Washington State University
School of Electrical Engineering and Computer Science
Fall 2021

CptS 440/540 Artificial Intelligence
Homework 7 – Solution
Due: October 21, 2021 (11:59pm pacific time)

General Instructions: Put your answers to the following problems into a PDF document and upload the document as your submission for Homework 7 for the course CptS 440 Pullman (all sections of CptS 440 and 540 are merged under the CptS 440 Pullman section) on the Canvas system by the above deadline. Note that you may submit multiple times, but we will only grade the most recent entry submitted before the deadline.

1. Consider the following full joint probability distribution to help us determine when the Cougars will win. Compute the following probabilities. Show your work.

Win		true		false	
Uniform		crimson	gray	crimson	gray
Weather	clear	0.18	0.08	0.06	0.08
	cloudy	0.08	0.10	0.07	0.09
	rainy	0.05	0.09	0.08	0.04

   a. P(Win=true, Uniform=crimson, Weather=clear)?
   b. P(Weather=clear)?
   c. P(Uniform=crimson)?
   d. P(Win=true | Weather=clear)?
   e. P(Win=true | Weather=cloudy $\vee$ Weather=rainy)?

   Solution:
   a. P(Win=true, Uniform=crimson, Weather=clear)
      = 0.18 (table lookup)
   b. P(Weather=clear)
      = 0.18 + 0.08 + 0.06 + 0.08 = 0.40 (sum of 1st row)
   c. P(Uniform=crimson)
      = 0.18 + 0.08 + 0.05 + 0.06 + 0.07 + 0.08 = 0.52 (sum of 1st and 3rd columns).
   d. P(Win=true | Weather=clear)
      = P(Win=true, Weather=clear) / P(Weather=clear) = (0.18 + 0.08) / 0.40 = 0.65
   e. P(Win=true | Weather=cloudy $\vee$ Weather=rainy)
      = P(Win=true $\wedge$ (Weather=cloudy $\vee$ Weather=rainy) / P(Weather=cloudy $\vee$ Weather=rainy)

$= P((\text{Win=true} \wedge \text{Weather=cloudy}) \vee (\text{Win=true} \wedge \text{Weather=rainy}) /$
$\quad P(\text{Weather=cloudy} \vee \text{Weather=rainy})$
$= [P(\text{Win=true} \wedge \text{Weather=cloudy}) + P(\text{Win=true} \wedge \text{Weather=rainy}) - P(\text{Win=true} \wedge$
$\quad \text{Weather=cloudy} \wedge \text{Weather=rainy})] / P(\text{Weather=cloudy} \vee \text{Weather=rainy})$

Since Weather cannot be both cloudy and rainy,

$$P(\text{Win=true} \wedge \text{Weather=cloudy} \wedge \text{Weather=rainy}) = 0$$

$= [P(\text{Win=true} \wedge \text{Weather=cloudy}) + P(\text{Win=true} \wedge \text{Weather=rainy})] /$
$\quad P(\text{Weather=cloudy} \vee \text{Weather=rainy})$
$= [P(\text{Win=true} \wedge \text{Weather=cloudy}) + P(\text{Win=true} \wedge \text{Weather=rainy})] /$
$\quad [P(\text{Weather=cloudy}) + P(\text{Weather=rainy}) - P(\text{Weather=cloudy} \wedge \text{Weather=rainy})]$

Again, since Weather cannot be both cloudy and rainy,

$$P(\text{Weather=cloudy} \wedge \text{Weather=rainy}) = 0$$

$= [P(\text{Win=true} \wedge \text{Weather=cloudy}) + P(\text{Win=true} \wedge \text{Weather=rainy})] /$
$\quad [P(\text{Weather=cloudy}) + P(\text{Weather=rainy})]$
$= [(0.08 + 0.10) + (0.05 + 0.09)] /$
$\quad [(0.08 + 0.10 + 0.07 + 0.09) + (0.05 + 0.09 + 0.08 + 0.04)]$
$= 0.32 / (0.34 + 0.26)$
$= 0.53$

2. Consider the problem with three Boolean random variables: Win, Practice, Healthy. Assume you know only the following information:
   - $P(\text{Win=true}) = 0.7$
   - $P(\text{Practice=true} \wedge \text{Healthy=true} \mid \text{Win=true}) = 0.8$
   - $P(\text{Practice=true} \wedge \text{Healthy=true} \mid \text{Win=false}) = 0.4$

   Using Bayes rule and normalization, compute $\mathbf{P}(\text{Win} \mid \text{Practice=true} \wedge \text{Healthy=true})$. Note the "**P**" is boldfaced, so we want a distribution.

   Solution:

   First, let's compute for Win=true:

   $P(\text{Win=true} \mid \text{Practice=true} \wedge \text{Healthy=true})$
   $= \alpha P(\text{Practice=true} \wedge \text{Healthy=true} \mid \text{Win=true}) * P(\text{Win=true})$
   $= \alpha(0.8)(0.7) = \alpha(0.56)$

   Next, let's compute for Win=false:

   $P(\text{Win=false} \mid \text{Practice=true} \wedge \text{Healthy=true})$
   $= \alpha P(\text{Practice=true} \wedge \text{Healthy=true} \mid \text{Win=false}) * P(\text{Win=false})$
   $= \alpha(0.4)(0.3) = \alpha(0.12)$

   Normalizing by $\alpha = 1 / (0.56+0.12) = 1.47$

   $\mathbf{P}(\text{Win} \mid \text{Practice=true} \wedge \text{Healthy=true}) = \langle 0.82, 0.18 \rangle$

3. Suppose we have the 3x3 Wumpus world shown below. Your agent visited locations (1,1), (2,1), and (1,2), and perceived breezes in (2,1) and (1,2). The agent then takes a calculated risk, moves to (3,1), but unfortunately encounters a pit. Given this information, we want to compute the probability of a pit in (2,2). You may use $p_{x,y}$ and $\neg p_{x,y}$ as shorthand notation for $Pit_{x,y}$=true and $Pit_{x,y}$=false, respectively. Similarly, you may use $b_{x,y}$ and $\neg b_{x,y}$ as shorthand notation for $Breeze_{x,y}$=true and $Breeze_{x,y}$=false, respectively. Specifically,

B, OK		
¬B, OK	B, OK	P

   a. Define the sets: *breeze*, *known*, *frontier* and *other*.
   b. Following the method in the textbook and lecture, compute the probability *distribution* $\mathbf{P}(Pit_{2,2} \mid breeze, known)$. Show your work.

Solution:

a. $breeze = \{\neg b_{1,1}, b_{2,1}, b_{1,2}\}$

$known = \{\neg p_{1,1}, \neg p_{2,1}, \neg p_{1,2}, p_{3,1}\}$

$frontier = \{Pit_{3,2}, Pit_{1,3}\}$

$other = \{Pit_{2,3}, Pit_{3,3}\}$

b. $\mathbf{P}(Pit_{2,2} \mid breeze, known)$

$= \mathbf{P}(P_{2,2}, breeze, known) / \mathbf{P}(breeze, known)$

$= \alpha\, \mathbf{P}(P_{2,2}, breeze, known)$

$= \alpha \sum_{frontier} \sum_{other} \mathbf{P}(P_{2,2}, breeze, known, frontier, other)$

$= \alpha \sum_{frontier} \sum_{other} \mathbf{P}(breeze \mid P_{2,2}, known, frontier, other)\, \mathbf{P}(P_{2,2}, known, frontier, other)$

Since *breeze* is independent of *other* given $P_{2,2}$, *known* and *frontier*:

$= \alpha \sum_{frontier} \sum_{other} \mathbf{P}(breeze \mid P_{3,1}, known, frontier)\, \mathbf{P}(P_{3,1}, known, frontier, other)$

$= \alpha \sum_{frontier} \mathbf{P}(breeze \mid P_{2,2}, known, frontier) \sum_{other} \mathbf{P}(P_{2,2}, known, frontier, other)$

Since $P_{2,2}$, *known, frontier, other* are independent of each other:

$= \alpha \sum_{frontier} \mathbf{P}(breeze \mid P_{2,2}, known, frontier) \sum_{other} \mathbf{P}(P_{2,2})\, P(known)\, P(frontier)\, P(other)$

$= \alpha\, \mathbf{P}(P_{2,2})\, P(known) \sum_{frontier} \mathbf{P}(breeze \mid P_{2,2}, known, frontier)\, P(frontier) \sum_{other} P(other)$

Letting $\alpha' = \alpha * P(known)$, and since $\sum_{other} P(other) = 1$:

$= \alpha'\, \mathbf{P}(P_{2,2}) \sum_{frontier} \mathbf{P}(breeze \mid P_{2,2}, known, frontier)\, P(frontier)$

$= \alpha' < P(p_{2,2})\, [\, \sum_{frontier} P(breeze \mid p_{2,2}, known, frontier)\, P(frontier)\,]\,,$

$\quad P(\neg p_{2,2})\, [\, \sum_{frontier} P(breeze \mid \neg p_{2,2}, known, frontier)\, P(frontier)\,] >$

$= \alpha' < P(p_{2,2})\, [\, P(breeze \mid p_{2,2}, known, p_{3,2}, p_{1,3})\, P(p_{3,2}, p_{1,3}) +$

$\quad P(breeze \mid p_{2,2}, known, p_{3,2}, \neg p_{1,3})\, P(p_{3,2}, \neg p_{1,3}) +$

$\quad P(breeze \mid p_{2,2}, known, \neg p_{3,2}, p_{1,3})\, P(\neg p_{3,2}, p_{1,3}) +$

$\quad P(breeze \mid p_{2,2}, known, \neg p_{3,2}, \neg p_{1,3})\, P(\neg p_{3,2}, \neg p_{1,3})\,]\,,$

$\quad P(\neg p_{3,1})\, [\, P(breeze \mid \neg p_{2,2}, known, p_{3,2}, p_{1,3})\, P(p_{3,2}, p_{1,3}) +$

$\quad P(breeze \mid \neg p_{2,2}, known, p_{3,2}, \neg p_{1,3})\, P(p_{3,2}, \neg p_{1,3}) +$

$\quad P(breeze \mid \neg p_{2,2}, known, \neg p_{3,2}, p_{1,3})\, P(\neg p_{3,2}, p_{1,3}) +$

$\mathbf{P}(breeze \mid \neg p_{2,2}, known, \neg p_{3,2}, \neg p_{1,3})\, P(\neg p_{3,2}, \neg p_{1,3})\ ]\ >$

$= \alpha' < (0.2)\ [\ (1.0)(0.2)(0.2) + (1.0)(0.2)(0.8) + (1.0)(0.8)(0.2) + (1.0)(0.8)(0.8)\ ]\ ,$

$(0.8)\ [\ (1.0)(0.2)(0.2) + (0.0)(0.2)(0.8) + (1.0)(0.8)(0.2) + (0.0)(0.8)(0.8)\ ]\ >$

$= \alpha' < (0.2)\,(1.0), (0.8)(0.2) >$

$= \alpha' < 0.2, 0.16 >$

$= < 0.56, 0.44 >$

4. *CptS 540 Students Only*. Suppose an oracle tells the agent in Question 3 that there is a breeze in (3,3). Will this change the probability of a pit in (2,2)? Justify your answer.

   Solution:

   No. Since a breeze in (3,3) is caused by a pit in (2,3) or (3,2), or both, and a pit being in these locations does not affect any of the original breeze information, then a breeze in (3,3) does not change $P(pit_{2,2})$.