Notes

2 Coordinating the newsvendor

With the standard wholesale-price contract, it is shown that the retailer does not order enough inventory to maximize the supply chain's total profit because the retailer ignores the impact of his action on the supplier's profit. Hence, coordination requires that the retailer be given an **incentive** to increase his order

Types of contracts to coordinate the supply chain and arbitrarily divide its profit: - Buyback contracts - revenue-sharing contracts - quantity-flexibility contracts - sales-rebate contracts - quantity-discount contracts

2.1 Model and analysis

$\mu = E[D]$ is the mean of demand. The supplier's production cost per unit is c_s and the retailer's marginal cost per unit is c_r, $c_s + c_r < p$. c_r is incurred upon procuring a unit. Goodwill penalty cost g_r and the analogous cost for the supplier is g_s. Let $c = c_s + c_r$ and $g = g_s + g_r$. v is net of any salvage expenses.

The details of the negotiation process is not explored.

Each firm is risk neutral. Full information.

- voluntary compliance - forced compliance - The approach taken in this section is to assume forced compliance but to check if the supplier has an incentive to deviate from the proposed contractual terms.

$$
\begin{aligned}
S(q) &= E[\min(q, D)] = q(1 - F(q)) + \int_0^q y f(y)dy = q - \int_0^q F(y)dy \\
I(q) &= E[(q - D)^+] = q - S(q) \\
L(q) &= E[(D - q)^+] = \mu - S(q)
\end{aligned}
$$

where $I(q)$ is the expected leftover inventory and $L(q)$ is the lost-sales function.

The retailer's profit function is

$$
\begin{aligned}
\pi_r(q) &= pS(q) + vI(q) - g_r L(q) - c_r q - T \\
&= (p - v + g_r)S(q) - (c_r - v)q - g_r\mu - T,
\end{aligned}
\tag{Retailer}
$$

the supplier's profit function is

$$\pi_s(q) = g_s S(q) - c_s q - g_s \mu + T, \tag{Supplier}$$

and the supply chain's profit function is

$$\Pi(q) = \pi_r(q) + \pi_s(q) = (p - v + g)S(q) - (c - v)q - g\mu \tag{2.1}$$

Let q^o be a supply chain optimal order quantity, we have

$$S'(q^o) = \overline{F}(q^o) = \frac{c - v}{p - v + g} \tag{2.2}$$

since F is strictly increasing and thus Π is strictly concave and the optimal order quantity is unique.

Let $q_r^* = \arg\max \pi_r(q)$

2.2 The wholesale-price contract

Let $T_w(q, w) = wq$. Since $\pi_r(q, w)$ is strictly concave in q, we have

$$(p - v + g_r)S'(q_r^*) - (w + c_r - v) = 0. \tag{2.3}$$

Since $S'(q)$ is decreasing, $q_r^* = q^o$ only when

$$w = (\frac{p - v + g_r}{p - v + g})(c - v) - (c_r - v).$$

It shows that $w \leq c_s$, i.e., coordinates only if the supplier earns a **nonpositive** profit. Thus the wholesale-price contract is generally **not considered** a coordinating contract.

From Equation 2.3 we have

$$F(q_r^*) = 1 - \frac{w + c_r - v}{p - v + g_r}$$

It's obvious that there is a one-for-one mapping between w and q_r^*, then we have

$$w(q) = (p - v + g_r)\overline{F}(q) - (c_r - v),$$

the unique wholesale price that induces the retailer to order q_r^* units. Then we have the supplier's profit function:

$$\pi_s(q, w(q)) = g_s S(q) + (w(q) - c_s)q - g_s \mu, \tag{2.4}$$

from this we know that the *compliance regime* **does not** matter with this contract: for a fixed w no less than c_s the supplier's profit is nondecreasing in q.

We have the supplier's marginal profit:

$$\begin{aligned}\frac{\partial \pi_s(q, w(q))}{\partial q} &= g_s S'(q) + w(q) - c_s + w'(q)q \\ &= (p - v + g_r)\overline{F}(q)\left(1 + \frac{g_s}{p - v + g_r} - \frac{qf(q)}{\overline{F}(q)}\right) - (c - v)\end{aligned}$$

$\pi_s(q, w(q))$ is decreasing in q if $qf(q)/\overline{F}(q)$ is increasing. This type of demand distributions are called increasing generalized failure rate (**IGFR**) distributions.

Similarly, from Retailer we have

$$\begin{aligned}\pi_r(q, w(q)) &= (p - v + g_r)S(q) - (c_r - v)q - g_r v - w(q)q \\ &= (p - v + g_r)\left(S(q) - \overline{F}(q)q\right) - g_r v\end{aligned}$$

then we have

$$\frac{\partial \pi_r(q, w(q))}{\partial q} = (p - v + g_r)f(q)q > 0,$$

so the supplier can increase the retailer's profit by reducing the price. The supply chain's profit is increasing in q for $[q_s^*, q^o]$ and so is the retailer's profit. Hence, **an increase in retail power can actually improve supply chain performance.**

Define the efficiency of the contract, $\Pi(q_s^*)/\Pi(q^o)$ and $\pi_s(q_s^*, w(q_s^*))/\Pi(q_s^*)$, the supplier's profit share. For a broad set of demand distributions, the argument that the retailer is being compensated for **the risk that demand and supply do no match** holds, where both measures approach 1 with the variation approach 0.[**?**]

Two-period version of the model which has excess inventory and demand updating. *Push* and *pull* strategies. Advanced purchase discount $w_1 < w_2$. The supply chain effciency is substantially higher. There exist conditions in which advanced purchase discounts coordinate the supply chain and arbitrarily allocate its profit. **TBD**

2.3 The buyback contract

With a buyback contract the supplier charges the retailer w per unit puchased, but pays the retailer b per unit remaining at the end of the season:

$$T_b(q, w, b) = wq - bI(q) = bS(q) + (w - b)q.$$

See [**?**] for detail. An important **implicit** assumption is that the supplier is able to verify the number of remaining units and the cost of such monitoring does not negate the benefits created by the contract.

The retailer's profit now is:

$$\pi_{\mathrm{r}}\left(q, w_{\mathrm{b}}, b\right)=\left(p-v+g_{\mathrm{r}}-b\right) S(q)-\left(w_{\mathrm{b}}-b+c_{\mathrm{r}}-v\right) q-g_{\mathrm{r}} \mu$$

Consider $\{w_b, b\}$ such that for $\lambda \geq 0$,

$$p-v+g_r-b=\lambda(p-v+g) \tag{2.5}$$
$$w_b-b+c_r-v=\lambda(c-v) \tag{2.6}$$

A Comparing with Equation 2.1 leads to:

$$\begin{aligned}\pi_r(q, w_b, b) &= \lambda(p-v+g)S(q)-\lambda(c-v)q-g_r\mu \\ &= \lambda\Pi(q)+\mu(\lambda g-g_r).\end{aligned} \tag{2.7}$$

The supplier's profit function is

$$\pi_s(q, w_b, b)=(1-\lambda)\Pi(q)-\mu(\lambda g-g_r).$$

So the buyback contract **coordinates** with voluntary compliance as long as $\lambda \leq 1$. When $\lambda = 1$ (or $\lambda = 0$), the q^o is optimal for the supplier (or retailer), but so is every other quantity since the profit function is not related with q. Hence, coordination is possible but no longer the uniuqe Nash equilibrium.

The λ parameter acts to allocate the supply chain's profit between the two firms. The retailer earns the entire supply chain profit $\pi_r(q^o, w_b, b) = \Pi(q^o)$ when

$$\lambda=\frac{\Pi(q^o)+\mu g_r}{\Pi(q^o)+\mu g} \leq 1 \tag{2.8}$$

and the supplier $\pi_s(q^o, w_b, b) = \Pi(q^o)$, when

$$0 \leq \lambda=\frac{\mu g_r}{\Pi(q^o)+\mu g}. \tag{2.9}$$

So **every** possible profit allocation is feasible with this set of coordinating contracts, assuming $\lambda = 0$ and $\lambda = 1$ are considered feasible.

Note 1. The coordination of the supply chain requires the **simultaneous adjustment** of both the wholesale price w_b and the buyback rate b. This has implications for the bargaining process, e.g., never negotiate those parameters sequentially.

Note 2. Stock rebalancing in centralized system and decentralized system.

2.4 The revenue-sharing contract

With a revenue-sharing contract the supplier charges w_r per unit purchased plus the retailer gives the supplier a percentage of his revenue. Let ϕ be the fraction of revenue that retailer keeps.

The transfer payment with revenue sharing is

$$\begin{aligned}T_r(q, w_r, \phi) &= w_r q + (1-\phi)(vI(q) + pS(q)) \\ &= (w_r + (1-\phi)v)q + (1-\phi)(p-v)S(q)\end{aligned}$$

The retailer's profit function is

$$\pi_{\mathrm{r}}\left(q, w_{\mathrm{r}}, \phi\right) = \left(\phi(p-v) + g_{\mathrm{r}}\right) S(q) - \left(w_{\mathrm{r}} + c_{\mathrm{r}} - \phi v\right) q - g_{\mathrm{r}} \mu$$

Now consider the set of revenue-sharing contracts, $\{w_{\mathrm{r}}, \phi\}$, such that $\lambda \geq 0$ and

$$\begin{aligned}\phi(p-v) + g_{\mathrm{r}} &= \lambda(p-v+g) \\ w_{\mathrm{r}} + c_{\mathrm{r}} - \phi v &= \lambda(c-v)\end{aligned}$$

Now we have

$$\begin{aligned}\pi_r(q, w_r, \phi) &= \lambda\Pi(q) + \mu(\lambda g - g_r) \\ \pi_s(q, w_r, \phi) &= (1-\lambda)\Pi(q) - \mu(\lambda g - g_r).\end{aligned} \tag{2.10}$$

It's obvious that Equation 2.8 and Equation 2.9 provides the same λ.

From Equation 2.10 and Equation 2.7 we find similarity. Consider a coordinating buyback contract $\{w_b, b\}$. The retailer pays $w_b - b$ for each unit purchased and an additional b per unit sold. With revenue sharing the retailer pays $w_r + (1-\phi)v$ and $(1-\phi)(p-v)$. Now they are equivalent when

$$\begin{aligned}w_b - b &= w_r + (1-\phi)v \\ b &= (1-\phi)(p-v)\end{aligned}$$

Note 3. Their path will diverge in more complex settings.

2.5 The quantity-flexibility contract

With a quantity-flexibility contract, the supplier charges w_q per unit purchased but then compensates the retailer for his losses on unsold units. The retailer receives a credit from the supplier at the end of the season equal to $(w_q + c_r - v)\min(I, \delta q)$, where I is the leftover and $\delta \in [0,1]$ a contract parameter. It **fully** protects the retailer on **a portion of** the retailer's order whereas the buyback contract gives **partial** protection on the retailer's **entire order**.

Now the transfer payment is

$$T_q(q, w_q, \delta) = w_q q - (w_q + c_r - v)\int_{(1-\delta)q}^{q} F(y)dy$$

Note 4. Need to be checked.

The retailer's profit function is

$$\begin{aligned}\pi_{\mathrm{r}}\left(q, w_q, \delta\right) &= \left(p-v+g_{\mathrm{r}}\right) S(q)-\left(c_{\mathrm{r}}-v\right) q-T_q\left(q, w_q, \delta\right)-\mu g_{\mathrm{r}} \\ &= \left(p-v+g_{\mathrm{r}}\right) S(q)-\left(w_q+c_{\mathrm{r}}-v\right) q \\ &\quad+\left(w_q+c_{\mathrm{r}}-v\right) \int_{(1-\delta) q}^q F(y) \mathrm{d} y-\mu g_{\mathrm{r}} .\end{aligned}$$

To achieve supply chain coordination it is necessary that

$$(p-v+g_r)S'(q^o)-(w_q+c_r-v)\left(1-F(q^o)+(1-\delta)F((1-\delta)q^o)\right)=0. \tag{2.11}$$

Let $w_q(\delta)$ be the wholesale price that satisfies Equation 2.11:

$$w_q(\delta)=\frac{\left(p-v+g_{\mathrm{r}}\right)\left(1-F\left(q^{\mathrm{o}}\right)\right)}{1-F\left(q^{\mathrm{o}}\right)+(1-\delta) F\left((1-\delta) q^{\mathrm{o}}\right)}-c_{\mathrm{r}}+v$$

$w_q(\delta)$ is indeed a coordinating wholesale price if the retailer's profit function is concave:

$$\begin{aligned}\frac{\partial^2 \pi_{\mathrm{r}}\left(q, w_q(\delta), \delta\right)}{\partial q^2} &= -\left(p+g_{\mathrm{r}}-w_q(\delta)-c_{\mathrm{r}}\right) f(q)-\left(w_q(\delta)+c_{\mathrm{r}}-v\right)\left(1+(1-\delta)^2 f((1-\delta) q)\right) \\ &\leq 0\end{aligned}$$

which holds when $v-c_{\mathrm{r}} \leq w_q(\delta) \leq p+g_{\mathrm{r}}-c_{\mathrm{r}}$. That range is satisfied with $\delta \in[0,1]$ because

$$\begin{aligned}w_q(0) &= \left(p-v+g_{\mathrm{r}}\right) \bar{F}\left(q^{\mathrm{o}}\right)+v-c_{\mathrm{r}} \\ w_q(1) &= p+g_{\mathrm{r}}-c_{\mathrm{r}}\end{aligned}$$

and $w_q(\delta)$ is increasing in δ.

Now we consider supplier's profit function:

$$\pi_{\mathrm{s}}\left(q, w_q(\delta), \delta\right)=g_{\mathrm{s}} S(q)+\left(w_q(\delta)-c_{\mathrm{s}}\right) q-\left(w_q(\delta)+c_{\mathrm{r}}-v\right) \int_{(1-\delta) q}^q F(y) \mathrm{d} y-\mu g_{\mathrm{s}}$$

and

$$\begin{aligned}\frac{\partial \pi_{\mathrm{s}}\left(q, w_q(\delta), \delta\right)}{\partial q} =& g_{\mathrm{s}}(1-F(q))+\left(w_q(\delta)-c_{\mathrm{s}}\right)-\left(w_q(\delta)+c_{\mathrm{r}}-v\right)(F(q) \\ &-(1-\delta) F((1-\delta) q)) \\ =& g_{\mathrm{s}}(1-F(q))-c+v+\left(w_q(\delta)+c_{\mathrm{r}}-v\right)(1-F(q) \\ &+(1-\delta) F((1-\delta) q))\end{aligned}$$

The supplier's first-order condition at q° is satisfied:

$$\frac{\partial \pi_{\mathrm{s}}\left(q^{\mathrm{o}}, w_q(\delta), \delta\right)}{\partial q}=g_{\mathrm{s}}\left(1-F\left(q^{\mathrm{o}}\right)\right)-c+v+\left(p-v+g_{\mathrm{r}}\right)\left(1-F\left(q^{\mathrm{o}}\right)\right)=0$$

Note 5. See Equation 2.2

However, the sign of the second-order condition at q° is ambiguous,

$$\frac{\partial^2 \pi_{\mathrm{s}}\left(q, w_q(\delta), \delta\right)}{\partial q^2} = -w_q(\delta)\left(f(q) - (1-\delta)^2 f((1-\delta)q)\right) - g_{\mathrm{s}} f(q)$$

Hence, supply chain coordination under **voluntary compliance** is **not assured** with a quantity-flexibility contract even if the wholesale price is $w_q(\delta)$. It's **achieved** under **forced compliance** since then the supplier's action is **not relevant**.

Note 6. There are some conditions that makes q^o a local maximum, e.g., $\mu = 10$, $\sigma = 1$, $p = 10$, $c_s = 1$, $c_r = 0$, $g_r = g_s = v = 0$ and $\delta = 0.1$.

Assuming a $(w_q(\delta), \delta)$ **quantity-flexibility contract coordinates the channel.** When $\delta = 0$, for the retailer we have

$$\begin{aligned}
\pi_r(q, w_q(0), 0) &= (p - v + g_r)S(q) - \left(\frac{p - v + g_r}{p - v + g}\right)(c - v)q^o - \mu g_r \\
&= \Pi(q^o) + g_s\left(\mu - S(q^o) + \overline{F}(q^o)q^o\right) \\
&\geq \Pi(q_o)
\end{aligned}$$

When $\delta = 1$, for the supplier we have

$$\begin{aligned}
\pi_s(q, w_q(1), 1) &= g_s S(q^o) + (p + g_r - c)q^o - (p + g_r - v)\int_0^q F(y)dy - \mu g_s \\
&= \Pi(q^o) + \mu g_r \\
&\geq \Pi(q^o)
\end{aligned}$$

Since the profit function is continuous in δ, all possible allocation of $\Pi(q^o)$ are possible.

2.6 The sales-rebate contract

With a sales-rebate contract the supplier charges ws per unit purchased but then gives the retailer an r rebate per unit sold above a threshold t. The transfer payment with the sales-rebate contract is

$$T_{\mathrm{s}}\left(q, w_{\mathrm{s}}, r, t\right) = \begin{cases} w_{\mathrm{s}} q & q < t \\ \left(w_{\mathrm{s}} - r\right) q + r\left(t + \int_t^q F(y)\mathrm{d}y\right) & q \geq t \end{cases}$$

Note 7. $T = w_s q - rE[(\min(q, D) - t)^+]$

For this contract to achieve supply chain coordination, q^o must at least be a local maximum:

$$\frac{\partial \pi_{\mathrm{r}}\left(q^{\mathrm{o}}, w_{\mathrm{s}}, r, t\right)}{\partial q} = \left(p - v + g_{\mathrm{r}}\right)\bar{F}\left(q^{\mathrm{o}}\right) - \left(c_{\mathrm{r}} - v\right) - \frac{\partial T_{\mathrm{s}}\left(q^{\mathrm{o}}, w_{\mathrm{s}}, r, t\right)}{\partial q} = 0 \tag{2.12}$$

If $q^o \leq t$, the above leads to $w_s = c_s - g_s\overline{F}(q^o) \leq c_s$, which is not acceptable to the supplier. So assume $q^o > t$. Then from Equation 2.12 we have

$$w_s(r) = (p - v + g_r + r)\overline{F}(q^o) - c_r + v \tag{2.13}$$

Thus, we have the retailer's profit function

$$\begin{aligned}\pi_{\mathrm{r}}\left(q, w_{\mathrm{s}}(r), r, t\right) =& \Pi(q) + g_{\mathrm{s}}\left(\mu - S(q) + q\bar{F}\left(q^{\mathrm{o}}\right)\right) - rq\bar{F}\left(q^{\mathrm{o}}\right) \\ &+ \begin{cases} 0 & q < t \\ rq - r\left(t + \int_t^q F(y)\mathrm{d}y\right) & q \geq t \end{cases}\end{aligned}$$

and

$$\begin{aligned}\pi_{\mathrm{r}}\left(q^{\mathrm{o}}, w_{\mathrm{s}}(r), r, t\right) =& \Pi\left(q^{\mathrm{o}}\right) + g_{\mathrm{s}}\left(\mu - S\left(q^{\mathrm{o}}\right) + q^{\mathrm{o}}\bar{F}\left(q^{\mathrm{o}}\right)\right) \\ &+ r\left(q^{\mathrm{o}}F\left(q^{\mathrm{o}}\right) - t - \int_t^{q^{\mathrm{o}}} F(y)\mathrm{d}y\right)\end{aligned}$$

With $t = 0$ the retailer earns more than $\Pi\left(q^{\circ}\right)$, so q° is surely optimal. With $t = q^{\mathrm{o}}$, the retailer's profit function is decreasing for $t \geq q^{\mathrm{o}}; \bar{q}$ is at least as good for the retailer as q^{o}. Given that $\pi_{\mathrm{r}}\left(q^{\mathrm{o}}, w_{\mathrm{s}}(r), r, t\right)$ is decreasing in t, there must exist some t in the range $[0, q^{\mathrm{o}}]$ such that $\pi_{\mathrm{r}}\left(q^{\mathrm{o}}, w_{\mathrm{s}}(r), r, t\right) = \pi_{\mathrm{r}}\left(\bar{q}, w_{\mathrm{s}}(r), r, t\right)$, i.e., there are coordinating contracts such that q^o is preferred by the retailer over $\overline{q}$.

Note 8. Why there must exist some t in $[0, q^o]$ such that $\pi_{\mathrm{r}}\left(q^{o}, w_{\mathrm{s}}(r), r, t\right) = \pi_{\mathrm{r}}\left(\bar{q}, w_{\mathrm{s}}(r), r, t\right)$?

Note 9. It's easy to check there are a set of contracts that generate any allocation of supply chain's profit.

Now consider the supplier's production decision. The supplier's profit function in this type of contract is

$$\pi_{\mathrm{s}}\left(q, w_{\mathrm{s}}(r), r, t\right) = -g_{\mathrm{s}}(\mu - S(q)) - c_{\mathrm{s}}q + T_{\mathrm{s}}\left(q, w_{\mathrm{s}}(r), r, t\right)$$

For $q > t$

$$\begin{aligned}\frac{\partial \pi_{\mathrm{s}}\left(q, w_{\mathrm{s}}(r), r, t\right)}{\partial q} &= g_{\mathrm{s}}\bar{F}(q) - c_{\mathrm{s}} + w_{\mathrm{s}}(r) - r + rF(q) \\ &= \left(r - g_{\mathrm{s}}\right)\left(F(q) - F\left(q^{\mathrm{o}}\right)\right)\end{aligned}$$

To have q^o a local maximum for the supplier, we should have $r < g_s$ for $q \leq q^o$ which leads to that $w_s(r) \leq c_s$ and the supplier cannot earn a positive profit. Thus we must have $r > g_s$, but this implies the supplier loss money for each unit delivered to the retailer above t by Equation 2.13:

$$w_s(r) - r = c_s - v - g_s\overline{F}(q^o) - rF(q^o) < c_s.$$

Thus the sales-rebate contract does not coordinate the supply chain with voluntary compliance.

2.7 The quantity-discount contract

This section considers an "all unit" quantity discount, i.e., $T_d(q) = w_d(q)q$ where $w_d(q)$ is decreasing in q.

Note 10. There are many types of quantity discounts. See Moorthy (1987) for a more detailed explanation for why many coordinating quantity discount schedules exist. See Kolay and Shaffer (2002) for a discussion on different types of quantity discounts.

The retailer's profit function is then

$$\pi_r(q, w_d(q)) = (p - v + g_r)S(q) - (w_d(q) + c_r - v)q - g_r\mu.$$

One technique to obtain coordination is to choose the payment schedule such that the retailer's profit equals a constant fraction of the supply chain's profit. To be specific, let

$$w_d(q) = ((1-\lambda)(p - v + g) - g_s)\left(\frac{S(q)}{q}\right) + \lambda(c - v) - c_r + v.$$

The above is decreasing in q if $\lambda \leq \overline{\lambda}$, where

$$\overline{\lambda} = \frac{p - v + g_r}{p - v + g},$$

since $S(q)/q$ is decreasing in q. The retailer's profit function is now

$$\begin{aligned}\pi_r(q, w_d(q)) &= \lambda(p - v + g)S(q) - \lambda(c - v)q - g_r\mu \\ &= \lambda(\Pi(q) + g\mu) - g_r\mu\end{aligned}$$

Hence q^o is optimal for both the retailer and the supplier. The parameter λ acts to allocate the supply chain's profit between the two firms, however, it has an upperbound that prevents too much profit from being allocated to the retailer. The $w_d(q)$ can still coordinate even if $\lambda > \overline{\lambda}$ but then the $w_d(q)$ will be increasing in q, i.e., a quantity-premium contract.

2.8 Discussion

Revenue sharing and quantity discounts always coordinate the supplier's action with voluntary compliance, quantity-flexibility contracts generally, but not always, coordinate the supplier's action and sales-rebate contracts never do.

he coordinating revenue-sharing contracts do not depend on the demand distribution, but do depend on the retailer's marginal cost.

Note 11. Read Inducing Forecast Revelation through Restricted Returns

3 Coordinating the newsvendor with *price-dependent* demand

3.1 Model and analysis

Now the retailer chooses his price in addition to his order quantity. Let $F(q|p)$ be the distribution function of demand, where p is the retail price. Assume $\frac{\partial F(q|p)}{\partial p} > 0$. To obtain initial insights, assume the retailer sets his **price** *at the same time as* his **stocking decision** and the price is **fixed** throughout the season.

Note 12. van Mieghem and Dada (1999). A hybrid model. The retailer chooses q, then observes a demand signal and then chooses price.

The integrated channel's profit is

$$\Pi(q,p) = (p - v + g)S(q,p) - (c - v)q - g\mu$$

where $S(q,p)$ is expected sales given the stocking quantity q and the price p, and similarly, we have

$$S(q,p) = q - \int_0^q F(y|p)dy$$

Note 13. The integrated channel profit function need not be concave nor unimodal (Petruzzi & Dada 1999)

Let $p^o(q)$ be the supply chain optimal price for a given q. The necessary condition for coordination is

$$\frac{\partial \Pi(q, p^o(q))}{\partial p} = S(q, p^o(q)) + (p^o(q) - v + g)\frac{\partial S(q, p^o(q))}{\partial p} = 0. \tag{3.1}$$

Note 14. Either not satisfy the first-order condition or fail to coordinate the quantity decision.

Consider the **quantity-flexibility** contract. The retailer's profit function is

$$\begin{aligned}\pi_r(q,p,w_q,\delta) = &(p - v + g_r)S(q,p) - (w_q + c_r - v)q \\ &+(w_q + c_r - v)\int_{(1-\delta)q}^{q} F(y|p)dy - \mu g_r\end{aligned}$$

For price coordination the first-order condition must hold,

$$\begin{aligned}\frac{\partial \pi_{\mathrm{r}}\left(q, p^{\mathrm{o}}(q), w_q, \delta\right)}{\partial p} =& S\left(q, p^{\mathrm{o}}(q)\right) + \left(p^{\mathrm{o}}(q) - v + g_{\mathrm{r}}\right) \frac{\partial S\left(q, p^{\mathrm{o}}(q)\right)}{\partial p} \\ &+ \left(w_q + c_{\mathrm{r}} - v\right) \int_{(1-\delta)q}^{q} \frac{\partial F\left(y \mid p^{\mathrm{o}}(q)\right)}{\partial p}\, \mathrm{d}y \\ =& 0 \end{aligned} \tag{3.2}$$

The second term in Equation 3.2 is no smaller than the second term in Equation 3.1[1], so the above holds only if the third term is nonpositive. But the third term is nonnegative as $w_q + c_r - v \geq 0$, so with a coordinating w_q, the coordination of price can only occur if $g_s = 0$ and either $w_q = v - c_r$ or $\delta = 0$. **Neither** is desirable. With $w_q = v - c_r$, then supplier has $w_q < c_s$[2] which is not acceptable. With $\delta = 0$ the contract degenerates to just a wholesale-price contract, so the retailer's quantity action is not optimal. Hence, the quantity-flexibility contract does not coordinate the newsvendor with price-dependent demand.

The **sales-rebate** contract does not fare better:

$$\begin{aligned}\frac{\partial \pi_r(q, p^o(q), w_s, r, t)}{\partial p} =& S(q, p^o(q)) + (p^o(q) - v + g_r)\frac{\partial S(q, p^o(q))}{\partial p} \\ &- r\int_t^q \frac{\partial F(y|p^o(q))}{\partial p} dy\end{aligned}$$

Since the last term is negative when $r > 0$ and $t < q$, we know that the retailer prices below the optimal price[3]. Coordination might be achieved if there is something to induce the retailer to a higher price.

Now consider a **buyback** contract. The retailer's profit function is

$$\pi_r(q, p, w_b, b) = (p - v + g_r - b)S(q, p) - (w_b - b + c_r - v)q - g_r\mu.$$

For coordination we must have the first-order condition:

$$\frac{\partial \pi_r(q, p^o(q), w_b, r, t)}{\partial p} = S(q, p^o(q)) + (p^o(q) - v + g_r - b)\frac{\partial S(q, p^o(q))}{\partial p} = 0. \tag{3.3}$$

But comparing with Equation 3.1 it holds only if $b = -g_s < 0$ which violates that $b \geq 0$[4]. Therefore, a buyback contract does not coordinate the newsvendor with price-dependent demand.

The buyback contract fails to coordinate in this setting because the parameters of the coordinating contracts depend on the price: from Equation 2.5 and Equation 2.6, the coordinating parameters are

$$\begin{aligned} b &= (1-\lambda)(p - v + g) - g_s \\ w_b &= \lambda c_s + (1-\lambda)(p + g - c_r) - g_s. \end{aligned}$$

[1] The assumption of $\partial F(q|p)/\partial p > 0$.

[2] Why? An assumption?

[3] The above derivative is negative. Why it means that the retailer prices below the optimal price?

[4] If $g_s = 0$, then $w_b = c_s$ and $b_s = 0$ which means that the supplier earns no positive profit.

For a fixed λ, the coordianting buyback rate and wholesale price are linear in p. Hence, the buyback contract coordiantes the newsvendor with price-dependent demand if b and w_b are made **contingent** on the retail price chosen, or if b and w_b are chosen **after** the retailer commits to a price (but before the retailer chooses q). This is the **price-discount-sharing** contract[5], which is called a "bill back" in practice. The retailer gets a lower wholesale price if the retailer reduces his price, i.e., the supplier shares in the cost of a price discount with the retailer. Then we have the retailer profit function:

$$\begin{aligned}\pi_r(q,p,w_b,b) &= \lambda(p-v+g)S(q,p) - \lambda(c-v)q - g_r\mu \\ &= \lambda(\Pi(q,p)+g\mu) - g_r\mu\end{aligned}$$

Hence, for the retailer as well ass the supplier, $\{q^o, p^o\}$ is optimal for $\lambda \in [0,1]$.

Now consider the **revenue-sharing** contract. The retailer's profit is

$$\pi_r(q,p,w_r,\phi) = (\phi(p-v)+g_r)S(q,p) - (w_r+c_r-\phi v)q - g_r\mu.$$

Coordination require

$$\frac{\pi_r(q,p^o(q),w_r,\phi)}{\partial p} = S(q,p^o(q)) + (p^o(q)-v+g_r/\phi)\frac{\partial S(q,p^o(q))}{\partial p} = 0. \tag{3.4}$$

- Consider $g_r = g_s = 0$. In this situation,

  $$\frac{\partial \pi_r(q,p,w_r,\phi)}{\partial p} = \frac{\partial \Pi(q,p)}{\partial p}$$

  with **any** revenue-sharing contract. Thus, the retailer chooses $p^o(q)$ no matter which revenue-sharing contract is chosen. Now revenue sharing is able to coordinate the retailer's quantity decision with precisely the same set of contracts used when the retailer prices is fixed.

  Recall that with the *fixed price* newsvendor **revenue sharing** and **buybacks** are equivalent. Here, the contracts produce different outcomes because with a buyback the retailer's share of revenue $(1-b/p)$ depends on the price, whereas with revenue sharing it is independent of the price, by definition[6]. However, the **price contingent buyback** contract (**price-discount** contract) is equivalent to revenue sharing: if $g_r = g_s = 0$, the coordinating revenue-sharing contract yield

  $$\pi_r(q,p,w_r,\phi) = \lambda\Pi(q,p)$$

  from Equation 2.10. And the price contingent buyback contract yield the same profit for any quantity and price from Equation 2.7,

  $$\pi_r(q,p,b(p),w_b(p)) = \lambda\Pi(q,p).$$

- Consider either $g_r > 0$ or $g_s > 0$.

[5]Bernstein and Federgruen (2000)
[6]The above partial derivative