Důkazy kombinatorických identit

1.

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{p}{r}\binom{p-r}{q-r} = \binom{p}{q}\binom{q}{r}$$

2.

$$\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$$

3.

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \ldots + \binom{n}{k}a^{n-k}b^k + \ldots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

4.

$$2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \ldots + \binom{n}{n}$$

5.

$$0 = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \ldots + (-1)^n\binom{n}{n}$$

6.

$$\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \binom{n}{2}^2 + \binom{n}{3}^2 + \ldots + \binom{n}{n}^2$$

$$\binom{2n}{n} = \binom{k}{0}\binom{2n-k}{n} + \binom{k}{1}\binom{2n-k}{n-1} + \binom{k}{2}\binom{2n-k}{n-2} + \ldots + \binom{k}{k}\binom{2n-k}{n-k}$$

$$\binom{m+p}{m} = \binom{p}{0}\binom{m}{m} + \binom{p}{1}\binom{m}{m-1} + \binom{p}{2}\binom{m}{m-2} + \ldots + \binom{p}{m}\binom{m}{0}$$

7.

$$\binom{p}{q} = \binom{p-1}{q-1} + \binom{p-2}{q-1} + \binom{p-3}{q-1} + \ldots + \binom{q}{q-1} + \binom{q-1}{q-1}$$

8.

$$n \cdot 2^{n-1} = 1\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \ldots + (n-1)\binom{n}{n-1} + n\binom{n}{n}$$