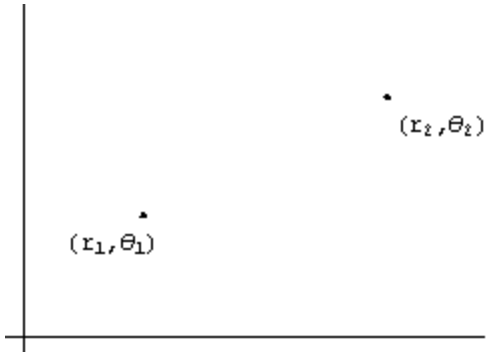
The Distance Formula For Polar Coordinates

Suppose we are given two points in polar coordinates, (r_1, θ_1) and (r_2, θ_2). We want to show that the distance between the two points is $D = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_1 - \theta_2)}$.



Recall that the distance formula for rectangular coordinates is given by the equation $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Using the fact that $x = r\cos\theta$ and $y = r\sin\theta$, we can derive the desired distance formula.

$$
\begin{aligned}
D &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{(r_2\cos\theta_2 - r_1\cos\theta_1)^2 + (r_2\sin\theta_2 - r_1\sin\theta_1)^2} \\
&= \sqrt{r_2^2\cos^2\theta_2 - 2r_1r_2\cos\theta_1\cos\theta_2 + r_1^2\cos^2\theta_1 + r_2^2\sin^2\theta_2 - 2r_1r_2\sin\theta_1\sin\theta_2 + r_1^2\sin^2\theta_1} \\
&= \sqrt{r_2^2\cos^2\theta_2 + r_2^2\sin^2\theta_2 - 2r_1r_2\cos\theta_1\cos\theta_2 - 2r_1r_2\sin\theta_1\sin\theta_2 + r_1^2\cos^2\theta_1 + r_1^2\sin^2\theta_1} \\
&= \sqrt{r_2^2(\cos^2\theta_2 + \sin^2\theta_2) - 2r_1r_2(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) + r_1^2(\cos^2\theta_1 + \sin^2\theta_1)} \\
&= \sqrt{r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2) + r_1^2} \\
&= \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}
\end{aligned}
$$

And so, we have shown the desired formula. Notice that the order of θ_1 and θ_2 does not matter, since $\cos(\theta_1 - \theta_2) = \cos(\theta_2 - \theta_1)$ because cosine is an even function.