ccc

§5. Vzájemná poloha dvou rovin

V.5.1.: Věta o vzájemné poloze dvou rovin daných obecnými rovnicemi Nechť $\rho : ax + by + cz + d = 0, \sigma : ex + fy + gz + h = 0$ jsou roviny. Pak platí:

- $\rho = \sigma \Leftrightarrow \exists k \in \mathbb{R} : (a, d, ci, d) = k \cdot (e, f, g, d)$
- $\rho \parallel \sigma \wedge \rho \neq \sigma \Leftrightarrow \exists k \in \mathbb{R} : (a, b, c) = k \cdot (e, f, g) \wedge d \neq k \cdot h$
- $\rho \nparallel \sigma \Leftrightarrow \forall k \in \mathbb{R} : (a, b, c) \neq k \cdot (e, f, g)$.

Př: Určete vzájemnou polohu dvou rovin:
$\rho : 2x + 3y + 4z + 5 = 0$
$\sigma : x - y - z + 1 = 0$
$\overrightarrow{n_\rho} = (2; 3; 4)$
$\overrightarrow{n_\sigma} = (1; -1; -1)$
vektory jsou lin. nezávislé: $\rho \nparallel \sigma$

Určení průsečnice rovin ρ, σ (hledáme parametrickou rovnici přímky v E_3):
volíme $z = t; t \in \mathbb{R}$:
$2x + 3y + 4t + 5 = 0$
$x - y - t + 1 = 0$
$\Rightarrow 5x + t + 8 = 0 \Rightarrow x = -\frac{8}{5} - \frac{t}{5}$
$y = x - t + 1 \Rightarrow y = -\frac{3}{5} - \frac{6}{5}t.$
Průsečnice: $\left\{\left[-\frac{8}{5} - \frac{1}{5}t; -\frac{3}{5} - \frac{6}{5}t; t\right] | t \in \mathbb{R}\right\}$

V.5.2.: Věta o vzájemné poloze dvou rovin daných parametrickými rovnicemi:
Nechť $\rho(A, \overrightarrow{u}, \overrightarrow{v}), \sigma(B, \overrightarrow{k}, \overrightarrow{l})$ jsou roviny. Pak platí:

- $\rho = \sigma \Leftrightarrow \dim \left\langle \overrightarrow{u}, \overrightarrow{v}, \overrightarrow{k}, \overrightarrow{l} \right\rangle = 2 \wedge \dim \left\langle \overrightarrow{u}, \overrightarrow{v}, \overrightarrow{k}, \overrightarrow{l}, \overrightarrow{AB} \right\rangle = 2/$
- $\rho \parallel \sigma \wedge \rho \neq \sigma \Leftrightarrow \dim \left\langle \overrightarrow{u}, \overrightarrow{v}, \overrightarrow{k}, \overrightarrow{l} \right\rangle = 2 \wedge \dim \left\langle \overrightarrow{u}, \overrightarrow{v}, \overrightarrow{k}, \rightarrow\overrightarrow{AB} \right\rangle = 3$
- $\rho \nparallel \sigma \Leftrightarrow \dim \left\langle \overrightarrow{u}, \overrightarrow{v}, \overrightarrow{k}, \overrightarrow{l}, \overrightarrow{AB} \right\rangle = 3.$

Př: Určete vzájemnou polohu rovin ρ a $\sigma : \rho = \{[1 + t_1 + 2t_2; 2t_1 + 3t_2; -2 - 2t_1 + t_2]; t_1, t_2 \in \mathbb{R}\}$
$\Rightarrow \rho(A = [1; 0; -2]; \overrightarrow{u} = (1; 2; -2); \overrightarrow{v} = (2; 3; 1))$
$\sigma = \{[r_1; -3 + r_2; 1 + 4r_1 - r_2]; r_1, r_2 \in \mathbb{R}\}$
$\Rightarrow \sigma(B = [0; -3; 1]; \overrightarrow{k} = (1; 0; 4); \overrightarrow{l} = (0; 1; -1))$
$\Rightarrow \overrightarrow{AB} = (-1; -3; 3)$

$$\begin{pmatrix} 1 & 2 & -2 \\ 2 & 3 & 1 \\ 1 & 0 & 4 \\ 0 & 1 & -1 \\ -1 & -3 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -2 \\ 0 & -1 & 5 \\ 0 & -2 & 6 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -2 \\ 0 & -1 & 5 \\ 0 & 0 & 1 \end{pmatrix}$$

$\dim < \overrightarrow{u}, \overrightarrow{v}, \overrightarrow{k}, \overrightarrow{l} >= 3 \Rightarrow$ roviny jsou různaběžné.

Rovnice průsečnice ρ, σ: – porovnání souřadnic ρ a σ:
$1 + t_1 + 2t_2 = r_1$
$2t_1 + 3t_0 = +3 + r_2$
$-2 - 2t_1 + t_1 = 1 + 4r_1 - r_2$
soustava 3 rovnic o 4 neznámých, po vyjádření z 2. a 3.rovnice $t = r_1 = t$, odsud a z 1. rovnice $t_1 = -1 - t, r_2 = t + 1$.
Dosazením do rovnice roviny ρ :

$$p = \{[t; -2 + t; 3t] | t \in \mathbb{R}\}$$

Př: 182/19,20

Rozhodněte, jakou maji roviny vzájemnou polohu a určete průsečnice:

$$\begin{aligned} \rho &: 2x - 3y + z - 4 = 0 \\ \sigma &: 4x + y - 5z + 3 = 0 \\ \tau &: x + 2y - z + 1 = 0 \\ \varphi &: -4x + 6y - 2z + 5 = 0 \\ \alpha &: 3x - y - x + 5 = 0 \\ \beta &: x + y + z - 7 = 0 \end{aligned}$$

- ρ a σ:
  Nerovnoběžné:
  $\bar{A} = \left(\begin{array}{ccc|c} 2 & -3 & 1 & 4 \\ 4 & 1 & -5 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & -3 & 1 & 4 \\ 0 & 7 & -7 & -11 \end{array}\right) \sim \left(\begin{array}{ccc|c} 14 & 0 & -14 & -5 \\ 0 & 7 & -7 & -11 \end{array}\right)$
  Průsečnice:
  $$\left\{ \left[\frac{-5 + 14a}{14}; \frac{-11 + 7a}{7}; a \right] : a \in \mathbb{R} \right\}$$
- ρ a τ:
  Nerovnoběžné:
  $\left(\begin{array}{ccc|c} 2 & -3 & 1 & 4 \\ 1 & 2 & -1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 2 & -3 & 1 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & -7 & 3 & 6 \end{array}\right) \sim$
  $\left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 7 & -3 & -6 \end{array}\right) \sim \left(\begin{array}{ccc|c} 7 & 0 & -1 & 5 \\ 0 & 7 & -3 & -6 \end{array}\right)$
  Průsečnice:
  $$\left\{ \left[\frac{5 + 1a}{7}; \frac{-6 + 3a}{7}; a \right] : a \in \mathbb{R} \right\}$$
- σ a τ:
  Nerovnoběžné:
  $\left(\begin{array}{ccc|c} 4 & 1 & -5 & -3 \\ 1 & 2 & -1 & -1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 4 & 1 & -5 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & -7 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 7 & 1 & -1 \end{array}\right) \sim$
  $\left(\begin{array}{ccc|c} 7 & 0 & -9 & -5 \\ 0 & 7 & 1 & -1 \end{array}\right)$
  Průsečnice:
  $$\left\{ \left[\frac{-5 + 9a}{7}; \frac{-1 - 1a}{7}; a \right] : a \in \mathbb{R} \right\}$$
- ρ a φ:
  $(2, -3, 1) = -\frac{1}{2}(-4, 3, -2) \wedge -4 \cdot \frac{-1}{2} = -2 \neq 5$
  Rovnoběžné.

- σ a φ:
  Nerovnoběžné:
  $\left(\begin{array}{ccc|c} 4 & 1 & -5 & -3 \\ -4 & 6 & -2 & -5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 4 & 1 & -5 & -3 \\ 4 & -6 & 2 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 4 & 1 & -5 & -3 \\ 0 & -7 & 7 & 8 \end{array}\right) \sim$
  $\left(\begin{array}{ccc|c} 4 & 1 & -5 & -3 \\ 0 & 7 & -7 & -8 \end{array}\right) \sim \left(\begin{array}{ccc|c} 28 & 0 & -28 & -13 \\ 0 & 7 & -7 & -8 \end{array}\right)$
  Průsečnice:
  $$\left\{ \left[\frac{-13+28a}{28}; \frac{-8+7a}{7}; a \right] : a \in \mathbb{R} \right\}$$

- τ a φ:
  Nerovnoběžné:
  $\left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ -4 & 6 & -2 & -5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 4 & -6 & 2 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & -14 & 6 & 9 \end{array}\right) \sim$
  $\left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 14 & -6 & -9 \end{array}\right) \sim \left(\begin{array}{ccc|c} 7 & 0 & -1 & 2 \\ 0 & 14 & -6 & -9 \end{array}\right)$
  Průsečnice:
  $$\left\{ \left[\frac{2+1a}{7}; \frac{-9+6a}{14}; a \right] : a \in \mathbb{R} \right\}$$

- ρ a α:
  Nerovnoběžné:
  $\left(\begin{array}{ccc|c} 2 & -3 & 1 & 4 \\ 3 & -1 & -1 & -5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & -3 & 1 & 4 \\ 0 & 7 & -5 & -22 \end{array}\right) \sim \left(\begin{array}{ccc|c} 14 & 0 & -8 & -38 \\ 0 & 7 & -5 & -22 \end{array}\right) \sim$
  $\left(\begin{array}{ccc|c} 7 & 0 & -4 & -19 \\ 0 & 7 & -5 & -22 \end{array}\right)$
  Průsečnice:
  $$\left\{ \left[\frac{-19+4a}{7}; \frac{-22+5a}{7}; a \right] : a \in \mathbb{R} \right\}$$

- σ a α:
  Nerovnoběžné:
  $\left(\begin{array}{ccc|c} 4 & 1 & -5 & -3 \\ 3 & -1 & -1 & -5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 3 & -1 & -1 & -5 \\ 4 & 1 & -5 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 3 & -1 & -1 & -5 \\ 0 & 7 & -11 & 11 \end{array}\right) \sim$
  $\left(\begin{array}{ccc|c} 21 & 0 & -18 & -24 \\ 0 & 7 & -11 & 11 \end{array}\right) \sim \left(\begin{array}{ccc|c} 7 & 0 & -6 & -8 \\ 0 & 7 & -11 & 11 \end{array}\right)$
  Průsečnice:
  $$\mathrm{P} = \left\{ \left[\frac{-8+6a}{7}; \frac{11+11a}{7}; a \right] : a \in \mathbb{R} \right\}$$

- τ a α:
  Nerovnoběžné:
  $\left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 3 & -1 & -1 & -5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & -7 & 2 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 7 & -2 & 2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 7 & 0 & -3 & -11 \\ 0 & 7 & -2 & 2 \end{array}\right)$
  Průsečnice:
  $$\left\{ \left[\frac{-11+3a}{7}; \frac{2+2a}{7}; a \right] : a \in \mathbb{R} \right\}$$

- φ a α:
  Nerovnoběžné:
  $\left(\begin{array}{ccc|c} -4 & 6 & -2 & -5 \\ 3 & -1 & -1 & -5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 4 & -6 & 2 & 5 \\ 3 & -1 & -1 & -5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 3 & -1 & -1 & -5 \\ 4 & -6 & 2 & 5 \end{array}\right) \sim$

$$\left(\begin{array}{ccc|c} 3 & -1 & -1 & -5 \\ 0 & -14 & 10 & 35 \end{array}\right) \sim \left(\begin{array}{ccc|c} 3 & -1 & -1 & -5 \\ 0 & 14 & -10 & -35 \end{array}\right) \sim \left(\begin{array}{ccc|c} 42 & 0 & -24 & -105 \\ 0 & 14 & -10 & -35 \end{array}\right) \sim$$
$$\left(\begin{array}{ccc|c} 14 & 0 & -8 & -35 \\ 0 & 14 & -10 & -35 \end{array}\right)$$

Průsečnice:

$$\left\{ \left[\frac{-35+8a}{14}; \frac{-35+10a}{14}; a \right] : a \in \mathbb{R} \right\}$$

- ρ a β:
  Nerovnoběžné:
  $$\left(\begin{array}{ccc|c} 2 & -3 & 1 & 4 \\ 1 & 1 & 1 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 2 & -3 & 1 & 4 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -5 & -1 & -10 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & 5 & 1 & 10 \end{array}\right) \sim$$
  $$\left(\begin{array}{ccc|c} 5 & 0 & 4 & 25 \\ 0 & 5 & 1 & 10 \end{array}\right)$$
  Průsečnice:
  $$\left\{ \left[\frac{25-4a}{5}; \frac{10-1a}{5}; a \right] : a \in \mathbb{R} \right\}$$

- σ a β:
  Nerovnoběžné:
  $$\left(\begin{array}{ccc|c} 4 & 1 & -5 & -3 \\ 1 & 1 & 1 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 4 & 1 & -5 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -3 & -9 & -31 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & 3 & 9 & 31 \end{array}\right) \sim$$
  $$\left(\begin{array}{ccc|c} 3 & 0 & -6 & -10 \\ 0 & 3 & 9 & 31 \end{array}\right)$$
  Průsečnice:
  $$\left\{ \left[\frac{-10+6a}{3}; \frac{31-9a}{3}; a \right] : a \in \mathbb{R} \right\}$$

- τ a β:
  Nerovnoběžné:
  $$\left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 1 & 1 & 1 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & -1 & 2 & 8 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & -1 \\ 0 & 1 & -2 & -8 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 3 & 15 \\ 0 & 1 & -2 & -8 \end{array}\right)$$
  Průsečnice:
  $$\{[15-3a; -8+2a; a] : a \in \mathbb{R}\}$$

- φ a β:
  Nerovnoběžné:
  $$\left(\begin{array}{ccc|c} -4 & 6 & -2 & -5 \\ 1 & 1 & 1 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 4 & -6 & 2 & 5 \\ 1 & 1 & 1 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 4 & -6 & 2 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -10 & -2 & -23 \end{array}\right) \sim$$
  $$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & 10 & 2 & 23 \end{array}\right) \sim \left(\begin{array}{ccc|c} 10 & 0 & 8 & 47 \\ 0 & 10 & 2 & 23 \end{array}\right)$$
  Průsečnice:
  $$\left\{ \left[\frac{47-8a}{10}; \frac{23-2a}{10}; a \right] : a \in \mathbb{R} \right\}$$

- α a β:
  Nerovnoběžné:
  $$\left(\begin{array}{ccc|c} 3 & -1 & -1 & -5 \\ 1 & 1 & 1 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 3 & -1 & -1 & -5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & -4 & -4 & -26 \end{array}\right) \sim$$
  $$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 7 \\ 0 & 2 & 2 & 13 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 0 & 0 & 1 \\ 0 & 2 & 2 & 13 \end{array}\right)$$
  Průsečnice:
  $$\left\{ \left[\frac{1}{2}; \frac{13-2a}{2}; a \right] : a \in \mathbb{R} \right\}$$

Př: 183/21

$$\rho(A = [1,2,0], \overrightarrow{r} = (2,-1,1), \overrightarrow{s} = (-1,1,-1))$$
$$\sigma(B = [2,3,-1], \overrightarrow{t} = (-2,2,-2), \overrightarrow{u} = (2,-2,-2))$$
$$\tau(C = [4,3,2], \overrightarrow{m} = (-1,1,1), \overrightarrow{n} = (1,-2,-3))$$

Což mohu ekvivalentně převést na:

$$\rho(A = [1,2,0], \overrightarrow{r} = (2,-1,1), \overrightarrow{s} = (1,-1,1))$$
$$\sigma(B = [2,3,-1], \overrightarrow{t} = (1,-1,1), \overrightarrow{u} = (1,-1,-1))$$
$$\tau(C = [4,3,2], \overrightarrow{m} = (1,-1,-1), \overrightarrow{n} = (1,-2,-3))$$

$\overrightarrow{BA} = (-1,-1,1)$
$\overrightarrow{AC} = (3,1,2)$
$\overrightarrow{BC} = (2,0,3)$

- $\underline{\rho \text{ a } \phi}$:

$$\bar{A} = \begin{pmatrix} 2 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \\ 2 & -1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & -2 \\ 0 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix} \sim$$
$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow \dim \left\langle \overrightarrow{r}, \overrightarrow{s}, \overrightarrow{t}, \overrightarrow{u} \right\rangle = 3 \Rightarrow$ nerovnoběžné.

Průsečnice:

$$\left(\begin{array}{cccc|c} 2 & -1 & 2 & -2 & 1 \\ -1 & 1 & -2 & 2 & 1 \\ 1 & -1 & 2 & 2 & -1 \end{array}\right) \sim \left(\begin{array}{cccc|c} 2 & -1 & 2 & -2 & 1 \\ 1 & -1 & 2 & -2 & -1 \\ 1 & -1 & 2 & 2 & -1 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -1 & 2 & -2 & -1 \\ 1 & -1 & 2 & 2 & -1 \\ 2 & -1 & 2 & -2 & 1 \end{array}\right) \sim$$
$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & -2 & -1 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 1 & -2 & 2 & 3 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -1 & 2 & -2 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -2 & 2 & 3 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & -1 & 2 & -2 & -1 \\ 0 & 1 & -2 & 2 & 3 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \sim$$
$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & -2 & 2 & 3 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & -2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$$

$$u = 0 \wedge t \in \mathbb{R} \Rightarrow p = \{[2-2t; 3+2t; -1-2t] | t \in \mathbb{R}\}$$

- $\underline{\rho \text{ a } \tau}$:

$$\begin{pmatrix} 2 & -1 & 1 \\ -1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -2 & -3 \end{pmatrix} \sim \begin{pmatrix} 2 & -1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \\ 1 & -2 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 1 & -1 & -1 \\ 1 & -2 & -3 \\ 2 & -1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & -2 \\ 0 & -1 & -4 \\ 0 & 1 & -1 \end{pmatrix} \sim$$
$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 4 \\ 0 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 4 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & -5 \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$$

$\Rightarrow \dim \langle \overrightarrow{r}, \overrightarrow{s}, \overrightarrow{m}, \overrightarrow{n} \rangle = 3 \Rightarrow$ nerovnoběžné.

- <u>φ a τ:</u>

$$\begin{pmatrix}1 & -1 & 1\\ 1 & -1 & -1\\ 1 & -1 & -1\\ 1 & -2 & -3\end{pmatrix} \sim \begin{pmatrix}1 & -1 & 1\\ 0 & 0 & -2\\ 0 & 0 & -2\\ 0 & -1 & -4\end{pmatrix} \sim \begin{pmatrix}1 & -1 & 1\\ 0 & 0 & 1\\ 0 & 0 & 1\\ 0 & 1 & 4\end{pmatrix} \sim \begin{pmatrix}1 & -1 & 1\\ 0 & 1 & 4\\ 0 & 0 & 1\\ 0 & 0 & 1\end{pmatrix} \sim \begin{pmatrix}1 & -1 & 1\\ 0 & 1 & 4\\ 0 & 0 & 1\end{pmatrix}$$

$\Rightarrow \dim \left\langle \overrightarrow{t}, \overrightarrow{u}, \overrightarrow{m}, \overrightarrow{n} \right\rangle = 3 \Rightarrow$ nerovnoběžné.