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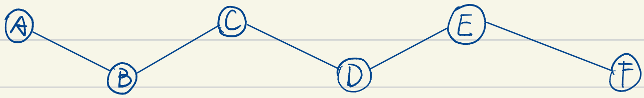
Exercise 1

2. (a)

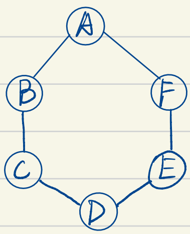


In this case, node B, C, D, and E are "gatekeepers"

4 nodes / 6 nodes > $\frac{1}{2}$

Without loss of generality, I pick node B to explain.
node B has neighbors A and C that are not connected by an edge. To reach A to C or C to A, B must be traversed.

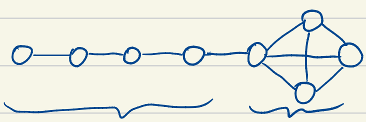
(b)



In this case, all nodes are local gatekeepers but not gatekeepers.

Without loss of generality, for example, node A has neighbors B and F that are not connected by an edge; however, every pair of nodes, including B and F, can be connected by a path that does not go through A.

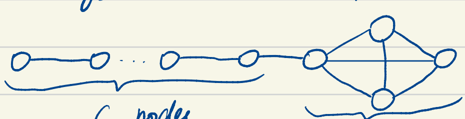
3. (a)



4 nodes a SCC with n nodes

As shown above, the diameter of the graph is 5.
As n tends to infinity, the average distance of the graph tends to 1.
Obviously, the diameter is more than three times as large as the average distance.

(b)



C nodes a SCC with n nodes.

As the graph shown above, a SCC with n nodes and one extanded path with C nodes.
The diameter of the graph is C+1, as long as n >> c, the diameter will be c times larger than average distance but never hit c+1 times.