§7. Příčka mimoběžek

Def: Nechť p, q jsou 2 mimoběžné přímky. Přímka r, která je různoběžná s oběma přímkami p, q, se nazývá *příčka mimoběžek p, q* .

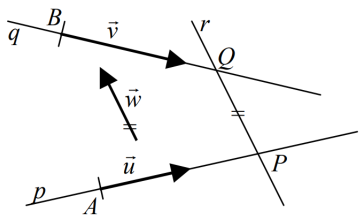
Pozn: Nechť $p(A, \overrightarrow{u}); q(B, \overrightarrow{v})$ jsou přímky. Pak pro příčku mimoběžek $r(Q, \overrightarrow{w})$ platí: $\overrightarrow{w} \in \left\langle \overrightarrow{u}, \overrightarrow{w}, \overrightarrow{AB} \right\rangle$.

A) Nalezení příčky r mimoběžek p, q, která je rovnoběžná s daným vektorem

Pozn: Dáno: $p(A, \overrightarrow{u}); q(B, \overrightarrow{v}); \overrightarrow{w} \neq \overrightarrow{0}$

Rozbor:

1) $\overrightarrow{w} \in \langle \overrightarrow{u}, \overrightarrow{v} \rangle \Rightarrow$ 0 řešení
2) $\overrightarrow{w} \notin \langle \overrightarrow{u}, \overrightarrow{v} \rangle \Rightarrow$ 1 řešení



$$\left.\begin{array}{l} P = A + k \cdot \overrightarrow{u} \\ Q = B + l \cdot \overrightarrow{v} \end{array}\right\} \Rightarrow \overrightarrow{PQ} = Q - P = B + l\overrightarrow{v} - A - k\overrightarrow{u}$$

$\overrightarrow{PQ} = x\overrightarrow{w}$

$\Rightarrow B + l \cdot \overrightarrow{v} - A - k\overrightarrow{u} = x\overrightarrow{w}$

$\underline{\overrightarrow{AB} = x\overrightarrow{w} + k\overrightarrow{u} - l\overrightarrow{v}}$

Př: Jsou dány mymoněžky $p(A, \overrightarrow{u}), q(B, \overrightarrow{v})$, a vektor $\overrightarrow{w}$:
$A[1; -2; 5]; B[-1; 1; -5]; \overrightarrow{u}(1; 3; -1), \overrightarrow{v}(1; 1; 2), \overrightarrow{w}(1; 1; 4)$
Najděte příčku p, q, která je rovnoběžná s $\overrightarrow{w}$.

$\overrightarrow{AB} = (-2; 3; -10) \Rightarrow (-2; 3; -10) = x(1; 1; 4) - l(1; 1; 2) + k(1; 3; -1)$. Hledáme body P, Q, pro které platí: $P = A + k \cdot \overrightarrow{u}; Q = B + l \cdot \overrightarrow{v}$ a zároveň $\overrightarrow{PQ} = x \cdot \overrightarrow{w}$.
$\Rightarrow x \cdot \overrightarrow{w} = Q - P = B - A + l\overrightarrow{v} - k\overrightarrow{u}$
$k\overrightarrow{u} - l\overrightarrow{v} + x\overrightarrow{w} = \overrightarrow{AB} = (-2, 3; -10)$.

$$\bar{A} = \left(\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 3 & -1 & 1 & 3 \\ -1 & -2 & 4 & -10 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 3 & -1 & 1 & 3 \\ 1 & 2 & -4 & 10 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 1 & 2 & -4 & 10 \\ 3 & -1 & 1 & 3 \end{array}\right) \sim$$
$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 3 & -5 & 12 \\ 0 & 2 & -2 & 9 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 2 & -2 & 9 \\ 0 & 3 & -5 & 12 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 2 & -2 & 9 \\ 0 & 0 & -4 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & -2 \\ 0 & 2 & -2 & 9 \\ 0 & 0 & 4 & 3 \end{array}\right) \sim$$
$$\left(\begin{array}{ccc|c} 2 & 0 & 0 & 5 \\ 0 & 2 & -2 & 9 \\ 0 & 0 & 4 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & 0 & 5 \\ 0 & 4 & 0 & 21 \\ 0 & 0 & 4 & 3 \end{array}\right)$$

$k = \frac{5}{2} \Rightarrow P = \left[\frac{7}{2}; \frac{11}{2}; \frac{5}{2}\right] \Rightarrow \overleftrightarrow{PQ} = \left\{ \left[\frac{7}{2} + t; \frac{11}{2} + t; \frac{5}{2} + 4t\right] | t \in \mathbb{R} \right\}$

Př: Cvičení 1:
Jsou dány mymoněžky $p(A, \overrightarrow{u}), q(B, \overrightarrow{v})$, a vektor $\overrightarrow{w}$:
$A[10; -7; 0]; B[-3; 5; 0]; \overrightarrow{u}(5; 4; 1), \overrightarrow{v}(2; 1; 1), \overrightarrow{w}(8; 7; 1)$
Najděte příčku p, q, která je rovnoběžná s $\overrightarrow{w}$.

$$k\overrightarrow{u} - l\overrightarrow{v} + x\overrightarrow{w}\overrightarrow{AB} = (-13; 12; 0).\ \bar{A} = \left(\begin{array}{ccc|c} 5 & -2 & 8 & -13 \\ 4 & -1 & 7 & 12 \\ 1 & -1 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 4 & -1 & 7 & 12 \\ 5 & -2 & 8 & -13 \end{array}\right) \sim$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 3 & 3 & 12 \\ 0 & 3 & 3 & -13 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 4 \\ 0 & 3 & 3 & -13 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & -25 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{array}\right)$$

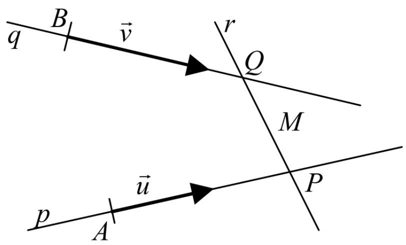
Soustava nemá řešení $\Rightarrow$ hledaná příčka neexistuje.

B) Nalezení příčky r mimoběžek p, q, která prochází bodem M

Pozn: Dáno: $p(A, \overrightarrow{u}); q(B, \overrightarrow{v})$, bod M.

Rozbor:

1) $M \in p \cap q \Rightarrow$ nekonečně mnoho řešení
2) $M \notin p \cap q \Rightarrow$
   (a) jedna přímka je rovnoběžná s rovinou, která je dána druhou přímkou a bodem $M \Rightarrow 0$ řešení.
   (b) ani jedna přímka není rovnoběžná s rovinou, která je dána druhou přímkou a bodem $M \Rightarrow 1$ řešení.



$P = A + k\overrightarrow{u}$
$Q = B + l\overrightarrow{v}$
$\overrightarrow{MP} = x \cdot \overrightarrow{MQ}$
$A + k\overrightarrow{u} = x(\overrightarrow{MB} + l\overrightarrow{v})$
$-k\overrightarrow{u} + m\overrightarrow{v} + x\overrightarrow{MB} = \overrightarrow{MA}$, kde $m = x \cdot l$

$\Rightarrow$ 3 rovnice o 3 neznámých k, m, x.

Př: Jsou dány mimoběžky $p(A, u), q(B, v)$, bod M. Nalezněte příčku mimoběžek p, q, procházející bodem M.
$A[1; 5; 2]; B[0; -1; 1]; M[0; 1; -5]; \overrightarrow{u}(1; 2; 1), \overrightarrow{v}(3; 1; 0)$

$\overrightarrow{MA} = (1; 4; 7)$
$\overrightarrow{MB} = (0; -2; 6)$

$$\left(\begin{array}{ccc|c} -1 & 3 & 0 & 1 \\ -2 & 1 & -2 & 4 \\ -1 & 0 & 6 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -3 & 0 & -1 \\ 2 & -1 & 2 & -4 \\ 1 & 0 & -6 & -7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -3 & 0 & -1 \\ 1 & 0 & -6 & -7 \\ 2 & -1 & 2 & -4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -3 & 0 & -1 \\ 0 & 3 & -6 & -6 \\ 0 & 5 & 2 & -2 \end{array}\right) \sim$$
$$\left(\begin{array}{ccc|c} 1 & -3 & 0 & -1 \\ 0 & 1 & -2 & -2 \\ 0 & 5 & 2 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -3 & 0 & -1 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 12 & 8 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -3 & 0 & -1 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 3 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -6 & -7 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 3 & 2 \end{array}\right) \sim$$
$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 3 & 0 & -2 \\ 0 & 0 & 3 & 2 \end{array}\right)$$

$$\mathrm{P} = \left\{ \left[-3; -\frac{2}{3}; \frac{2}{3} \right] \right\}$$

$$k = -3; m = -\frac{2}{3}; x = \frac{2}{3}$$

$P = [1; 5; 2] - 3(1; 2; 1) = [-2; -1; -1]$
$Q = [0; -1; 1] - 1(3; 1; 0) = [-3; -2; 1]$
$\overleftrightarrow{PQ} = \{[t; 1+t; -5-2t] | t \in \mathbb{R}\}.$

Př: Jsou dány mimoběžky $p(A, u), q(B, v)$, bod M. Nalezněte příčku mimoběžek p, q, procházející bodem M.
$A[3; 1; 2]; B[-2; 1; 0]; M[-\frac{1}{2}; \frac{5}{2}; 0]; \overrightarrow{u}(1; 2; -1), \overrightarrow{v}(3; -1; 1)$

$\overrightarrow{MA} = (\frac{7}{2}, -\frac{3}{2}; 2)$
$\overrightarrow{MB} = (-\frac{3}{2}; -\frac{3}{2}; 0)$

$P = A + k\overrightarrow{u}$
$Q = B + l\overrightarrow{v}$
$\overrightarrow{MP} = x \cdot \overrightarrow{MQ}$
$A + k\overrightarrow{u} = x(\overrightarrow{MB} + l\overrightarrow{v})$
$-k\overrightarrow{u} + m\overrightarrow{v} + x\overrightarrow{MB} = \overrightarrow{MA}$, kde $m = x \cdot l$

$$\bar{A} = \left(\begin{array}{ccc|c} -2 & 6 & -3 & 7 \\ -4 & -2 & -3 & -3 \\ -1 & 1 & 0 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & -6 & 3 & -7 \\ 4 & 2 & 3 & 3 \\ 1 & -1 & 0 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & -2 \\ 2 & -6 & 3 & -7 \\ 4 & 2 & 3 & 3 \end{array}\right) \sim$$
$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & -2 \\ 0 & -4 & 3 & -3 \\ 0 & 6 & 3 & 11 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & -2 \\ 0 & 4 & -3 & 3 \\ 0 & 6 & 3 & 11 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -1 & 0 & -2 \\ 0 & 4 & -3 & 3 \\ 0 & 0 & 15 & 13 \end{array}\right) \sim \left(\begin{array}{ccc|c} 4 & 0 & -3 & -5 \\ 0 & 4 & -3 & 3 \\ 0 & 0 & 15 & 13 \end{array}\right) \sim$$
$$\left(\begin{array}{ccc|c} 20 & 0 & 0 & -12 \\ 0 & 20 & 0 & 28 \\ 0 & 0 & 15 & 13 \end{array}\right) \sim \left(\begin{array}{ccc|c} 5 & 0 & 0 & -3 \\ 0 & 5 & 0 & 7 \\ 0 & 0 & 15 & 13 \end{array}\right)$$

$$k = -\frac{3}{5}; m = \frac{7}{5}; x = \frac{13}{15}$$

$P = [3 - \frac{3}{5}; 1 - \frac{6}{5}; 2 + \frac{3}{5}] = [\frac{12}{5}; -\frac{1}{5}; \frac{13}{5}]$
$Q = [-2 + \frac{21}{5}; 1 - \frac{7}{5}; 0 + \frac{7}{5}] = [\frac{11}{5}; -\frac{2}{5}; \frac{7}{5}]$
$\overrightarrow{QP} = [\frac{1}{5}; \frac{1}{5}; \frac{6}{5}]$

$$\overleftrightarrow{PQ} = \left\{ \left[[\frac{12}{5} + k; -\frac{1}{5} + k; \frac{13}{5} + 6k \right] | k \in \mathbb{R} \right\}$$

C) Nalezení osy o mimoběžek p, q

Def: Nechť p, q jsou mimoběžné přímky. Pak příčka mimoběžek o, která je kolmá k přímkám p i q, se nazývá osa mimoběžek p, q.

Př: nalezněte osu mimoběžek p, q.
$p = \{[8+t; 5+2t; 8-t] | t \in \mathbb{R}\}$ $q = \{[-4-7r; 3+2r; 4+3r], r \in \mathbb{R}\}$

Hledáme osu $o(P, \overrightarrow{w})$; $\overrightarrow{w} = (w_1, w_2, w_3)$, máme dáno:
$A[8; 5; 8]$, $\overrightarrow{u}(1; 2; -1)$
$B[-4; 3; 4]$, $\overrightarrow{u}(-7; 2; 3)$

$p \perp o : \overrightarrow{u} \cdot \overrightarrow{w} = w_1 + 2w_2 - w_3 = 0 \Rightarrow w_2 = \frac{w_1}{2}$
$q \perp o : \overrightarrow{v} \cdot \overrightarrow{w} = -7w_1 + 2w_2 + 3w_3 = 0 \Rightarrow w_3 = 2w_1$

w_1 – libovolný (jedná se jen o násobek) $\Rightarrow \overrightarrow{w} = (2; 1; 4)$.

tento vektor lze také zistat jako vektorový součin $\overrightarrow{u} \times \overrightarrow{v}$. Nyní hledáme příčku p, q rovnoběžnou s $\overrightarrow{w}$.

$$\left.\begin{array}{l} P = A + k \cdot \overrightarrow{u} \\ Q = B + l \cdot \overrightarrow{v} \end{array}\right\} \Rightarrow \overrightarrow{PQ} = Q - P = B + l\overrightarrow{v} - A - k\overrightarrow{u}$$

$\overrightarrow{PQ} = x\overrightarrow{w}$
$\Rightarrow B + l \cdot \overrightarrow{v} - A - k\overrightarrow{u} = x\overrightarrow{w}$
$\underline{\overrightarrow{AB} = x\overrightarrow{w} + k\overrightarrow{u} - l\overrightarrow{v}}$

$k\overrightarrow{u} - l\overrightarrow{v} + x\overrightarrow{w} = \overrightarrow{AB} = (-12; -2; -4)$.

$$\bar{A} = \left(\begin{array}{ccc|c} 1 & 7 & 2 & -12 \\ 2 & -2 & 1 & -2 \\ -1 & -3 & 4 & -4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 7 & 2 & -12 \\ 2 & -2 & 1 & -2 \\ 1 & 3 & -4 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 7 & 2 & -12 \\ 1 & 3 & -4 & 4 \\ 2 & -2 & 1 & -2 \end{array}\right) \sim$$
$$\left(\begin{array}{ccc|c} 1 & 7 & 2 & -12 \\ 0 & -4 & -6 & 16 \\ 0 & -16 & -3 & 22 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 7 & 2 & -12 \\ 0 & 2 & 3 & -8 \\ 0 & 16 & 3 & -22 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 7 & 2 & -12 \\ 0 & 2 & 3 & -8 \\ 0 & 0 & -21 & 42 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 7 & 2 & -12 \\ 0 & 2 & 3 & -8 \\ 0 & 0 & 1 & -2 \end{array}\right) \sim$$
$$\left(\begin{array}{ccc|c} 2 & 0 & -17 & 32 \\ 0 & 2 & 3 & -8 \\ 0 & 0 & 1 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & 0 & -2 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 1 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -2 \end{array}\right)$$

$k = 1 \Rightarrow P = [7; 3; 9]$

$$o = \overleftrightarrow{PQ} = \{[7+2t; 3+t; 9+4t] | t \in \mathbb{R}\}$$

Př: Úkol 3
Nalezněte příčku mimoběžek, která je rovnoběžná s rovinami ρ, σ.

$p : A[-5; 2; 2]$; $\overrightarrow{u} = (2; 0; 1)$
$q : z - 2 = 0 \wedge 5x - 8y + 9z + 100 = 0 \Rightarrow B[\frac{-118}{5}; 0; 2]$; $\overrightarrow{v}(8; 5; 0)$

$\rho \{[3 + 3r + s; 2r; 2s]; r, s \in \mathbb{R}\} \Rightarrow \rho : 2x - 3y - 3z = 6$
$\sigma : x - 4u - 3z + 12 = 0$

$$\bar{A} = \left(\begin{array}{ccc|c} 2 & -3 & -3 & 6 \\ 1 & -4 & -3 & -12 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -4 & -3 & -12 \\ 2 & -3 & -3 & 6 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & -4 & -3 & -12 \\ 0 & 5 & 3 & 30 \end{array}\right) \sim$$
$$\left(\begin{array}{ccc|c} 5 & 0 & -3 & 60 \\ 0 & 5 & 3 & 30 \end{array}\right)$$

$\Rightarrow \overrightarrow{w}(3; -3; 5)$

$$\left.\begin{array}{l} P = A + k\cdot\overrightarrow{u} \\ Q = B + l\cdot\overrightarrow{v} \end{array}\right\} \Rightarrow \overrightarrow{PQ} = Q - P = B + l\overrightarrow{v} - A - k\overrightarrow{u}$$

$\overrightarrow{PQ} = x\overrightarrow{w}$
$\Rightarrow B + l\cdot\overrightarrow{v} - A - k\overrightarrow{u} = x\overrightarrow{w}$
$\overrightarrow{AB} = x\overrightarrow{w} + k\overrightarrow{u} - l\overrightarrow{v}$

$k\overrightarrow{u} - l\overrightarrow{v} + x\overrightarrow{w} = \overrightarrow{AB} = (\frac{-93}{5}; -2; 0)$. $\bar{A} = \left(\begin{array}{ccc|c} 10 & -40 & 15 & -93 \\ 0 & -5 & -10 & -40 \\ 15 & -93 & 0 & -5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 10 & -40 & 15 & -93 \\ 0 & 1 & 2 & 8 \\ 15 & -93 & 0 & -5 \end{array}\right) \sim$
$\left(\begin{array}{ccc|c} 10 & -40 & 15 & -93 \\ 15 & -93 & 0 & -5 \\ 0 & 1 & 2 & 8 \end{array}\right) \sim \left(\begin{array}{ccc|c} 10 & -40 & 15 & -93 \\ 0 & -66 & -45 & 269 \\ 0 & 1 & 2 & 8 \end{array}\right) \sim \left(\begin{array}{ccc|c} 10 & -40 & 15 & -93 \\ 0 & 66 & 45 & -269 \\ 0 & 1 & 2 & 8 \end{array}\right) \sim$
$\left(\begin{array}{ccc|c} 10 & -40 & 15 & -93 \\ 0 & 1 & 2 & 8 \\ 0 & 66 & 45 & -269 \end{array}\right) \sim \left(\begin{array}{ccc|c} 10 & -40 & 15 & -93 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & -87 & -797 \end{array}\right) \sim \left(\begin{array}{ccc|c} 10 & -40 & 15 & -93 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 87 & 797 \end{array}\right) \sim$
$\left(\begin{array}{ccc|c} 10 & 0 & 95 & 227 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 87 & 797 \end{array}\right) \sim \left(\begin{array}{ccc|c} 870 & 0 & 0 & -55966 \\ 0 & 87 & 0 & -898 \\ 0 & 0 & 87 & 797 \end{array}\right) \sim \left(\begin{array}{ccc|c} 435 & 0 & 0 & -27983 \\ 0 & 87 & 0 & -898 \\ 0 & 0 & 87 & 797 \end{array}\right)$

$k = \frac{27983}{435} \Rightarrow P = [-5 + 2\frac{27983}{435}; 2; 2 + \frac{27983}{435}]$

$$o = \overleftrightarrow{PQ} = \left\{\left[-5 + 2\frac{27983}{435} + 3k; 2 - 3k; 2 + \frac{27983}{435} + 5k\right] | k \in \mathbb{R}\right\}$$

Př: úkol 4:
Nalezněte příčku mimoběžek, která leží v rovině ρ:

$p: x + y = 2 \wedge 2x + z = 5.$
$q: x + 2y = 1 \wedge -3y + z = 2$

$\rho: x + 2y - x = -2$

Určím $P = p\cap\rho$: $\bar{A} = \left(\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 1 & 1 & 0 & 2 \\ 2 & 0 & 1 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & -1 & 1 & 4 \\ 0 & -4 & 3 & 9 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 1 & -1 & -4 \\ 0 & 4 & -3 & -9 \end{array}\right) \sim$
$\left(\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 6 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 7 \end{array}\right)$

$$P = [-1; 3; 7]$$

Určím $Q = q\cap\rho$: $\bar{A} = \left(\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ 0 & -3 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 1 & 2 & 0 & 1 \\ 0 & 3 & -1 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 0 & 1 & 3 \\ 0 & 3 & -1 & -2 \end{array}\right) \sim$
$\left(\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 3 & 0 & -1 & -2 \\ 0 & 3 & -1 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 3 & 0 & 0 & 1 \\ 0 & 3 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right)$

$$Q = \left[\frac{1}{3}; \frac{1}{3}; 3\right]$$

$\overrightarrow{PQ} = (\frac{4}{3}; -\frac{8}{3}; -4)$

$$\overleftrightarrow{PQ} = \{[-1 + 4t; 3 - 8t; 7 - 12t] | t \in \mathbb{R}\}$$