

$$1) a) \overrightarrow{AB} (4; 1; -1) \quad \overrightarrow{CD} = (-1; -1; 1) \quad 2) \overrightarrow{u} = (-4; 1; 3) \quad \overrightarrow{v} = (2; 2; 1)$$

$$A [3; 1; 1] \quad C [1; 2; 2] \quad A [-4; 1; 12] \quad B [-3; -5; 1]$$

$$\overrightarrow{AC} = (-2; 1; 1) \quad E [9; 3; 3] \quad \overrightarrow{AB} = (7; -6; -11)$$

$$k \cdot \overrightarrow{AB} + l \cdot \overrightarrow{u} + m \cdot \overrightarrow{CD} = \overrightarrow{AC}$$

$$\left(\begin{array}{ccc|c} 4 & 2 & -1 & -2 \\ 1 & -2 & -1 & 1 \\ -1 & 1 & 1 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 16 & 3 & -6 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & -1 & 1 \\ & 1 & 3 & 14 \end{array} \right) \sim \left(\begin{array}{ccc|c} 3 & 1 & 3 & 5 \\ & 1 & 3 & 14 \end{array} \right)$$

$$k = \frac{5}{3} \quad m = \frac{14}{3}$$

$$P = \left[\frac{29}{3}; \frac{8}{3}; -\frac{2}{3} \right] \quad Q = \left[\frac{17}{3}; \frac{20}{3}; -\frac{8}{3} \right]$$

$$b) \overrightarrow{w} = (2; 2; -2) \sim (-1; -1; 1) = \overrightarrow{CD}$$

\Rightarrow linearly independent.

$$c) k \cdot \overrightarrow{AB} + \overrightarrow{w} = \overrightarrow{AE}$$

$$k \cdot \overrightarrow{CD} + m \cdot \overrightarrow{w} = \overrightarrow{CE}$$

$$k \cdot \overrightarrow{CD} - m \cdot \overrightarrow{AB} = \overrightarrow{CE} - m \cdot \overrightarrow{AE}$$

$$k \cdot \overrightarrow{CD} + \overrightarrow{AB} + m \cdot \overrightarrow{AE} = \overrightarrow{CE}$$

$$\left(\begin{array}{ccc|c} -1 & 4 & 6 & 8 \\ -1 & 1 & 2 & 1 \\ 1 & -1 & 2 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 4 & -6 & -8 \\ & 3 & 4 & 7 \\ & 3 & 8 & 9 \end{array} \right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & -4 & -6 & -8 \\ & 3 & 4 & 7 \\ & 4 & 2 & 2 \end{array} \right) \sim \left(\begin{array}{ccc|c} 3 & 3 & 2 & 5 \\ & 3 & 2 & 5 \end{array} \right)$$

$$k = \frac{5}{3} \quad \frac{5}{3} = -\frac{1}{2} k \Rightarrow k = -\frac{10}{3}$$

$$Q = \left[-\frac{2}{3}; \frac{1}{3}; \frac{11}{3} \right]$$

$$P = \left[\frac{29}{3}; -\frac{7}{3}; -\frac{7}{3} \right] \quad Q = \left[\frac{43}{3}; \frac{16}{3}; \frac{4}{3} \right]$$

$$\overrightarrow{u} \cdot \overrightarrow{v} = (-5; 10; -10) \sim (-1; 2; -2)$$

$$k \cdot \overrightarrow{u} + l \cdot \overrightarrow{v} + m \cdot \overrightarrow{w} = \overrightarrow{AB}$$

$$\left(\begin{array}{ccc|c} -4 & -1 & 2 & 7 \\ 1 & 2 & 2 & -6 \\ 3 & -2 & 1 & -11 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 2 & -6 \\ 0 & -9 & -5 & -6 \\ 0 & 7 & -10 & 23 \end{array} \right) \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 2 & 2 & -6 \\ & 7 & 10 & -23 \\ & -45 & 135 & \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & -2 \\ & 1 & 1 & -3 \end{array} \right)$$

$$k = -2 \quad m = 3$$

$$P = [4; -1; 6] \quad Q = [3; 1; 4]$$

$$3) \overrightarrow{AB} = (9; -2; -4) \quad \overrightarrow{CD} = (6; 4; 0)$$

$$A [-6; 1; 1] \quad C [-2; -1; 1]$$

$$\overrightarrow{v} = \overrightarrow{AB} \times \overrightarrow{CD} = (16; -24; 48)$$

$$\sim (2; 3; 6)$$

$$2(-6) + 3 \cdot 1 + 6 \cdot 1 + d = 0 \Rightarrow d = 3$$

$$\alpha: 2x + 3y + 6z + 3 = 0$$

$$\rho(p, q) = \rho(\alpha, q)$$

$$\frac{|2(-2) + 3(-1) + 6(1) + 3|}{\sqrt{2^2 + 3^2 + 6^2}} = \frac{2}{\sqrt{49}} = \frac{2}{7}$$