

# Recurrent Neural Network

Instructor: Seunghoon Hong

# Course overview

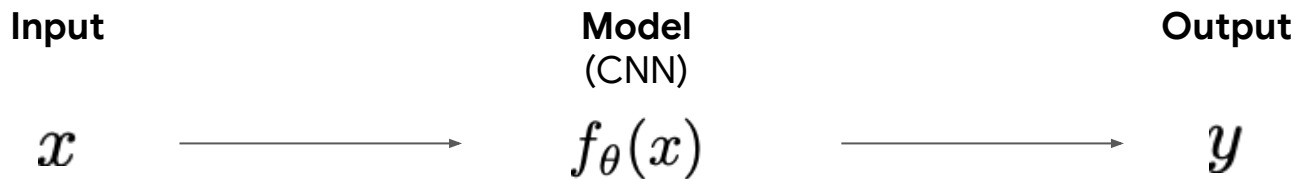
- ❑ Image classification
- ❑ Object detection
- ❑ Semantic segmentation
- ❑ Pose estimation
- ❑ Visualization
- ❑ Style transfer
- ❑ Adversarial attacks
- ❑ Text classification
- ❑ Machine translation
- ❑ Image captioning
- ❑ Visual question answering
- ❑ Image generation
- ❑ Text generation
- ❑ Text-to-image synthesis
- ❑ Img-to-img translation
- ❑ Unpaired img-to-img translation
- ❑ Interactive drawing
- ❑ Search algorithms
- ❑ Markov decision processes
- ❑ Reinforcement learning

**We are here!**



# Recap: Visual recognition with CNN

- Learning to associate input to pre-defined, task-specific labels



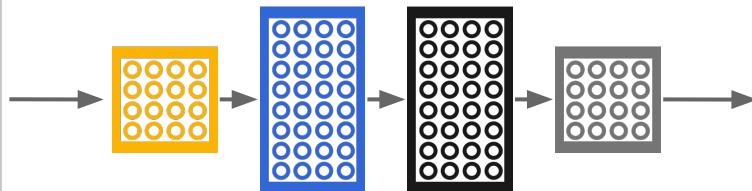
# Recap: Visual recognition with CNN

- Learning to associate input to pre-defined, task-specific labels
- Examples: **classification**

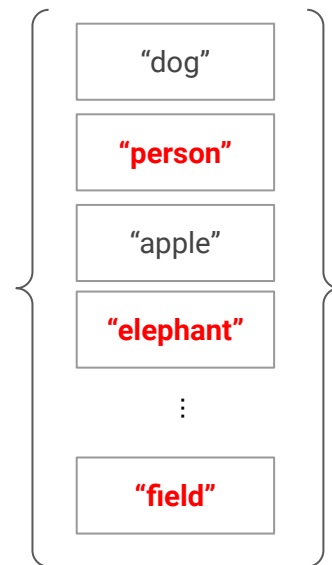
input  $x$



model  
 $f_{\theta}(x)$



output  $y$



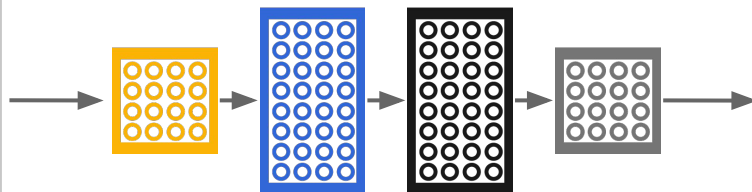
# Recap: Visual recognition with CNN

- Learning to associate input to pre-defined, task-specific labels
- Examples: **detection**

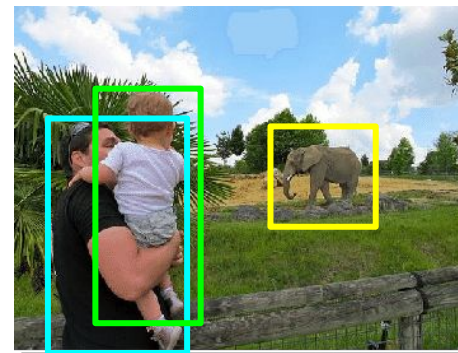
input x



$$f_{\theta}(x)$$



output y



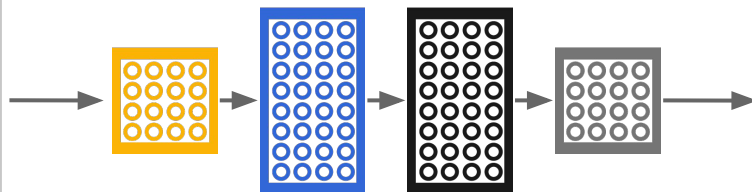
# Recap: Visual recognition with CNN

- Learning to associate input to pre-defined, task-specific labels
- Examples: **pose estimation**

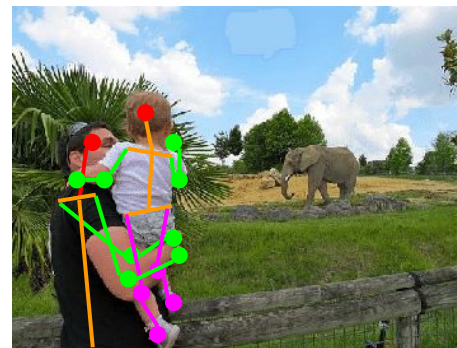
input x



$$f_{\theta}(x)$$



output y



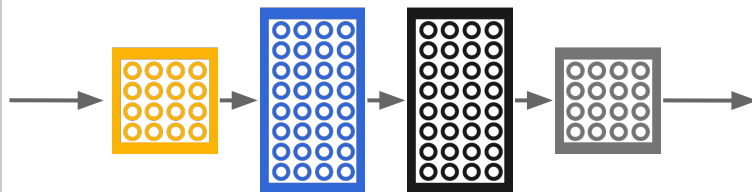
# Recap: Visual recognition with CNN

- Learning to associate input to pre-defined, task-specific labels
- Examples: **segmentation**

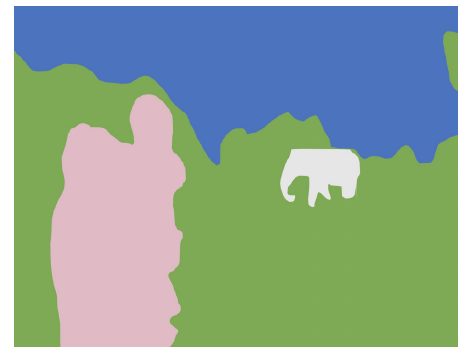
input x



$$f_{\theta}(x)$$



output y



# Modeling sequences

What if we want to deal with a **sequential** data?



# Today's agenda

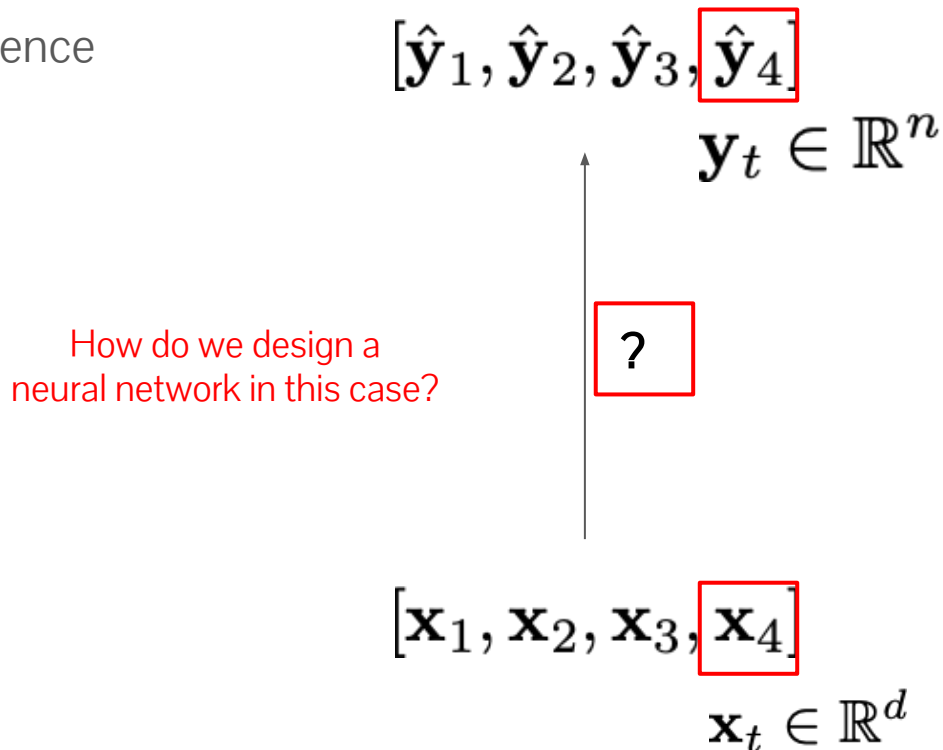
- RNN basics
- Backpropagation through time
- The vanishing/exploding gradients problem
- Advanced RNNs

# Today's agenda

- **RNN basics**
- Backpropagation through time
- The vanishing/exploding gradients problem
- Advanced RNNs

# Modeling sequences with feedforward network

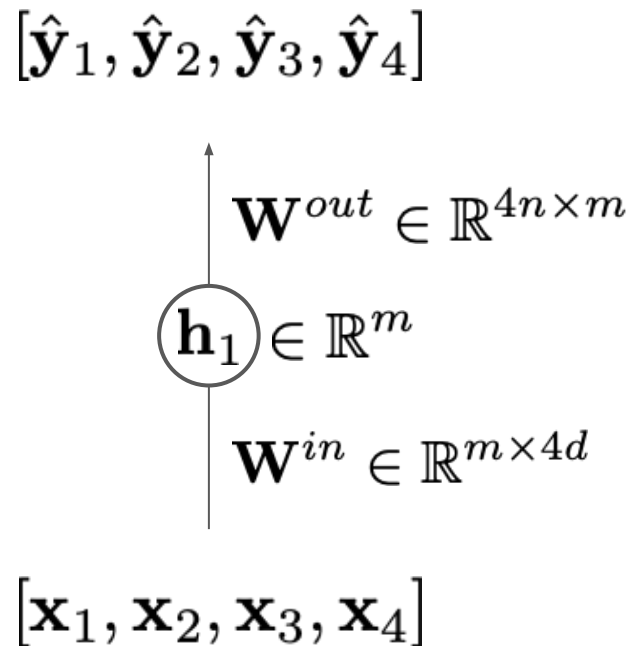
Simple case: modeling a fixed-size sequence



# Modeling sequences with feedforward network

Simple case: modeling a fixed-size sequence

- Option 1: MLP

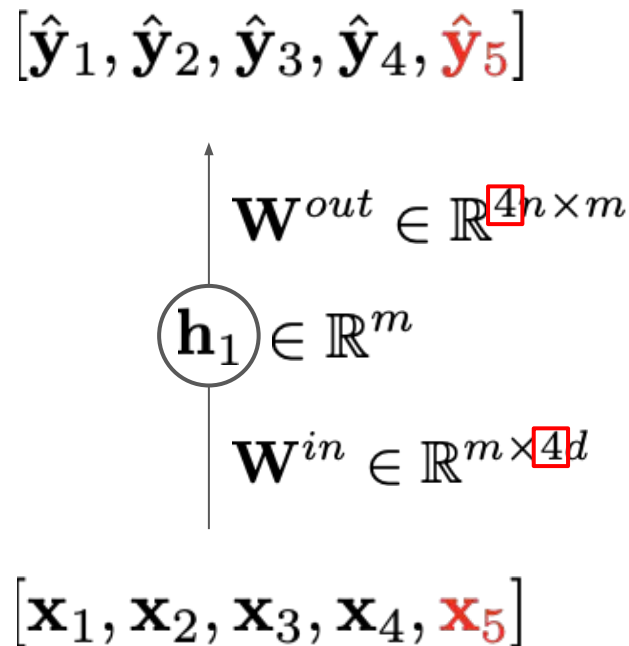


where  $\mathbf{x}_t \in \mathbb{R}^d, \mathbf{h}_t \in \mathbb{R}^m, \mathbf{y}_t \in \mathbb{R}^{n_2}$

# Modeling sequences with feedforward network

Simple case: modeling a fixed-size sequence

- Option 1: MLP
- What if we change the sequence length?
  - We cannot reuse the same network for handling sequences in different length!
  - It is because the network parameter bounds the length of the sequences!

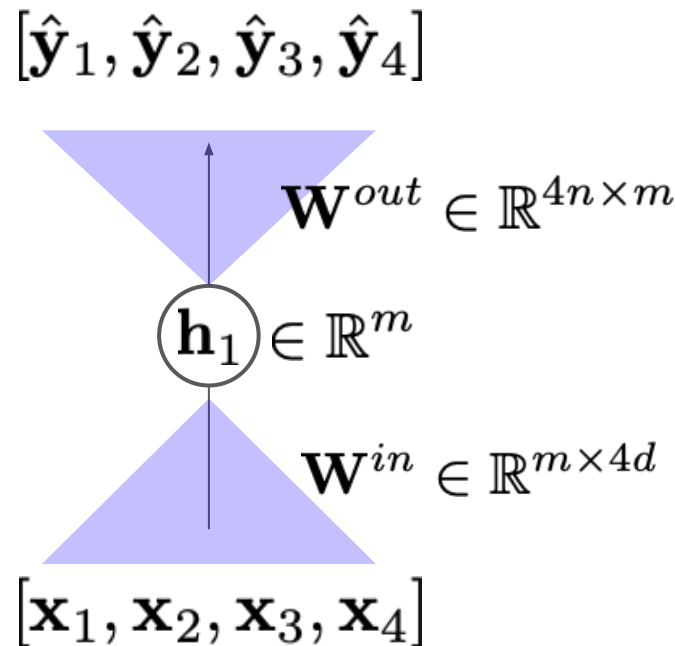


where  $\mathbf{x}_t \in \mathbb{R}^d, \mathbf{h}_t \in \mathbb{R}^m, \mathbf{y}_t \in \mathbb{R}^n$

# Modeling sequences with feedforward network

Simple case: modeling a fixed-size sequence

- Option 2: CNN

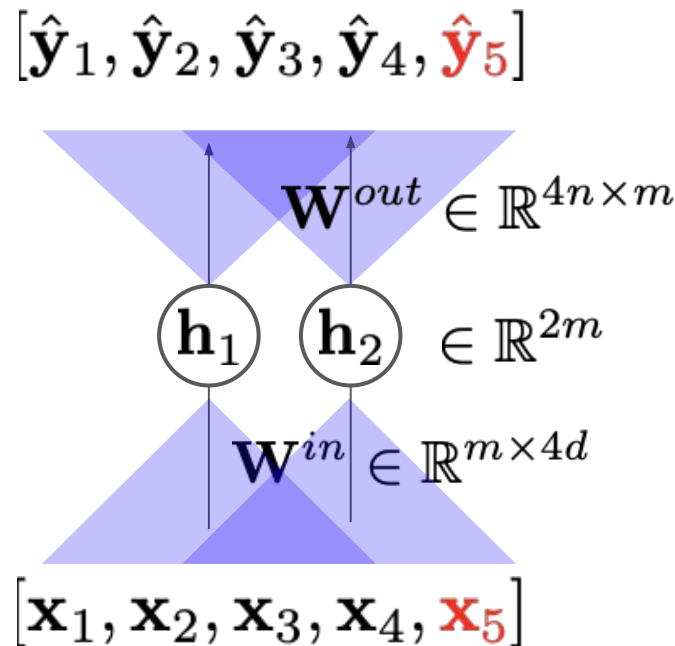


where  $\mathbf{x}_t \in \mathbb{R}^d, \mathbf{h}_t \in \mathbb{R}^m, \mathbf{y}_t \in \mathbb{R}^{n \times 4}$

# Modeling sequences with feedforward network

Simple case: modeling a fixed-size sequence

- Option 2: CNN
- What if we change the sequence length?
  - We can reuse the same network by sliding convolution filters over the sequence
- Problems?
  - The hidden representation grows with the length of the sequence!
  - The receptive field is fixed!



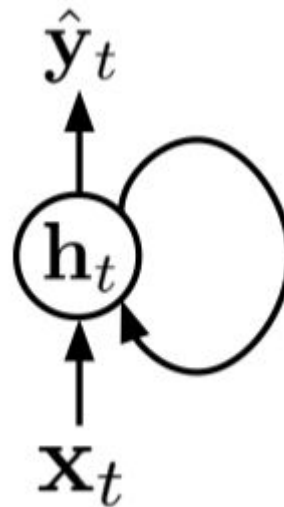
where  $\mathbf{x}_t \in \mathbb{R}^d, \mathbf{h}_t \in \mathbb{R}^m, \mathbf{y}_t \in \mathbb{R}^{n_1}$

# Modeling sequences efficiently

How can we model sequences in an efficient way?

Use recursion!

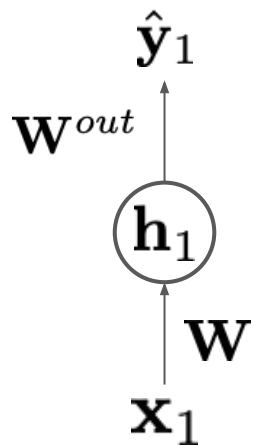
- A model that reads each input one at a time
- Parameters can simply be reused (or shared) for each input in a recurrent computation





# Recurrent Neural Network (RNN)

Unrolling RNNs through time

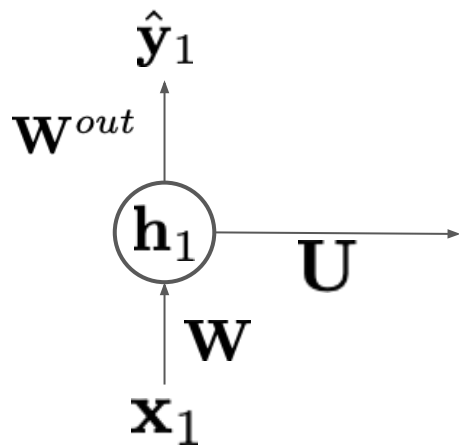


$$\mathbf{h}_1 = \sigma(\mathbf{W}\mathbf{x}_1 + \mathbf{b})$$

$$\hat{\mathbf{y}}_1 = \mathbf{W}^{out}\mathbf{h}_1$$

# Recurrent Neural Network (RNN)

Unrolling RNNs through time

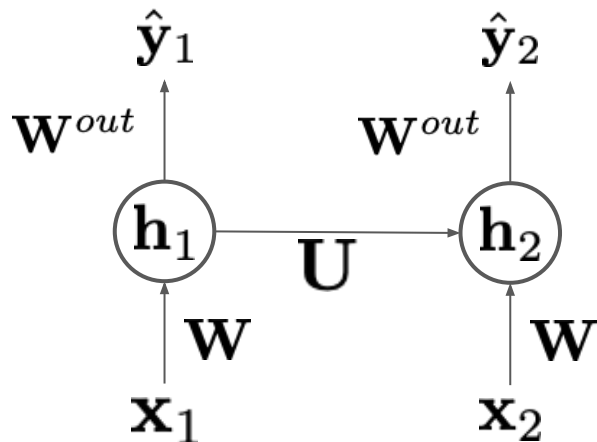


$$\mathbf{h}_1 = \sigma(\mathbf{W}\mathbf{x}_1 + \mathbf{b})$$

$$\hat{\mathbf{y}}_1 = \mathbf{W}^{out}\mathbf{h}_1$$

# Recurrent Neural Network (RNN)

Unrolling RNNs through time



$$h_1 = \sigma(Wx_1 + b)$$

$$\hat{y}_1 = W^{out}h_1$$

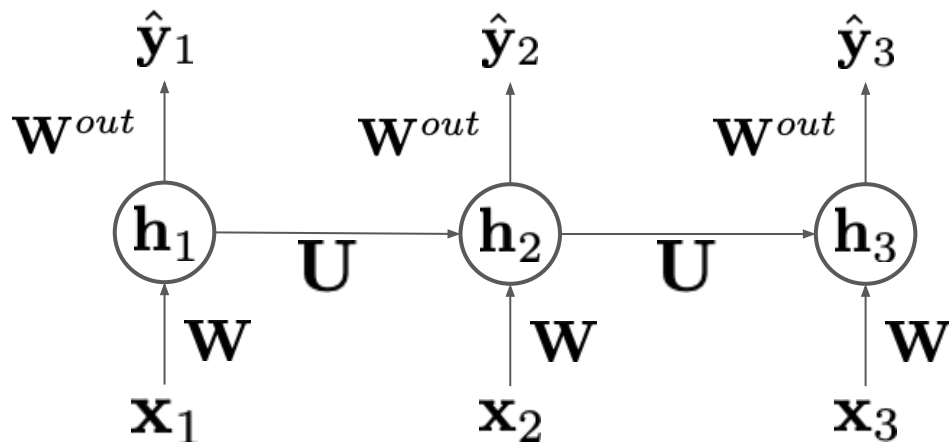
$$h_2 = \sigma(\boxed{Uh_1} + Wx_2 + b)$$

$$\hat{y}_2 = W^{out}h_2$$

We have a temporal connection that models a temporal dependency!

# Recurrent Neural Network (RNN)

Unrolling RNNs through time



$$\mathbf{h}_1 = \sigma(\mathbf{W}\mathbf{x}_1 + \mathbf{b})$$

$$\hat{\mathbf{y}}_1 = \mathbf{W}^{out}\mathbf{h}_1$$

$$\mathbf{h}_2 = \sigma(\mathbf{U}\mathbf{h}_1 + \mathbf{W}\mathbf{x}_2 + \mathbf{b})$$

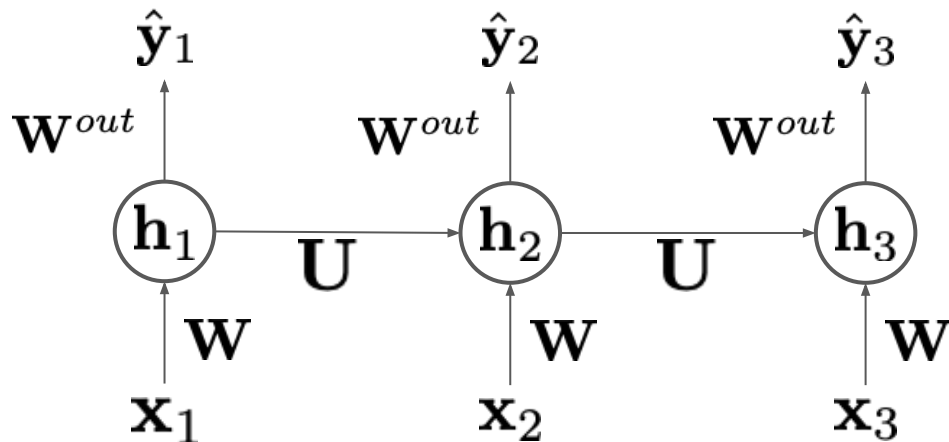
$$\hat{\mathbf{y}}_2 = \mathbf{W}^{out}\mathbf{h}_2$$

$$\mathbf{h}_3 = \sigma(\mathbf{U}\mathbf{h}_2 + \mathbf{W}\mathbf{x}_3 + \mathbf{b})$$

$$\hat{\mathbf{y}}_3 = \mathbf{W}^{out}\mathbf{h}_3$$

# Recurrent Neural Network (RNN)

Unrolling RNNs through time



In general, for any  $t \geq 1$ ,

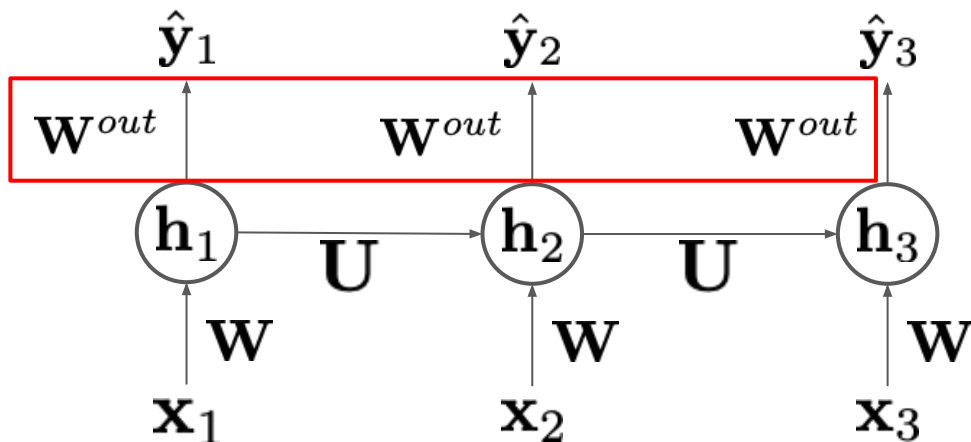
$$\mathbf{h}_t = \sigma(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t + \mathbf{b})$$

$$\hat{\mathbf{y}}_t = \mathbf{W}^{out}\mathbf{h}_t$$

$$\mathbf{h}_0 = \mathbf{0}$$

# Recurrent Neural Network (RNN)

Unrolling RNNs through time



Weights are shared over time!

In general, for any  $t \geq 1$ ,

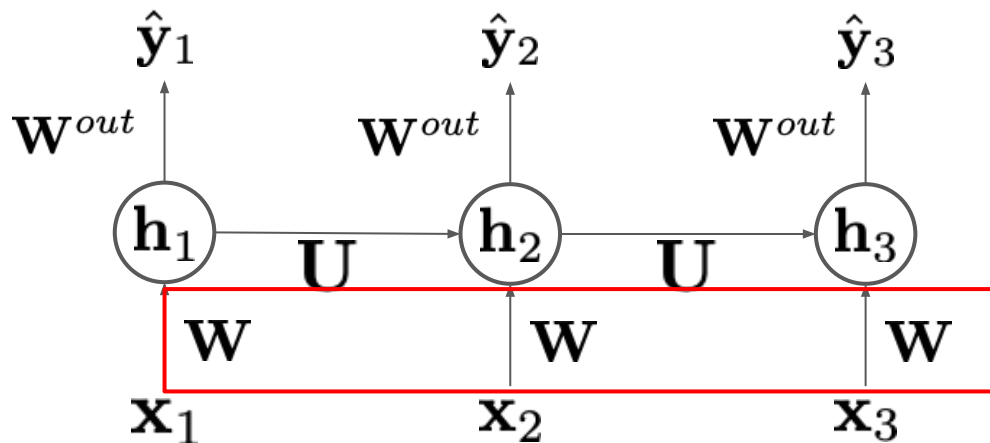
$$\mathbf{h}_t = \sigma(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t + \mathbf{b})$$

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$$\mathbf{h}_0 = \mathbf{0}$$

# Recurrent Neural Network (RNN)

Unrolling RNNs through time



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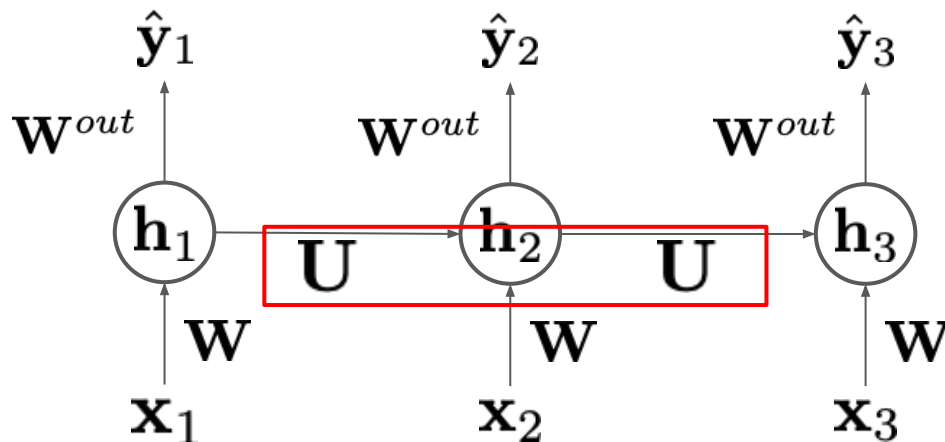
$$\mathbf{h}_t = \sigma(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t + \mathbf{b})$$

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# Recurrent Neural Network (RNN)

Unrolling RNNs through time



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$$\mathbf{h}_t = \sigma(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t + \mathbf{b})$$

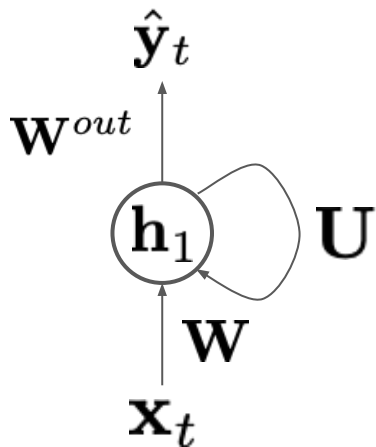
$$\hat{\mathbf{y}}_t = \mathbf{W}^{out}\mathbf{h}_t$$

$$\mathbf{h}_0 = \mathbf{0}$$



# Recurrent Neural Network (RNN)

Unrolling RNNs through time



In general, for any  $t \geq 1$ ,  
 $\mathbf{h}_t = \sigma(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t + \mathbf{b})$   
 $\hat{\mathbf{y}}_t = \mathbf{W}^{out}\mathbf{h}_t$

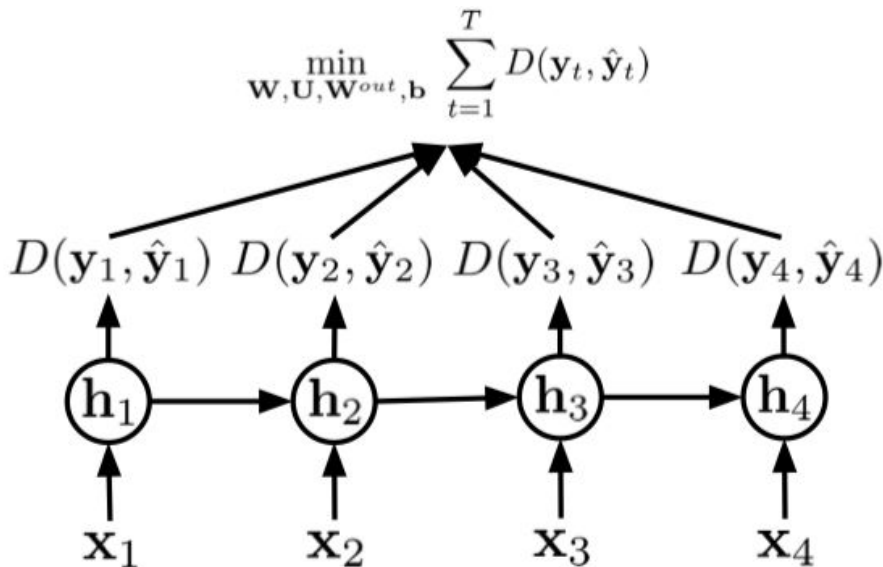
Quiz: size of weights? ( $\mathbf{x}_t \in \mathbb{R}^d$ ,  $\mathbf{h}_t \in \mathbb{R}^m$ ,  $\mathbf{y}_t \in \mathbb{R}^n$ )

$$\mathbf{W} = \mathbb{R}^{m \times d} \quad \mathbf{U} = \mathbb{R}^{m \times m} \quad \mathbf{W}^{out} = \mathbb{R}^{n \times m}$$

# Recurrent Neural Network (RNN)

## Loss computation

- Compute loss (if any) for each step and aggregate
- Gradient flows through all unrolled steps in RNN



# Today's agenda

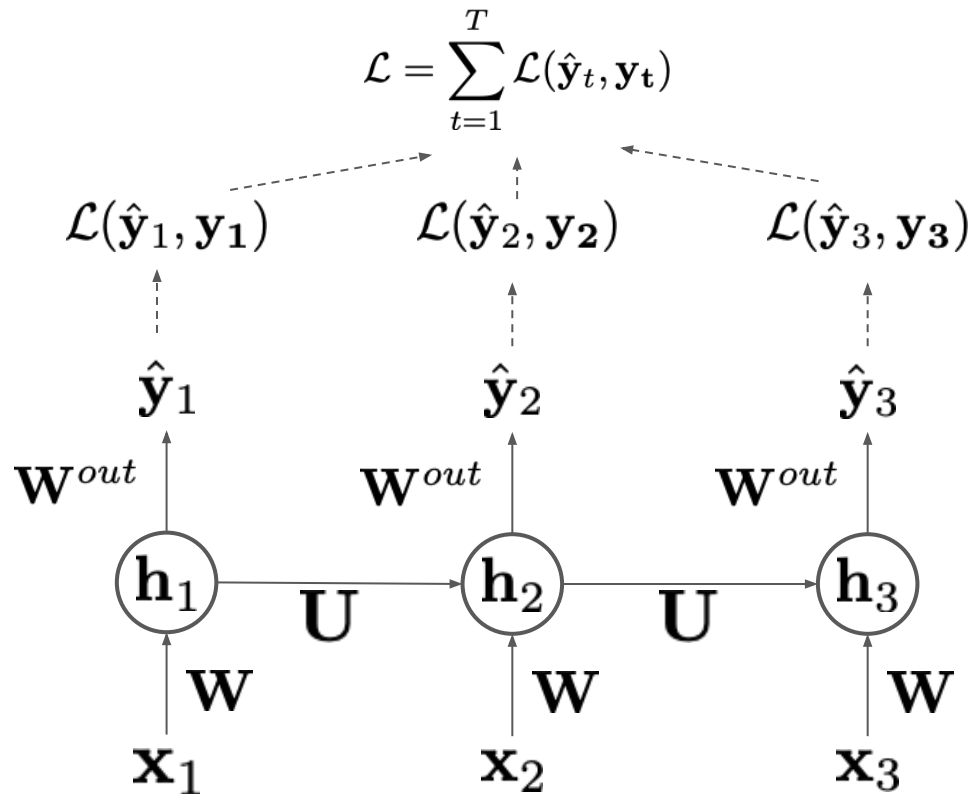
- RNN basics
- **Backpropagation through time**
- The vanishing/exploding gradients problem
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# Backpropagation in RNN

Forward propagation

$$\mathbf{h}_t = \sigma(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t + \mathbf{b})$$

$$\hat{\mathbf{y}}_t = \mathbf{W}^{out}\mathbf{h}_t$$



# Backpropagation in RNN

Forward propagation

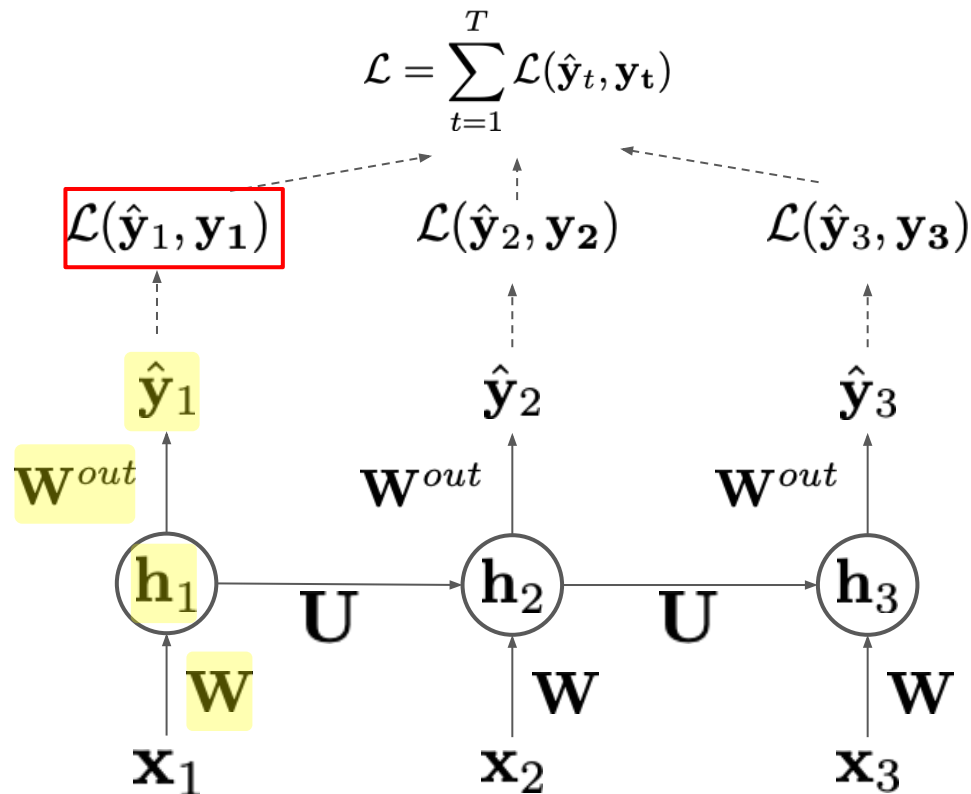
$$\mathbf{h}_t = \sigma(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t + \mathbf{b})$$

$$\hat{\mathbf{y}}_t = \mathbf{W}^{out}\mathbf{h}_t$$

Before we derive gradients, let's think about how the gradient would flow

If we want to reduce  $\mathcal{L}(\hat{\mathbf{y}}_1, \mathbf{y}_1)$ , which variables should we update?

$\hat{\mathbf{y}}_1, \mathbf{W}^{out}, \mathbf{h}_1, \mathbf{W}$

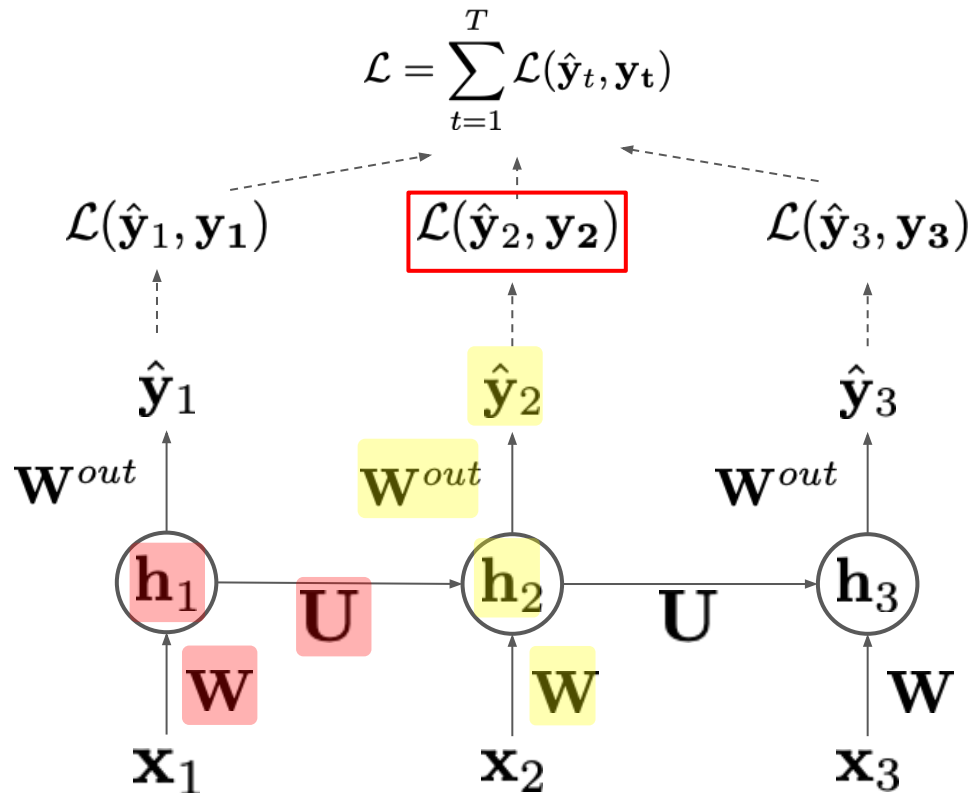


# Backpropagation in RNN

Forward propagation

$$\mathbf{h}_t = \sigma(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t + \mathbf{b})$$

$$\hat{\mathbf{y}}_t = \mathbf{W}^{out}\mathbf{h}_t$$



Before we derive gradients, let's think about how the gradient would flow

If we want to reduce  $\mathcal{L}(\hat{\mathbf{y}}_1, \mathbf{y}_1)$ , which variables should we update?

$$\hat{\mathbf{y}}_1, \mathbf{W}^{out}, \mathbf{h}_1, \mathbf{W}$$

If we want to reduce  $\mathcal{L}(\hat{\mathbf{y}}_2, \mathbf{y}_2)$ , which variables should we update?

$$\hat{\mathbf{y}}_2, \mathbf{W}^{out}, \mathbf{h}_2, \mathbf{W}, \underbrace{\mathbf{U}, \mathbf{h}_1, \mathbf{W}}$$

These contribute to the value of  $\mathbf{h}_2$

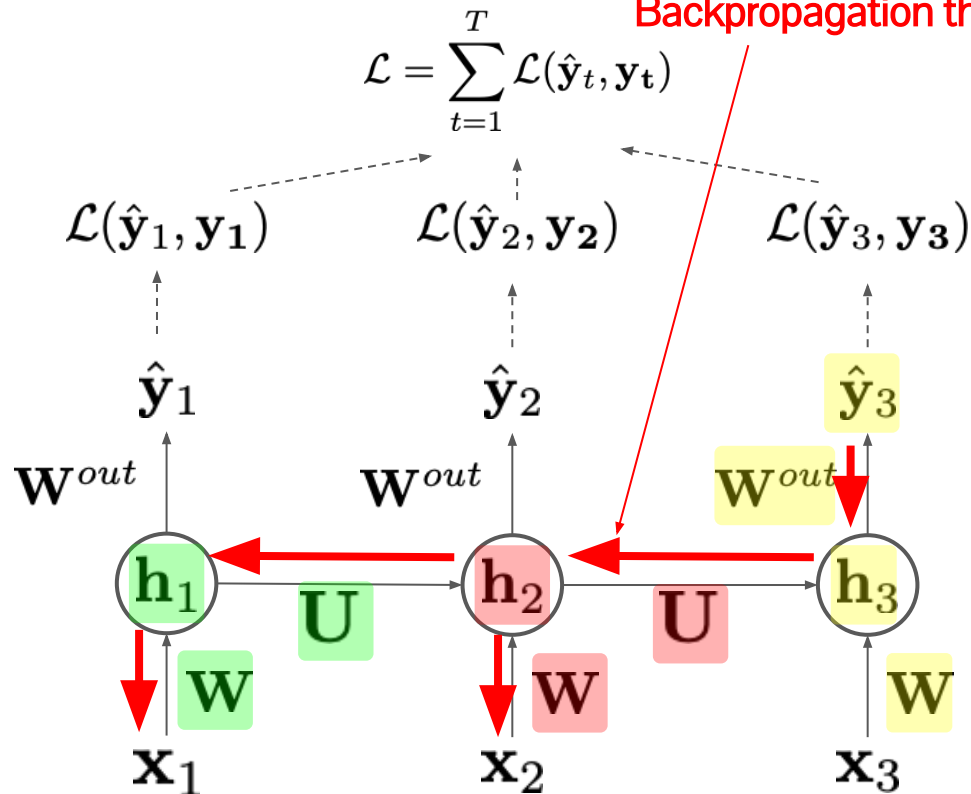
# Backpropagation in RNN

Forward propagation

$$\mathbf{h}_t = \sigma(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t + \mathbf{b})$$

$$\hat{\mathbf{y}}_t = \mathbf{W}^{out}\mathbf{h}_t$$

Backpropagation through time!



Before we derive gradients, let's think about how the gradient would flow

If we want to reduce  $\mathcal{L}(\hat{\mathbf{y}}_1, \mathbf{y}_1)$ , which variables should we update?

$$\hat{\mathbf{y}}_1, \mathbf{W}^{out}, \mathbf{h}_1, \mathbf{W}$$

If we want to reduce  $\mathcal{L}(\hat{\mathbf{y}}_2, \mathbf{y}_2)$ , which variables should we update?

$$\hat{\mathbf{y}}_2, \mathbf{W}^{out}, \mathbf{h}_2, \mathbf{W}, \mathbf{U}, \mathbf{h}_1, \mathbf{W}$$

If we want to reduce  $\mathcal{L}(\hat{\mathbf{y}}_3, \mathbf{y}_3)$ , which variables should we update?

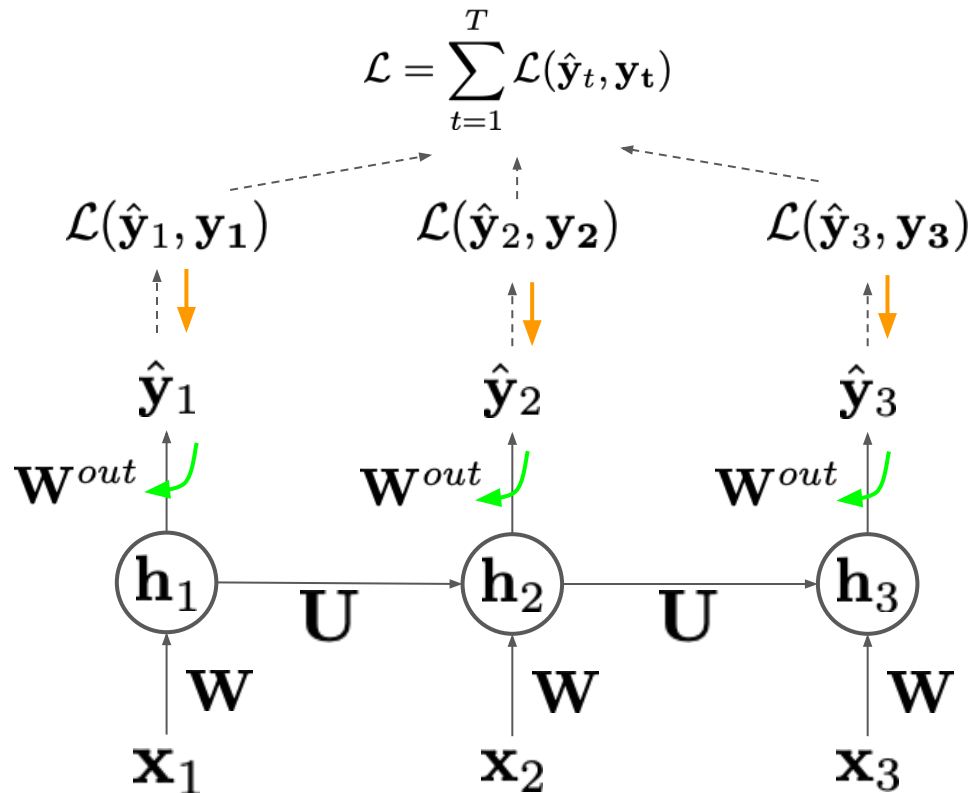
$$\hat{\mathbf{y}}_3, \mathbf{W}^{out}, \mathbf{h}_3, \mathbf{W}, \mathbf{U}, \mathbf{h}_2, \mathbf{W}, \mathbf{U}, \mathbf{h}_1, \mathbf{W}$$

# Backpropagation through time

Forward propagation

$$\mathbf{h}_t = \sigma(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t + \mathbf{b})$$

$$\hat{\mathbf{y}}_t = \mathbf{W}^{out}\mathbf{h}_t$$



$$\frac{\partial \mathcal{L}}{\partial W_{n,m}^{out}} = \sum_t \frac{\partial \mathcal{L}(\hat{y}_{t,n}, y_{t,n})}{\partial \hat{y}_{t,n}} \frac{\partial \hat{y}_{t,n}}{\partial W_{n,m}^{out}}$$

A (n,m)th element of  $\mathbf{W}^{out}$

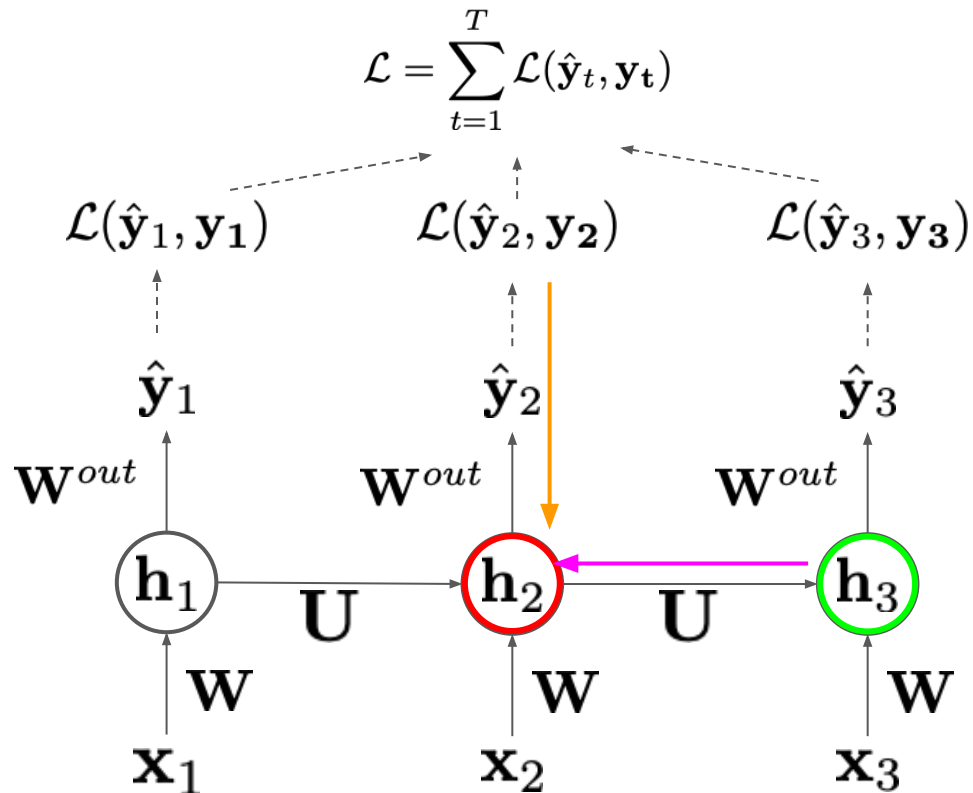


# Backpropagation through time

Forward propagation

$$\mathbf{h}_t = \sigma(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t + \mathbf{b})$$

$$\hat{\mathbf{y}}_t = \mathbf{W}^{out}\mathbf{h}_t$$



Let  $\mathcal{L}_t = \sum_{t'=t}^T \mathcal{L}(\hat{\mathbf{y}}_{t'}, \mathbf{y}_{t'})$  (note:  $\mathcal{L}_1 = \mathcal{L}$ )

$$\begin{aligned} \frac{\partial \mathcal{L}_t}{\partial h_{t,m}} &= \frac{\partial \mathcal{L}(\hat{\mathbf{y}}_t, \mathbf{y}_t)}{\partial h_{t,m}} + \frac{\partial \mathcal{L}_{t+1}}{\partial h_{t,m}} \\ &= \frac{\partial \mathcal{L}(\hat{\mathbf{y}}_t, \mathbf{y}_t)}{\partial h_{t,m}} + \sum_{m'} \frac{\partial h_{t+1,m'}}{\partial h_{t,m}} \frac{\partial \mathcal{L}_{t+1}}{\partial h_{t+1,m'}} \end{aligned}$$

It's applied recursively over temporal horizon

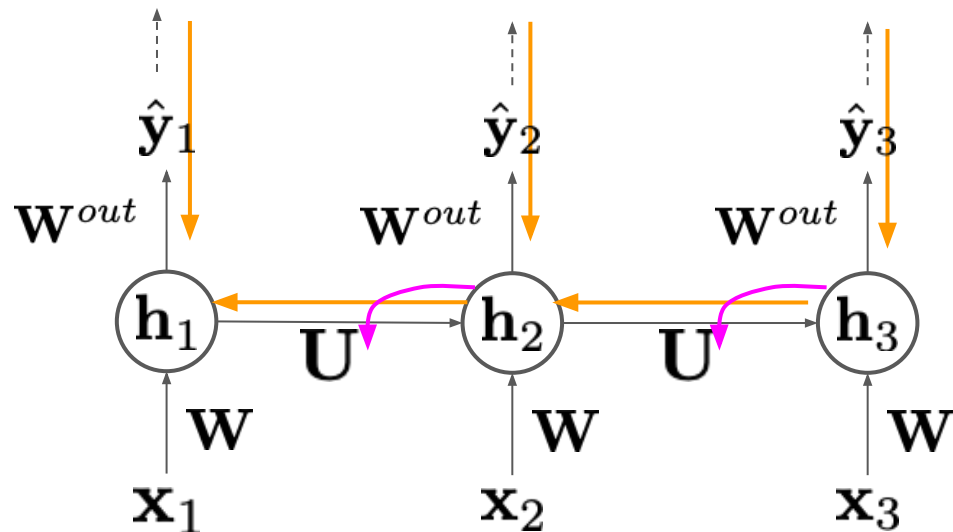
# Backpropagation through time

$$\mathcal{L} = \sum_{t=1}^T \mathcal{L}(\hat{\mathbf{y}}_t, \mathbf{y}_t)$$

$$\mathcal{L}(\hat{\mathbf{y}}_1, \mathbf{y}_1)$$

$$\mathcal{L}(\hat{\mathbf{y}}_2, \mathbf{y}_2)$$

$$\mathcal{L}(\hat{\mathbf{y}}_3, \mathbf{y}_3)$$



Forward propagation

$$\mathbf{h}_t = \sigma(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t + \mathbf{b})$$

$$\hat{\mathbf{y}}_t = \mathbf{W}^{out}\mathbf{h}_t$$

$$\mathcal{L}_t = \sum_{t'=t}^T \mathcal{L}(\hat{\mathbf{y}}_{t'}, \mathbf{y}_{t'})$$

$$\frac{\partial \mathcal{L}}{\partial U_{m,m'}} = \sum_t \frac{\partial \mathcal{L}_t}{\partial h_{t,m}} \frac{\partial h_{t,m}}{\partial U_{m,m'}}$$

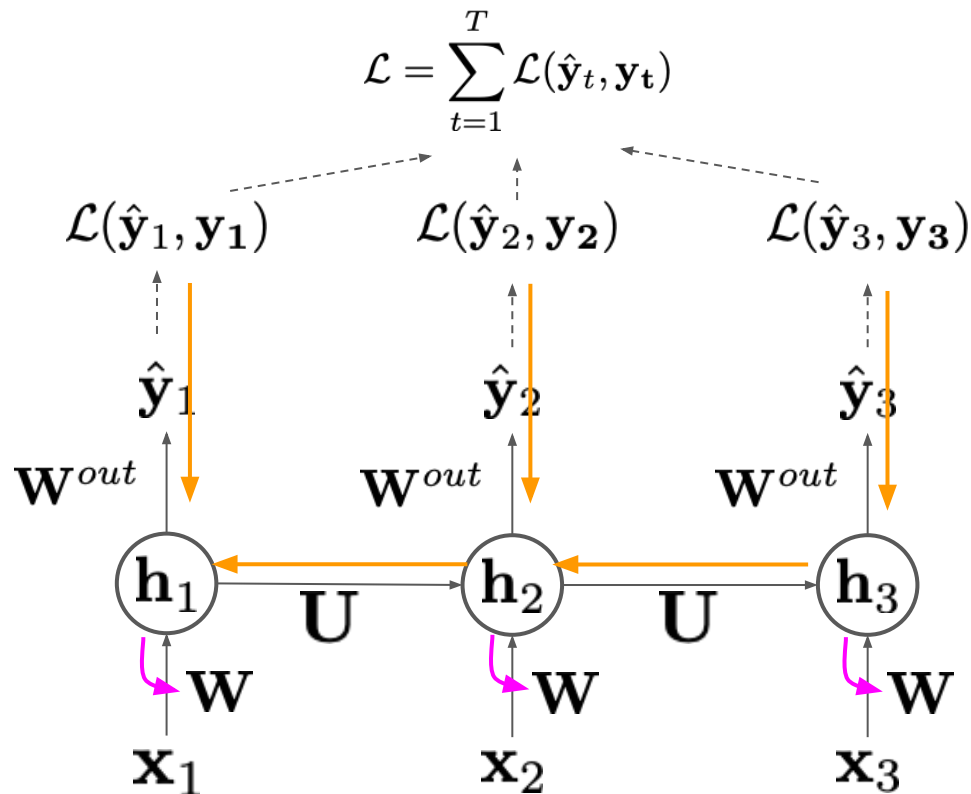
# Backpropagation through time

Forward propagation

$$\mathbf{h}_t = \sigma(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t + \mathbf{b})$$

$$\hat{\mathbf{y}}_t = \mathbf{W}^{out}\mathbf{h}_t$$

$$\mathcal{L}_t = \sum_{t'=t}^T \mathcal{L}(\hat{\mathbf{y}}_{t'}, \mathbf{y}_{t'})$$



$$\frac{\partial \mathcal{L}}{\partial W_{m,d}} = \sum_t \frac{\partial \mathcal{L}_t}{\partial h_{t,m}} \frac{\partial h_{t,m}}{\partial W_{m,d}}$$

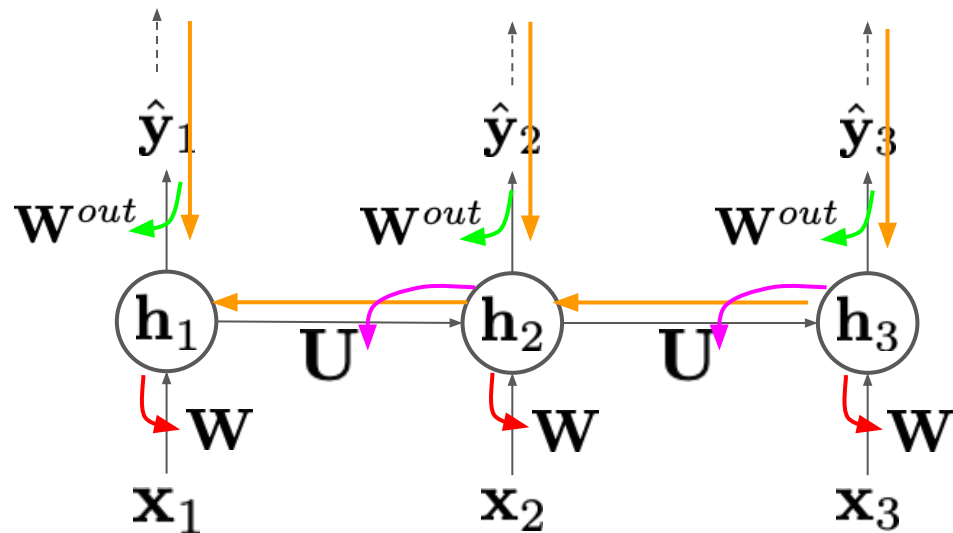
# Backpropagation through time

$$\mathcal{L} = \sum_{t=1}^T \mathcal{L}(\hat{\mathbf{y}}_t, \mathbf{y}_t)$$

$$\mathcal{L}(\hat{\mathbf{y}}_1, \mathbf{y}_1)$$

$$\mathcal{L}(\hat{\mathbf{y}}_2, \mathbf{y}_2)$$

$$\mathcal{L}(\hat{\mathbf{y}}_3, \mathbf{y}_3)$$



Forward propagation

$$\mathbf{h}_t = \sigma(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t + \mathbf{b})$$

$$\hat{\mathbf{y}}_t = \mathbf{W}^{out}\mathbf{h}_t$$

$$\mathcal{L}_t = \sum_{t'=t}^T \mathcal{L}(\hat{\mathbf{y}}_{t'}, \mathbf{y}_{t'})$$

Putting all together:

$$\frac{\partial \mathcal{L}}{\partial W_{n,m}^{out}} = \sum_t \frac{\partial \mathcal{L}(\hat{\mathbf{y}}_{t,n}, \mathbf{y}_{t,n})}{\partial \hat{\mathbf{y}}_{t,n}} \frac{\partial \hat{\mathbf{y}}_{t,n}}{\partial W_{n,m}^{out}}$$

$$\frac{\partial \mathcal{L}}{\partial h_{t,m}} = \frac{\partial \mathcal{L}(\hat{\mathbf{y}}_t, \mathbf{y}_t)}{\partial h_{t,m}} + \sum_{m'} \frac{\partial h_{t+1,m'}}{\partial h_{t,m}} \frac{\partial \mathcal{L}_{t+1}}{\partial h_{t+1,m'}}$$

$$\frac{\partial \mathcal{L}}{\partial U_{m,m'}} = \sum_t \frac{\partial \mathcal{L}_t}{\partial h_{t,m}} \frac{\partial h_{t,m}}{\partial U_{m,m'}}$$

$$\frac{\partial \mathcal{L}}{\partial W_{m,d}} = \sum_t \frac{\partial \mathcal{L}_t}{\partial h_{t,m}} \frac{\partial h_{t,m}}{\partial W_{m,d}}$$

# Today's agenda

- RNN basics
- Backpropagation through time
- **The vanishing/exploding gradients problem**
- Advanced RNNs

# What can go wrong in BPTT?

- The hidden-to-hidden connections in standard RNN can cause gradient vanishing during backprop of the loss in the last step.

$$\frac{\partial \mathcal{L}_T}{\partial \mathbf{h}_T} = \frac{\partial \mathcal{L}_T}{\partial \mathbf{h}_T} \frac{\partial \mathbf{h}_T}{\partial \mathbf{h}_t}$$

Note:

$$\mathbf{h}_t = \sigma(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t + \mathbf{b})$$

$$= \frac{\partial \mathcal{L}_T}{\partial \mathbf{h}_T} \prod_{i=t+1}^T \frac{\partial \mathbf{h}_i}{\partial \mathbf{h}_{i-1}} = \frac{\partial \mathcal{L}_T}{\partial \mathbf{h}_T} \prod_{I=t+1}^T \text{diag}(\mathbf{h}_i(\mathbf{1} - \mathbf{h}_i)) \mathbf{U}$$

# What can go wrong in BPTT?

- The hidden-to-hidden connections in standard RNN can cause gradient vanishing during backprop of the loss in the last step.
- This is caused by gradients with respect to the activation function being multiplied through time!

$$\frac{\partial \mathcal{L}_T}{\partial \mathbf{h}_T} = \frac{\partial \mathcal{L}_T}{\partial \mathbf{h}_T} \frac{\partial \mathbf{h}_T}{\partial \mathbf{h}_t}$$

Note:

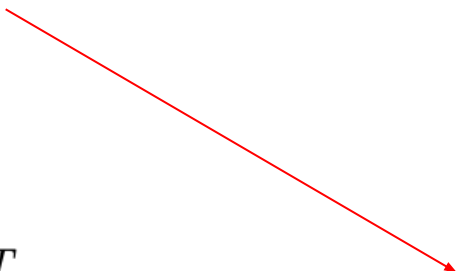
$$\mathbf{h}_t = \sigma(\mathbf{U}\mathbf{h}_{t-1} + \mathbf{W}\mathbf{x}_t + \mathbf{b})$$

$$= \frac{\partial \mathcal{L}_T}{\partial \mathbf{h}_T} \prod_{i=t+1}^T \frac{\partial \mathbf{h}_i}{\partial \mathbf{h}_{i-1}} = \frac{\partial \mathcal{L}_T}{\partial \mathbf{h}_T} \prod_{I=t+1}^T \text{diag}(\mathbf{h}_i(1 - \mathbf{h}_i)) \mathbf{U}$$

Gradient of sigmoid causes vanishing!  
It is always  $< 1$ , which can be multiplied  
over time to very small value!

# What can go wrong in BPTT?

- The hidden-to-hidden connections in standard RNN can cause gradient vanishing during backprop of the loss in the last step.
- This is caused by gradients with respect to the activation function being multiplied through time!
- Gradient can also explode if norm of  $U$  is large

$$\begin{aligned}\frac{\partial \mathcal{L}_T}{\partial \mathbf{h}_T} &= \frac{\partial \mathcal{L}_T}{\partial \mathbf{h}_T} \frac{\partial \mathbf{h}_T}{\partial \mathbf{h}_t} \\ &= \frac{\partial \mathcal{L}_T}{\partial \mathbf{h}_T} \prod_{i=t+1}^T \frac{\partial \mathbf{h}_i}{\partial \mathbf{h}_{i-1}} = \frac{\partial \mathcal{L}_T}{\partial \mathbf{h}_T} \prod_{I=t+1}^T \text{diag}(\mathbf{h}_i(\mathbf{1} - \mathbf{h}_i)) \mathbf{U}\end{aligned}$$




# Techniques to prevent vanishing/exploding gradients

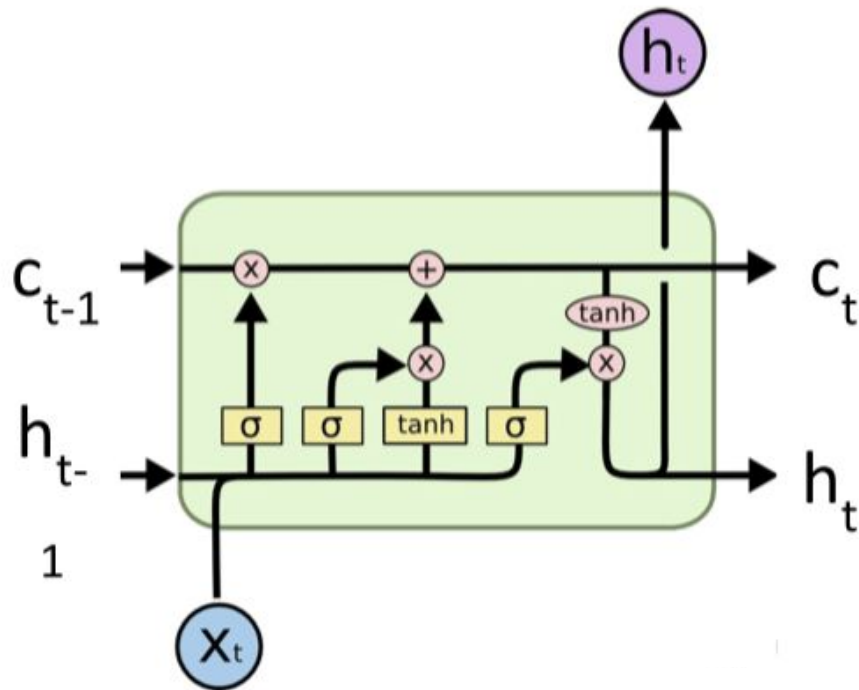
- Initializing weights  $\mathbf{U}$  to be orthogonal
- Clip gradients if the value is too large

# Today's agenda

- RNN basics
- Backpropagation through time
- The vanishing/exploding gradients problem
- **Advanced RNNs**

# RNNs addressing vanishing/exploding gradients

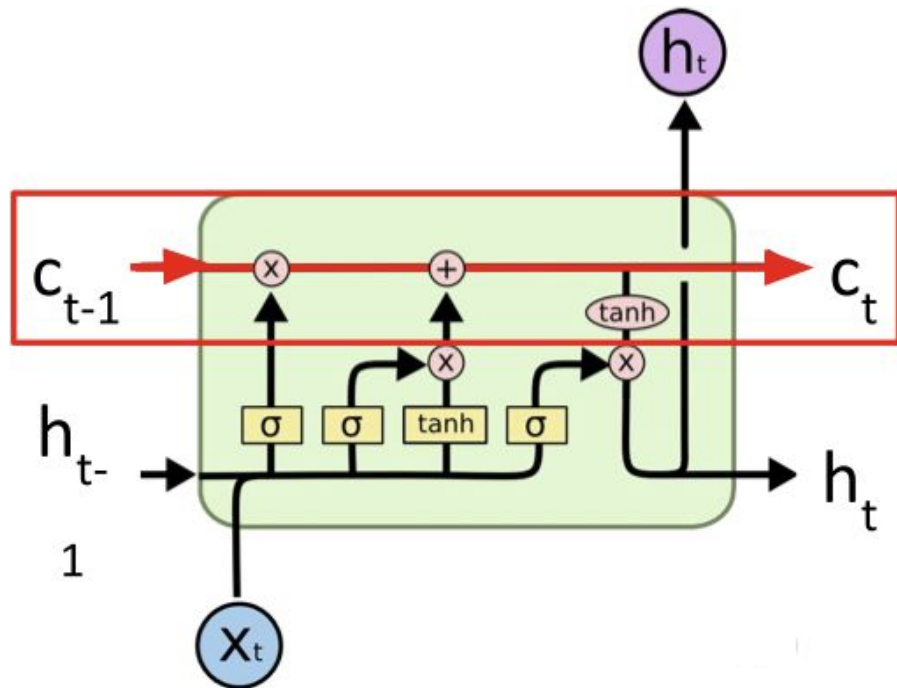
## Long Short-Term Memory (LSTM)



- A recurrent neural network variety designed to retain long-term dependencies.
- Helps dealing with both the vanishing and exploding gradient problem

# RNNs addressing vanishing/exploding gradients

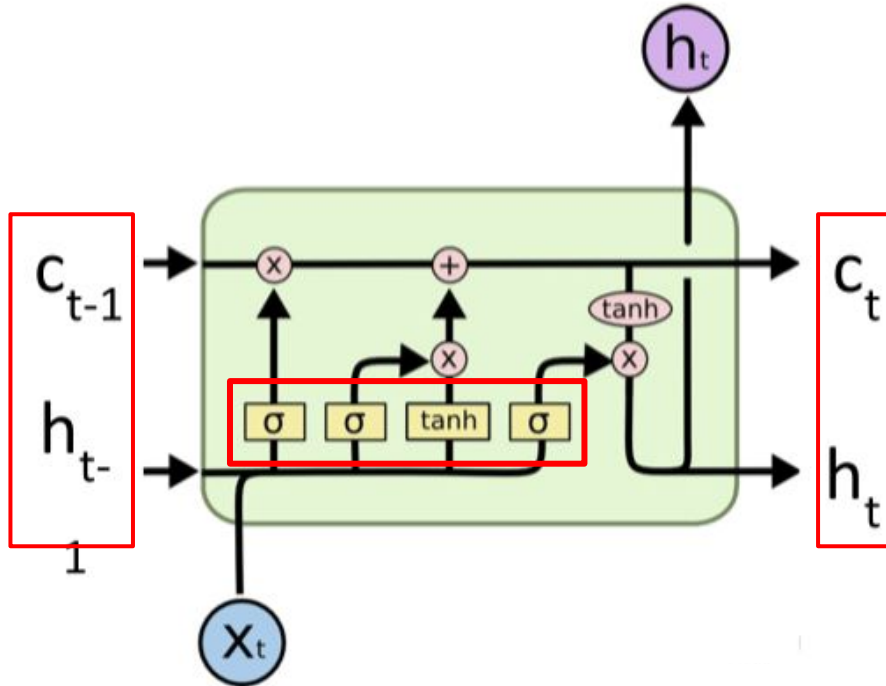
## Long Short-Term Memory (LSTM)



- A recurrent neural network variety designed to retain long-term dependencies.
- Helps dealing with both the vanishing and exploding gradient problem
- The key idea is an additive connection of previous memories passed through time

# Long Short-Term Memory (LSTM)

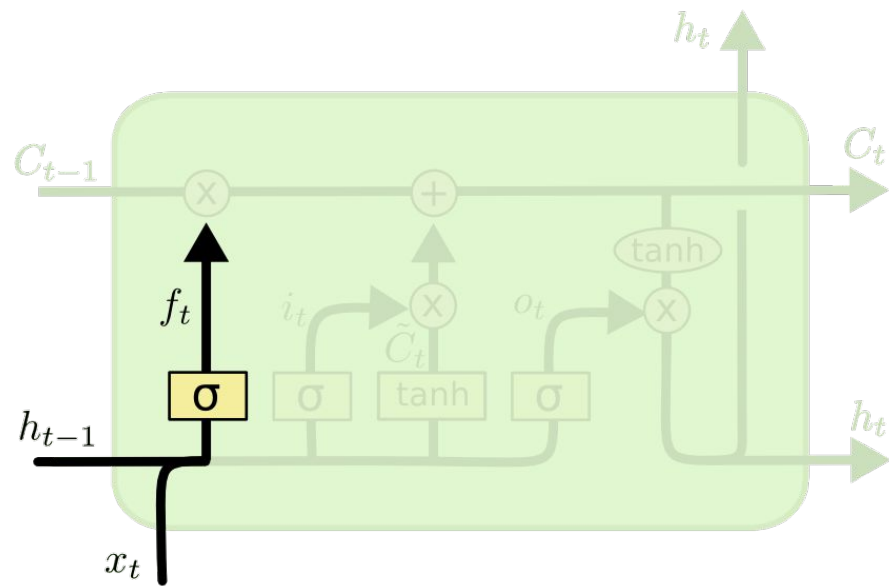
- Overview



- Information is passed through two variables
- There are **four switch variables** that determines how the information flows through time

# Long Short-Term Memory (LSTM)

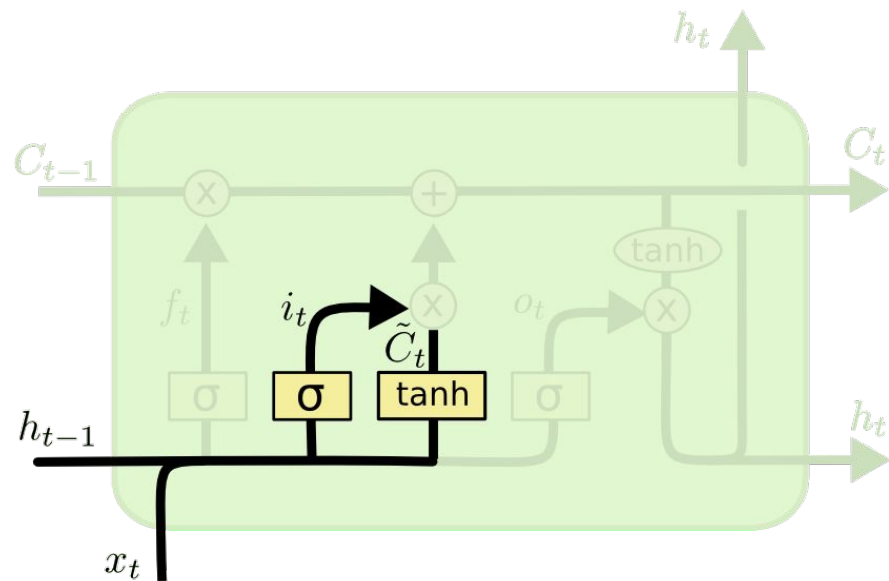
- The forget gate allows LSTM to choose to zero out part of previous memories and let others through.



$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$

# Long Short-Term Memory (LSTM)

- The input gate behaves similar to forget gates with new inputs

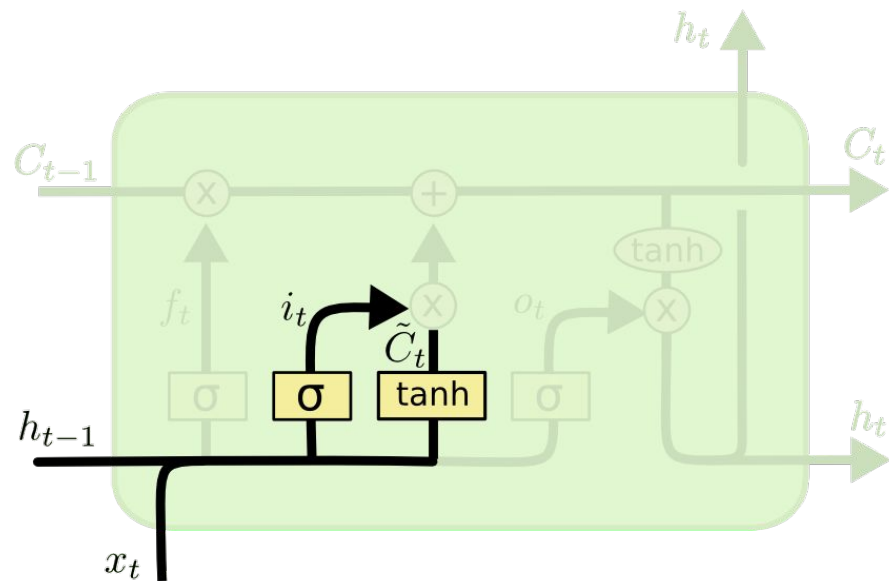


$$i_t = \sigma (W_i \cdot [h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

# Long Short-Term Memory (LSTM)

- The input gate behaves similar to the forget gate with new inputs.
- New information is computed from the current input and previous hidden units.



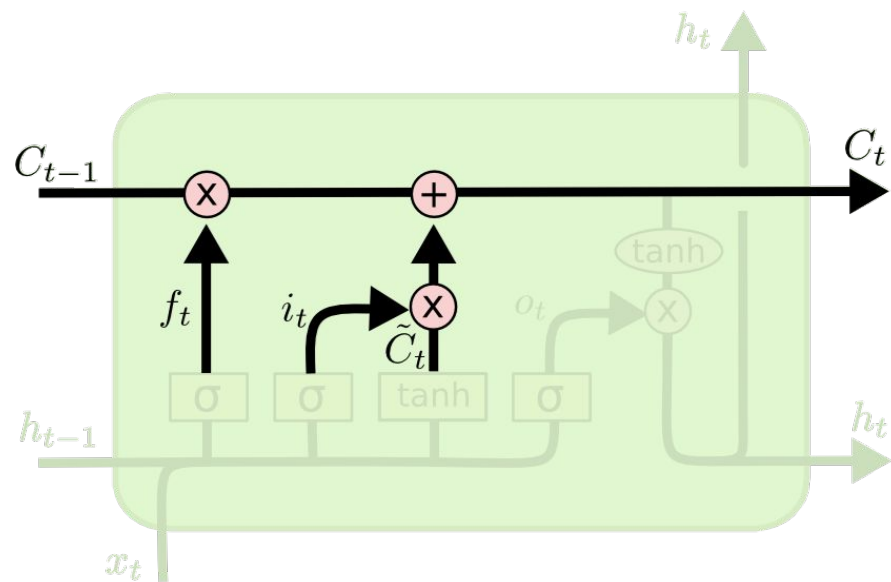
$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$



# Long Short-Term Memory (LSTM)

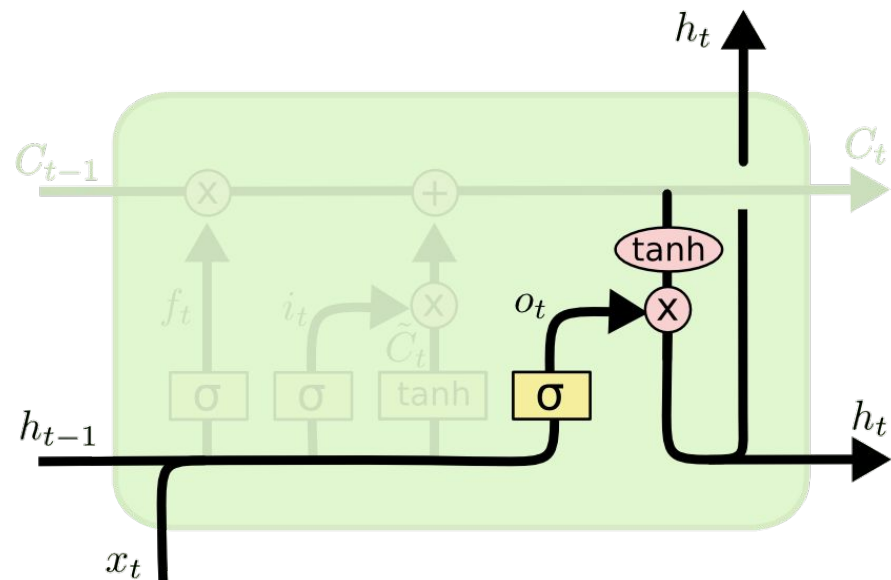
- Memories to be passed are computed using the forget gate on the previous memories and the input gate on the current information found in the sequence



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

# Long Short-Term Memory (LSTM)

- An output gate is computed to choose information from the current memories for the next hidden state in the LSTM.

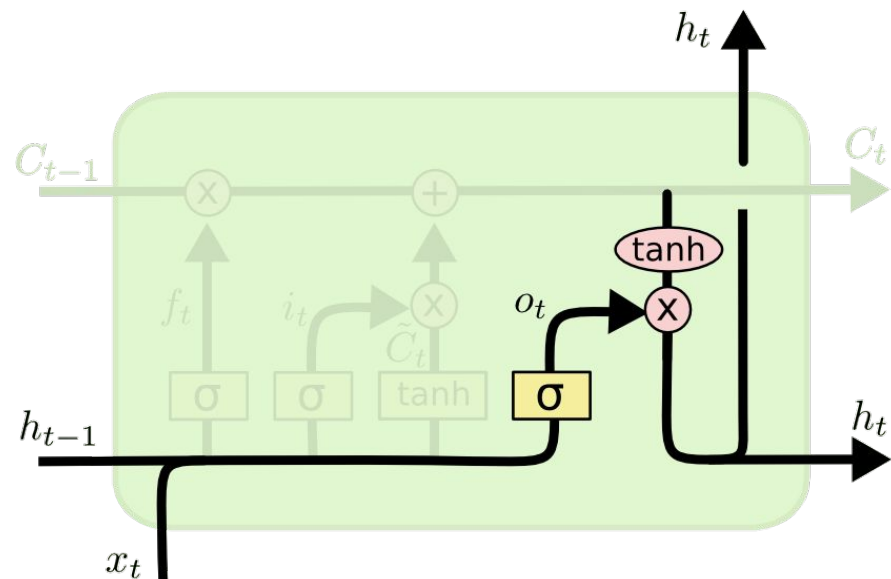


$$o_t = \sigma (W_o [h_{t-1}, x_t] + b_o)$$

$$h_t = o_t * \tanh (C_t)$$

# Long Short-Term Memory (LSTM)

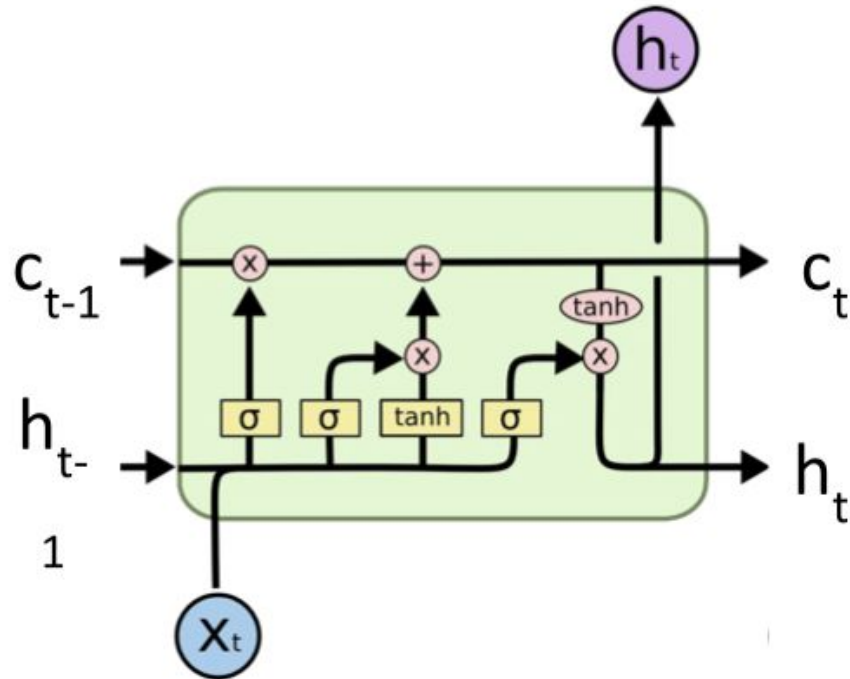
- An output gate is computed to choose information from the current memories for the next hidden state in the LSTM.
- Next hidden state is computed from the current memories and gate.



$$o_t = \sigma (W_o [h_{t-1}, x_t] + b_o)$$

$$h_t = o_t * \tanh (C_t)$$

# Long Short-Term Memory (LSTM)



$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$

$$i_t = \sigma(W_i \cdot [h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] + b_C)$$

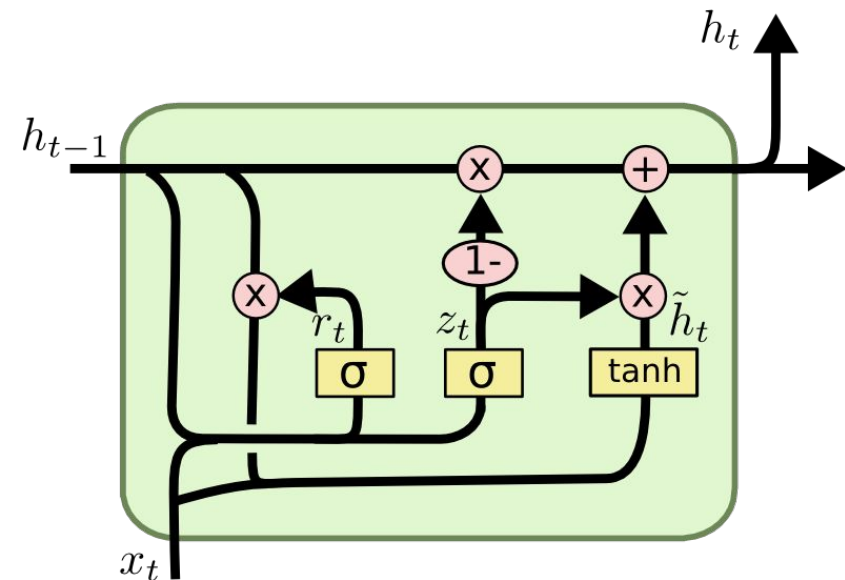
$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

$$o_t = \sigma(W_o \cdot [h_{t-1}, x_t] + b_o)$$

$$h_t = o_t * \tanh(C_t)$$

# Gated Recurrent Unit (GRU)

- A simplified variation of LSTM



$$z_t = \sigma(W_z \cdot [h_{t-1}, x_t])$$

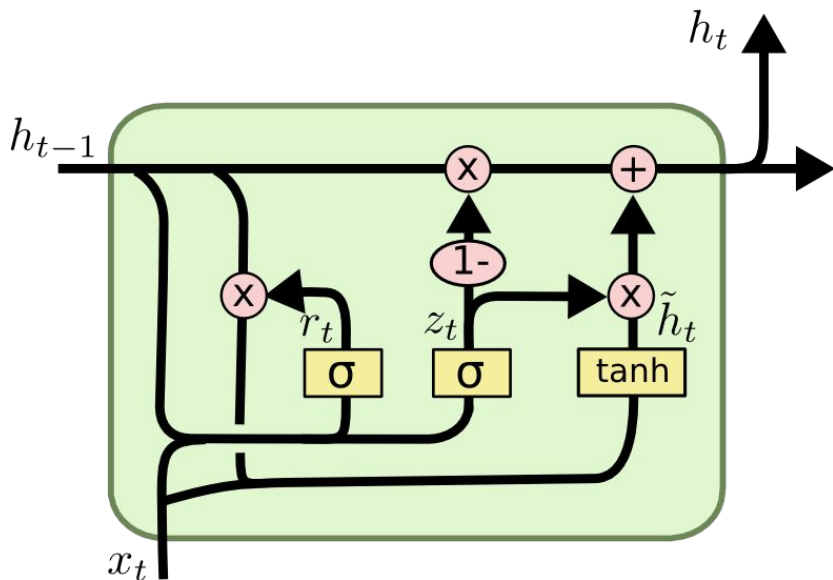
$$r_t = \sigma(W_r \cdot [h_{t-1}, x_t])$$

$$\tilde{h}_t = \tanh(W \cdot [r_t * h_{t-1}, x_t])$$

$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

# Gated Recurrent Unit (GRU)

- A simplified variation of LSTM
- The forget, input and output gates are simplified into a single gate



$$z_t = \sigma(W_z \cdot [h_{t-1}, x_t])$$

$$r_t = \sigma(W_r \cdot [h_{t-1}, x_t])$$

$$\tilde{h}_t = \tanh(W \cdot [r_t * h_{t-1}, x_t])$$

$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

# Next

- RNNs for sequence modeling
  - Language model