48) $\vec{BA}=(-3;6)\sim(-1;2)$

$\vec{BC}=(3;1)$

$\beta=\arccos\frac{-3+2}{\sqrt{5}\sqrt{10}}=\arccos\left(-\frac{\sqrt{2}}{10}\right)$

$|\angle \vec{AB};\vec{BC}|=\arccos\left(\cancel{\frac{\sqrt{3}}{6}}\right)\left(+\frac{\sqrt{2}}{10}\right)$

$|\angle \vec{AB};\overset{(1;0)}{x}|=\arccos\left(\frac{|-1|}{\sqrt{5}\cdot 1}\right)=\arccos\left(\frac{\sqrt{5}}{5}\right)$

$T=\left[\frac{6}{3};\frac{11}{3}\right]=\left[2;\frac{11}{3}\right]$

$\vec{TA}=\left(-3;\frac{11}{3}\right)\sim(-9;11)$

$\vec{TB}=\left(0;-\frac{7}{3}\right)\sim(0;-1)$

$|\angle ATB|=\arccos\frac{11}{\sqrt{202}\cdot 1}=\arccos\left(\frac{11\sqrt{202}}{202}\right)$

49) a) $\vec{u}=(a;1)$

$\vec{v}=(1;2)$

$\vec{u}\cdot\vec{v}=a+2=0\Rightarrow a=-2$

b) $\frac{\sqrt{2}}{2}=\frac{|a+2|}{\sqrt{5}\sqrt{a^2+1}}$

$\sqrt{10}\cdot\sqrt{a^2+1}=|2a+4|$

$\cancel{10\ 1a}$

$10a^2+10=|4a^2-16a+16|$

$5a^2+5-|2a^2-8a+8|=0$

$\cancel{5a^2+5-|(a-2)^2|=0}$

$3a^2+8a-3$

$a\in\left\{-3;\frac{1}{3}\right\}$

50) $P[6;1]$

$Q[-2;7]$

$X[x;0]$

$\vec{XP}=(6-x;1)$

$\vec{XQ}=(-2-x;7)$

$\vec{XP}\cdot\vec{XQ}=-12-4x+x^2+7=0$

$x^2-4x-5=0$

$(x-5)\cdot(x+1)=0$

a) $x=5$: $\vec{XP}=(1;1)$ $p: x-y-5=0$

$\vec{XQ}=(-7;7)$ $q: x+y-5=0$

$x=-1$: $\vec{XP}=(7;1)$ $p: x-7y+1=0$

$\vec{XQ}=(-1;7)$ $q: 7x+y+7=0$

51) $\vec{r}=(1;1)$

a) $(1+i)\cdot e^{i\frac{\pi}{3}}=\frac{1-\sqrt{3}}{2}+\frac{1+\sqrt{3}}{2}i$

$r=\{[-1+(1-\sqrt{3})t;3+(1+\sqrt{3})t]\mid t\in\mathbb{R}\}$

b) $(1+i)\cdot e^{-i\frac{\pi}{3}}=\frac{1+\sqrt{3}}{2}+\frac{1-\sqrt{3}}{2}i$

$r=\{[-1+(1+\sqrt{3})t;3+(1-\sqrt{3})t]\mid t\in\mathbb{R}\}$

52) $\vec{r}=\left(\frac{\sqrt{3}}{3};-1\right)\Rightarrow\frac{\sqrt{3}}{3}-i$

a) $\left(\frac{\sqrt{3}}{3}-i\right)\cdot e^{-i\frac{\pi}{6}}=-\frac{2\sqrt{3}}{3}i$

$\vec{u}=(0;1)\Rightarrow p: y=0x+2$

b) $\left(\frac{\sqrt{3}}{3}-i\right)\cdot e^{i\frac{\pi}{6}}=1-\frac{\sqrt{3}}{3}i$

$\vec{u}=\left(1;-\frac{\sqrt{3}}{3}\right)=(\sqrt{3};-1)\Rightarrow p: y=\sqrt{3}x+2$

$k\in\{0;\sqrt{3}\}$

62) $A[-3;13]$

$\vec{LK} = (5;10) \sim (1;2)$

$p: 2x - y + 4 = 0$

$\rho(A,p) = \dfrac{|-6-13+4|}{\sqrt{5}} = \dfrac{\sqrt{5}\cdot 15}{5} = \underline{\underline{3\sqrt{5}}}$

65) Určím rovnoběžky s q ve vzdálenosti 3:

$3 = \dfrac{|-4+c|}{\sqrt{25+144}} \Rightarrow 3\cdot 13 = |-4+c|$ $\quad c = 9 \vee c = -17$

$q_1: 5x + 12y + 9 = 0 \qquad A = [17; -\frac{19}{3}]$

$\left(\begin{array}{cc|c} 5 & 12 & 9 \\ 1 & 3 & -2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 3 & -2 \\ 0 & -3 & 19 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & & 17 \\ & 3 & -19 \end{array}\right)$

$q_2: 5x + 12y - 17 = 0 \qquad A = [-9; \frac{7}{3}]$

$\left(\begin{array}{cc|c} 5 & 12 & -17 \\ 1 & 3 & -2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 3 & -2 \\ & -3 & -7 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & & -9 \\ & 3 & 7 \end{array}\right)$

83) c) $p \cap r = \{P\} = [-\frac{7}{4}; \frac{3}{4}]$

$\left(\begin{array}{cc|c} 3 & -1 & -6 \\ 1 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & -1 \\ & 4 & -3 \end{array}\right) \sim \left(\begin{array}{cc|c} 4 & & -7 \\ & 4 & 3 \end{array}\right)$

$\vec{n} = (3; -1) \Rightarrow \vec{n'} = (-1; 3)$

$p': -x + 3y + c = 0$

$\frac{7}{4} + \frac{9}{4} + c = 0 \Rightarrow c = -\frac{16}{4} = -4$

$\underline{p: -x + 3y - 4 = 0}$

Čistopis:

71) $p: ax + by + c = 0$

$a \neq 0$ [Dk: SPORem: $by \pm 2\sqrt{2} = 0 \Rightarrow A \notin p$]

BÚNO $a = 1$.

$A \in p: -2 - 6b + c = 0 \Rightarrow c = 2 + 6b$

$\rho([0,0]; p) = 2\sqrt{2} \Rightarrow 2\sqrt{2} = \dfrac{|0+0+2+6b|}{\sqrt{1+b^2}}$

$8(1+b^2) = 4 + 24b + 36b^2$

$28b^2 - 24b + 4 = 0$

$b = -1 \vee b = \frac{1}{7}$

$b = -1 \Rightarrow c = -4 \Rightarrow p: x - y - 4 = 0$

$b = \frac{1}{7} \Rightarrow c = \frac{20}{7} \Rightarrow q: 7x + y + 20 = 0$

76) a) p odděluje A, B:

$A + B \in P$ (z podobnosti $\triangle AA_0 (A \to B)$ a $\triangle BB_0 (A \to B)$)

$[2;2] \in p \wedge M[4;6] \in p$

$\vec{v}(2;4) \sim (1;2) \Rightarrow \underline{p: 2x - y - 2 = 0}$

b) p neodděluje A, B:

$\overleftrightarrow{AB} \parallel p$:

$\vec{v} = (16; -16) \sim (1; -1)$

$\underline{\underline{p: x + y - 10 = 0}}$

97) Ortocentrické čtverice: Hledám ortocentrum ABV

$\overrightarrow{AB} = (9; -3) \sim (3; 1)$

$n_{AB} = 3x + y - 6 = 0$

$\overrightarrow{AV} = (8; -4) \sim (2; -1)$

$n_{AV} = 2x - y - 10 = 0$

$$\left(\begin{array}{cc|c} 3 & 1 & 6 \\ 2 & -1 & 10 \end{array}\right) \sim \left(\begin{array}{cc|c} 2 & -1 & 10 \\ 0 & 5 & -18 \end{array}\right) \sim$$

$$\left(\begin{array}{cc|c} 5 & & 16 \\ & 5 & 18 \end{array}\right)$$

$C = \left[\frac{16}{5}; -\frac{18}{5}\right]$