Quantum Computing Assignment

Submitted by Jishnu M
Under the guidance of Dr.Shailesh S

Department of Computer Science
CUSAT

January 2023

COCHIN UNIVERSITY OF SCIENCE AND TECHNOLOGY

Quantum Teleportation

- Suppose that Alice wants to send quantum information to Bob. Specifically, suppose she wants to send the qubit state $|\Psi> = \alpha|0> + \beta|1>$. This entails passing on information about α and β to Bob.
- There exists a theorem in quantum mechanics that states that you cannot simply make an exact copy of an unknown quantum state. This is known as the no-cloning theorem. As a result of this, we can see that Alice can't simply generate a copy of $|\Psi>$ and give the copy to Bob.
- However, by taking advantage of two classical bits and an entangled qubit pair, Alice can transfer her state $|\Psi>$ to Bob. We call this teleportation because, in the end, Bob will have $|\Psi>$ and Alice won't anymore.

The Quantum Teleportation Protocol

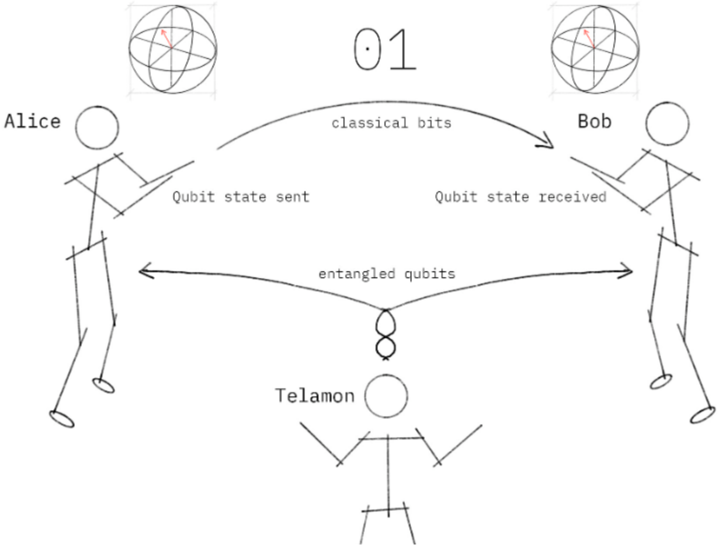


Figure:

Quantum Teleportation Protocol

- To transfer a quantum bit, Alice and Bob must use a third party (Telamon) to send them an entangled qubit pair.
- Alice then performs some operations on her qubit, sends the results to Bob over a classical communication channel, and Bob then performs some operations on his end to receive Alice's qubit.

Step 1

1. A third party, Telamon, creates an entangled pair of qubits and gives one to Bob and one to Alice. The pair Telamon creates is a special pair called a Bell pair.
2. In quantum circuit language, the way to create a Bell pair between two qubits is to first transfer one of them to the X-basis ($|+\rangle$ and $|\rangle$) using a Hadamard gate, and then to apply a CNOT gate onto the other qubit controlled by the one in the X-basis.

Step 2

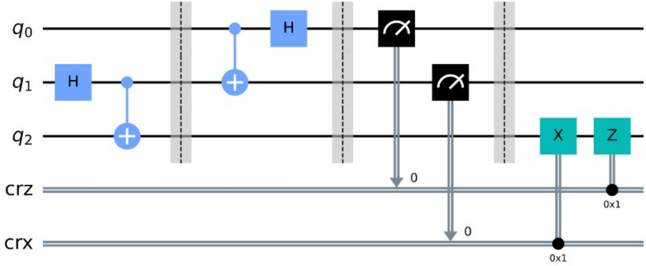
1. Alice applies a CNOT gate to q_1, controlled by $|\psi>$(the qubit she is trying to send Bob).
2. Then Alice applies a Hadamard gate to $|\psi>$. In our quantum circuit, the qubit ($|\psi>$) Alice is trying to send is q_0.

Step 3

1. Next, Alice applies a measurement to both qubits that she owns, q_1 and $|\psi>$, and stores this result in two classical bits.
2. Alice then sends these two bits to Bob.

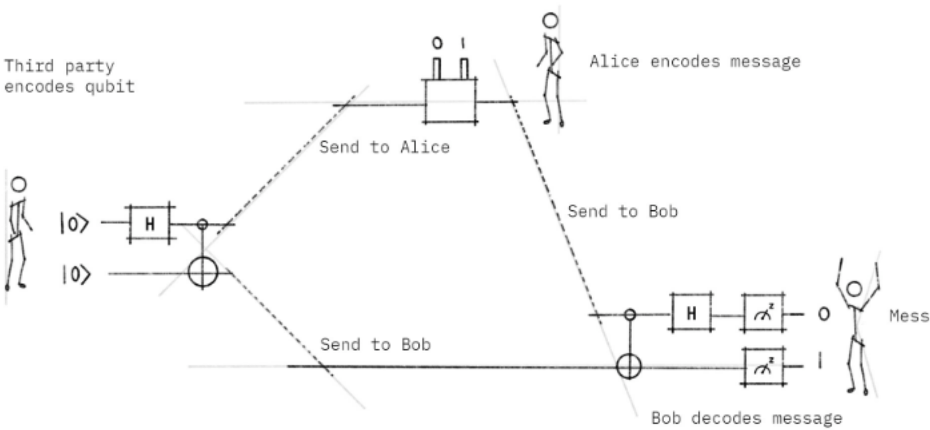
Step 4

- Bob, who already has the qubit q_2, then applies the following gates depending on the state of the classical bits:
  00 $\rightarrow$ Do nothing
  01 $\rightarrow$ Apply X gate
  10 $\rightarrow$ Apply Z gate
  11 $\rightarrow$ Apply ZX gate



- At the end of this protocol, Alice's qubit has now teleported to Bob.

Super Dense Coding



Super Dense Coding

- Quantum teleportation and superdense coding are closely related, to avoid confusion we need to clarify the difference.
- Superdense coding is a procedure that allows someone to send two classical bits to another party using just a single qubit of communication.
- The teleportation protocol can be thought of as a flipped version of the superdense coding protocol, in the sense that Alice and Bob merely "swap their equipment."

Step 1

- The process starts with a third party, who we'll call Charlie.
- Two qubits are prepared by Charlie in an entangled state. He initially starts the 2 qubits in the basis state $|0\rangle$.
- He applies Hadamard gate (H) to the first qubit to create superposition. He then applies CNOT gate (CX) using the first qubit as a control and the second as the target.
- This is the entangled state (Bell pair).

Step 2

- Charlie sends the first qubit to Alice and the second qubit to Bob.
- The goal of the protocol is for Alice to send 2 classical bits of information to Bob using her qubit.
- But before she does, she needs to apply a set of quantum gates to her qubit depending on the 2 bits of information she wants to send:

Encoding Rules for Superdense Coding (Alice protocol):

Intended Message	Applied Gate	Resulting State $(\cdot \frac{1}{\sqrt{2}})$
00	I	$\|00\rangle + \|11\rangle$
01	X	$\|10\rangle + \|01\rangle$
10	Z	$\|00\rangle - \|11\rangle$
11	ZX	$-\|10\rangle + \|01\rangle$

- Thus if she wants to send a 00, she does nothing to her qubit (apply the identity (I) gate).
- If she wants to send a 01, then she applies the X gate.
- Depending on what she wants to send, she applies the appropriate gate, then sends her qubit to Bob for the final step in the process.

Reference

1. https://qiskit.org/textbook/ch-algorithms/teleportation.html
2. https://qiskit.org/textbook/ch-algorithms/superdense-coding.html

Thank you!