CptS 591: Elements of Network Science

Signed Networks

(Networks with positive and negative edges)

Basic model of positive and negative relationships

- Consider an undirected complete graph
- Label each edge with either + or –
  - Positive: friends
  - Negative: enemies
- Examine sets of three nodes A, B and C

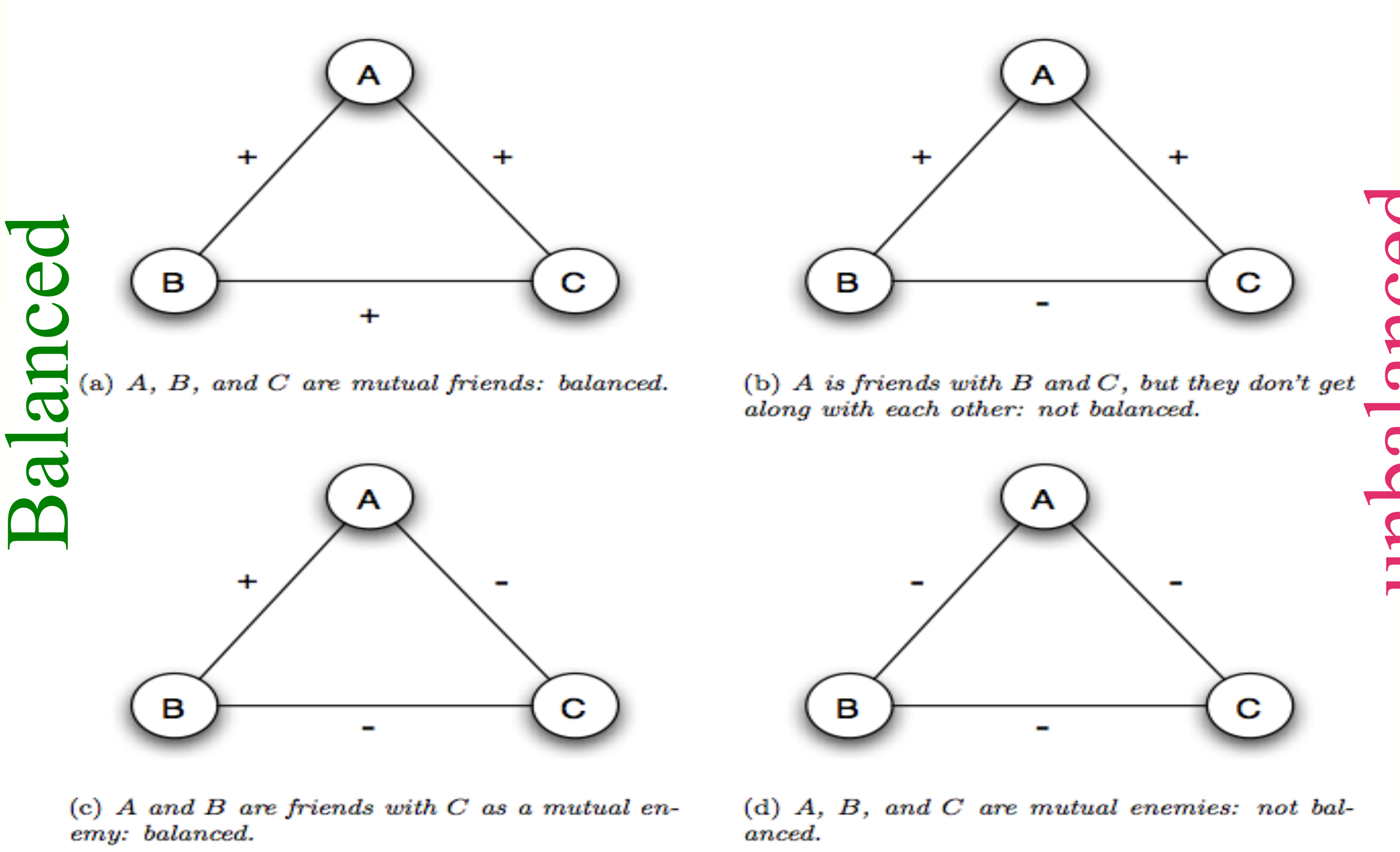
Theory of structural balance

- Social psychology
  - Heider (1940s)
  - Generalizations and extension: Cartwright and Harary (1950s)

- Other areas where signed graphs arise (see Wikipedia page on signed graphs)
  - Topological graph theory
  - Group theory
  - Ising model (computation of ground state energy)
  - Correlation clustering

Structural balance

Balanced



(a) A, B, and C are mutual friends: balanced.

(b) A is friends with B and C, but they don't get along with each other: not balanced.

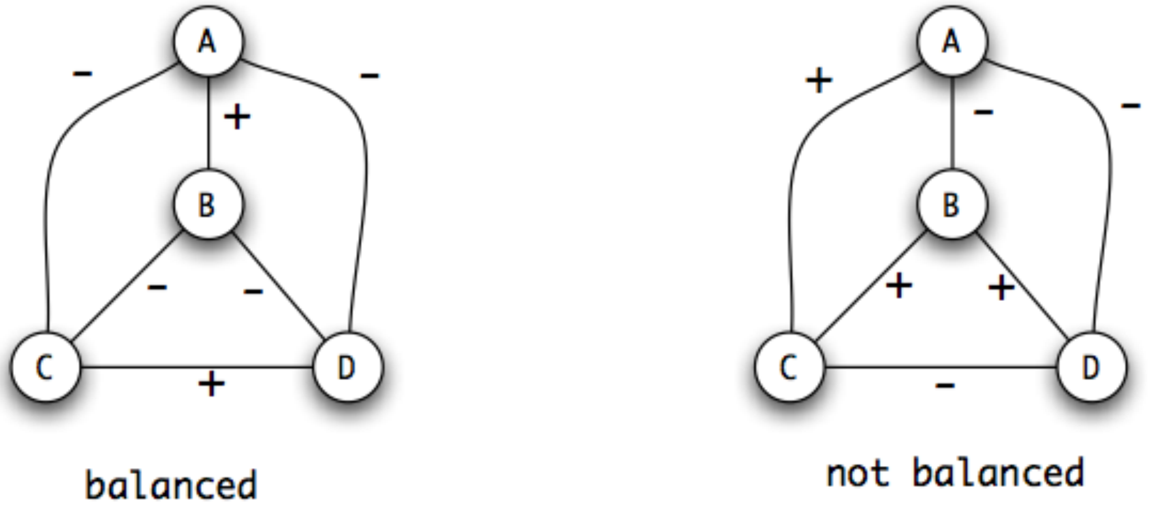
(c) A and B are friends with C as a mutual enemy: balanced.

(d) A, B, and C are mutual enemies: not balanced.

unbalanced

Defining structural balance for networks

- A labeled complete graph is balanced if every one of its triangles has:
  - All 3 edges labeled + or
  - Exactly 1 edge labeled +



Characterizing the structure of balanced networks

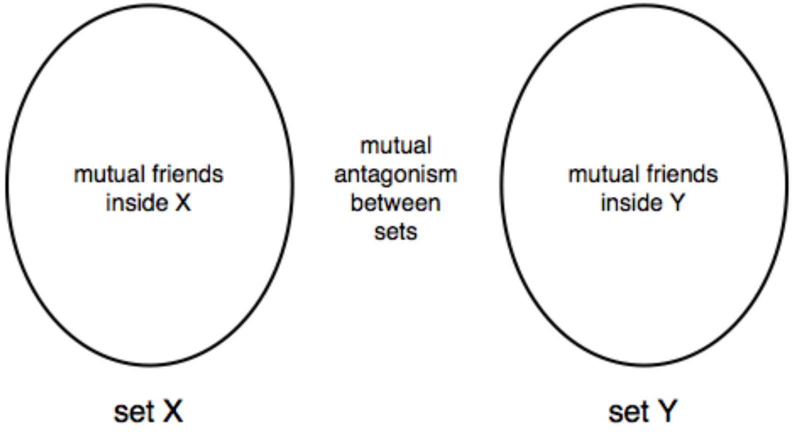
- The Balance Theorem (Harary, 1953)

  If a labeled complete graph is balanced, then either

  (1) all edges are positive, or else

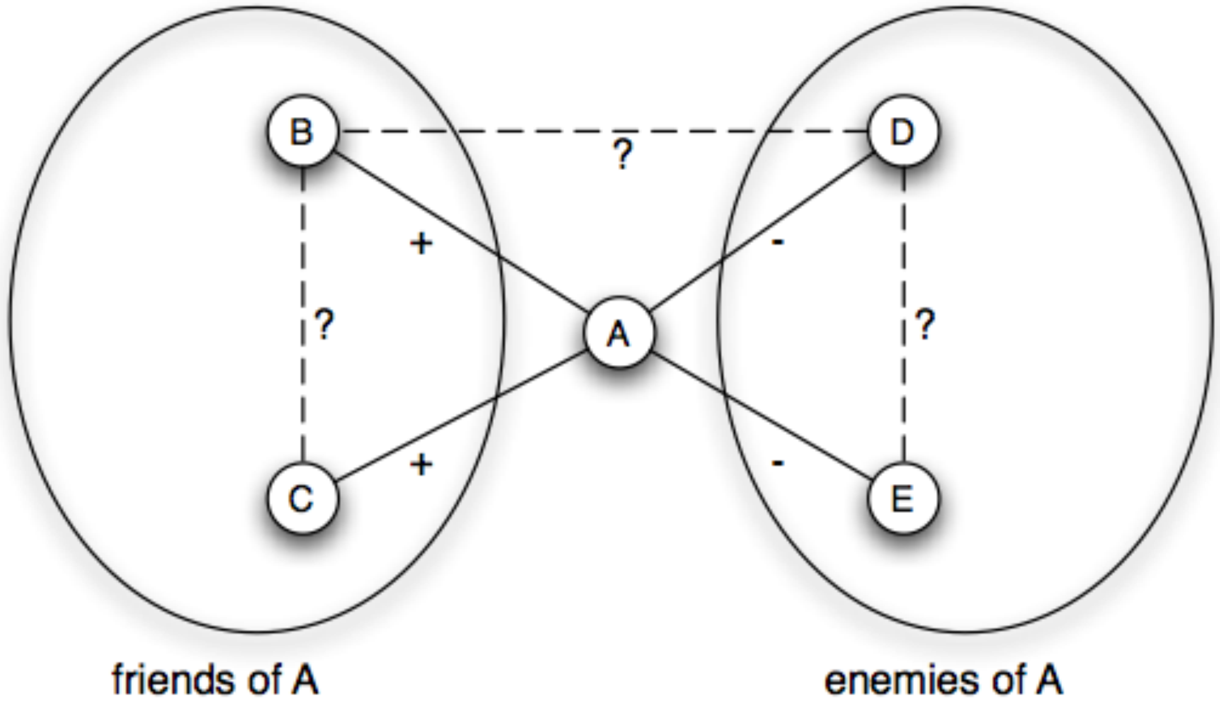
  (2) the nodes can be divided in to two groups, X and Y, such that every edge in X is positive, every edge in Y is positive, and every edge running between X and Y is negative.

Local property (balance) → Global property (battling factions)



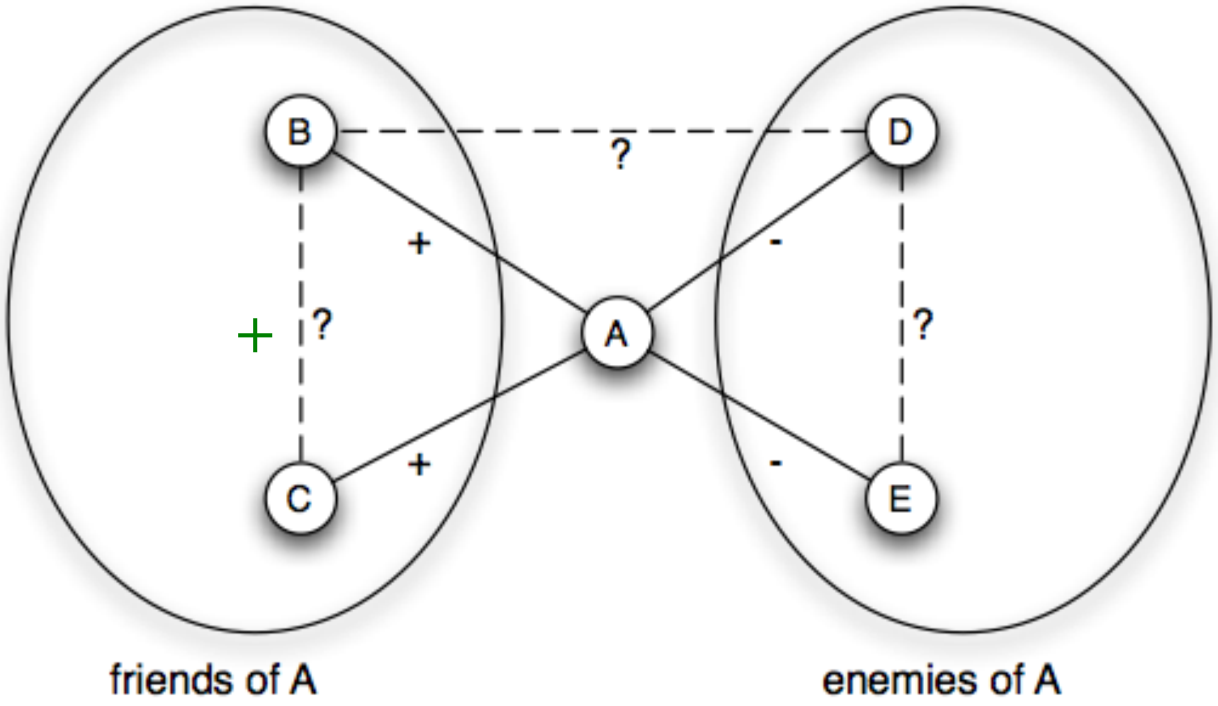
Proving the Balance Theorem

Given: a balanced graph. Show that the structure in the BT is attained.



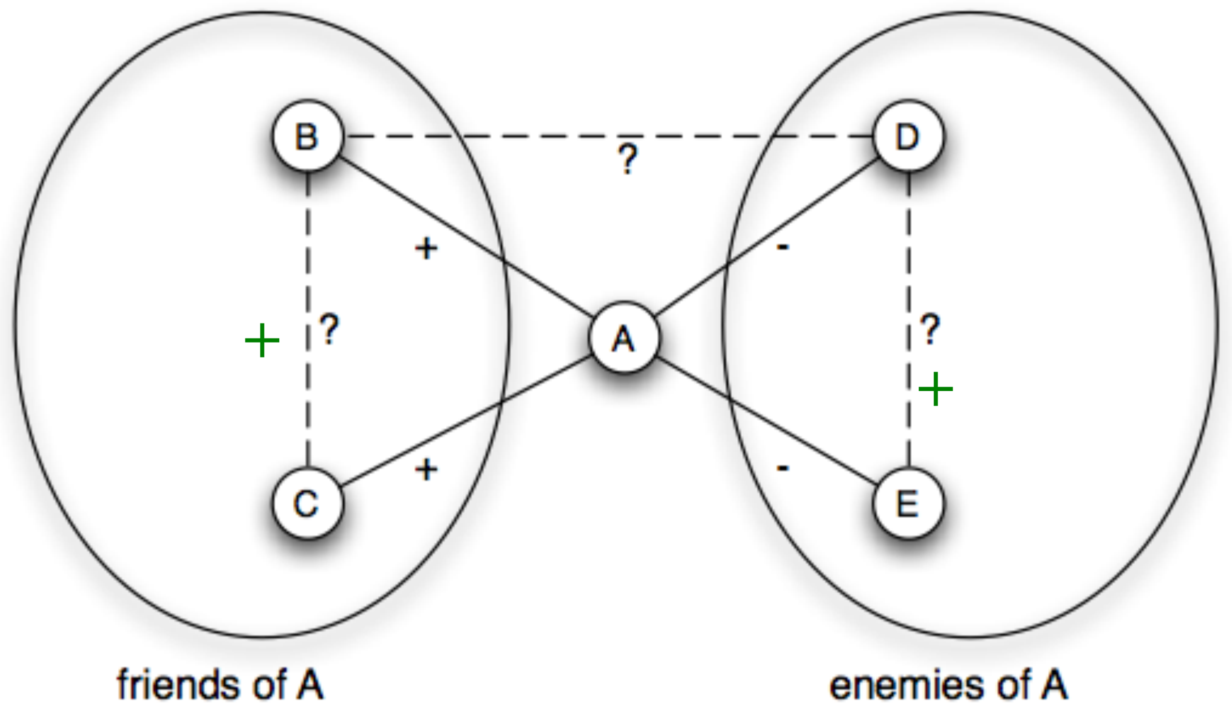
Proving the Balance Theorem

Given: a balanced graph. Show that the structure in the BT is attained.



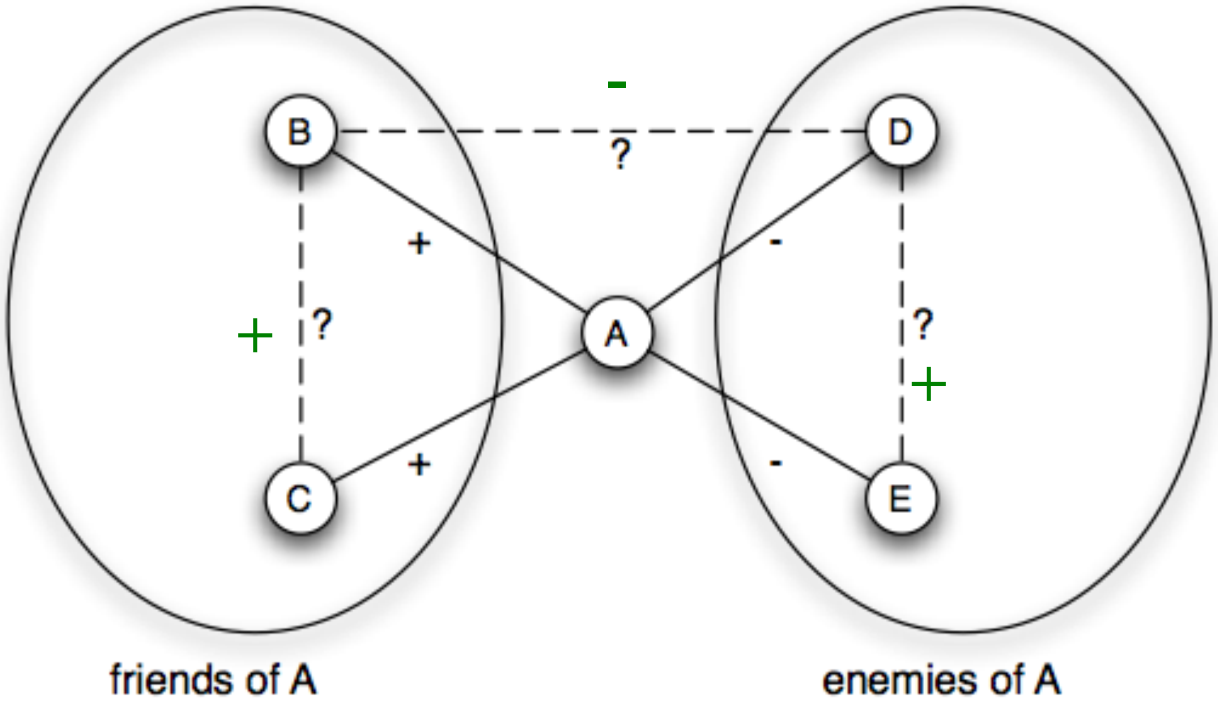
Proving the Balance Theorem

Given: a balanced graph. Show that the structure in the BT is attained.



Proving the Balance Theorem

Given: a balanced graph. Show that the structure in the BT is attained.



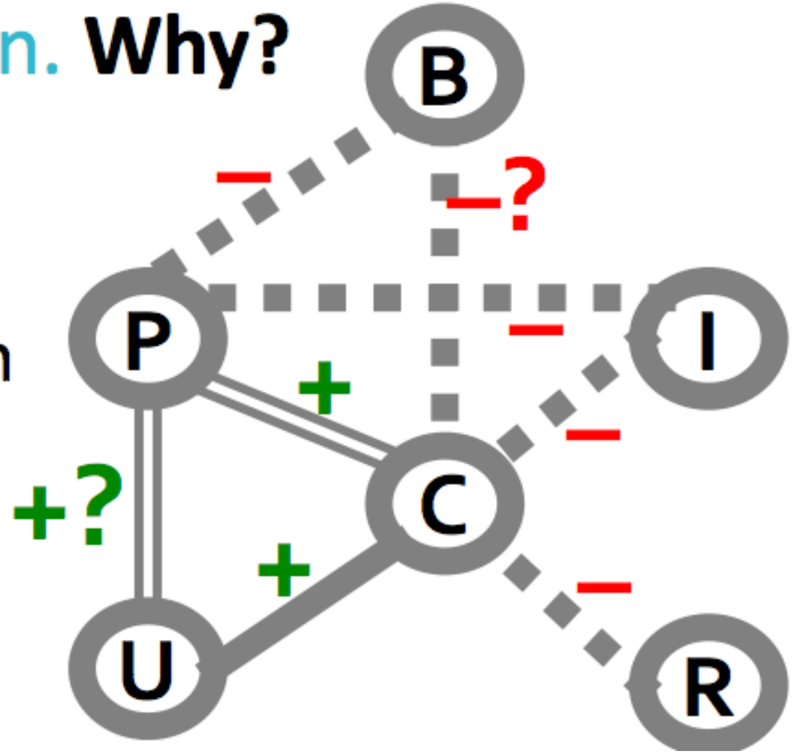
Applications of Structural balance

- Dynamic process
  - Antal, Krapivsky and Redner (2006) study a model in which one starts with a random labeling and then repeatedly look for a triangle that is not balanced and flip one of its label to make it balanced.
  - The mathematics involved here resembles the mathematical models one uses for certain physical systems as they reconfigure to minimize their energy.
- Complex systems
  - Analysis of signed directed graphs has found applications in formal causal reasoning about behavior of complex causal systems (Puccia and Levins, 1986; Dambacher et al, 2002)
- Data clustering
  - Correlation clustering looks for natural clustering of data by similarity. The data points are represented as vertices of a graph, with positive edge joining similar items and a negative edge joining dissimilar items.
- Two other examples (further discussed in next few slides):
  - International Relations
  - Online social media sites

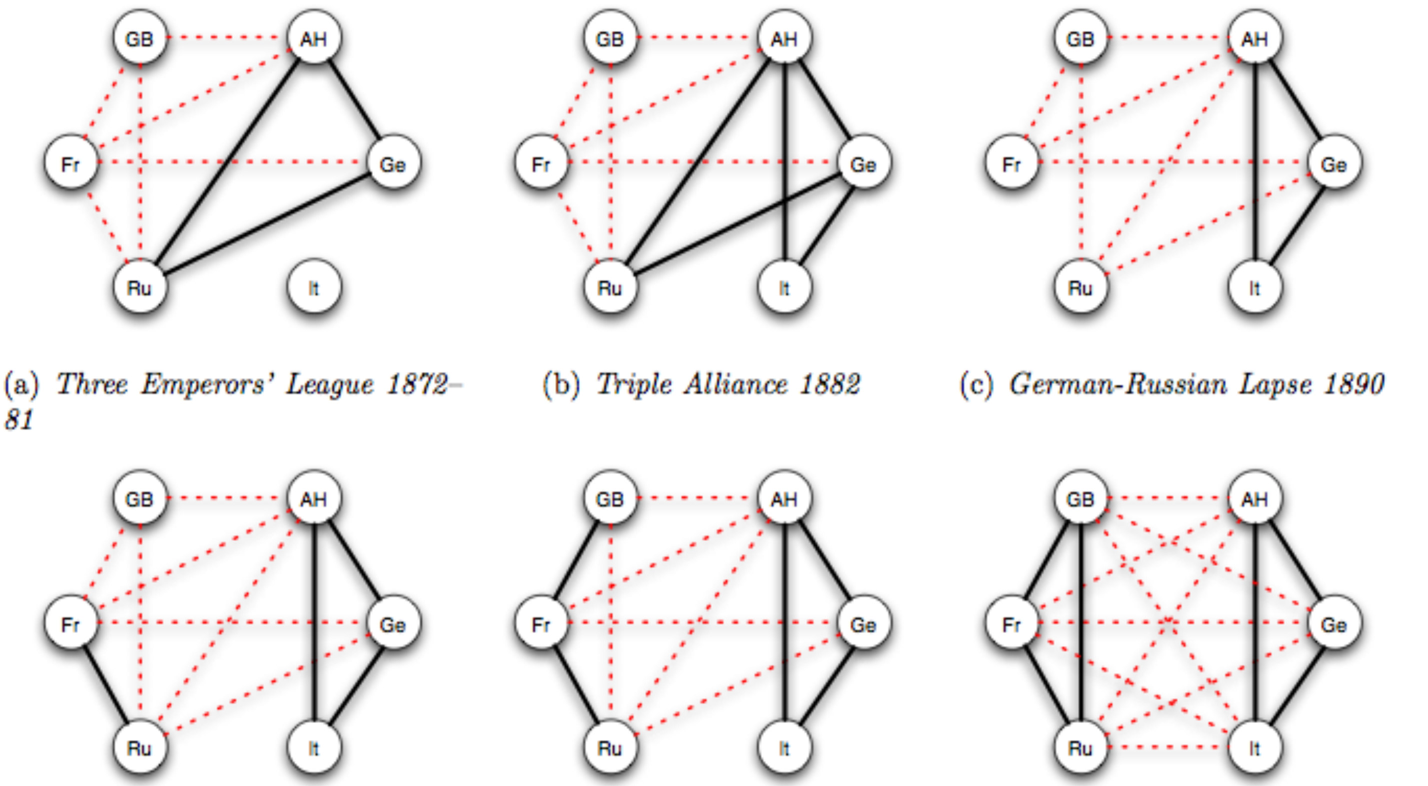
International relations: Bangladesh session

Separation of Bangladesh from Pakistan in 1971: US supports Pakistan. **Why?**

- USSR was enemy of China
- China was enemy of India
- India was enemy of Pakistan
- US was friendly with China
- China vetoed Bangladesh from U.N.



International Relations: Evolution of alliance in Europe, 1872--1907



(a) *Three Emperors' League 1872–81*

(b) *Triple Alliance 1882*

(c) *German-Russian Lapse 1890*

(d) *French-Russian Alliance 1891–94*

(e) *Entente Cordiale 1904*

(f) *British Russian Alliance 1907*

Trust, Distrust and Online Ratings

- Slashdot
  - Technology news site where users can designate each other as friend or foe
- Epinions
  - Product rating site where a user can evaluate different products and also express trust or distrust of other users

- Guha et al (2004) analyze a network of user evaluations on Epinions.
  - Showed how the trust-distrust dichotomy in online ratings has both similarities and differences with the friend-enemy dichotomy in structural balance theory.
  - One difference is because they work with directed graphs.
  - Another, more subtle difference, appears when we look at how triangles behave?

Trust and distrust: triangle behavior

- Example: User A trusts user B, and user B trusts user C.

Then, natural to expect that A trusts C.

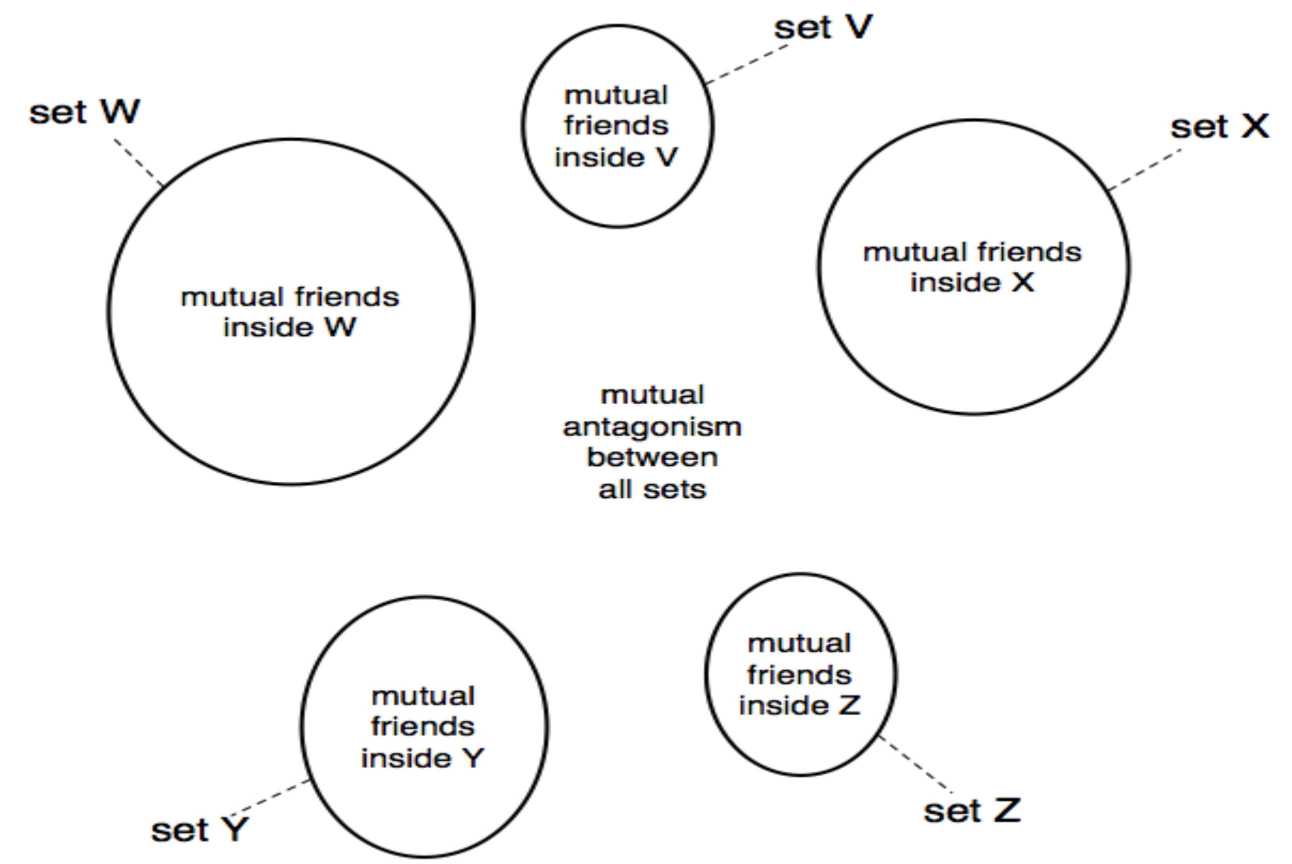
- But what if A distrusts B and B distrusts C?
  - Should A trust or distrust C?
  - There are intuitively appealing arguments in both directions.
  - If distrust expresses anomisity, then Balance Theory would suggest that A should trust C.
  - If distrust expresses “more knowledge” (status), then we may expect A distrusts C, perhaps even more strongly than she distrusts B.

- Lesovek, Huttenlocher, Kleinberg (2010) take this a step further and formulate the Status Theory

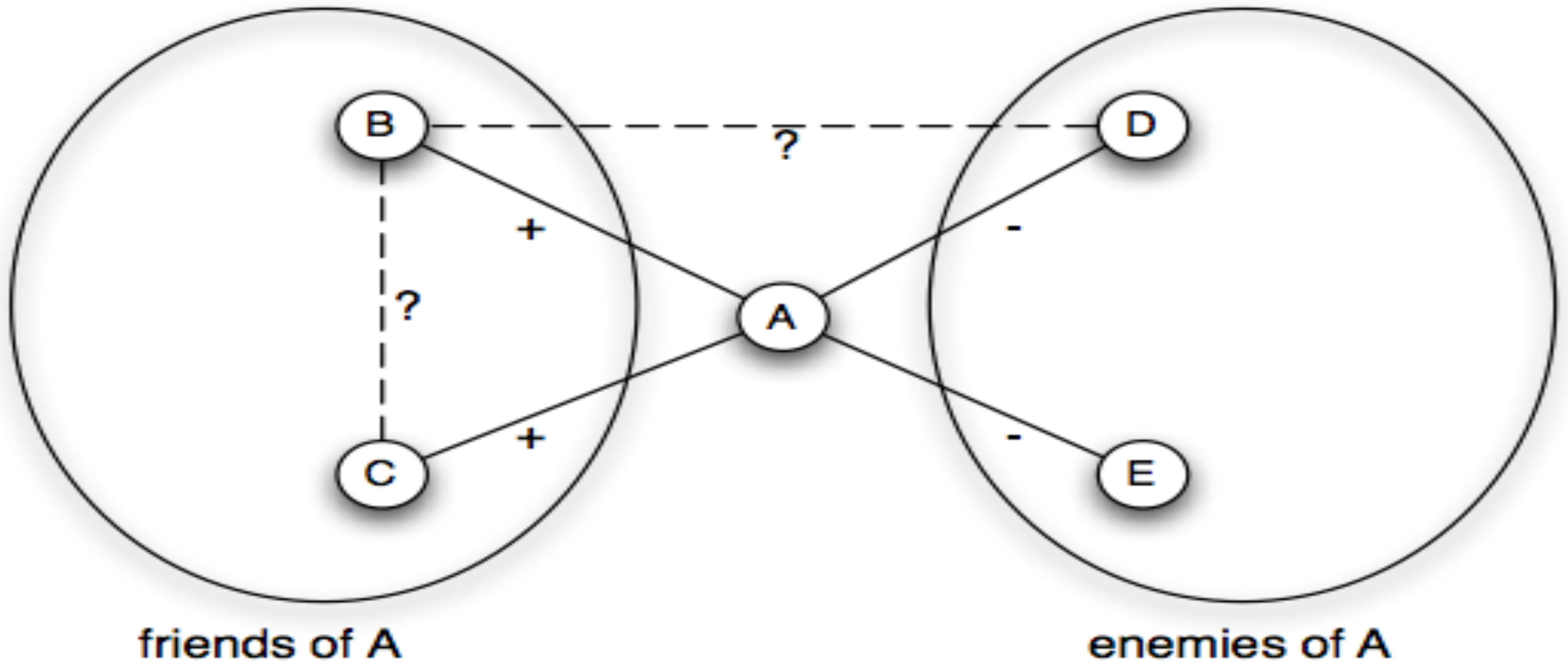
Weaker form of Structural Balance

- In the basic model, imbalance comes in two kinds:
  1. A triangle with 2 + edges and 1 – edge
  2. A triangle with 3 – edges
- In many settings the factor in the first is significantly stronger than the second
- Weakly Balanced Network: no set of three nodes such that the edges among them consists of exactly 2 positive edges and one negative edge.
- Characterization: if a labeled complete graph is weakly balanced, then its nodes can be divided into groups in such a way that every two nodes belonging to the same group are friends, and every two nodes belonging to different groups are enemies.

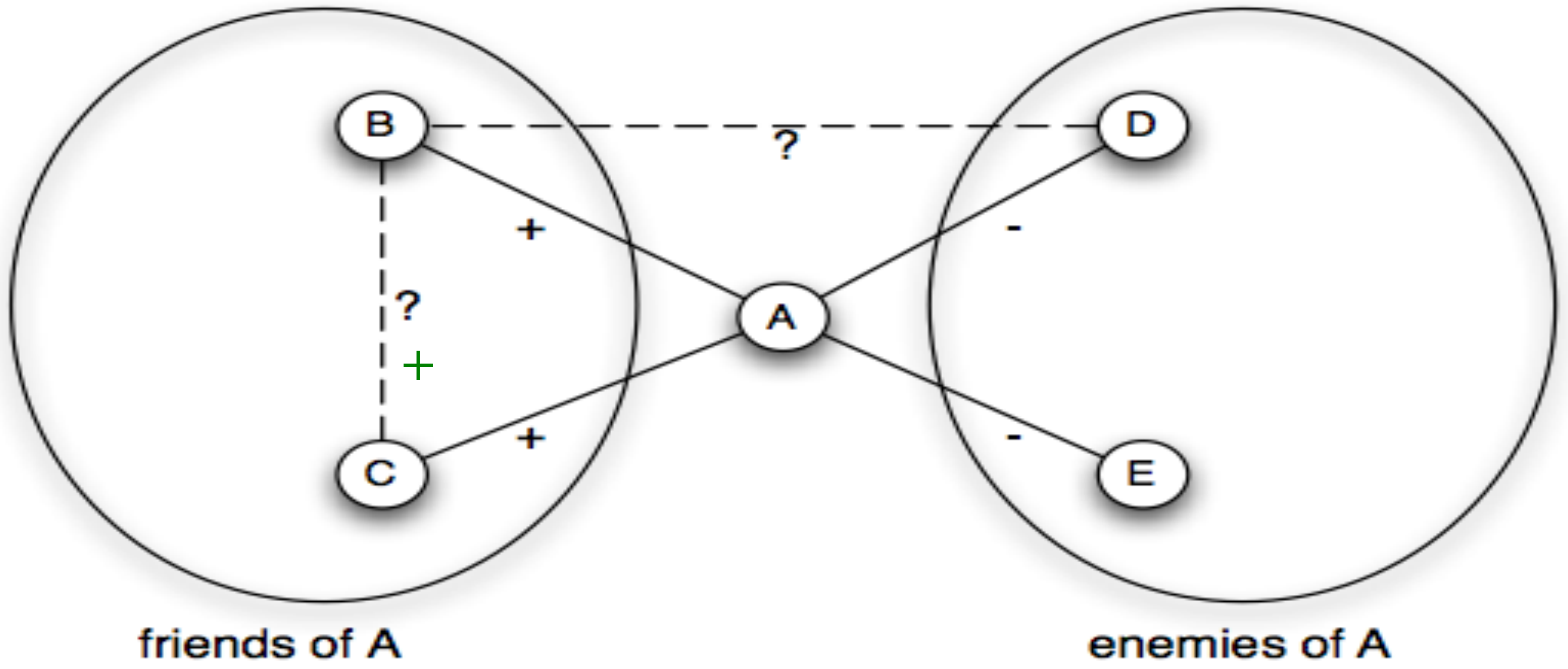
Weakly balanced network



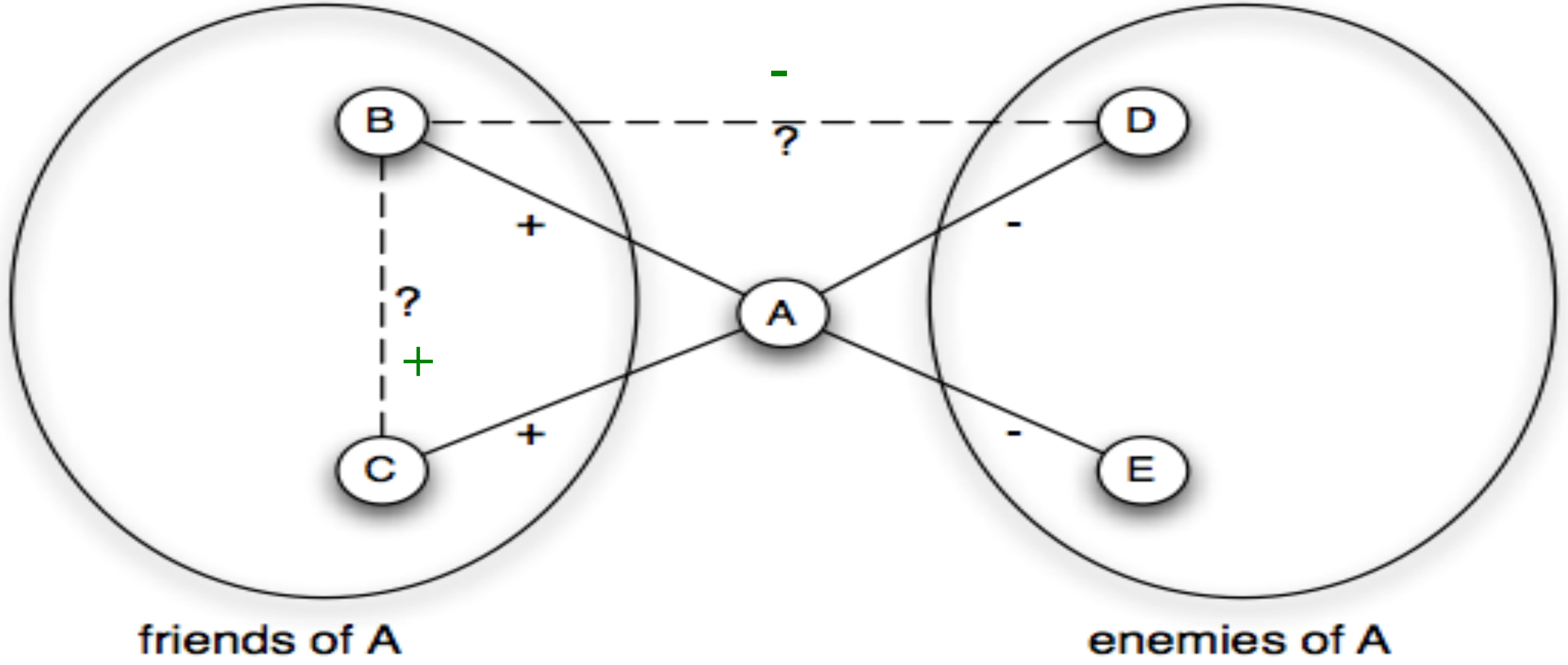
Weakly balanced network: analysis



Weakly balanced network: analysis

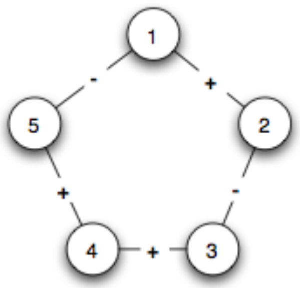


Weakly balanced network: analysis

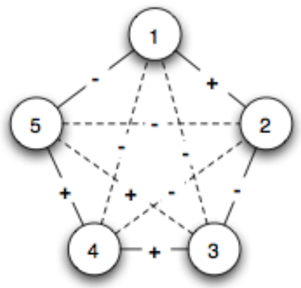


Balance in general (noncomplete) networks

Option 1: Local View
Fill in the missing edges to achieve balance

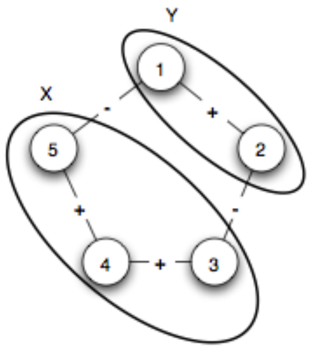


(a) *A graph with signed edges.*



(b) *Filling in the missing edges to achieve balance.*

Option 2: Global View
Divide the network into two mutually opposed set of friends



(c) *Dividing the graph into two sets.*

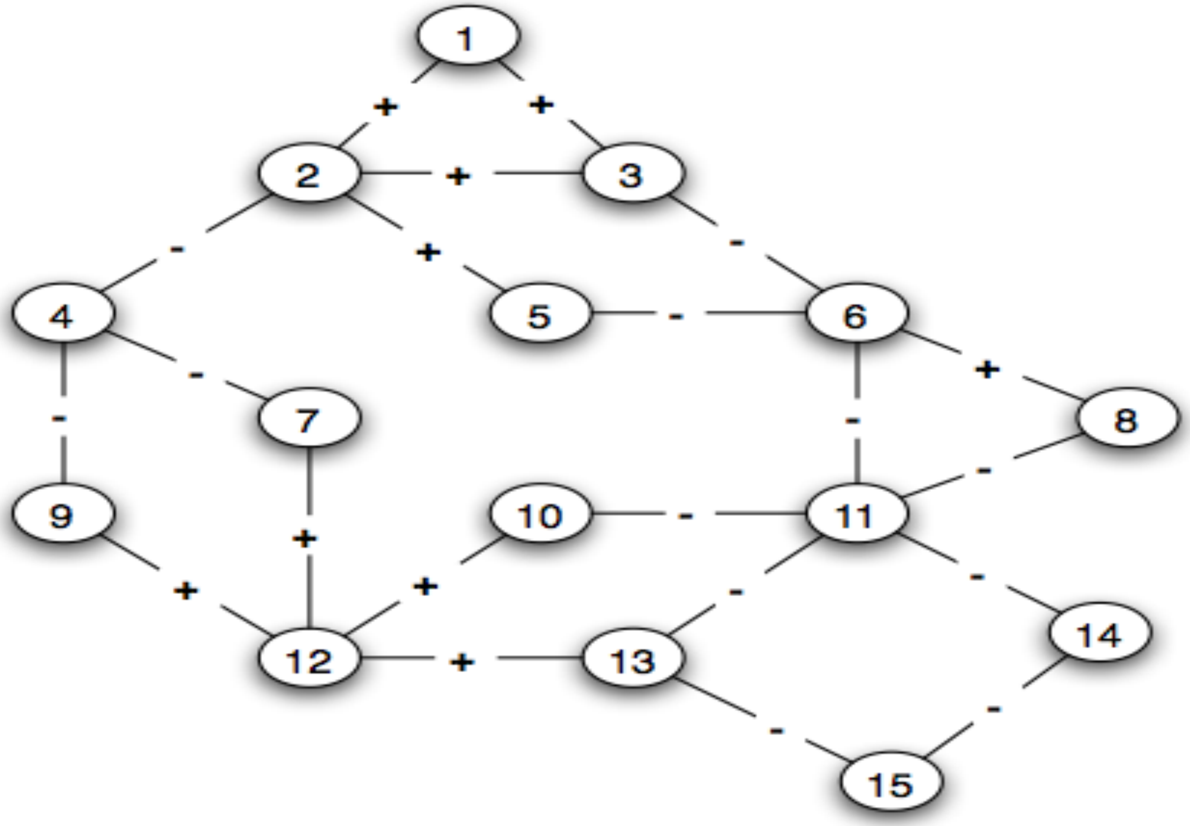
The two definitions are equivalent.

When is a signed network balanced?

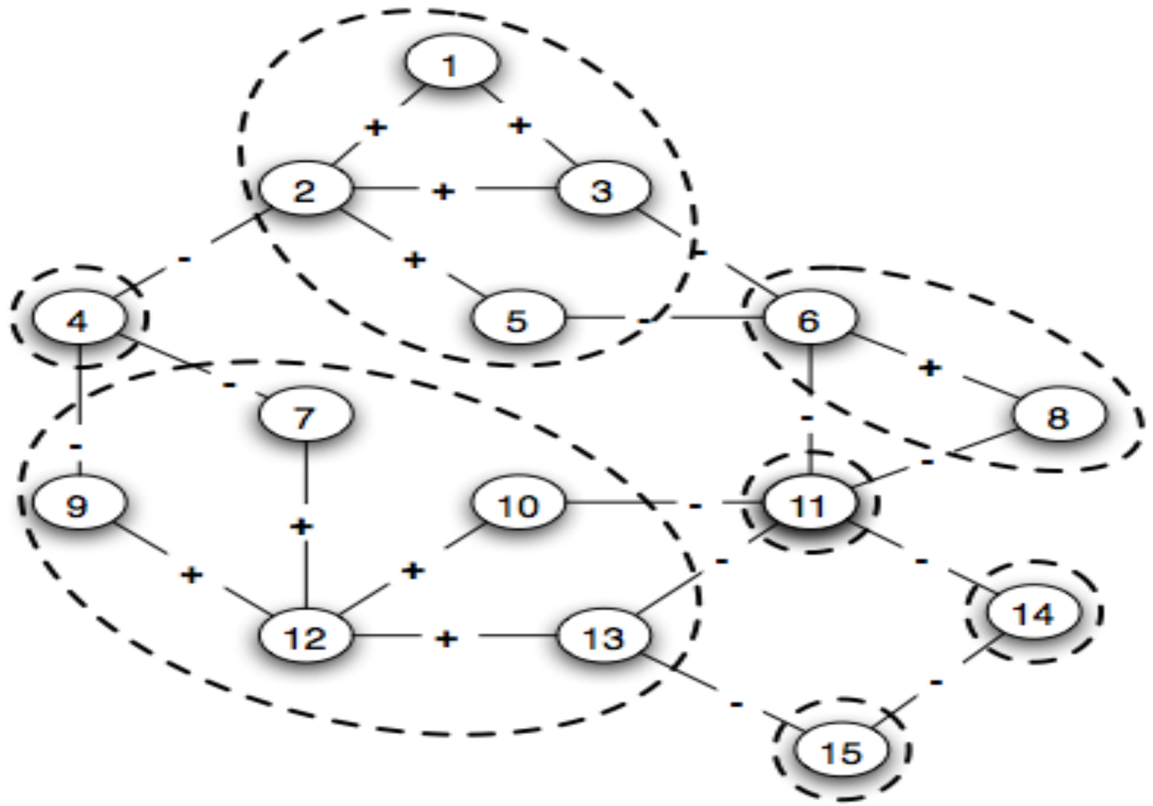
- A graph is balanced iff it contains no cycle with an odd number of negative edges.

- How to compute this?
  - Find connected components on + edges
    - If we find a component of nodes on + edges that contains a – edge ➔ unbalanced
    - For each component create a supernode
    - Connect components A and B if there is a negative edge between the members
    - Assign supernodes to *sides* (Left or Right) using BFS

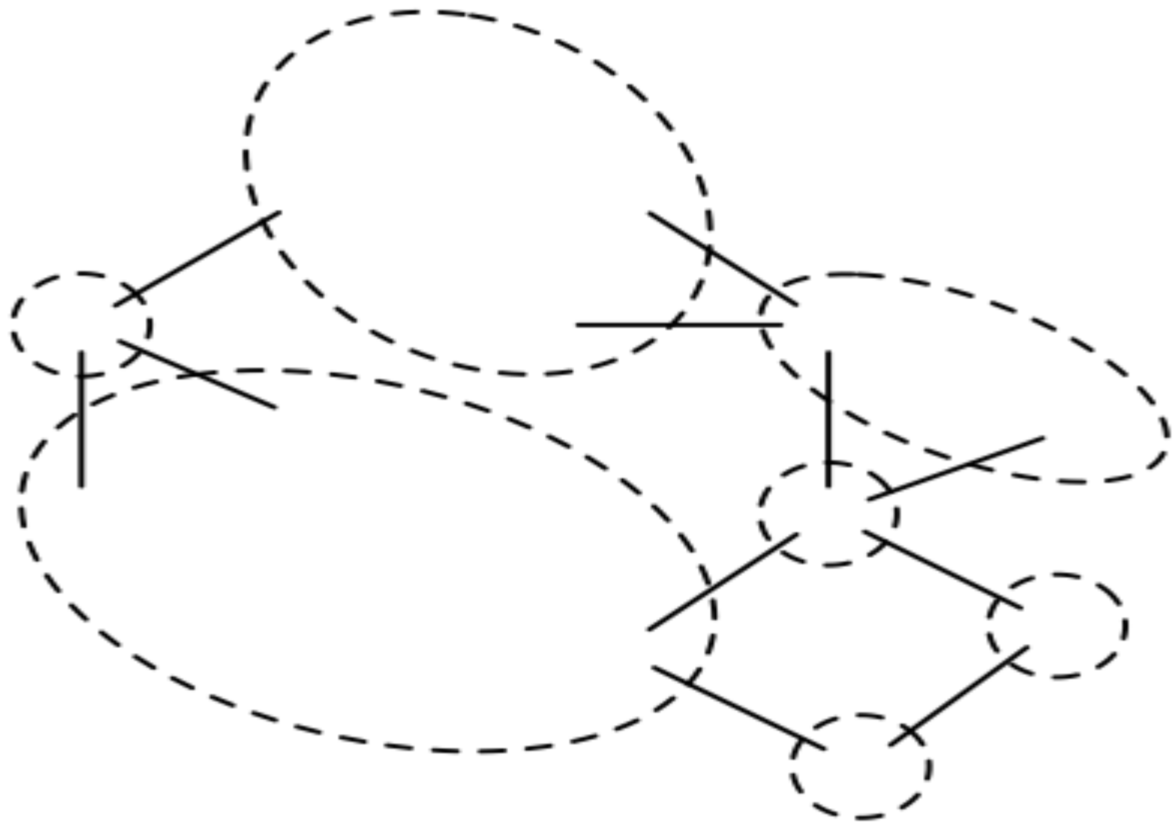
When is a signed network balanced?



Connected components on + edges

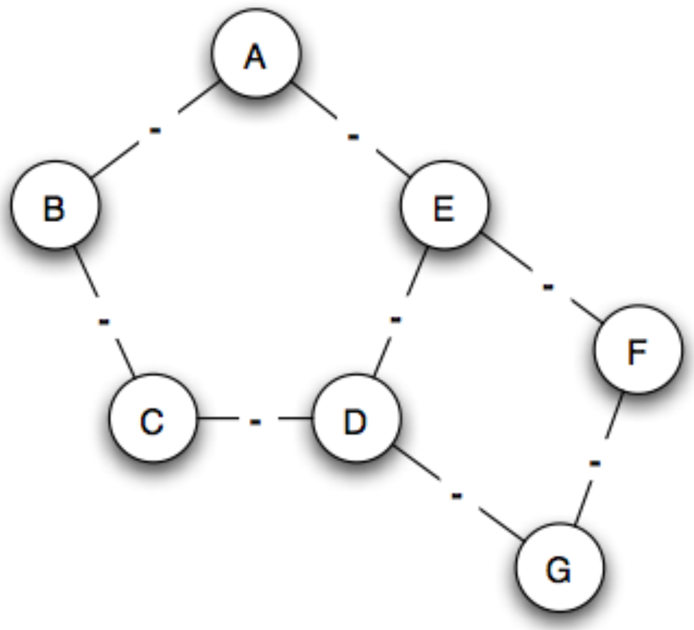


Reduced Graph (on super nodes)



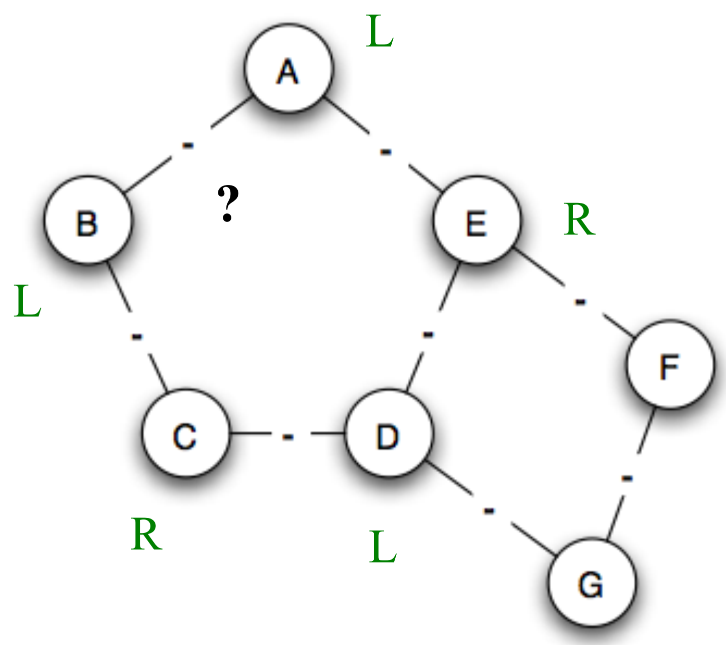
BFS on Reduced Graph

- Using BFS assign each node a *side*
- Graph is *unbalanced* if any two super nodes are assigned the *same side*

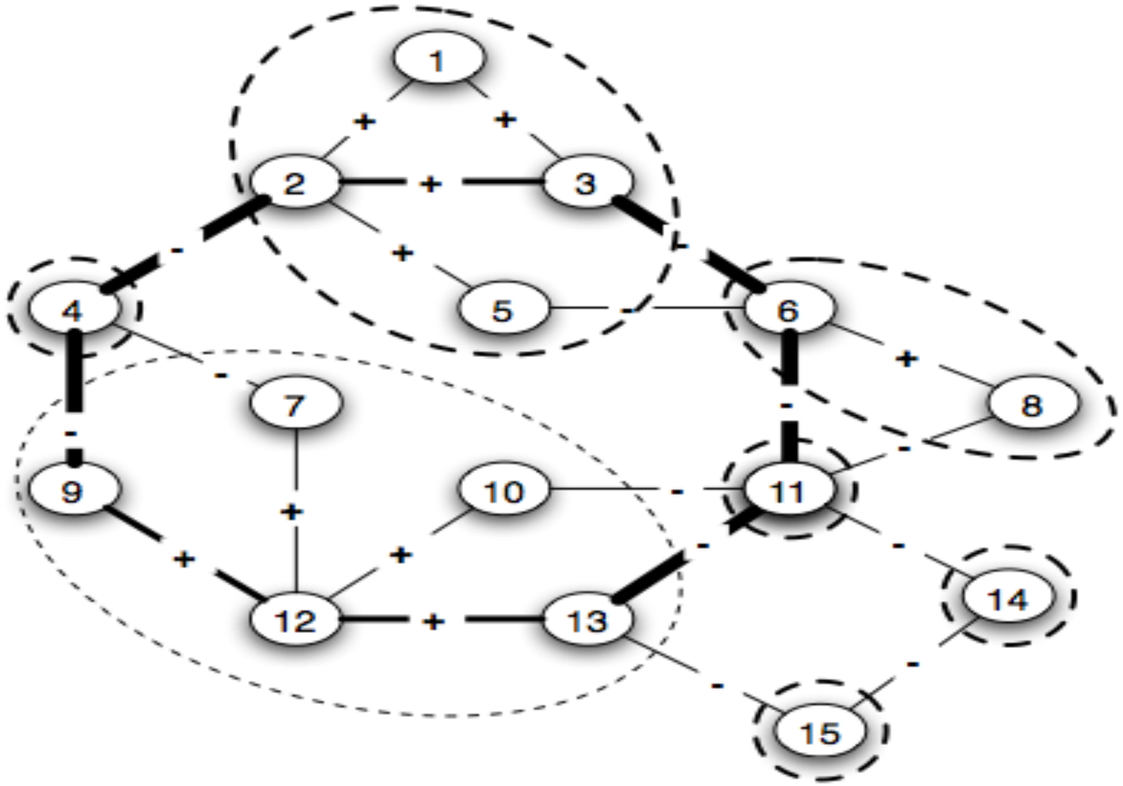


BFS on Reduced Graph

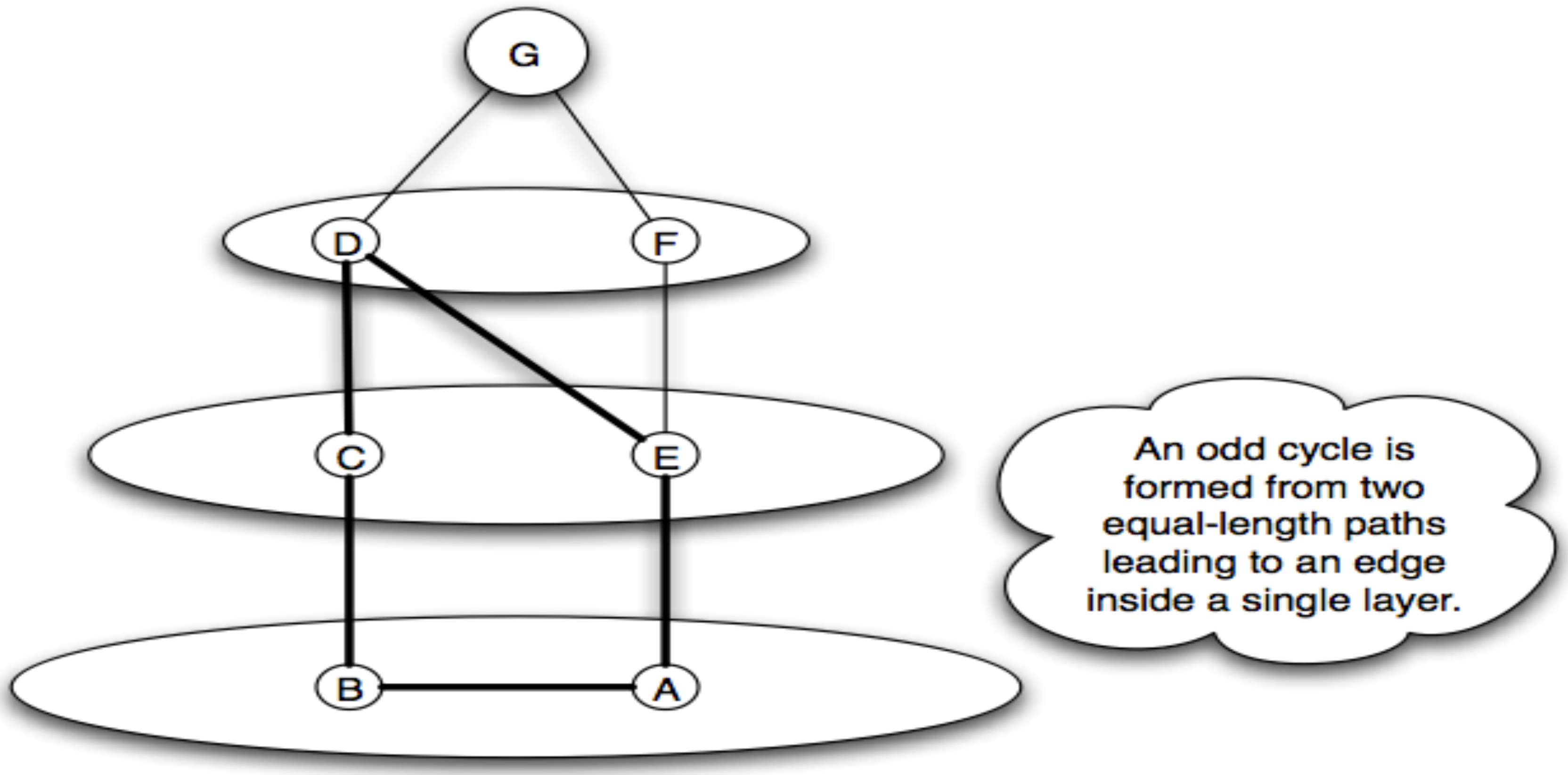
- Using BFS assign each node a *side*
- Graph is *unbalanced* if any two super nodes are assigned the *same side*



The cycle in the original graph



Bipartite graph detection: BFS



Further Reading

- Chapter 5 of Easley-Kleinberg