a) $\cos 2x = \cos^2 x - \sin^2 x = \cos^2 x - \sin^2 x(cos^2 x + \sin^2 x) = \cos^4 x - \sin^4 x$

b) $1 - \frac{1}{2}\sin^2 2x = 1 - \frac{1}{2}(2\sin x \cos x)^2 = \sin^2 x + \cos^2 - 2\sin^2 x \cos^2 x = \sin^2 x - 2\sin^2 x \cos^2 x + \cos^2 = \sin^2 x - 2\sin^2 x(1 - \sin^2 x + 1) - \sin^2 x = \sin^2 x - 2\sin^2 x + 2\sin^4 x + 1 - \sin^2 x 2 sin^4 x - 2\cos^2 x + 1 = sin^4 x + \left(sin^4 x - 2\cos^2 x + 1\right) = \sin^4 x + \cos^4 x$

c) Nechť $a = \frac{\alpha}{2}$.

$1 + \cos 2a = 1 + \cos^2 a - \sin^2 a = 1 + \cos^2 a - 1 + \cos^2 a = 2\cos^2 a$

d) Nechť $a = \frac{\alpha}{2}$.

$2\sin^2 a = 2(1 - \cos^2 a) = 2 - 2\cos^2 a = 1 + \sin^2 a - \cos^2 a = 1 - \cos 2a$

d) Nechť $a = \frac{\alpha}{2}$.

$2\sin 2a + \sin 4a = 4\sin a \cos a + 2\sin 2a \cos 2a = 4\sin a \cos a + 4\sin a \cos a \cos 2a = 4\sin a \cos a(1 + \cos^2 a - \sin^2 a) = 2\sin 2a(1 + \cos^2 a - 1 + \cos^2 a) = 2\sin 2a(2\cos^2 a) = 4\sin 2a \cos^2 a$

$$\frac{2x+2}{x^5 - x^4 + 2x^3 - 2x^2 + x - 1} = \frac{2x+2}{(x-1)(x^2+1)(x^2+1)} = \frac{a}{x-1} + \frac{bx+c}{x^2+1} + \frac{dx+e}{(x^2+1)^2} =$$

$$= \frac{ax^4 + 2ax^2 + a + bx^4 - bx^3 + bx^2 - bx + cx^3 - cx^2 + cx - c + dx^2 - dx + ex - e}{(x-1)(x^2+1)(x^2+1)} =$$

$$= \frac{(a+b)x^4 + -(b+c)x^3 + (2a+b-x+d)x^2 + (-b+c-d+e)x + a - c - e}{(x-1)(x^2+1)(x^2+1)}$$

$a + b = 0$
$b + c = 0$
$2a + b - x + d = 0$
$-b + c - d + e = 2$
$a - c - e = 0$

$$\frac{1}{x-1} + \frac{-x-1}{x^2+1} + \frac{2x}{(x^2+1)^2}$$