Term Structure Analysis

A. Construct Spot Curve Using Bond Price

We used 15 different treasury bonds with maturity ranging from 0.5 to 30 years to calculate the par value coupon rate. Then we used linear and cubic interpolation to derive the full coupon rate for every half year from now till 30 years later. Then we used bootstrap method to generate the spot rate with the same time period. As a result, we have a total of 60 data points on the spot curve.

The following figure is generated by bond price using Matlab:

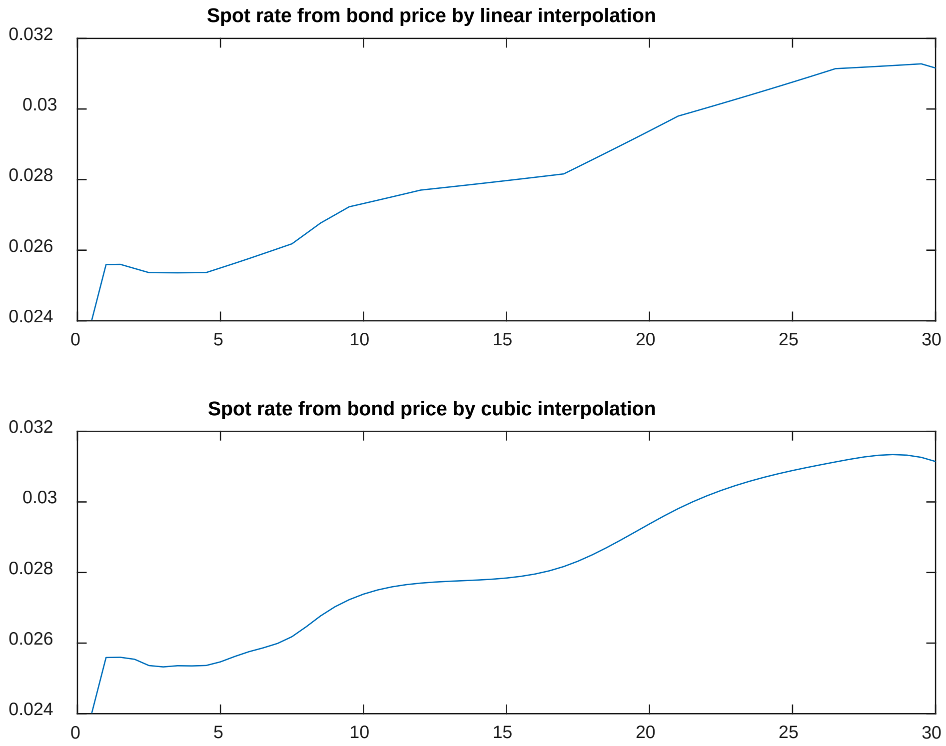

Figure 1

B. Construct Spot Curve Using Current Swap Rate

After that, we used current swap rate to derive spot curve. Firstly, we downloaded 1 to 30 years' swap rates from Bloomberg terminal. Then we used linear and cubic interpolation in order to get the half year swap rate within the 30-year period. Using iteration, we calculated the discount factor for all data points. And using the formula of discount factor and yield, we calculated spot rate.

The following figure is generated by swap rate using Matlab:

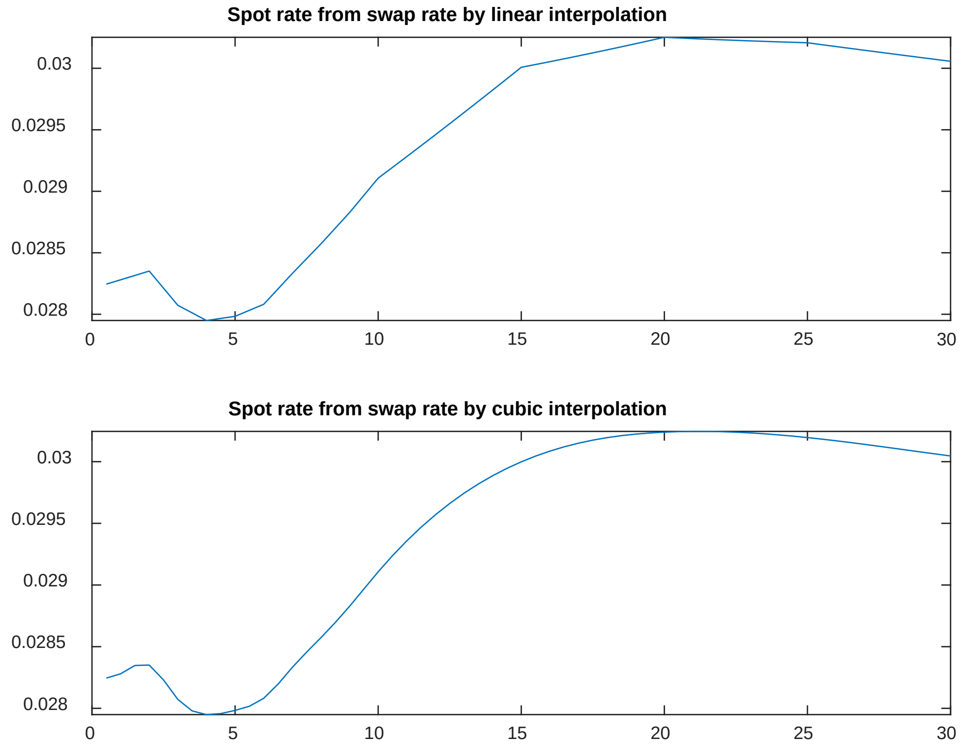

Figure 2

Now we have generated four 30-year-spot curves starting from now using bond price and swap rate with the application of linear and cubic interpolations.

C. Identify Potentially Mispriced Securities

Using the yield curve we established before with the linear and cubic interpolation from bond, linear and cubic interpolation from swap. We can calculate the intrinsic value of certain bonds. We take 6 bonds with different maturity and coupon rate to check if there exists mispriced bonds in the market.

Formula is PRICE(settlement, maturity, rate, yld, redemption, frequency, [basis]) price(2019/02/07,2019/08/15,0.08125,0.024,100,4). The following table is generated from this formula.

Maturity	Coupon	Actual_price	Linear_bond	Cubic_bond	Linear_swap	Cubic_swap
8/15/2019	0.0813	102.9922	102.9610	102.9610	102.7376	102.7376
8/15/2020	0.0875	102.9922	109.2122	109.2122	108.7867	108.7818
2/15/2025	0.0763	102.9922	128.0788	128.0826	126.6021	126.6019
8/15/2029	0.0613	102.9922	130.8345	130.7373	128.9508	128.9049
2/15/2042	0.0313	102.9922	101.6182	101.2994	101.6991	101.6815
8/15/2044	0.0313	102.9922	100.6379	100.4840	101.8778	101.8940

Table 1

Linear_bond	Cubic_bond	Linear_swap	Cubic_swap
0.0312	0.0312	0.2546	0.2546
-6.2200	-6.2200	-5.7946	0.4643
-25.0866	-25.0904	-23.6099	1.4528
-27.8423	-27.7451	-25.9586	2.1264
1.3740	1.6928	1.2931	0.5998
2.3543	2.5082	1.1144	0.0748

Table 2

This table 2 tells the difference between actual and interpolated price (actual price – interpolated price). The bond with maturity of 2019/8/15 is underpriced using all interpolation methods. Using linear and cubic interpolation from bond and linear method from swap, it’s shown that bonds with the maturity 2020/8/15, 2025/02/15 and 2029/08/15 are overpriced. The bond with maturity of 2042/02/15 and 2044/08/15 are both overpriced for all interpolation methods.

D. Perform Principal Component Analysis for Treasure Yield Curve

We retrieved data of Daily Treasury Yield Curve Rate from treasury.gov for two different time periods, 1996-2006 and 2007-2017. We deleted data with maturity 1 month, 2 month and 20 years due to null values. Then we performed Principal Component Analysis for the two different time periods in order to generate new component, which can be used to better interpret yield variations. Each principal component is a linear combination of all the explanatory variables. For the analysis on Daily Treasury Yield Curve Rate within 1996-2006, we found that the first principal component can explain 94.0686% of the data. Together, the first two principal components can explain 99.5287% of the variations. For the analysis on Daily Treasury Yield Curve Rate within 2007-2017, we found that the first principal component can explain 89.9508% of the data. Together, the first two principal components can explain 98.8115% of the variations.

To sum it up, the first principal component for the two time periods can both explain about 90% of the variations. Hence, yield curves for different maturity would most likely shift in parallel. As a result, we could hedge using duration because the linear relation for the yield change and the maturity. So, we just need to choose one bond to hedge the interest rate risk under PCA method. If you want to hedge in a more precise way, we could further use convexity method to hedge the second order risk for the change of yield curve.

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A. Construct Spot Curve Using Bond Price

```
%1. input the existing bond yiled and maturity
bond_data = xlsread('Bond.xlsx','sheet1');
t_bond = bond_data(:,1)';
y_bond = bond_data(:,2)';
%2. get the full maturity => prepare input for interpolation
int_bond = 0.5:0.5:t_bond(end);
%3. linear interpolation
l_y_bond = interp1(t_bond,y_bond,int_bond);
%4. cubic interpolatioz
c_y_bond = spline(t_bond,y_bond,int_bond);
%5. bootstrap for spot rate curve
spot_linear_bond = [y_bond(1:2),ones(1,length(int_bond)-2) * -1];
spot_cubic_bond = [y_bond(1:2),ones(1,length(int_bond)-2) * -1];
for i = 3 : length(int_bond)
    left_value_linear = 0;
    left_value_cubic = 0;
    for j = 1 : i-1
        left_value_linear = left_value_linear + l_y_bond(i)/2 * 100 /
 (1 + spot_linear_bond(j) / 2)^j;
        left_value_cubic = left_value_cubic + c_y_bond(i)/2 * 100 / (1
 + spot_cubic_bond(j) / 2)^j;
    end
    spot_linear_bond(i) = (((100 + l_y_bond(i)/2 * 100) / (100 -
 left_value_linear))^(1 / i) - 1) * 2;
    spot_cubic_bond(i) = (((100 + c_y_bond(i)/2 * 100) / (100 -
 left_value_cubic))^(1 / i) - 1) * 2;
end
%6. plot the yiled curve
figure(1)
subplot(2,1,1)
plot(int_bond,spot_linear_bond)
title('Spot rate from bond price by linear interpolation')
subplot(2,1,2)
plot(int_bond,spot_cubic_bond)
title('Spot rate from bond price by cubic interpolation')
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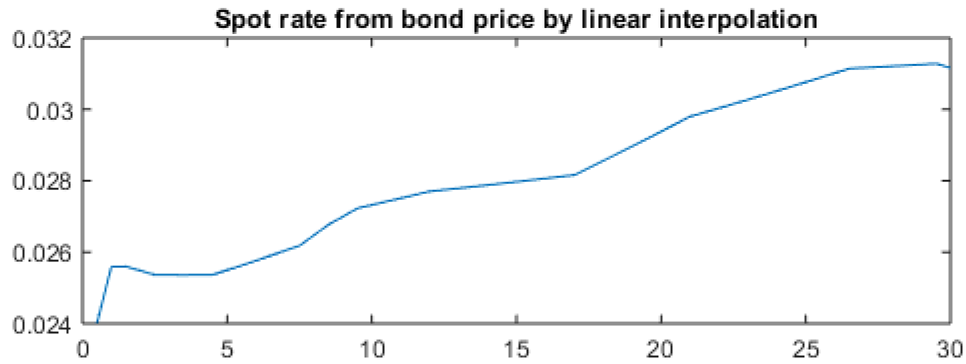
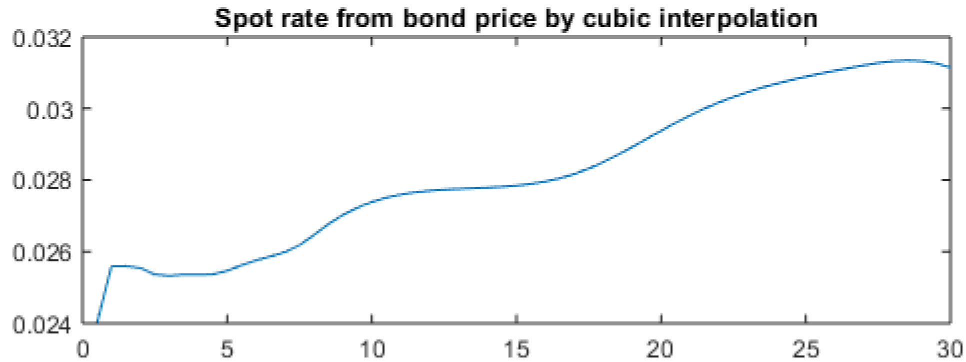
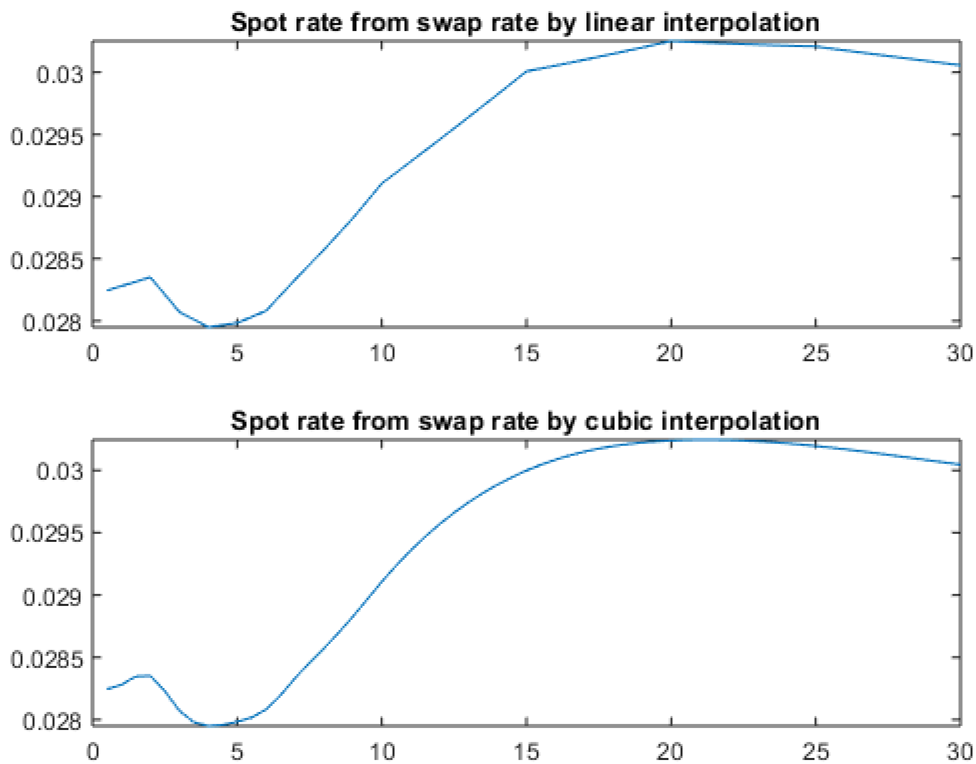




B. Construct Spot Curve Using Current Swap Rate

```
%1. input the existing swap yiled and maturity
swap_data = xlsread('Swap.xlsx','sheet1');
t_swap = swap_data(:,1);
y_swap = swap_data(:,2) / 100;
%2. interpolate the swap yield data
int_swap = 0.5:0.5:t_swap(end);
l_y_swap = interp1([0.5,t_swap'],[y_swap(1)-(y_swap(2)-
y_swap(1))/2,y_swap'],int_swap);
c_y_swap = spline([0.5,t_swap'],[y_swap(1)-(y_swap(2)-
y_swap(1))/2,y_swap'],int_swap);
%3. calculate discount factor
df_l_swap = ones(1,length(int_swap)) * -1;
df_l_swap(1) = 100 / (100 + l_y_swap(1) * 100 / 2);
df_c_swap = ones(1,length(int_swap)) * -1;
df_c_swap(1) = 100 / (100 + c_y_swap(1) * 100 / 2);
for i = 2:length(int_swap)
    df_l_swap(i) = (100 - repmat(l_y_swap(i) * 100 / 2, 1, i-1) *
 df_l_swap(1:i-1)') / (100 + l_y_swap(i) * 100 / 2);
    df_c_swap(i) = (100 - repmat(c_y_swap(i) * 100 / 2, 1, i-1) *
 df_c_swap(1:i-1)') / (100 + c_y_swap(i) * 100 / 2);
end
```

```
%4. calculte spot rate
sr_l_swap = ones(1,length(df_l_swap)) * -1;
sr_c_swap = ones(1,length(df_c_swap)) * -1;
for i = 1:length(df_l_swap)
    sr_l_swap(i) = 2 * ((1 / df_l_swap(i))^(1 /(2 * int_swap(i))) -
 1);
    sr_c_swap(i) = 2 * ((1 / df_c_swap(i))^(1 /(2 * int_swap(i))) -
 1);
end
%5. plot the yiled curve
figure(2)
subplot(2,1,1)
plot(int_swap,sr_l_swap)
title('Spot rate from swap rate by linear interpolation')
subplot(2,1,2)
plot(int_swap,sr_c_swap)
title('Spot rate from swap rate by cubic interpolation')
```



D. Perform Principal Component Analysis for Treasure Yield Curve

```
%1. input the treasury data into workspace
load x
%2. calculte the pca
t_data_1 = data_1;
```

```
[coee_1,score_1,latent_1,tsquared_1,explained_1] = pca(t_data_1);
t_data_2 = data_2;
[coee_2,score_2,latent_2,tsquared_2,explained_2] = pca(t_data_2);
```