§1. Mocniny, mocninná funkce

***17.** $\left[\frac{a^2-b^2}{(x-y)^n}\right]^m \cdot \frac{[(x^2-y^2)^m]^n}{(a+b)^m} \cdot \left[\frac{a-b}{(x+y)^n}\right]^m$ (*m*, *n* jsou čísla přirozená).

$$\frac{(a+b)^m(a-b)^m(x-y)^{mn}(x+y)^{nm}(a-b)^m}{(x-y)^{nm}(a+b)^m(x+y)^{nm}} = (a-b)^{2m}$$

***18.** $\frac{a^{2x+3y}\cdot b^{4x-5y}}{a^{5x-y}\cdot b^{3x+y}} : \frac{a^{4x+5y}\cdot b^{2x-4y}}{a^{8x+2y}\cdot b^{x+2y}}$,

x, *y* jsou čísla přirozená, $x > 2y$.

$$a^{2x+3y-5x+y-4x-5y+8x+2y}b^{4x-5y-3x-y-2x+4y+x+2y} = a^{x+y}b^0 = a^{x+y}$$

***37.** $\left(\frac{a^x+a^{-x}}{b^y+b^{-y}}\right)^{-1} \cdot \left(\frac{a^x-a^{-x}}{b^y-b^{-y}}\right)^2 \cdot \left(\frac{a^{2x}-1}{b^{2y}-1}\right)^{-2} : \left(\frac{a^{2x}+1}{b^{2y}+1}\right)^{-1}$; *x*, *y* jsou čísla celá.

Nechť $k = a^x$ a $l = b^y$:

$$\frac{(l+\frac{1}{l})(k^2-2+\frac{2}{k^2})(l^2-2l+1)(k^2+1)}{(k+\frac{1}{k})(l^2-2+\frac{1}{l^2})(k^2-2k+1)(l^2+1)} = \frac{(l^3-2l^2+2l-2+\frac{1}{l})(k^4-k^2+\frac{1}{k^2})}{(k^3-kl^2+2k-2+\frac{1}{k})(l^4-l^2+\frac{1}{l^2})} =$$

$$= \frac{(b^{3y}-2b^{2y}+2b^y-2+\frac{1}{b^y})(a^{4x}-a^{2x}+\frac{1}{a^{2x}})}{(a^{3x}-2a^{2x}+2a^x-2+\frac{1}{a^x})(b^{4y}-b^{2y}+\frac{1}{b^{2y}})}$$

***d)** $\sqrt[n-1]{a^n} - x\sqrt[n-1]{a} - (a-x)\sqrt[n-1]{x}$;

$$a^{\frac{n}{n-1}} - xa^{\frac{1}{n-1}} - ax^{\frac{1}{n-1}} + xx^{\frac{1}{n-1}} = aa^{\frac{1}{n-1}} - xa^{\frac{1}{n-1}} - ax^{\frac{1}{n-1}} + xx^{\frac{1}{n-1}} = (a-\sqrt{n-1}x)(\sqrt[n-1]{a}-x)$$

***c)** $\frac{12}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$;

$$\frac{12}{\sqrt{2}+\sqrt{3}-\sqrt{5}} \cdot \frac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{\sqrt{2}+\sqrt{3}+\sqrt{5}} = \frac{12(\sqrt{2}+\sqrt{3}+\sqrt{5})}{(\sqrt{2}+\sqrt{3})^2+5} = \frac{12(\sqrt{2}+\sqrt{3}+\sqrt{5})}{(5+\sqrt{12})^2+5} = \frac{12(\sqrt{2}+\sqrt{3}+\sqrt{5})}{5+\sqrt{12}+5} \cdot \frac{10-\sqrt{12}}{10-\sqrt{12}} =$$

***c)** $\frac{\sqrt{3}+1}{\sqrt{6}+\sqrt{2}-\sqrt{3}-1}$;

$$\frac{\sqrt{3}+1}{(\sqrt{2}-1)(\sqrt{3}+1)} \cdot \frac{\sqrt{2}-1}{\sqrt{2}+1} = 1$$

a) $\left[\left(a^{\frac{1}{3}}-x^{\frac{1}{3}}\right)^{-1}.(a-x)-\dfrac{a+x}{a^{\frac{1}{3}}+x^{\frac{1}{3}}}\right].2^{-1}.(ax)^{-\frac{1}{3}}$;

$$\frac{(a-x)(a+x)}{2\sqrt[3]{a}\sqrt[3]{x}(\sqrt[3]{a}-\sqrt[3]{x})(\sqrt[3]{a}+\sqrt[3]{x})}=\frac{a^2-x^2}{2a\sqrt[3]{x}-2x\sqrt[3]{a}}$$

***112.** $\left(\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)^3+\left(\dfrac{3}{2+\sqrt{3}}+3\sqrt{3}\right)^4$.

$$\left(\frac{5+2\sqrt{15}+3+5-2\sqrt{15}+3}{5-3}\right)^3+\left(\frac{6-3\sqrt{3}}{4-3}+3\sqrt{3}\right)^4=\left(\frac{16}{2}\right)^3+(6)^4=8^3+6^4$$

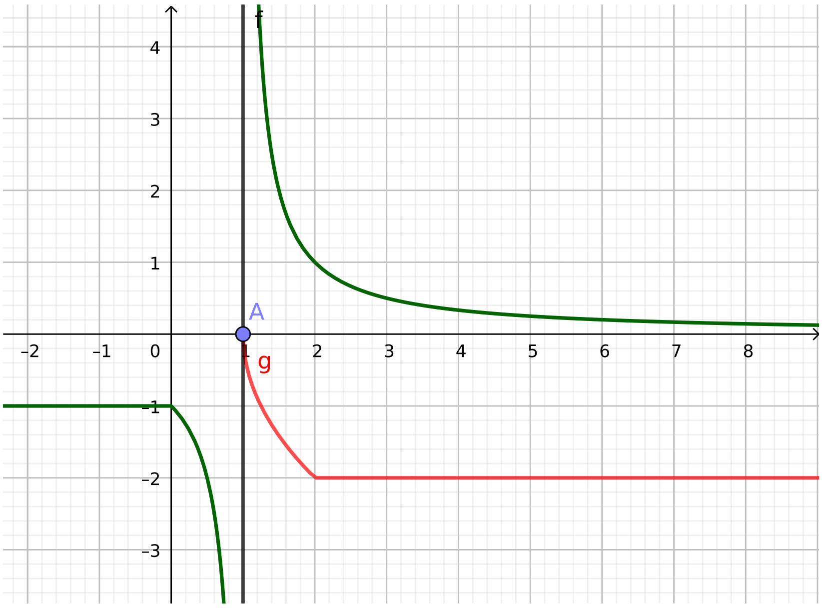
***128. Sestrojte graf funkce** a) $y=\dfrac{2}{x+|x|-2}$, $x\neq 1$;

b) $y=-\sqrt{2(x-|x-2|)}$.

1. Když $x>0$: $y=\frac{2}{2x-2}=\frac{1}{x-1}$
   Rovnoosá hyperbola se středem $[1;0]$.

   Když $x\leq 0$: $y=\frac{2}{-2}=-1$.

2. Když $x\geq 2$: $-\sqrt{2(x-x+2)}=-2$ Když $x\leq 2$: $-\sqrt{2(x+x-2)}=-2\sqrt{x+1}$
   Odmocniná funkce s počátkem $[1;0]$.



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