Table of Basic Integrals

Integrals of Rational Functions

To compute them use your mind
Or go to WolframAlpha.com

(1) $$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

(2) $$\int \frac{x}{a^2+x^2}dx = \frac{1}{2}\ln|a^2+x^2|$$

(3) $$\int \frac{x^2}{a^2+x^2}dx = x - a\tan^{-1}\frac{x}{a}$$

(4) $$\int \frac{x^3}{a^2+x^2}dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln|a^2+x^2|$$

(5) $$\int \frac{1}{ax^2+bx+c}dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}}$$

(6) $$\int \frac{1}{(x+a)(x+b)}dx = \frac{1}{b-a}\ln\frac{a+x}{b+x},\ a \neq b$$

(7) $$\int \frac{x}{(x+a)^2}dx = \frac{a}{a+x} + \ln|a+x|$$

(8) $$\int \frac{x}{ax^2+bx+c}dx = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}}$$

Integrals with Roots

(9) $$\int x\sqrt{x-a}\,dx = \begin{cases} \frac{2a}{3}(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2}, & \text{or} \\ \frac{2}{3}x(x-a)^{3/2} - \frac{4}{15}(x-a)^{5/2}, & \text{or} \\ \frac{2}{15}(2a+3x)(x-a)^{3/2} \end{cases}$$

(10) $$\int \frac{x}{\sqrt{x\pm a}}\,dx = \frac{2}{3}(x\mp 2a)\sqrt{x\pm a}$$

(11) $$\int \sqrt{\frac{x}{a-x}}\,dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a}$$

(12) $$\int \sqrt{\frac{x}{a+x}}\,dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x}+\sqrt{x+a}\right]$$

(13) $$\int x\sqrt{ax+b}\,dx = \frac{2}{15a^2}(-2b^2+abx+3a^2x^2)\sqrt{ax+b}$$

(14)
$$\int \sqrt{x(ax+b)}\,dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)} - b^2\ln\left|a\sqrt{x}+\sqrt{a(ax+b)}\right|\right]$$

(15)
$$\int \sqrt{x^3(ax+b)}\,dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right]\sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}}\ln\left|a\sqrt{x}+\sqrt{a(ax+b)}\right|$$

(16) $$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \pm \frac{1}{2}a^2\ln\left|x+\sqrt{x^2\pm a^2}\right|$$

(17) $$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$$

(18) $$\int x\sqrt{x^2 \pm a^2}\, dx = \frac{1}{3}\left(x^2 \pm a^2\right)^{3/2}$$

(19) $$\int \frac{1}{\sqrt{x^2 \pm a^2}}\, dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$$

(20) $$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\frac{x}{a}$$

(21) $$\int \frac{x}{\sqrt{x^2 \pm a^2}}\, dx = \sqrt{x^2 \pm a^2}$$

(22) $$\int \frac{x}{\sqrt{a^2 - x^2}}\, dx = -\sqrt{a^2 - x^2}$$

(23) $$\int \frac{x^2}{\sqrt{x^2 \pm a^2}}\, dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}a^2 \ln\left|x + \sqrt{x^2 \pm a^2}\right|$$

(24)
$$\int \sqrt{ax^2 + bx + c}\, dx = \frac{b + 2ax}{4a}\sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a^{3/2}} \ln\left|2ax + b + 2\sqrt{a(ax^2 + bx^+ c)}\right|$$

(25)
$$\int x\sqrt{ax^2 + bx + c}\, dx = \frac{1}{48a^{5/2}}\left(2\sqrt{a}\sqrt{ax^2 + bx + c}\left(-3b^2 + 2abx + 8a(c + ax^2)\right)\right.$$
$$\left. +3(b^3 - 4abc)\ln\left|b + 2ax + 2\sqrt{a}\sqrt{ax^2 + bx + c}\right|\right)$$

(26) $$\int \frac{1}{\sqrt{ax^2 + bx + c}}\, dx = \frac{1}{\sqrt{a}} \ln\left|2ax + b + 2\sqrt{a(ax^2 + bx + c)}\right|$$

(27)
$$\int \frac{x}{\sqrt{ax^2 + bx + c}}\, dx = \frac{1}{a}\sqrt{ax^2 + bx + c} - \frac{b}{2a^{3/2}} \ln\left|2ax + b + 2\sqrt{a(ax^2 + bx + c)}\right|$$

(28) $$\int \frac{dx}{(a^2 + x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 + x^2}}$$

Integrals with Logarithms

(29) $$\int \ln ax \; dx = x \ln ax - x$$

(30) $$\int x \ln x \; dx = \frac{1}{2}x^2 \ln x - \frac{x^2}{4}$$

(31) $$\int x^2 \ln x \; dx = \frac{1}{3}x^3 \ln x - \frac{x^3}{9}$$

(32) $$\int x^n \ln x \; dx = x^{n+1}\left(\frac{\ln x}{n+1} - \frac{1}{(n+1)^2}\right), \quad n \neq -1$$

(33) $$\int \frac{\ln ax}{x} \; dx = \frac{1}{2}(\ln ax)^2$$

(34) $$\int \frac{\ln x}{x^2} \; dx = -\frac{1}{x} - \frac{\ln x}{x}$$

(35) $$\int \ln(ax+b) \; dx = \left(x + \frac{b}{a}\right)\ln(ax+b) - x, a \neq 0$$

(36) $$\int \ln(x^2+a^2) \; dx = x \ln(x^2+a^2) + 2a \tan^{-1}\frac{x}{a} - 2x$$

(37) $$\int \ln(x^2-a^2) \; dx = x \ln(x^2-a^2) + a \ln\frac{x+a}{x-a} - 2x$$

(38)
$$\int \ln\left(ax^2+bx+c\right) \; dx = \frac{1}{a}\sqrt{4ac-b^2}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left(\frac{b}{2a}+x\right)\ln\left(ax^2+bx+c\right)$$

$$\int x \ln(ax+b)\ dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax+b) \tag{39}$$

$$\int x \ln\left(a^2 - b^2x^2\right)\ dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 - \frac{a^2}{b^2}\right)\ln\left(a^2 - b^2x^2\right) \tag{40}$$

$$\int (\ln x)^2\ dx = 2x - 2x\ln x + x(\ln x)^2 \tag{41}$$

$$\int (\ln x)^3\ dx = -6x + x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x \tag{42}$$

$$\int x(\ln x)^2\ dx = \frac{x^2}{4} + \frac{1}{2}x^2(\ln x)^2 - \frac{1}{2}x^2\ln x \tag{43}$$

$$\int x^2(\ln x)^2\ dx = \frac{2x^3}{27} + \frac{1}{3}x^3(\ln x)^2 - \frac{2}{9}x^3\ln x \tag{44}$$

Integrals with Exponentials

$$\int e^{ax}\ dx = \frac{1}{a}e^{ax} \tag{45}$$

(46)
$$\int \sqrt{x}e^{ax}\ dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\mathrm{erf}\left(i\sqrt{ax}\right),\ \text{where } \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$$

$$\int xe^x\ dx = (x-1)e^x \tag{47}$$

$$\int xe^{ax}\,dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax} \tag{48}$$

$$\int x^2e^{x}\,dx = \left(x^2 - 2x + 2\right)e^{x} \tag{49}$$

$$\int x^2e^{ax}\,dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax} \tag{50}$$

$$\int x^3e^{x}\,dx = \left(x^3 - 3x^2 + 6x - 6\right)e^{x} \tag{51}$$

$$\int x^ne^{ax}\,dx = \frac{x^ne^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,\mathrm{d}x \tag{52}$$

$$\int x^ne^{ax}\,dx = \frac{(-1)^n}{a^{n+1}}\Gamma[1+n, -ax], \text{ where } \Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}\,\mathrm{d}t \tag{53}$$

$$\int e^{ax^2}\,dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(ix\sqrt{a}\right) \tag{54}$$

$$\int e^{-ax^2}\,dx = \frac{\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(x\sqrt{a}\right) \tag{55}$$

$$\int xe^{-ax^2}\,dx = -\frac{1}{2a}e^{-ax^2} \tag{56}$$

$$\int x^2e^{-ax^2}\,dx = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}\mathrm{erf}(x\sqrt{a}) - \frac{x}{2a}e^{-ax^2} \tag{57}$$

Integrals with Trigonometric Functions

(58) $$\int \sin ax \; dx = -\frac{1}{a}\cos ax$$

(59) $$\int \sin^2 ax \; dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

(60) $$\int \sin^3 ax \; dx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a}$$

(61) $$\int \sin^n ax \; dx = -\frac{1}{a}\cos ax \;\; {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right]$$

(62) $$\int \cos ax \; dx = \frac{1}{a}\sin ax$$

(63) $$\int \cos^2 ax \; dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

(64) $$\int \cos^3 ax dx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a}$$

(65) $$\int \cos^p ax dx = -\frac{1}{a(1+p)}\cos^{1+p} ax \times {}_2F_1\left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax\right]$$

(66) $$\int \cos x \sin x \; dx = \frac{1}{2}\sin^2 x + c_1 = -\frac{1}{2}\cos^2 x + c_2 = -\frac{1}{4}\cos 2x + c_3$$

(67) $$\int \cos ax \sin bx \; dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b$$

(68) $$\int \sin^2 ax \cos bx \; dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)}$$

(69) $$\int \sin^2 x \cos x \; dx = \frac{1}{3}\sin^3 x$$

(70) $$\int \cos^2 ax \sin bx \; dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)}$$

(71) $$\int \cos^2 ax \sin ax \; dx = -\frac{1}{3a}\cos^3 ax$$

(72)
$$\int \sin^2 ax \cos^2 bx dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)}$$

(73) $$\int \sin^2 ax \cos^2 ax \; dx = \frac{x}{8} - \frac{\sin 4ax}{32a}$$

(74) $$\int \tan ax \; dx = -\frac{1}{a}\ln \cos ax$$

(75) $$\int \tan^2 ax \; dx = -x + \frac{1}{a}\tan ax$$

(76) $$\int \tan^n ax \; dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1\left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right)$$

(77) $$\int \tan^3 axdx = \frac{1}{a}\ln \cos ax + \frac{1}{2a}\sec^2 ax$$

(78) $$\int \sec x \; dx = \ln|\sec x + \tan x| = 2\tanh^{-1}\left(\tan\frac{x}{2}\right)$$

(79) $$\int \sec^2 ax \; dx = \frac{1}{a}\tan ax$$

(80) $$\int \sec^3 x \; dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x|$$

(81) $$\int \sec x \tan x \; dx = \sec x$$

(82) $$\int \sec^2 x \tan x \; dx = \frac{1}{2}\sec^2 x$$

(83) $$\int \sec^n x \tan x \; dx = \frac{1}{n}\sec^n x, n \neq 0$$

(84) $$\int \csc x \; dx = \ln\left|\tan\frac{x}{2}\right| = \ln|\csc x - \cot x| + C$$

(85) $$\int \csc^2 ax \; dx = -\frac{1}{a}\cot ax$$

(86) $$\int \csc^3 x \; dx = -\frac{1}{2}\cot x \csc x + \frac{1}{2}\ln|\csc x - \cot x|$$

(87) $$\int \csc^n x \cot x \; dx = -\frac{1}{n}\csc^n x, n \neq 0$$

(88) $$\int \sec x \csc x \; dx = \ln|\tan x|$$

Products of Trigonometric Functions and Monomials

$$\int x\cos x\ dx = \cos x + x\sin x \tag{89}$$

$$\int x\cos ax\ dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax \tag{90}$$

$$\int x^2\cos x\ dx = 2x\cos x + \left(x^2-2\right)\sin x \tag{91}$$

$$\int x^2\cos ax\ dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2-2}{a^3}\sin ax \tag{92}$$

$$\int x^n\cos x dx = -\frac{1}{2}(i)^{n+1}\left[\Gamma(n+1,-ix) + (-1)^n\Gamma(n+1,ix)\right] \tag{93}$$

$$\int x^n\cos ax\ dx = \frac{1}{2}(ia)^{1-n}\left[(-1)^n\Gamma(n+1,-iax) - \Gamma(n+1,ixa)\right] \tag{94}$$

$$\int x\sin x\ dx = -x\cos x + \sin x \tag{95}$$

$$\int x\sin ax\ dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2} \tag{96}$$

$$\int x^2\sin x\ dx = \left(2-x^2\right)\cos x + 2x\sin x \tag{97}$$

$$\int x^2\sin ax\ dx = \frac{2-a^2x^2}{a^3}\cos ax + \frac{2x\sin ax}{a^2} \tag{98}$$

$$\int x^n \sin x \; dx = -\frac{1}{2}(i)^n \left[\Gamma(n+1,-ix) - (-1)^n \Gamma(n+1,-ix)\right] \tag{99}$$

$$\int x \cos^2 x \; dx = \frac{x^2}{4} + \frac{1}{8}\cos 2x + \frac{1}{4}x \sin 2x \tag{100}$$

$$\int x \sin^2 x \; dx = \frac{x^2}{4} - \frac{1}{8}\cos 2x - \frac{1}{4}x \sin 2x \tag{101}$$

$$\int x \tan^2 x \; dx = -\frac{x^2}{2} + \ln \cos x + x \tan x \tag{102}$$

$$\int x \sec^2 x \; dx = \ln \cos x + x \tan x \tag{103}$$

Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x \; dx = \frac{1}{2}e^x(\sin x - \cos x) \tag{104}$$

$$\int e^{bx} \sin ax \; dx = \frac{1}{a^2+b^2}e^{bx}(b \sin ax - a \cos ax) \tag{105}$$

$$\int e^x \cos x \; dx = \frac{1}{2}e^x(\sin x + \cos x) \tag{106}$$

$$\int e^{bx} \cos ax \; dx = \frac{1}{a^2+b^2}e^{bx}(a \sin ax + b \cos ax) \tag{107}$$

$$\int x e^x \sin x \; dx = \frac{1}{2}e^x(\cos x - x \cos x + x \sin x) \tag{108}$$

$$\int x e^x \cos x \; dx = \frac{1}{2}e^x(x \cos x - \sin x + x \sin x) \tag{109}$$

Integrals of Hyperbolic Functions

(110) $$\int \cosh ax \; dx = \frac{1}{a} \sinh ax$$

(111) $$\int e^{ax} \cosh bx \; dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2}[a \cosh bx - b \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} + \frac{x}{2} & a = b \end{cases}$$

(112) $$\int \sinh ax \; dx = \frac{1}{a} \cosh ax$$

(113) $$\int e^{ax} \sinh bx \; dx = \begin{cases} \frac{e^{ax}}{a^2 - b^2}[-b \cosh bx + a \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} - \frac{x}{2} & a = b \end{cases}$$

(114) $$\int \tanh ax \, dx = \frac{1}{a} \ln \cosh ax$$

(115) $$\int e^{ax} \tanh bx \; dx = \begin{cases} \frac{e^{(a+2b)x}}{(a+2b)} {}_2F_1\left[1 + \frac{a}{2b}, 1, 2 + \frac{a}{2b}, -e^{2bx}\right] \\ \qquad -\frac{1}{a} e^{ax} {}_2F_1\left[1, \frac{a}{2b}, 1 + \frac{a}{2b}, -e^{2bx}\right] & a \neq b \\ \frac{e^{ax} - 2 \tan^{-1}[e^{ax}]}{a} & a = b \end{cases}$$

(116) $$\int \cos ax \cosh bx \; dx = \frac{1}{a^2 + b^2} [a \sin ax \cosh bx + b \cos ax \sinh bx]$$

(117) $$\int \cos ax \sinh bx \; dx = \frac{1}{a^2 + b^2} [b \cos ax \cosh bx + a \sin ax \sinh bx]$$

$$(118) \qquad \int \sin ax \cosh bx \; dx = \frac{1}{a^2 + b^2} \left[-a \cos ax \cosh bx + b \sin ax \sinh bx \right]$$

$$(119) \qquad \int \sin ax \sinh bx \; dx = \frac{1}{a^2 + b^2} \left[b \cosh bx \sin ax - a \cos ax \sinh bx \right]$$

$$(120) \qquad \int \sinh ax \cosh ax dx = \frac{1}{4a} \left[-2ax + \sinh 2ax \right]$$

$$(121) \qquad \int \sinh ax \cosh bx \; dx = \frac{1}{b^2 - a^2} \left[b \cosh bx \sinh ax - a \cosh ax \sinh bx \right]$$