CptS 591: Elements of Network Science
Spring 2021

Mid Term Exam (Take-home)
Released: Tuesday March 23, 9:30am
Due: by Wednesday March 24, 1pm (to be submitted on Canvas)

Instructions

- There are five problems in this exam; some of the problems have multiple parts. The weight each problem carries is shown in parenthesis. The points add up to 100. Read each problem carefully and give a brief but complete answer.
- Submit your solution as a single PDF file on Canvas by the deadline (Wed March 24, 1pm). You don't have to wait until the deadline to submit; you may submit any time you are ready before the deadline.
- You are welcome to either hand-write your solution and submit a scan (or a picture taken by your phone) or type-up your solution and submit the pdf, whichever you find easier. If you hand-write, please write legibly.
- The questions should be clear, but if you find any question that is ambigious, state the assumption you have made in interpreting the question and solve accordingly. You may also ask for clarification by email.
- Problem 4 is based on the lectures on spectral analysis we had on Feb 16 and Feb 18. You may refer to those notes and the references cited in the lectures notes of Feb 16 in solving the problem.

Problem 1 (20%). For each of the following statements, state whether the statement is True or False. If your answer is False, provide a brief justification. If your answer is True, no justification is needed.

a) The degree distribution of an Erdos-Renyi random graph is Poisson.

b) The degree distribution of most real-world networks follows a Power-Law.

c) Erdos-Renyi randm graphs and most real-world networks behave similarly in terms of average path length.

d) In a given network, the average local clustering coefficient is equal to the global clustering coefficient.

e) Let A be the adjacency matrix representation for an undirected graph G. The (i, j)th entry of the square matrix A^2 gives the number of common neighbors nodes i and j share.

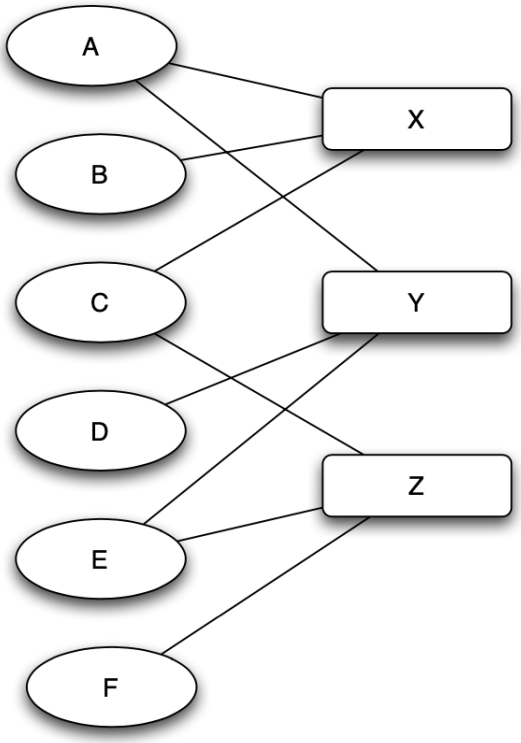

Figure 1: An affiliation network on six people labeled A–F and their membership in three organizations X, Y, and Z.

Problem 2 (25%). Consider the affiliation network in Figure 1, with six people labeled A–F, and three organizations (foci) labeled X, Y, and Z.

a) Draw the derived network on just the six people, joining two people when they share a focus.

b) In the resulting network on people, can you identify a sense in which the triangle on the nodes A, C, and E has a qualitatively different meaning than the other triangles that appear in the network? Explain.

Problem 3 (25%). Show the values that you get if you run two rounds of computing hub and authority values on the network of Web pages shown in Figure 2. (That is, the values computed by the k-step hub-authority computation discussed in Chapter 14 of the Kleinberg-Easley book when we choose the number of steps k to be 2.)

Show the values both before and after the final normalization step, in which we divide each authority score by the sum of all authority scores, and divide each hub score by the sum of all hub scores. (It is fine to write the normalized scores as fractions rather than decimals.)

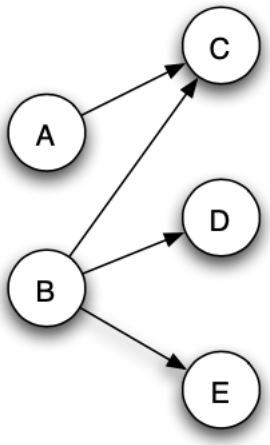

Figure 2: A network on five Web pages A–E.

Problem 4 (15%). Show that the Laplacian of the star graph on n vertices (the graph with the edge set $\{(1, u) : 2 \leq u \leq n\}$) has eigenvalue 0 with multiplicity 1, eigenvalue 1 with multiplicity $n - 2$, and eigenvalue n with multiplicity 1.

Problem 5 (15%). Suppose that a team of anthropologists is studying a set of three small villages that neighbor one another. Each village has 30 people, consisting of 2-3 extended families. Everyone in each village knows all the people in their own village, as well as the people in the other villages.

When the anthropologists build the social network on the people in all three villages taken together, they find that each person is friends with all the other people in their own village, and enemies with everyone in the two other villages. This gives them a network on 90 people (i.e., 30 in each village), with positive and negative signs on its edges.

According to the definitions we discussed in class on the topic of signed networks, is this network on 90 people balanced? Give a brief explanation for your answer.