1 Basic Reproductive Number

1.1

Using the notation of P_{SI}, P_{IR}, P_{RS} and setting $P_{RS} = 1$, examine the following setting: two conected nodes where the first is infected and the second isn't. The probability that the second node will move from susceptible to infected on the k_{th} turn is:

$$P\left(\text{Infected in the k turn}\right) = \left(1 - P_{SI}\right)^{k-1} P_{SI}\left(1 - P_{IR}\right)^{k}$$

and the total probability of the node being infected before the first is cured is:

$$\begin{aligned}
P &= \sum_{k=1}^{\infty}\left(1 - P_{SI}\right)^{k-1} P_{SI}\left(1 - P_{IR}\right)^{k} \\
&= \sum_{k=1}^{\infty}\left[\left(1 - P_{SI}\right)\left(1 - P_{IR}\right)\right]^{k-1} P_{SI}\left(1 - P_{IR}\right) \\
&= P_{SI}\left(1 - P_{IR}\right)\sum_{k=1}^{\infty}\left[\left(1 - P_{SI}\right)\left(1 - P_{IR}\right)\right]^{k-1} \\
&= P_{SI}\left(1 - P_{IR}\right)\sum_{k=0}^{\infty}\left[\left(1 - P_{SI}\right)\left(1 - P_{IR}\right)\right]^{k} \\
&= P_{SI}\left(1 - P_{IR}\right)\cdot\frac{1}{1 - \left(1 - P_{SI}\right)\left(1 - P_{IR}\right)} \\
&= P_{SI}\left(1 - P_{IR}\right)\cdot\frac{1}{P_{SI} + P_{IR} - P_{SI}P_{IR}}
\end{aligned}$$

1.2 Expected Infected Neighbours

Treating each neighbour as an independent bernoulli trial with probability of success being the result from 1.1, and given we have d neighbours then the infected vertex will infect

$$P_{SI}\left(1 - P_{IR}\right)\cdot\frac{d}{P_{SI} + P_{IR} - P_{SI}P_{IR}}$$

vertices on average.

1.3 Empiric Epidemic Spread With Infection Expectancy 2

Taking the expectating and plugging in $d = 4$ then equating to the desired expectation of 2 we get the relation:

$$P_{SI} = \frac{P_{IR}}{\left(1 - P_{IR}\right)}$$