Numerical Methods of Thermo-Fluid Dynamics I

Winter Semester 2017-2018

DELIVERABLE TASK II:
Numerical Solution of Boundary Layer Equation

Given: Monday, 27/11/2017
Deadline: 8/1/2018

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Deliverable Task II

The goal of the second deliverable task is to solve the boundary layer equation over a flat plate:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2} \,. \tag{2}$$

The Reynolds number is defined as $\mathrm{Re} = u_\infty L/\nu$, where ν is the kinematic viscosity, L is the plate length and u_∞ is the free-stream velocity of the outer flow. Here the velocity components u and v are dimensionless, $u = \bar{u}/u_\infty$ and $v = \bar{v}/u_\infty$, where $\bar{u}$ and $\bar{v}$ are the dimensional velocities. Lengths have been made dimensionless by using the plate length, e.g. $x = \bar{x}/L$. The boundary conditions are as follows:

$y = 0:$	$u = v = 0$	no-slip condition	(3)
$y \to \infty:$	$u \to 1\ (\bar{u} \to u_\infty)$	free outer flow.	(4)

- (5p) Discretise equation (1). One possible scheme for $\frac{\partial u}{\partial x}$ is as follows:

$$\left(\frac{\partial u}{\partial x}\right)_{i,j} = \frac{1}{2}\left(\frac{u_{i,j} - u_{i-1,j}}{\Delta x} + \frac{u_{i,j-1} - u_{i-1,j-1}}{\Delta x}\right) \tag{5}$$

  Discretise $\frac{\partial v}{\partial y}$, write the iterative formula for v based on your schemes, show the order of accuracy. Note that you could choose other schemes to approximate $\frac{\partial u}{\partial x}$.

- (10p) Discretise equation (2), write the iterative formula for u, give comments on your scheme (explicit/implicit, order of accuracy, stability constraint, etc.)

- (25p) Write a MATLAB/OCTAVE program that solves the boundary layer equation ($0 \le x \le 1$ and $0 \le y \le 2\delta$, where $\delta = \frac{5}{\sqrt{\mathrm{Re}}}$ is the boundary layer thickness at $x = 1$).

- (30p) Calculate the solution for $\mathrm{Re} = 10000$, generate 2 figures for the numerical results of u and v of the whole domain, explain your results.

- (30p) Consider $u_\infty = 10[m/s]$ and $L = 1[m]$. For $\mathrm{Re} = 10000$, compare your u and v velocity profiles obtained above at $x = 0.0005$ and $x = 0.5$ with the results you obtained from the Blasius equation in Exercise Sheet. Is there a good agreement between the two? Why? Explain the results and state your conclusions.

The Deliverable Task II contains one report and one matlab code file, which must be submitted to `nan.chen@fau.de` before **January 8th, 2018, 10:00am.**