CptS 540 Artificial Intelligence
HW8
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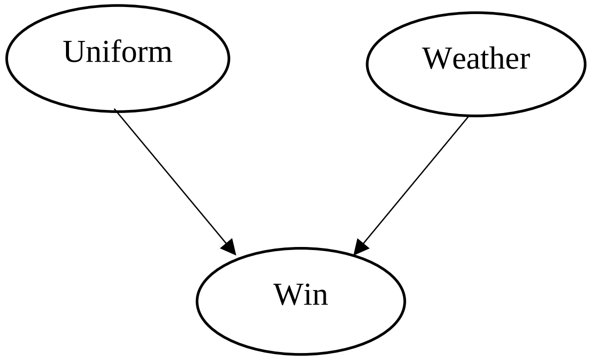
1.

Win		true		false	
Uniform		crimson	gray	crimson	gray
Weather	clear	0.18	0.08	0.06	0.08
	cloudy	0.08	0.10	0.07	0.09
	rainy	0.05	0.09	0.08	0.04

$P(Uniform = crimson) = 0.18 + 0.08 + 0.05 + 0.06 + 0.07 + 0.08 = 0.52$
$P(Uniform = gray) = 0.08 + 0.10 + 0.09 + 0.08 + 0.09 + 0.04 = 0.48$
$P(Weather = clear) = 0.18 + 0.08 + 0.06 + 0.08 = 0.40$
$P(Weather = cloudy) = 0.08 + 0.10 + 0.07 + 0.09 = 0.34$
$P(Weather = rainy) = 0.05 + 0.09 + 0.08 + 0.04 = 0.26$
$P(Win = true|uniform = crimson, Weather = clear)$
$= \frac{P(Win = true, Uniform = crimson, Weather = clear)}{P(Uniform = crimson, Weather = clear)}$
$= \frac{0.18}{0.18 + 0.06} = 0.75$
$P(Win = true|uniform = crimson, Weather = cloudy)$
$= \frac{P(Win = true, Uniform = crimson, Weather = cloudy)}{P(Uniform = crimson, Weather = cloudy)}$
$= \frac{0.08}{0.08 + 0.07} \approx 0.53$
$P(Win = true|uniform = crimson, Weather = rainy)$
$= \frac{P(Win = true, Uniform = crimson, Weather = rainy)}{P(Uniform = crimson, Weather = rainy)}$
$= \frac{0.05}{0.05 + 0.08} \approx 0.38$
Similarly, we get:
$P(Win = true|uniform = gray, Weather = clear) = 0.50$
$P(Win = true|uniform = gray, Weather = cloudy) \approx 0.53$
$P(Win = true|uniform = gray, Weather = rainy) \approx 0.69$
$P(Win = false|uniform = crimson, Weather = clear) = 0.25$
$P(Win = false|uniform = crimson, Weather = cloudy) \approx 0.47$
$P(Win = false|uniform = crimson, Weather = rainy) \approx 0.62$
$P(Win = false|uniform = gray, Weather = clear) = 0.50$
$P(Win = false|uniform = gray, Weather = cloudy) \approx 0.47$
$P(Win = false|uniform = gray, Weather = rainy) \approx 0.31$

Bayesian network is shown on the next page:

P(Uniform)	
crimson	gray
0.52	0.48



P(Weather)		
clear	cloudy	rainy
0.40	0.34	0.26

Uniform	Weather	P (Win | Uniform, Weather)	
		true	false
crimson	clear	0.75	0.25
crimson	cloudy	0.53	0.47
crimson	rainy	0.38	0.62
gray	clear	0.50	0.50
gray	cloudy	0.53	0.47
gray	rainy	0.69	0.31

2. For the brevity, let:
   U=Uniform, W=Weather, C=CallFriends, B=BuyJersey

   a. $P(U = crimson, W = clear, Win = true, C = true, B = true)$
   $= P(U = crimson) * P(W = clear) * P(Win = true|U = crimson, W = clear)$
   $* P(C = true|Win = true) * P(B = true|Win = true)$
   $= 0.6 * 0.3 * 0.9 * 0.7 * 0.6 = 0.06804$

   b. $P(C = true|U = gray, W = cloudy)$
   $= \alpha\, P(C = true, U = gray, W = cloudy)\ where\ \alpha = \dfrac{1}{P(U = gray, W = cloudy)}$
   $= \alpha < \sum_{Win} P(C = true, U = gray, W = cloudy, Win),$
   $\sum_{Win} P(C = false, U = gray, W = cloudy, Win) >$
   $= \alpha < P(U = gray) * P(W = cloudy) * \sum_{Win} P(Win|U = gray, W = cloudy) *$
   $P(C = true|Win),$
   $P(U = gray) * P(W = cloudy) * \sum_{Win} P(Win|U = gray, W = cloudy) *$
   $P(C = false|Win) >$
   $= \alpha < 0.4 * 0.4 * [P(Win = true|U = gray, W = cloudy) *$
   $P(C = true|Win = true) + P(Win = false|U = gray, W = cloudy) *$
   $P(C = true|Win = false)],$
   $0.4 * 0.4 * [P(Win = true|U = gray, W = cloudy) * P(C = false|Win = true)$

$+P(Win = false|U = gray, W = cloudy) * P(C = false|Win = false)] >$
$= \alpha < 0.4 * 0.4 * (0.4 * 0.7 + 0.6 * 0.2), 0.4 * 0.4 * (0.4 * 0.3 + 0.6 * 0.8) >$
$= \alpha < 0.064, 0.096 >$
$=< 0.4, 0.6 >$
Thus,

$$P(C = true|U = gray, W = cloudy) = 0.4$$

c. $P(U = crimson \mid C = true, B = true)$
$= \alpha < P(U = crimson, C = true, B = true), P(U = gray, C = true, B = true) >$
$= \alpha < \sum_W \sum_{Win} P(U = crimson, C = true, B = true, W, Win),$
$\sum_W \sum_{Win} P(U = gray, C = true, B = true, W, Win) >$
$= \alpha < \sum_W \sum_{Win} P(U = Crimson) * P(W) * P(Win|U = crimson, W) *$
$P(C = true|Win) * P(B = true|Win),$
$\sum_W \sum_{Win} P(U = gray) * P(W) * P(Win|U = gray, W) * P(C = true|Win) *$
$P(B = true|Win) >$
$= \alpha < P(U = crimson) * \{P(W = clear) *$
$[P(Win = true|U = crimson, W = clear) * P(C = true|Win = true) *$
$P(B = true|Win = true) + P(Win = false|U = crimson, W = clear) *$
$P(C = true|Win = false) * P(B = true|Win = false)] + P(W = cloudy) *$
$[P(Win = true|U = crimson, W = cloudy) * P(C = true|Win = true) *$
$P(B = true|Win = true) + P(Win = false|U = crimson, W = cloudy) *$
$P(C = true|Win = false) * P(B = true|Win = false)] + P(W = rainy) *$
$[P(Win = true|U = crimson, W = rainy) * P(C = true|Win = true) *$
$P(B = true|Win = true) + P(Win = false|U = crimson, W = rainy) *$
$P(C = true|Win = false) * P(B = true|Win = false)]\},$
$P(U = gray) * \{P(W = clear) * [P(Win = true|U = gray, W = clear) *$
$P(C = true|Win = true) * P(B = true|Win = true) +$
$P(Win = false|U = gray, W = clear) * P(C = true|Win = false) *$
$P(B = true|Win = false)] + P(W = cloudy) *$
$[P(Win = true|U = gray, W = cloudy) * P(C = true|Win = true) *$
$P(B = true|Win = true) + P(Win = false|U = gray, W = cloudy) *$
$P(C = true|Win = false) * P(B = true|Win = false)] + P(W = rainy) *$
$[P(Win = true|U = gray, W = rainy) * P(C = true|Win = true) *$
$P(B = true|Win = true) + P(Win = false|U = gray, W = rainy) *$
$P(C = true|Win = false) * P(B = true|Win = false)]\} >$

$$\begin{aligned} = \alpha < 0.6 \times \{&0.3 \times [0.9 \times 0.7 \times 0.6 + 0.1 \times 0.2 \times 0.3] \\ &+ 0.4 \times [0.6 \times 0.7 \times 0.6 + 0.4 \times 0.2 \times 0.3] \\ &+ 0.3 \times [0.4 \times 0.7 \times 0.6 + 0.6 \times 0.2 \times 0.3]\}, 0.4 \\ &\times \{0.3 \times [0.2 \times 0.7 \times 0.6 + 0.8 \times 0.2 \times 0.3] \\ &+ 0.4 \times [0.4 \times 0.7 \times 0.6 + 0.6 \times 0.2 \times 0.3] \\ &+ 0.3 \times [0.7 \times 0.7 \times 0.6 + 0.3 \times 0.2 \times 0.3]\} > \end{aligned}$$

$= \alpha < 0.1818, 0.08592 >$
$=< 0.68, 0.32 >$
Thus,

$$P(U = crimson \mid C = true, B = true) = 0.68$$

3. $\boldsymbol{P}(U) =< 0.6{,}0.4 >, Uniform = crimson$
   $\boldsymbol{P}(W) =< 0.3{,}0.4{,}0.3 >, Weather = cloudy$
   $\boldsymbol{P}(Win|U = crimson, W = cloudy) =< 0.6{,}0.4 >, Win = true$
   $\boldsymbol{P}(C|Win = true) =< 0.7{,}0.3 >, CallFriends = true$
   $\boldsymbol{P}(B|Win = true) =< 0.6{,}0.4 >, BuyJersey = true$
   Sample is [crimson, cloudy, true, true, true]
   The sampling probability for this event is:
   $S_{PS}(crimson, cloudy, true, true, true) = 0.6 \times 0.4 \times 0.6 \times 0.7 \times 0.6 = 0.06048$

4. If two independent events P(X) and P(Y) are information consistent with the full joint probability distribution, they should satisfy:
   $$\forall_{X,Y}\ P(X = x, Y = y) = P(X = x) * P(Y = y)$$
   In this case, a simple example can be given like:
   $P(Uniform = crimson) = 0.52$
   $P(Weather = clear) = 0.40$
   $P(Uniform = crimson, Weather = clear) = 0.18 + 0.06 = 0.24$
   $P(Uniform = crimson) * P(Weather = clear) = 0.52 * 0.40 = 0.208$
   Thus,
   $$P(Uniform = crimson, Weather = clear) \neq P(Uniform = crimson) * P(Weather = clear)$$
   Therefore, we can say *Uniform* and *Weather* are not information consistent with the full joint probability distribution.