' = \alpha * \mathrm{P}(known)$, and since $\sum_o \mathrm{P}(other) = 1$:
$= \alpha'\ \mathbf{P}(Pit_{2,2}) \sum_f \mathbf{P}\big(breeze \mid Pit_{2,2}, known, frontier\big)\ \mathrm{P}(frontier)$
$= \alpha' < \mathrm{P}\big(p_{2,2}\big)\big[\sum_f \mathbf{P}\big(breeze \mid p_{2,2}, known, frontier\big)\ \mathrm{P}(frontier)\big],$
$\mathrm{P}\big(\neg p_{2,2}\big)\big[\sum_f \mathbf{P}\big(breeze \mid \neg p_{2,2}, known, frontier\big)\ \mathrm{P}(frontier)\big] >$
Since $frontier = \{Pit_{1,3}, Pit_{3,2}\}$,
$= \alpha' < \mathrm{P}\big(p_{2,2}\big)[\mathrm{P}\big(breeze \mid p_{2,2}, known, p_{1,3}, p_{3,2}\big)\mathrm{P}\big(p_{1,3}, p_{3,2}\big) +$
$\mathrm{P}\big(breeze \mid p_{2,2}, known, p_{1,3}, \neg p_{3,2}\big)\mathrm{P}\big(p_{1,3}, \neg p_{3,2}\big) +$
$\mathrm{P}\big(breeze \mid p_{2,2}, known, \neg p_{1,3}, p_{3,2}\big)\mathrm{P}\big(\neg p_{1,3}, p_{3,2}\big) +$
$\mathrm{P}\big(breeze \mid p_{2,2}, known, \neg p_{1,3}, \neg p_{3,2}\big)\mathrm{P}\big(\neg p_{1,3}, \neg p_{3,2}\big)],$
$\mathrm{P}\big(\neg p_{2,2}\big)[\mathrm{P}\big(breeze \mid \neg p_{2,2}, known, p_{1,3}, p_{3,2}\big)\mathrm{P}\big(p_{1,3}, p_{3,2}\big) +$
$\mathrm{P}\big(breeze \mid \neg p_{2,2}, known, p_{1,3}, \neg p_{3,2}\big)\mathrm{P}\big(p_{1,3}, \neg p_{3,2}\big) +$
$\mathrm{P}\big(breeze \mid \neg p_{2,2}, known, \neg p_{1,3}, p_{3,2}\big)\mathrm{P}\big(\neg p_{1,3}, p_{3,2}\big) +$
$\mathrm{P}\big(breeze \mid \neg p_{2,2}, known, \neg p_{1,3}, \neg p_{3,2}\big)\mathrm{P}\big(\neg p_{1,3}, \neg p_{3,2}\big)] >$
$= \alpha' < (0.2)[(1 * 0.2 * 0.2) + (1 * 0.2 * 0.8) + (1 * 0.8 * 0.2) + (1 * 0.8 * 0.8)],$
$(0.8)[(1 * 0.2 * 0.2) + (1 * 0.2 * 0.8) + (0 * 0.8 * 0.2) + (0 * 0.8 * 0.8)] >$
$= \alpha' < 0.2, 0.16 >$
$\because \alpha' = \frac{1}{0.2+0.16} \approx 2.78$
$\therefore \mathbf{P}(Pit_{2,2} \mid breeze, known) = < 0.56, 0.44 >$

4. If we know that there is a breeze in (3,3), then the *frontier* set is $\{Pit_{1,3}, Pit_{3,2}, Pit_{2,3}\}$
   $\mathbf{P}(Pit_{2,2} \mid breeze, known)$
   $= \alpha' < \mathrm{P}\big(p_{2,2}\big)\big[\sum_f \mathbf{P}\big(breeze \mid p_{2,2}, known, frontier\big)\ \mathrm{P}(frontier)\big],$
   $\mathrm{P}\big(\neg p_{2,2}\big)\big[\sum_f \mathbf{P}\big(breeze \mid \neg p_{2,2}, known, frontier\big)\ \mathrm{P}(frontier)\big] >$
   Since $frontier = \{Pit_{1,3}, Pit_{3,2}, Pit_{2,3}\}$
   $= \alpha' < \mathrm{P}\big(p_{2,2}\big)[\mathrm{P}\big(breeze \mid p_{2,2}, known, p_{1,3}, p_{3,2}, p_{2,3}\big)\mathrm{P}\big(p_{1,3}, p_{3,2}, p_{2,3}\big) +$
   $\mathrm{P}\big(breeze \mid p_{2,2}, known, p_{1,3}, \neg p_{3,2}, p_{2,3}\big)\mathrm{P}\big(p_{1,3}, \neg p_{3,2}, p_{2,3}\big) +$
   $\mathrm{P}\big(breeze \mid p_{2,2}, known, \neg p_{1,3}, p_{3,2}, p_{2,3}\big)\mathrm{P}\big(\neg p_{1,3}, p_{3,2}, p_{2,3}\big) +$
   $\mathrm{P}\big(breeze \mid p_{2,2}, known, \neg p_{1,3}, \neg p_{3,2}, p_{2,3}\big)\mathrm{P}\big(\neg p_{1,3}, \neg p_{3,2}, p_{2,3}\big)$
   $\mathrm{P}\big(breeze \mid p_{2,2}, known, p_{1,3}, p_{3,2}, \neg p_{2,3}\big)\mathrm{P}\big(p_{1,3}, p_{3,2}, \neg p_{2,3}\big) +$
   $\mathrm{P}\big(breeze \mid p_{2,2}, known, p_{1,3}, \neg p_{3,2}, \neg p_{2,3}\big)\mathrm{P}\big(p_{1,3}, \neg p_{3,2}, \neg p_{2,3}\big) +$
   $\mathrm{P}\big(breeze \mid p_{2,2}, known, \neg p_{1,3}, p_{3,2}, \neg p_{2,3}\big)\mathrm{P}\big(\neg p_{1,3}, p_{3,2}, \neg p_{2,3}\big) +$
   $\mathrm{P}\big(breeze \mid p_{2,2}, known, \neg p_{1,3}, \neg p_{3,2}, \neg p_{2,3}\big)\mathrm{P}\big(\neg p_{1,3}, \neg p_{3,2}, \neg p_{2,3}\big)],$
   $\mathrm{P}\big(\neg p_{2,2}\big)[\mathrm{P}\big(breeze \mid \neg p_{2,2}, known, p_{1,3}, p_{3,2}, p_{2,3}\big)\mathrm{P}\big(p_{1,3}, p_{3,2}, p_{2,3}\big) +$
   $\mathrm{P}\big(breeze \mid \neg p_{2,2}, known, p_{1,3}, \neg p_{3,2}, p_{2,3}\big)\mathrm{P}\big(p_{1,3}, \neg p_{3,2}, p_{2,3}\big) +$
   $\mathrm{P}\big(breeze \mid \neg p_{2,2}, known, \neg p_{1,3}, p_{3,2}, p_{2,3}\big)\mathrm{P}\big(\neg p_{1,3}, p_{3,2}, p_{2,3}\big) +$
   $\mathrm{P}\big(breeze \mid \neg p_{2,2}, known, \neg p_{1,3}, \neg p_{3,2}, p_{2,3}\big)\mathrm{P}\big(\neg p_{1,3}, \neg p_{3,2}, p_{2,3}\big)$
   $\mathrm{P}\big(breeze \mid \neg p_{2,2}, known, p_{1,3}, p_{3,2}, \neg p_{2,3}\big)\mathrm{P}\big(p_{1,3}, p_{3,2}, \neg p_{2,3}\big) +$

$\text{P}\left(breeze \mid \neg p_{2,2}, known, p_{1,3}, \neg p_{3,2}, \neg p_{2,3}\right)\text{P}\left(p_{1,3}, \neg p_{3,2}, \neg p_{2,3}\right) +$
$\text{P}\left(breeze \mid \neg p_{2,2}, known, \neg p_{1,3}, p_{3,2}, \neg p_{2,3}\right)\text{P}\left(\neg p_{1,3}, p_{3,2}, \neg p_{2,3}\right) +$
$\text{P}\left(breeze \mid \neg p_{2,2}, known, \neg p_{1,3}, \neg p_{3,2}, \neg p_{2,3}\right)\text{P}\left(\neg p_{1,3}, \neg p_{3,2}, \neg p_{2,3}\right)] >$

$= \alpha' < (0.2)[(1 * 0.2 * 0.2 * 0.2) + (1 * 0.2 * 0.8 * 0.2) + (1 * 0.8 * 0.2 * 0.2) + (1 * 0.8 * 0.8 * 0.2) + (1 * 0.2 * 0.2 * 0.8) + (0 * 0.2 * 0.8 * 0.8) + (1 * 0.8 * 0.2 * 0.8) + (0 * 0.8 * 0.8 * 0.8)], (0.8)[(1 * 0.2 * 0.2 * 0.2) + (1 * 0.2 * 0.8 * 0.2) + (0 * 0.8 * 0.2 * 0.2) + (0 * 0.8 * 0.8 * 0.2) + (1 * 0.2 * 0.2 * 0.8) + (0 * 0.2 * 0.8 * 0.8) + (0 * 0.8 * 0.2 * 0.8) + (0 * 0.8 * 0.8 * 0.8)] >$

$= \alpha' < (0.2)(0.008 + 0.032 + 0.032 + 0.128 + 0.032 + 0.128), (0.8)(0.008 + 0.032 + 0.032) >$

$= \alpha' < 0.072, 0.0576 >$

$$\alpha' = \frac{1}{0.072 + 0.0576} \approx 7.72$$

$\mathbf{P}(Pit_{2,2} \mid breeze, known) =< 0.56, 0.44 >$

Thus, it will not change the probability of a pit in (2,2).