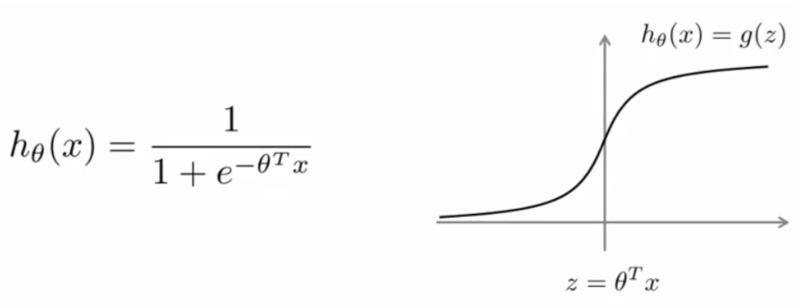
1. Optimization Objective

1. Logistic Regression

- Alternative view of logistic regression
    - Sigmoid



if y=1, then $h_\theta(x) \approx 1$ and $\Theta^T x \gg 0$

if y=0, then $h_\theta(x) \approx 0$ and $\Theta^T x \ll 0$

- Regularization
    - Greater number of features leads to overfitting
    - Regularization one way to avoid overfitting(high variance), other than reducing the number of features
    - Regularization decreases the affect of features
- Cost Function

$$J(\theta) = \frac{1}{m}\sum_{i=1}^m y^{(i)}(-\log(h_\theta(x^{(i)}))) + (1-y^{(i)})(-\log(1-h_\theta(x^{(i)}))) + \frac{\lambda}{2m}\sum_{j=1}^n \Theta_j^2$$

> A + λB: If λ is high, B gains more weight

2. Support Vector Machine

- Cost Function

  $\underset{\theta}{\min}=C\sum_{i=1}^my^{(i)}cost_1(\theta^Tx^{(i)})+(1-y^{(i)})cost_0(\theta^Tx^{(i)})+\sum_{j=1}^n\theta_j^2$

  > CA + B: If C is high, A gains more weight; where C = 1/λ

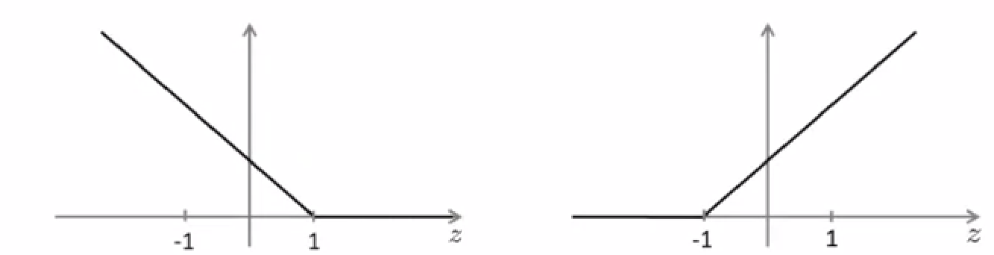
- Hypothesis

$$h_\theta(x)= \begin{cases} 1 & iff \ \theta^Tx \geq 0 \ 0 & \text{otherwise} \end{cases}$$

2. Large Margin Intuition

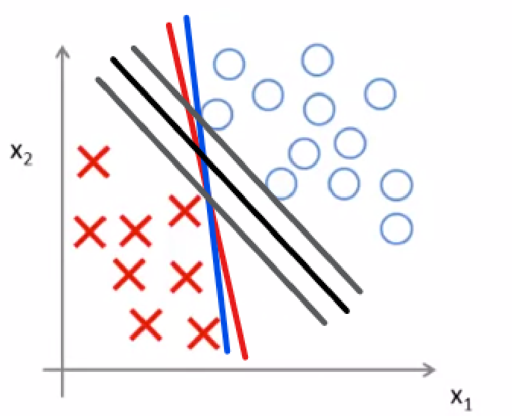
- Cost Function

$$J(\theta) = C\sum_{i=1}^{m} y^{(i)}\ \text{cost}_1(\theta^T x^{(i)}) + (1 - y^{(i)})\ \text{cost}_0(\theta^T x^{(i)}) + \frac{1}{2}\sum_{j=1}^{n} \Theta_j^2$$



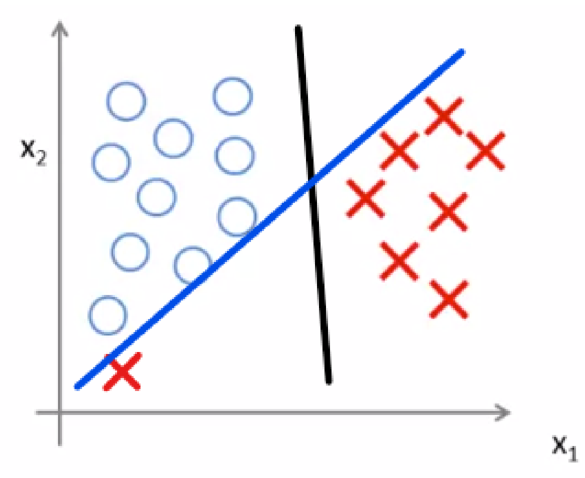
If $y = 1$, we want $\theta^T x \geq 1$ (not just ≥ 0)
If $y = 0$, we want $\theta^T x \leq -1$ (not just < 0)

- Parameter C
  - When C is very large, minimize the first term to zero, leaving the regularization term
  - Suppose they are as very large as 10,000
- Decision Boundary: Linearly Separable Case
  - There could be multiple potential lines, but the black one seems the most robust
  - Grey lines generate the largest minimun distants (margins)
  - SVM is also called **Large Margin Classifier**



- Large Margin Classifier in Presence of Outliers
  - **Large C**
    - Narrow margin (lower rate of misclassification)
    - The value of cost function will be sensitive with regard to the deviation of the value of θ^Tx from the threshold value
    - Therefore, margin will be adjusted more strictly such that cost-function-increasing data placed outside the margin
    - Decision boundary like the blue line will be changed drastically by an outlier
  - **Not too large C**

- Broad Margin (higher rate of misclassification)
- The value of cost function will be resistant with regard to the deviation of the value of θ^Tx from the threshold value
- Therefore, margin will be adjusted less strictly such that cost-function-increasing data may be placed inside the margin
- Decision boundary like the black one will not be changed drastically by an outlier

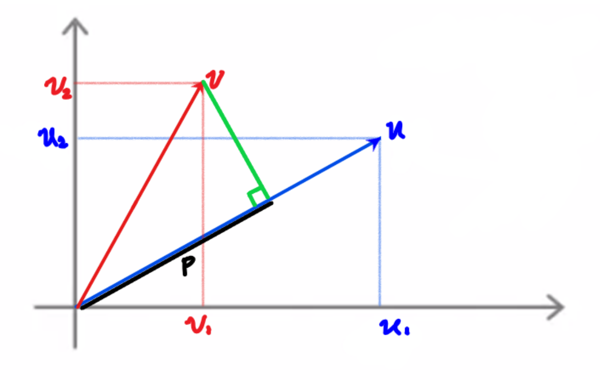


3. Mathmatics Behind Large Margin Classification

1. Vector Inner Product

- Vector

  $u = \begin{bmatrix}u1 \ u2 \end{bmatrix}$ $v = \begin{bmatrix}v1 \ v2 \end{bmatrix}$



- Norm of the Vector
  $||u||=length\ of\ vector \ u=\sqrt{u_1^2+u_2^2}$
  $p=length \ of \ projection \ of v \ onto \ u$
  $u^Tv = p \cdot ||u||=u_1v_1+u_2v_2$

  > p has a sign depending on the angle between vectors If the angle is greater than 90 degrees, then the sign of p is negative

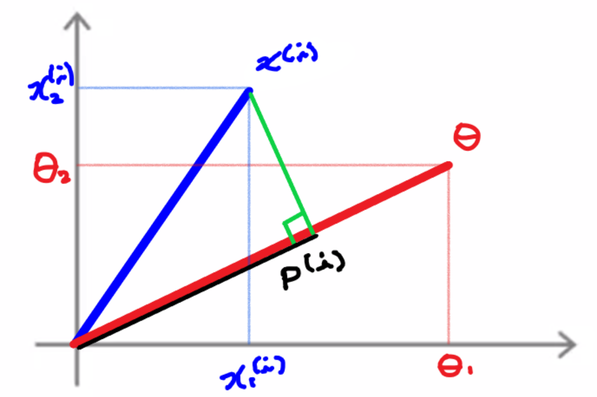
2. SVM Decision Boundary

- $\Large \underset{\theta}{{\min}}=\frac{1}{2}\sum_{j=1}^n\theta_j^2$

$\theta^Tx^{(i)} \geq 1 \ if \ y^{(i)} = 1$
$\theta^Tx^{(i)} \leq -1 \ if \ y^{(i)} = 0$

$Simplification: \theta_0 = 0, n = 2$
$\underset{\theta}{(\min)}=\frac{1}{2}\sum_{j=1}^n\theta_j^2=\frac{1}{2}(\theta_1^2+\theta_2^2)={\frac{1}{2}(\sqrt{\theta_1^2+\theta_2^2})^2}=\frac{1}{2}||\theta||^2$

- Inner product of $\theta^Tx^{(i)}$



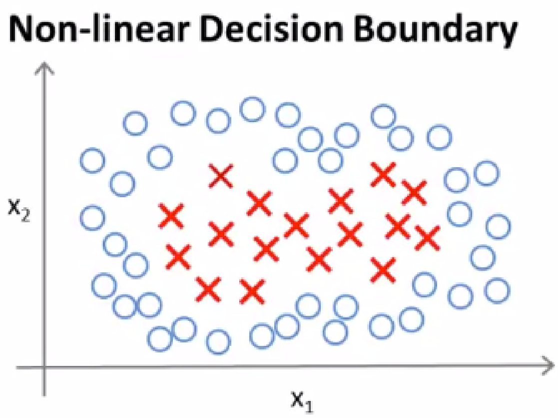
$\theta^Tx^{(i)}=p^{(i)}||\theta||$

  - As $\theta^Tx = 0$, θ is perpendicular with regard to x
  - If the margin is large, $p^{(i)}$ is little, causing $||\theta||$, as well as $J(\theta)$, to be greater
  - If the margin is small, $p^{(i)}$ is large, causing $||\theta||$, as well as $J(\theta)$, to be smaller

4. Kernels

1. Non-linear Decision Boundary

- Kernels allow us to make complex, non-linear classifiers using SVM



- $Predict \ y = 1 \ if$

$$ \theta_0 + \theta_1x_1 + \theta_2x_2 + \theta_3x_1x_2 + \theta_4x_1^2+ \theta_5x_2^2 + \cdots \geq 0 \ h_\theta(x) = \begin{cases} 1, & if \ \theta_0 + \theta_1x_1+ \cdot\cdot\cdot \geq 0 \ 0, & otherwise \end{cases} $$

- The expression can be denoted as:

$$ \theta_0 + \theta_1f_1 + \theta_2f_2 + \theta_3f_3 + \theta_4f_4 + \theta_5f_5 + \cdot\cdot\cdot $$

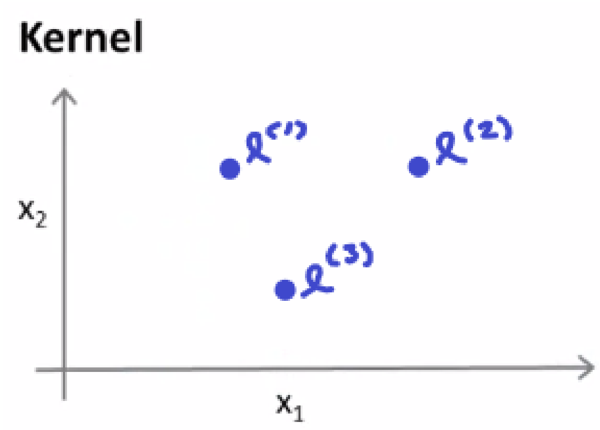
- The question: Is there a different / better choice of the features $f_1, \ f_2, \ f_3, \cdot\cdot\cdot \ ?$

2. Gaussian Kernel

1. Landmarks and Similarity

- Given x, compute new feature depending on proximity to landmarks $l^{(1)}, \ l^{(2)}, \ l^{(3)}$

> - One way to get the landmarks is to put them in the **exact same** locations as all the training examples
> - This gives us **m** landmarks, with one landmark per training example



$$ Given \ x: f_1=similarity(x,l^{(1)})=exp\left(-\frac{||x-l^{(1)}||^2}{2\sigma^2}\right) \ Given \ x: f_2=similarity(x,l^{(2)})=exp\left(-\frac{||x-l^{(2)}||^2}{2\sigma^2}\right) \ Given \ x: f_3=similarity(x,l^{(3)})=exp\left(-\frac{||x-l^{(3)}||^2}{2\sigma^2}\right) $$

- This gives us a **feature vector**, $f_{(i)}$ of all our features for example $x_{(i)}$
- $f_0=1$ to correspond with θ_0
- Training example $x_{(i)}$:

$$ x^{(i)} \rightarrow \left[\begin{matrix} f_1^{(i)}=similarity(x^{(i)},l^{(1)}) \ f_1^{(i)}=similarity(x^{(i)},l^{(2)}) \ \vdots \ f_m^{(i)}=similarity(x^{(i)},l^{(m)}) \ \end{matrix}\right], x^{(i)} \in \R^{n+1}; \text{where m is the size of data, n the number of features } $$

- Cost Function

$$ \underset{\theta}{\min}=C\sum_{i=1}^my^{(i)}cost_1(\theta^Tf^{(i)})+(1-y^{(i)})cost_0(\theta^Tf^{(i)})+\sum_{j=1}^m\theta_j^2 $$

2. Kernels and Similarity

- Gaussian Function as Similarity
  $\Large Given \ x^{(i)}: f_i=similarity(x^{(i)},l^{(i)})=exp\left(-\frac{||x^{(i)}-l^{(i)}||^2}{2\sigma^2}\right)=exp\left(-\frac{\sum_{j=1}^n(x_j-l_j^{(i)})^2}{2\sigma^2}\right)$

  > - Gaussian function computes the distance between vector x and $l^{(i)}$
  > - The formular, the choice of similarity function is called Gaussian Kernel

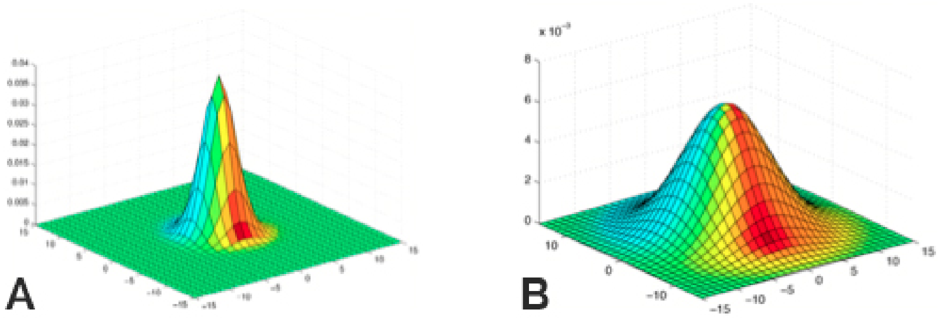
> - The kernel function, $similarity(x,l^{(i)})$, can also be denoted as $k(x, l^{(i)})$

- The value of the function ranges from 0 to 1, indicating the similarity

  $If \ x \approx l^{(i)}:f_1\approx exp\left(-\frac{\text{small number}}{2\sigma^2}\right) \approx 1$
  $If \ x \ far \ from \ l^{(i)}:f_1\approx exp\left(-\frac{\text{large number}}{2\sigma^2}\right) \approx 0$

- The distribution of the Gaussian function is affected by the parameter σ

  - **A**: If σ is small, the value of the function would decrease steeper as feature vector(x) gets further from vector l
  - **B**: If σ is small, the value of the function would decrease gentler as feature vector(x) gets further from vector l



- Given θ^T optimized by cost function, now θ^Tf can be evaluated with test data
  - Predict "1" when $\theta^Tf \geq 0$
  - Predict "0" otherwise

3. SVM with Kernels

- Hypothesis: Given x, compute features $f \in \R^{m+1}$ (Optionally plus one depending on the constant feature)

  - Predict "y=1" if $\theta^Tf \geq 0$

  - Training

    $\underset{\theta}{\min}=C\sum_{i=1}^my^{(i)}cost_1(\theta^Tf^{(i)})+(1-y^{(i)})cost_0(\theta^Tf^{(i)})+\sum_{j=1}^{\color{yellow}{m}}\theta_j^2$

    > - Note "m" in the second term for it should be the effective number of features of x
    > - Also in the second term, j starts from 1 because θ_0 is not regularized
    > - $\sum_{j=1}^m\theta_j^2 = \theta^T\theta$, but use $\theta^TM\theta$ instead
    > - M depends on the distance measure and it allows the SVM computation work more efficiently.

- SVM Parameters

  - $C(=\frac{1}{\Lambda})$

    - Large C (small λ): lower bias, high variance (overfitting)
    - Small C (large λ): higher bias, low variance (underfitting)

  - σ^2
    - Small σ^2: features f_i vary less smoothly; lower bias, higher variance (overfitting)
    - Large σ^2: features f_i vary more smoothely; higher bias, lower variance (underfitting)

5. SVM in Use

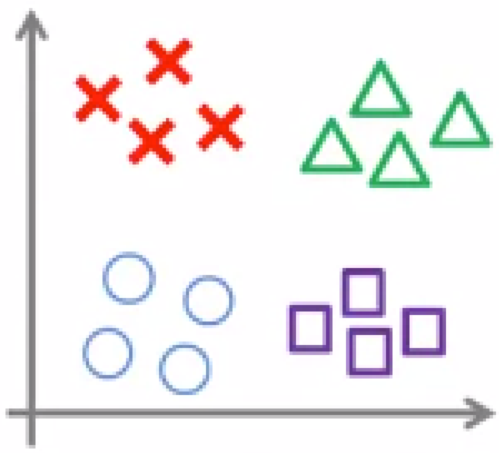
1. Things to be Specified

- C
- Kernel
  - No kernel (linear kernel)
  - Gaussian kernel

    > Choose σ^2 Perform feature scaling in advance Technical condition called "Mercer's Theorem" need to be satisfied

  - Many off-the-shelf kernels are available:
    - Polynomial kernel: $k(x, l)=(x^Tl+constant)^{degree}$
    - Many esoteric: string kernel, chi-square kernel, histogram intersection kernel etc

2. Multiclass Classification



$$ y\in{1, 2, 3, \ldots , K} $$

- Many SVM packages already have built-in multi-class classification functionally
- Otherwise, use one-vs-all method
  - Train K SVMs, one to distinguish $y=i$ from the rest, for $i=1, 2, \ldots, K$
  - Get $\theta^{(1)}, \theta^{(2)}, \ldots, \theta^{(K)}$
  - Pick class i with the largest $(\theta^{(i)})^Tx$ (hyperplane)
  - That the hyperplane is large "positive" value means that the data is for sure classified as 1

3. Logistic Regression vs. SVMs

$n=\text{number of features} (x \in \R^{n+1}), m = \text{number of training examples}$

- n > m: logistic regression or linear kernel
- small n & medium m: Gaussian kernel
- small n & large m: add features with logistic regression or linear kernel
- Neural Network likely to work well for most of these settings, but may be slower to train