Calibration of a Robot Hand-Eye System with a Concentric Circles Target

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***Abstract*—An approach for a robot hand-eye system calibration is presented using concentric circles target with one pair of orthogonal diameters. The geometric features of vanishing points on the orthogonal diameters are exploited to estimate the camera intrinsic and extrinsic parameters. One diameter is used to estimate the coefficients of the structured light plane. The structured light stripe projected on the circle center and one endpoint of each diameter are used to calculate the target position in the robot workspace and finally the transformation formula is used to obtain the hand-eye relationship. The measurement experimental results verify the calibration method with the measurement average absolute error 0.33mm and the average relative error 0.21%.**

I. INTRODUCTION

Linear structured light vision sensors have advantages of high measuring speed, high precision, robust to disturbance and low cost, so they are widely used to obtain three-dimensional (3D) information from images [1-5]. Linear structured light vision sensors are limited since only local surface information illuminated by the structured stripe can be obtained. In order to acquire the whole object 3D information, the linear structured light vision sensor is usually combined with other precision platforms, such as robots, to scan the object surface and online measurement. Ref. [6] gives the combined system of a linear structured light vision sensor and a coordinate measuring machine. Ref. [7] describes the combined system of linear structured light projector and a multi-joints measuring arm. Ref. [8] describes a measuring system consisting of a linear structured light vision sensor and a robot with six degree of freedom (6DOF).

When a vision sensor is mounted on a platform, it is necessary to know the relationship between the platform (hand, usually is a robot) and the vision sensor (eye). The processing of acquiring this relationship is referred to as the hand-eye calibration problem. Hand-eye calibration is important in at least two types of tasks: one is mapping sensor measurements into the robot workspace coordinate system; the other is allowing the robot to move the sensor precisely [9]. The traditional approach to calibrate a hand-eye system with the structured light vision sensor is based on a certain standard reference object. Ref. [7] and Ref. [8] use the center of a standard ball as the fix calibration point to calibrate the hand-eye relationship. Ref. [10] use two pieces of square with a round hole respectively to compute the camera intrinsic and to solve for the hand-eye calibration problem.

Our work is focused on a hand-eye system composed of a structured light vision sensor and a 6DOF robot. The proposed approach only requires the camera to observe a concentric circle target with a pair of known orthogonal diameters at a few different orientations. The target can be printed on a laser printer and attached to a planar surface. The proposed approach to calibrate the hand-eye system takes two separate stages: structured light vision sensor calibration and hand-eye calibration.

The structured light vision sensor calibration stage is to compute the camera intrinsic and extrinsic parameters based on the geometric characteristics of the vanishing points on the orthogonal diameters and compute the structured light plane by three points on one known diameter. While the camera is calibrated the target can be moved by hand in its view field and when the structured light plane is calibrated the known diameter for calibration on the target should be moved in the light plane. Those motions need not to be known. The hand-eye calibration stage is to estimate relationship between the structured light vision sensor and the robot end-effector. During this calibration process, the target should be fixed and the robot with the structured light vision sensor should be controlled to move, the orientation and the position of the end-effector should be known.

The remainder of this article is organized as follows. Section II states the components of the whole system and the models of the camera, linear structured light projector and hand-eye calibration problem. Section III describes the principle of each part from observing the calibration target. Section IV provides the experimental results and a segment measuring experiment results and Section V provides a short discussion.

II. HAND-EYE SYSTEM STRUCTURE AND MODELS

A. Hand-Eye System Structure

Fig.1 illustrates the structure of the hand-eye system.

Figure 1. A schematic diagram of a hand-eye system

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The two basic components of the hand-eye system are a 6DOF robot and a linear structured light vision sensor, which is composed of a structured light projector and a camera, displaced either laterally or vertically relative to each other in space.

All the coordinate systems in the whole system and the main coordinate's transformations between different coordinate systems are best described in Fig.2. Robot base coordinate system $\{o\}$ is the reference coordinate system. $\{w\}$ is the target coordinate system which is established on the calibration target. Its origin O_w is the circle center, x_w and y_w axes are the vectors of O_wA and O_wC respectively. $\{e\}$ is the end-effector coordinate system. $\{s\}$ is the computer image coordinate system in pixels. Its origin O_s is located in the upper-left of the image, u axis is the pixel column number and v axis is the pixel row number. $\{i\}$ is the image coordinate system in millimeters. Its origin O_i is the center of the image plane, x_i and y_i axes are parallel to u and v axes, respectively. $\{c\}$ is the camera coordinate system, whose origin O_c is the optical center, x_c , y_c axes are parallel to x_i and y_i axes, respectively, and z_c is the optical axis.

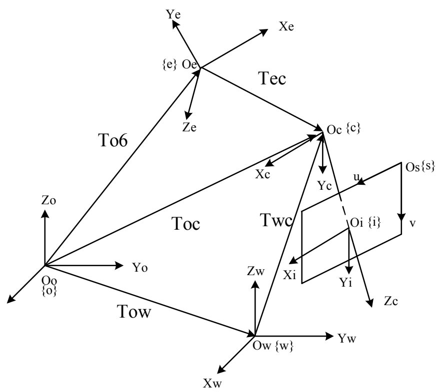

Figure 2. Coordinate systems and its mapping

T_{o6} is the transformation matrix of the end-effector coordinate system $\{e\}$ relative to the robot base coordinate system $\{o\}$. T_{ec} describes the transformation of the camera coordinate system $\{c\}$ to the end effector coordinate system $\{e\}$. T_{ow} represents the position of $\{w\}$ in the robot base in robot base coordinate system $\{o\}$. T_{wc} indicates the camera calibration system $\{c\}$ relative to the target coordinate system $\{w\}$. The inverse of T_{wc} is denoted by T_{cw}, which is camera extrinsic parameters matrix relative to the target coordinate system $\{w\}$.

B. *Hand-Eye System Models*

The camera is modelled by the pinhole. A space 3D point in the target coordinate system $\{w\}$ is denoted by $M_w(x_{Mw}, y_{Mw}, z_{Mw})^T$and its homogenous coordinates are $\tilde{M}_w(x_{Mw}, y_{Mw}, z_{Mw}, 1)^T$. Map this point to a two-dimensional (2D) point on the image plane. Its pixel coordinates are $m_s(u, v)^T$and its homogenous coordinates are $\tilde{m}_s(u, v, 1)^T$. The relationship between a 3D point M and its 2D image point m is given by

$$\mu\tilde{m}_s = P\tilde{M}_w \tag{1}$$

where $P = K\left[R_{cw} \quad t_{cw}\right]$ is called the perspective projection matrix. $K = \begin{bmatrix} \alpha & 0 & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$ is called the camera intrinsic matrix, where α and β are scale factors in u and v axes, respectively, and (u_0, v_0) are the coordinates of the principal point. R_{cw} and t_{cw} are the rotation matrix and translation vector which relates the target coordinate system $\{w\}$ to the camera coordinate system $\{c\}$. The camera calibration is to estimate the parameters K, R_{cw} and t_{cw}.

Fig.3 describes the structured light sensor. The light stripe emits from the projector and forms a space light plane in the camera coordinate system $\{c\}$, so the projector can be described by a 3D plane equation

$$ax_c + by_c + cz_c + d = 0 \tag{2}$$

The projector calibration is to estimate the coefficients a, b, c and d.

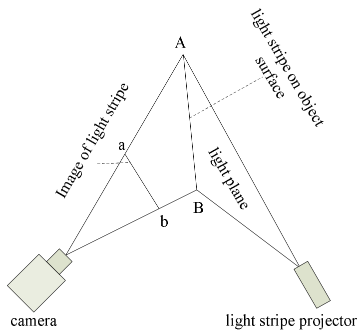

Figure 3. Structured light sensor

When a vision sensor is mounted on the end-effector of a robot, the rigid body transformation from the camera coordinate system $\{c\}$ to the end-effector coordinate system $\{e\}$is represented by matrix T_{ec}, which is the hand-eye relationship matrix. The determining of T_{ec} is called hand-eye calibration.

III. CALIBRATION PRINCPLE

The approach to calibrate the system takes four stages: camera intrinsic parameters calibration, structured light plane calibration, the camera extrinsic parameters calibration and the hand-eye calibration.

A. *Solving Camera Intrinsic Parameters*

Fig.4 illustrates the principle of the camera calibration.

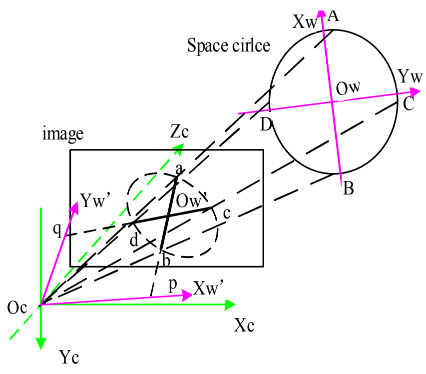

Figure 4. The camera calibration principle

Given an image of the calibration target, the vanishing points p and q on the orthogonal diameters AB and CD respectively can be estimated by the harmonic conjugate theory. Let's denote the unit focal length coordinates of the vanishing points p and q in the camera coordinate system $\{c\}$ by $p_c(\frac{u_p-u_0}{\alpha}, \frac{v_p-v_0}{\beta}, 1)^T$ and $q_c(\frac{u_q-u_0}{\alpha}, \frac{v_q-v_0}{\beta}, 1)^T$, respectively. From the definition of vanishing points, we can infer O_cp_c is orthogonal to O_cq_c [12]. By the Pythagorean Theorem, we have

$$\|p_cq_c\|^2 = \|O_cp_c\|^2 + \|O_cq_c\|^2 \tag{3}$$

Substituting the unit focal length coordinates of p_c and q_c into (3) gives

$$\frac{1}{\alpha^2}(u_0-u_p)(u_0-u_q) + \frac{1}{\beta^2}(v_0-v_p)(v_0-v_q) + 1 = 0 \tag{4}$$

If n images of the target are observed, n equations of (4) are given. Subtract $ith(1 \le i \le n)$ equation of (4) from $jth(1 \le j \le n, j \ne i)$ equation of (4), we have

$$\begin{aligned}&\frac{1}{\alpha^2}[(u_{jp}+u_{jq}-u_{ip}-u_{iq})u_0 + (u_{ip}u_{iq}-u_{jp}u_{jq})] + \\ &\frac{1}{\beta^2}[(v_{jp}+v_{jq}-v_{ip}-v_{iq})v_0 + (v_{ip}v_{iq}-v_{jp}v_{jq})] = 0\end{aligned} \tag{5}$$

Let $x' = u_0$, $y' = v_0\frac{\alpha^2}{\beta^2}$, $z' = \frac{\alpha^2}{\beta^2}$ be the intermediate variables, when $n \geqslant 4$, $n-1$ equations are available, by stacking $n-1$ equations as (5), we have

$$H\phi = g \tag{6}$$

where H is a $(n-1)\times 3$ matrix; ϕ is a 3×1 vector of the intermediate variables; g is a $(n-1)\times 1$ vector. Using the least square method, we have

$$\phi = (H^TH)^{-1}H^Tg$$

The intermediate variables x', y' and z' can be uniquely extract, so

$$u_0 = x', v_0 = \frac{y'}{z'}$$

Take the u_0, v_0, z' into (4), α, β, can be computed.

B. Structured Light Plane Calibration

The method in Ref. [13] is used to calibrate the structured light plane. The biggest circle's diameter AB is used to calibrate the plane and $l_1 = l_2 = R = \frac{\|AB\|}{2}$. Fig.4 shows the structured light plane calibration principle.

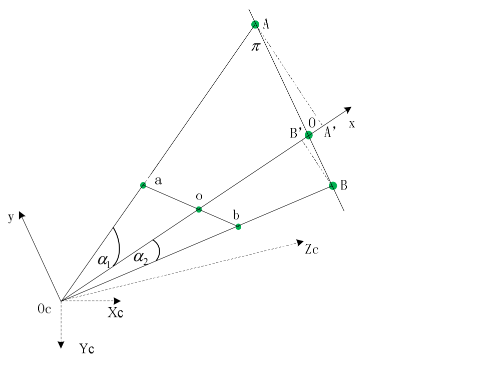

Figure 5. The structured light plane calibration principle

$a_c(\frac{u_a-u_0}{\alpha}, \frac{v_a-v_0}{\beta}, 1)^T$, $o_c(\frac{u_o-u_0}{\alpha}, \frac{v_o-v_0}{\beta}, 1)^T$ and $b_c(\frac{u_b-u_0}{\alpha}, \frac{v_b-v_0}{\beta}, 1)^T$ are the unit focal length coordinates of image points a, o, b in the camera coordinate system $\{c\}$ respectively. α_1 and α_2 are the angle formed by O_ca_c and O_co_c, O_co_c and O_cb_c respectively. We have

$$\alpha 1 = cos^{-1}(\frac{O_ca_c \cdot O_co_c}{\|O_ca_c\|\|O_co_c\|})$$
$$\alpha 2 = cos^{-1}(\frac{O_cb_c \cdot O_co_c}{\|O_cb_c\|\|O_co_c\|})$$

A coordinate system in the plane O_cAOB is established, its origin is O_c, x axis is coincided with O_cO, and rotating x axis $90°$ counterclockwise to form the y axis. By similar triangles, straight slope formula and the distance between two points, the coordinates of A, O and B in the O_cxy coordinate system can be figured out

$$\begin{cases} x_B = \sqrt{\frac{(2Rtan\alpha_1)^2}{(2tan\alpha_1 tan\alpha_2)^2 + (tan\alpha_1 - tan\alpha_2)^2}} \\ y_B = -x_B tan\alpha_2 \\ x_A = \frac{tan\alpha_2}{tan\alpha_1}x_B \\ y_A = x_B tan\alpha_2 \\ x_O = \frac{tan\alpha_1 + tan\alpha_2}{2tan\alpha_1}x_B \\ y_O = 0 \end{cases} \tag{7}$$

The distances from the origin O_c of the camera coordinate system $\{c\}$ to A, O, B respectively are

$$D_{Ac} = \|O_cA\| = \sqrt{x_A^2 + y_A^2}$$
$$D_{Oc} = \|O_cO\| = \sqrt{x_O^2 + y_O^2}$$
$$D_{Bc} = \|O_cB\| = \sqrt{x_B^2 + y_B^2}$$

Given that the camera has been calibrated, distances from the origin O_c of the camera coordinate system $\{c\}$ to the image points a, o, b respectively are

$$D_{ac} = \|O_ca\| = \sqrt{(\frac{u_a-u_0}{\alpha})^2 + (\frac{v_a-v_0}{\beta})^2 + 1}$$
$$D_{oc} = \|O_co\| = \sqrt{(\frac{u_o-u_0}{\alpha})^2 + (\frac{v_o-v_0}{\beta})^2 + 1}$$
$$D_{bc} = \|O_cb\| = \sqrt{(\frac{u_b-u_0}{\alpha})^2 + (\frac{v_b-v_0}{\beta})^2 + 1}$$

So by the similar triangular, the coordinates of A, O, B in the camera coordinate system $\{c\}$ are

$$\begin{aligned} x_{Ac} &= \frac{D_{Ac}(u_a-u_0)}{D_{ac}\alpha}, y_{Ac} = \frac{D_{Ac}(v_a-v_0)}{D_{ac}\beta}, z_{Ac} = \frac{D_{Ac}}{D_{ac}} \\ x_{Oc} &= \frac{D_{Oc}(u_o-u_0)}{D_{oc}\alpha}, y_{Oc} = \frac{D_{Oc}(v_o-v_0)}{D_{oc}\beta}, z_{Oc} = \frac{D_{Oc}}{D_{oc}} \\ x_{Bc} &= \frac{D_{Bc}(u_b-u_0)}{D_{bc}\alpha}, y_{Bc} = \frac{D_{Bc}(v_b-v_0)}{D_{bc}\beta}, z_{Bc} = \frac{D_{Bc}}{D_{bc}} \end{aligned} \tag{8}$$

The target of AOB is moved in the light plane at least twice and different points lying on the light plane are $A_c(x_{Acm}, y_{Acm}, z_{Acm})$, $O_c(x_{Ocm}, y_{Ocm}, z_{Ocm})$, $B_c(x_{Bcm}, y_{Bcm}, z_{Bcm})$, $m = 1, 2, \cdots, n(n \geqslant 2)$ is the number of the image. The least square method is exploited to identify the structured light plane equation's coefficients

$$\psi = (L^TL)^{-1}L^TD \tag{9}$$

Where $\psi = \begin{pmatrix} a & b & c \end{pmatrix}^T$ is the vector of the coefficients; L is $n \times 3$ matrix; D is $n \times 1$ vector of the coefficient d. Owing to the light plane cannot passing through the origin of the camera coordinate system, d can be chosen as any real number except zero.

C. Camera Extrinsic Parameters Calibration

A transition coordinate system $\{w'\}$ is established, whose origin is $\{O_c\}$, x_w', y_w' axes are O_cp_c, O_cq_c respectively and z_w' axis is the cross product of x_w', y_w' axes. Basing on the definition of the vanishing points and the geometric property of the vanishing points on the orthogonal diameters, the vectors $O_c p_c$ and $O_c q_c$ are parallel to $O_w A$ and $O_w C$ respectively. So x'_w and y'_w axes being parallel to x_w and y_w axes respectively can be inferred.

Given the camera has been calibrated and the unit focal length coordinates of the vanishing points of p and q in the camera coordinate system $\{c\}$ can be figured out. The following process shows how the rotation matrix R_{cw} is calculated.

- Computer and normalize to unit length normal the vector $n = \begin{pmatrix} n_1 & n_2 & n_3 \end{pmatrix}^T$ of z'_w axis using the computed unit focal length coordinates of p and q in the camera coordinate system $\{c\}$
  $$n = \frac{O_c p \times O_c q}{\|O_c p \times O_c q\|}$$
- Computer the angle η between n and $z_c = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}^T$ axis of the camera coordinate system $\{c\}$
  $$\eta^{-1} = cos^{-1}(n \cdot z_c) = cos^{-1} n(3)$$
- Computer and normalize to unit length normal the vector δ that is orthogonal to n and z_c
  $$\delta = \frac{n \times z_c}{\|n \times z_c\|}$$
- Computer the rotation matrix $R'_{w'c}$. The matrix can be derived from Rodrigues formula[11]
  $$R_{w'c} = cos(\eta) I + (1 - cos(\eta)) \delta \delta^T + sin(\eta) [\delta]_\times$$
  where $[\delta]_\times$ is the skew-symmetric matrix formed by δ.

The positions of vanishing points p and q on the vector of $O_{w'} a$ and $O_{w'} c$ are of four possibilities total, which are shown in Fig.5.

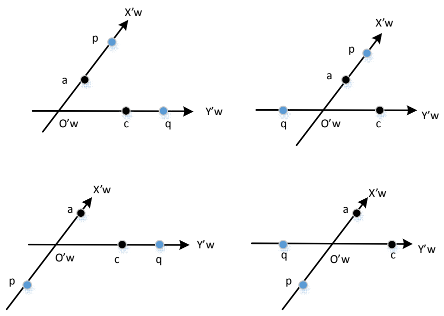

Figure 6. The position relationship between the vanish points and the projection of the target coordinates axes.

- If vanishing points p and q lie on the direction of the vector $O'_w a$ and the direction of the vector $O'_w c$ respectively, we have
  $$R_{w'c} = R'_{w'c} = \begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix}$$
- If vanishing point p lie on the direction of the vector $O'_w a$ and vanishing point q lies on the opposite direction of the vector $O'_w c$, we have
  $$R_{w'c} = \begin{bmatrix} r_1 & -r_2 & -r_3 \end{bmatrix}$$
- If vanishing point p lie on the opposite direction of the vector $O'_w a$ and vanishing point q lies on the direction of the vector $O'_w c$, we have
  $$R_{w'c} = \begin{bmatrix} -r_1 & r_2 & -r_3 \end{bmatrix}$$
- If vanishing points p and q lie on the opposite direction of the vector $O'_w a$ and the opposite direction of the vector $O'_w c$ respectively, we have
  $$R_{w'c} = \begin{bmatrix} -r_1 & -r_2 & r_3 \end{bmatrix}$$

Because of the error between the numerical simulation and real results of the camera intrinsic parameters and vanishing points p and q, the $R_{w'c}$ is not a unit orthogonal matrix. The singular value decomposition is adopted to find the optimal solution of $R_{w'c}$, the detailed process see Ref. [14].

$$R_{w'c} = USV^T$$
$$R_{wc} = R_{w'c^* = UV^T}$$

where U and V^T are 3×3 orthogonal matrix and S is a 3×3 diagonal matrix, whose all elements on the primary diagonal are the singular values of $R_{w'c}$, so

$$R_{cw} = R_{wc}^T$$

The structured light is projected on the circle center O, its coordinates in the camera coordinate system are given by

$$\begin{cases} a x_{Oc} + b y_{Oc} + c z_{Oc} + d = 0 \\ \frac{u_{o'} - u_0}{\alpha} = \frac{x_{Oc}}{z_{Oc}} \\ \frac{v_{o'} - v_0}{\beta} = \frac{y_{Oc}}{z_{Oc}} \end{cases} \tag{10}$$

So $t_{cw} = (x_{Oc}, y_{Oc}, z_{Oc})^T$, and the camera extrinsic parameters are

$$T_{cw} = \begin{bmatrix} R_{cw} & t_{cw} \\ 0^T & 1 \end{bmatrix} \tag{11}$$

D. Hand-Eye Calibration

First, move the robot to project the structured light stripe on to the points O, A and C respectively. The images are obtained and the corresponding positions and orientations of the end-effector are read from the robot controller. Let the coordinates of O, A and C in the robot base coordinate system $\{o\}$ are $O_o(x_{Oo}, y_{Oo}, z_{Oo})^T$, $A_o(x_{Ao}, y_{Ao}, z_{Ao})^T$, $C_o(x_{Co}, y_{Co}, z_{Co})^T$ respectively. Transition coordinate systems $\{o'\}$, $\{a'\}$ and $\{c'\}$ are established. Their origins are the points O, A and C respectively on the calibration target and their orientations are consistent with the robot base coordinate system. Let's denote the translation matrix between $\{o'\}$ and $\{o\}$, $\{a'\}$ and $\{o\}$, $\{c'\}$ and $\{o\}$ by $T_{oo'}$, $T_{oa'}$ and $T_{oc'}$, they are

$$T_{oo'} = \begin{bmatrix} 1 & 0 & 0 & x_{Oo} \\ 0 & 1 & 0 & y_{Oo} \\ 0 & 0 & 1 & z_{Oo} \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_{oa'} = \begin{bmatrix} 1 & 0 & 0 & x_{Ao} \\ 0 & 1 & 0 & y_{Ao} \\ 0 & 0 & 1 & z_{Ao} \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and }$$
$$T_{oc'} = \begin{bmatrix} 1 & 0 & 0 & x_{Co} \\ 0 & 1 & 0 & y_{Co} \\ 0 & 0 & 1 & z_{Co} \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

From Fig. 2, the transform equations for points O, A and C from camera coordinate system $\{c\}$ to robot base coordinate system $\{o\}$ are

$$T_{ocO} = T_{o6O} T_{ec} = T_{oo'} T_{wcO}$$
$$T_{ocA} = T_{o6A} T_{ec} = T_{oa'} T_{wcA}$$
$$T_{ocC} = T_{o6C} T_{ec} = T_{oc'} T_{wcC}$$

So the hand-eye relationship can be expressed by

$$T_{ec} = T_{o6O}^{-1} T_{oo'} T_{wcO}$$
$$T_{ec} = T_{o6A}^{-1} T_{oa'} T_{wcA}$$
$$T_{ec} = T_{o6C}^{-1} T_{oc'} T_{wcC}$$

The original pose of end-effector for points O, A and C are denoted by T_{o6i0} $(i = O, A, C)$ and the corresponding transformation matrices between the camera coordinate system and pattern coordinate system are denoted by T_{wci0} $(i = O, A, C)$. Hence

$$T_{ec} = T_{o6O0}^{-1} T_{oo'} T_{wcO0}$$
$$T_{ec} = T_{o6A0}^{-1} T_{oa'} T_{wcA0}$$
$$T_{ec} = T_{o6C0}^{-1} T_{oc'} T_{wcC0}$$

When the position and the orientation of the end-effector change, the transformations between the robot base coordinate system $\{o\}$ and the camera coordinate system $\{c\}$ are expressed by the above T_{ec}s as follows

$$T_{ocO}^{j} = T_{o6O}^{j} T_{o6O0}^{-1} T_{oo'} T_{wcO0}$$
$$T_{ocA}^{j} = T_{o6A}^{j} T_{o6A0}^{-1} T_{oa'} T_{wcA0}$$
$$T_{ocC}^{j} = T_{o6C}^{j} T_{o6C0}^{-1} T_{oc'} T_{wcC0}$$

$j = 1, 2, \cdots, n$ is the moving order number of the end-effector.

Let $\tilde{O}_c^j(x_{Oc}^j, y_{Oc}^j, z_{Oc}^j, 1)^T$, $\tilde{A}_c^j(x_{Ac}^j, y_{Ac}^j, z_{Ac}^j, 1)^T$, $\tilde{C}_c^j(x_{Cc}^j, y_{Cc}^j, z_{Cc}^j, 1)^T$ be the homogeneous coordinates of O, A and C of order j in the camera coordinate system $\{c\}$, which can be calculated as structured light plane has been calibrated. Therefore, the coordinates of O, A and C in the robot base coordinate system $\{o\}$ can be calculated by the following transformation equation

$$\begin{bmatrix} x_{io} \\ y_{io} \\ z_{io} \\ 1 \end{bmatrix} = T_{o6i}^{j} T_{o6i0}^{-1} T_{oi'} T_{wci0} \begin{bmatrix} x_{ic}^j \\ y_{ic}^j \\ z_{ic}^j \\ 1 \end{bmatrix} \tag{12}$$

Set

$$T_{o6i}^{j} T_{o6i0}^{-1} = \begin{bmatrix} R_{o6i0}^{j} & t_{o6i0}^{j} \\ 0 & 1 \end{bmatrix},\ T_{wci0} = \begin{bmatrix} R_{wci0} & t_{wci0} \\ 0 & 1 \end{bmatrix},$$

$$T_{oi'} = \begin{bmatrix} I & i_{oi'} \\ 0 & 1 \end{bmatrix}, \quad (x_{ic}^j, y_{ic}^j, z_{ic}^j, 1)^T = \begin{bmatrix} i_c^j & 1 \end{bmatrix}^T.$$

Through (12) we have

$$\begin{bmatrix} i_{oi'} \\ 1 \end{bmatrix} = \begin{bmatrix} R_{o6i0}^{j} R_{wci0} & R_{o6i0}^{j} t_{wci0} + R_{o6i0}^{j} i_{oi'} + t_{o6i0}^{j} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_c^j \\ 1 \end{bmatrix}$$

So

$$(I - R_{o6i0}^{j}) i_{oi'} = R_{o6i0}^{j} R_{wci0} i_c^j + R_{o6i0}^{j} t_{wci0} + t_{o6i0}^{j} \tag{13}$$

This will be written

$$A^j i_{oi'} = mi^j$$

for $n$images，$A^j = (I - R_{oci0}^{j})$ is a $3(n-1) \times 3$ matrix and $mi_j = R_{o6i0}^{i} R_{wic0} i_c^j + R_{o6i0}^{i} t_{wci0}$ is a column vector of $3(n-1)$ dimension. Adopt the least square method, we have

$$i_{oi'} = (A^{jT} A^j)^{-1} A^{jT} mi^j$$

$i = O, A, C$, $j = 1, 2, \cdots, n$, $i' = o', a', c'$, so

$$T_{oo'} = \begin{bmatrix} 1 & 0 & 0 & O_o(1) \\ 0 & 1 & 0 & O_o(2) \\ 0 & 0 & 1 & O_o(2) \\ 0 & 0 & 0 & 1 \end{bmatrix}, T_{oa'} = \begin{bmatrix} 1 & 0 & 0 & A_o(1) \\ 0 & 1 & 0 & A_o(2) \\ 0 & 0 & 1 & A_o(3) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and $T_{oc'} = \begin{bmatrix} 1 & 0 & 0 & C_o(1) \\ 0 & 1 & 0 & C_o(2) \\ 0 & 0 & 1 & C_o(3) \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

$O_o = T_{oo'}(1:3,4)$, $A_o = T_{oa'}(1:3,4)$, $C_o = T_{oc'}(1:3,4)$ are the coordinates of O, A and C in the robot base coordinate system $\{c\}$. O are the origin of the pattern coordinate system $\{w\}$ and A, C lie on x_w and y_w respectively. Therefore, T_{ow} can be calculated by the principle described in Ⅲ (C) part. The hand-eye relationship matrix is

$$T_{ec} = T_{o6O}^{-1} T_{ow} T_{wcO}$$

IV. EXPERIMENTAL RESULTS

The proposed approach has been tested on the real data. The camera to be calibrated is a Daheng DH-HV5051UC-ML CCD camera. The image resolution is 1024×768. Linear structured light projector is a self-made in the author's lab. The robot is 6DOF MOTOMAN DX100 ML. We use 14 circles to form the concentric circles pattern and the biggest diameter and the smallest diameter are 160mm and 50mm respectively.

A. *Experimental Results*

8 images of the target are taken in different position to calibrate the camera intrinsic parameters matrix, which is shown in Table I.

TABLE I. CAMERA INTRINSIC MATRIX

K	1939.77	0	574.69
	0	1937.73	475.58
	0	0	1

The structured light stripe is projected on to the known length diameter AOB. The target is moved 7 times maintaining the diameter AOB in the light plane. The structured light plane equation in the coordinate system is

$$4.2678x_c + 9.3291y_c - 0.8124z_c + 1000 = 0$$

The hand-eye relationship matrix is shown in Table II.

TABLE II. HAND-EYE MATRIX

T_{ec}	0.7234	0.5888	0.3605	-338.0554
	-0.1897	-0.3326	0.9238	-433.5400
	0.6639	-0.7367	-0.1289	-34.8966
	0	0	0	1

B. *Segment Measurement*

The accuracy of this hand-eye system is test by measuring the biggest diameter, whose length is 160mm. Project the light strip on the pattern cross the circle center, the intersection point of the light and the biggest circle are E and F, their coordinates in the robot base coordinate system can be calculated by

$$\begin{cases} ax_{cE} + by_{cE} + cz_{cE} + d = 0 \\ \frac{u_{e'} - u_0}{\alpha} = \frac{x_{cE}}{z_{cE}} \\ \frac{v_{e'} - v_0}{\beta} = \frac{y_{cE}}{z_{cE}} \end{cases}$$

$$E_o = T_{oe} T_{ec} E_c$$

$$\begin{cases} ax_{cF} + by_{cF} + cz_{cF} + d = 0 \\ \frac{u_{f'} - u_0}{\alpha} = \frac{x_{cF}}{z_{cF}} \\ \frac{v_{f'} - v_0}{\beta} = \frac{y_{cF}}{z_{cF}} \end{cases}$$

$$F_o = T_{oe} T_{ec} F_c$$

The length of EF are shown in Table III.

TABLE III. THE LENGTH OF EF

Real value (mm)	Measurement value(mm)	Absolute error (mm)	Relative error
160	159.83	0.17	0.11%
160	159.67	0.33	0.21%
160	160.37	0.37	0.23%
160	160.42	0.42	0.26%
160	159.80	0.20	0.13%
160	159.54	0.46	0.29%
160	160.82	0.82	0.51%
160	160.11	0.11	0.07%
160	159.91	0.09	0.06%
Average	160.05	0.33	0.21%

From the above data, we can see the measurement average absolute error and the average relative error of this system are 0.33mm and 0.21%, respectively. These experimental results can meet the requirements of millimeter level measurement.

V. CONCLUSION

This paper described a linear structured light robot hand-eye system calibration method using a concentric circles pattern with two known orthogonal diameters. We first calibrate the camera intrinsic and extrinsic parameters using the vanishing points of the two diameters and then use the known diameter of the biggest circle to calibrate the light plane in the camera coordinate system. Finally, fix the calibration pattern and use the circle center to calibrate the hand-eye relationship. The results show that this system can achieve the requirement of the basic error limit of the measurement tools with smallest measurement unit is millimeter and its accuracy meet the requirements of millimeter level measurement.

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