Winter Semester 2017-2018

DELIVERABLE TASK III:

Lid driven cavity flow

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Contents

1. Choosing the grid:

We used the staggered grid to discretize the domain in order to get a better pressure velocity coupling and reduce the oscillation in pressure. This will improve convergence and conserve kinetic energy.

The time step has to be chosen according to the step size of the grid. The stability condition for the explicit scheme is given by

$$\Delta t < Re * \min(\Delta x^2, \Delta y^2)/2$$

After testing different time steps and grid sizes. The following values were chosen for $Re = 0.1$:

$$\Delta t = 0.00001, \qquad \Delta x = \Delta y = h = 0.05$$

2. Matlab Code:

```
clc
clear
close all

SS=get(0,'ScreenSize');SW=SS(3);SH=SS(4);

% main parameters
h=0.05;
dt=0.00001;
Re=[0.1 1];
Vel=1;
xy= 0:h:1;

% velocity matrix
u = zeros(length(xy),length(xy)+1);
v = zeros(length(xy)+1,length(xy));

uu=zeros(2,length(xy)+1);
vv=zeros(2,length(xy));

F = zeros(length(xy),length(xy)+1);
G = zeros(length(xy)+1,length(xy));

% pressure matrix
P = zeros(length(xy)+1,length(xy)+1);

% result vector
```

```
nElements=length(xy)-1;
s=nElements*nElements;

B = zeros(s,1);

% boundary conditions
F(1,:)=0;
F(length(xy),:)=0;

G(:,1)=0;
G(:,length(xy))=0;

% preparing the diagonal vectors for the coefficient matrix
nn=1:s;

d00=-4/h^2*ones(1,s);
d00(nn/nElements<1 | nn/nElements>nElements-1 | mod(nn,nElements)==0 | mod(nn,nElements)==1)=-3/h^2;
d00(1)=-2/h^2;
d00(end)=-2/h^2;
d00(nElements)=-2/h^2;
d00(s-nElements+1)=-2/h^2;

d01=1/h^2*ones(1,s-1);
d01(mod(1:s-1,nElements)==0)=0;

d20=1/h^2*ones(1,s-nElements);

% coefficient matrix
P1=sparse(diag(d00,0)+diag(d01,-1)+diag(d01,1)+diag(d20,nElements)+diag(d20,-nElements));

% time marching
for k=1:2
    for t=0:length(0:dt:0.1)

        % calculating F
        for i= 2: length(xy)-1
            for j= 2:length(xy)
                F(i,j)= u(i,j) + dt*((u(i+1,j) -2*u(i,j)+u(i-1,j)+u(i,j+1)-2*u(i,j)+u(i,j-1))*1/(Re(k)*h^2)-
1/h*((u(i,j) +u(i+1,j)).^2/4-(u(i,j)+u(i-1,j)).^2/4+ (u(i,j+1)+u(i,j))*(v(i,j)+v(i+1,j))/4- (u(i,j)+u(i,j-1))*(v(i,j-
1)+v(i+1,j-1))/4));
            end
        end
```

```
        % calulating G
        for i= 2: length(xy)
            for j= 2:length(xy)-1
                G(i,j)= v(i,j) + dt*((v(i+1,j) -2*v(i,j)+v(i-1,j)+v(i,j+1)-2*v(i,j)+v(i,j-1))*1/(Re(k)*h^2) -
1/h*((v(i,j)+v(i,j+1)).^2/4-(v(i,j)+v(i,j-1)).^2/4 + (u(i,j)+u(i,j+1))*(v(i+1,j)+v(i,j))/4 - (u(i-1,j+1)+ u(i-
1,j))*(v(i,j)+v(i-1,j))/4));
            end
        end

        index=1;
        for i=2:length(xy)
            for j=2: length(xy)
                B(index,1) = (1/(h*dt))*(F(i,j)-F(i-1,j)+G(i,j)-G(i,j-1));
                index=index+1;
            end
        end
        B(mod(1:s-1,nElements)==0 | mod(1:s-1,nElements)==1)=B(mod(1:s-1,nElements)==0 | mod(1:s-1,nElements)==1)-
1/h^2; B(1)=0;

        P_result= P1\B;

        P(2:length(xy),2:length(xy))=reshape(P_result,length(xy)-1,length(xy)-1)';

        P(2:length(xy),1)= P(2:length(xy),2);
        P(2:length(xy),length(xy)+1)= P(2:length(xy),length(xy));
        P(1,2:length(xy))= P(2,2:length(xy));
        P(length(xy)+1,2:length(xy))= P(length(xy),2:length(xy));

        for i=2:length(xy)-1
            for j=2:length(xy)
                u(i,j) = F(i,j)-dt/h*(P(i+1,j)-P(i,j));
            end
        end

        for i=2:length(xy)
            for j=2:length(xy)-1
                v(i,j)= G(i,j)-dt/h*(P(i,j+1)-P(i,j));
            end
        end
```

```
        % updating boundary conditions for the new time step
        u(:,1)= -u(:,2);
        u(2:end-1,end)= 2*Vel-u(2:end-1,end-1);

        v(1,:)= -v(2,:);
        v(end,:)= -v(end-1,:);

        % plotting resutls
        if((t==2001 || t== 4001 || t==6001 || t==8001 || t==length(0:dt:0.1)) && k==1)

            figure('Name',['Navier stokes equation for Pressure contour and Veloicity Streamline at t=',num2str((t-
1)/100000)],'NumberTitle','off','OuterPosition',[0 30 SW SH-30])
            subplot(1,2,1);
            contourf(xy(2:end),xy(2:end),P(2:end-1,2:end-1)',50); colorbar
            xlabel(' Spatial co-ordinate (x)')
            ylabel('Spatial co-ordinate (y)')
            title(['Pressure contour plot for Navier stokes equation at t=',num2str((t-1)/100000)])

            subplot(1,2,2);
            quiver(xy(2:end),xy(2:end),u(2:end,2:end-1)',v(2:end-1,2:end)');
            streamline(xy(2:end),xy(2:end),u(2:end,2:end-1)',v(2:end-1,2:end)',h:0.2:1,h:0.2:1)
            axis([-0.1 1.1 -0.1 1.1])
            xlabel(' Spatial co-ordinate (x)')
            ylabel('Spatial co-ordinate (y)')
            title(['Streamline diagram for Navier stokes equation at t=',num2str((t-1)/100000)])

        end

    end

   %saving comparison vectors
   uu(k,:)=u(floor(length(xy)/2),:);
   vv(k,:)=v(floor((length(xy)+1)/2),:);
end

% calculating mean horizontal velocities to align them with the boundaries of the study domain
 uMean=uu;
 uMean(:,end)=[];
 uMean=[[0;0],uMean];
 uMean=(uMean+uu)./2;
 uMean(:,1)=[];
```

```
% plotting comparison
figure('Name','Horizontal and vertical velocity distributions at x=0.5 and t=0.1 for Re=0.1 and
Re=1','NumberTitle','off','OuterPosition',[0 30 SW SH-30]);
subplot(1,2,1);
plot(linspace(0,1,length(xy)),uMean(1,:)); hold on
plot(linspace(0,1,length(xy)),uMean(2,:));
legend('u @ Re=0.1','u @ Re=1','Location','NorthWest');
title('Comparing horizontal velocities at Re=0.1 and Re=1')
xlabel('Spatial co-ordinate (y)');
ylabel('Horizontal dimentionless velocity (u)');
grid

subplot(1,2,2);
plot(xy,vv(1,:) ); hold on
plot(xy,vv(2,:) );
legend('v @ Re=0.1','v @ Re=1','Location','NorthWest');
title('Comparing vertical velocities at Re=0.1 and Re=1')
xlabel('Spatial co-ordinate (y)');
ylabel('Vertical dimentionless velocity (v)');
grid
```

3. Simulation Results:

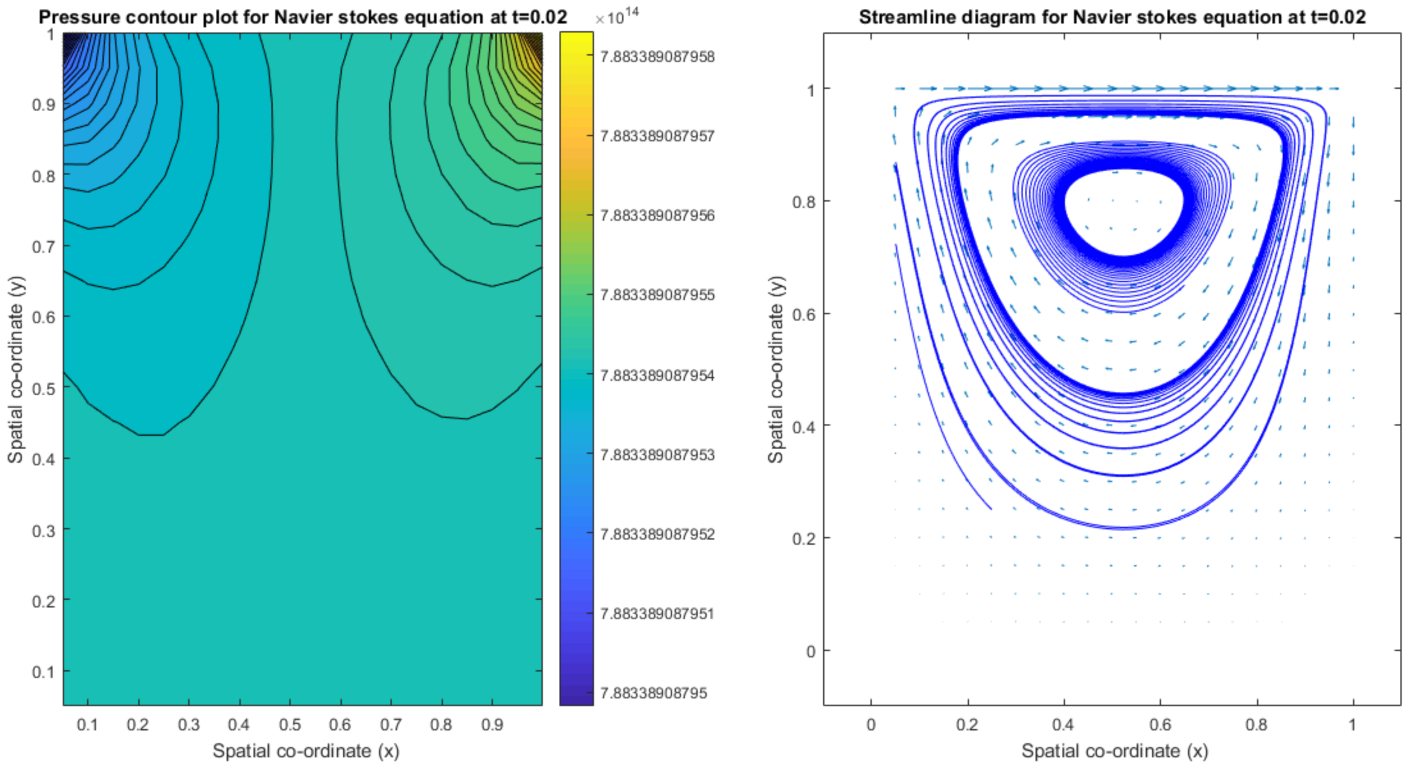
As the top lid moves it induces motion also in the fluid below it due to the shear forces. Since the lid is moving from left to right, the first layer of fluid moves with it and this causes the left most part to depart from the left wall creating a low pressure region and vice versa for the right side where the fluid layer is compressed against the right wall creating a high pressure region.

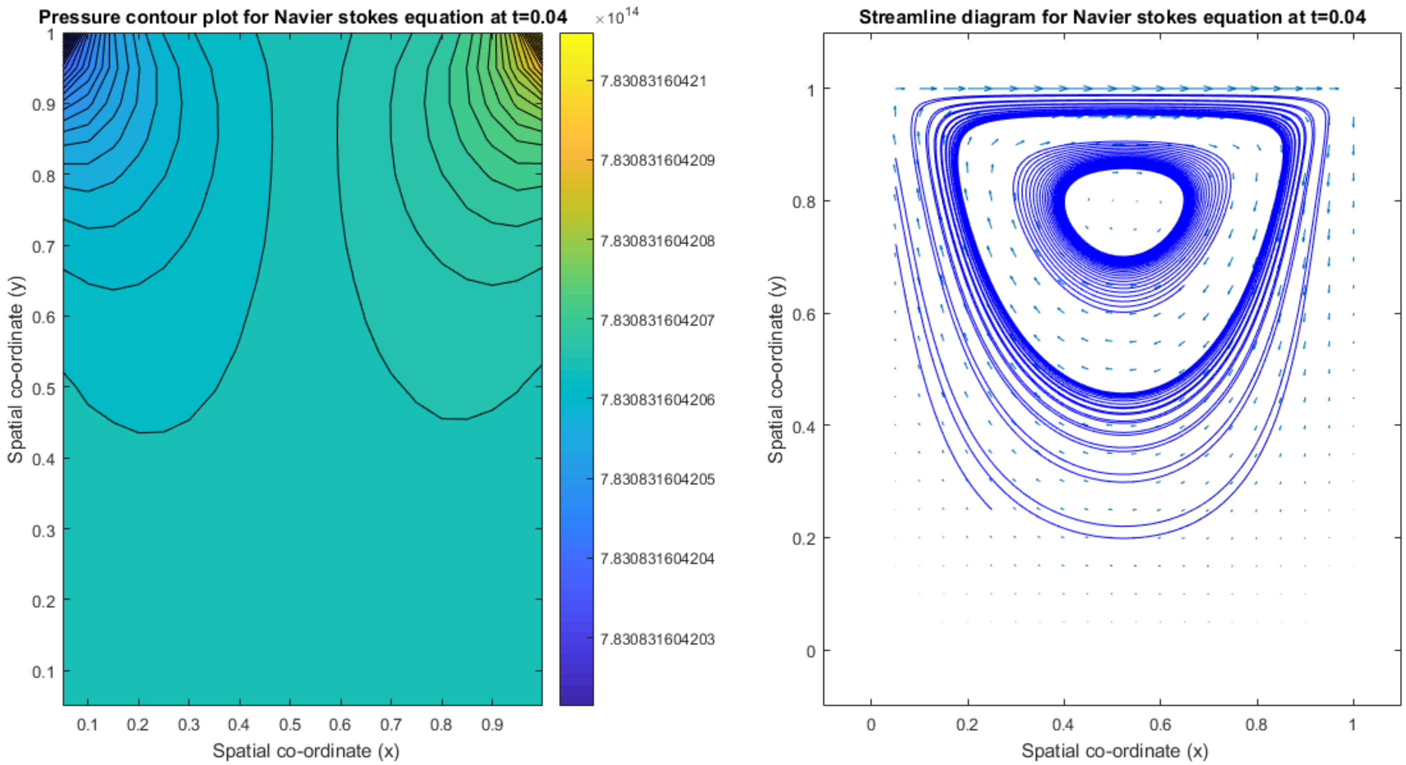
It is noticed that in general the pressure values decrease with time, this is due to the fact that the stationary fluid has an inertia that resists the newly induced motion of the lid. At $t = 0$ the resistance is largest and after the fluid start to move it keep reducing till it reaches a steady state.

Due to the no slip condition, the first layer of fluid will have the same velocity (speed and direction) of the lid, however, when it comes nearer to the right wall (high pressure region) it will start to decelerate till it reaches zero horizontal velocity at the wall and the fluid will have to change direction to down, this will induce a swirl motion in clock wise direction inside the cavity.

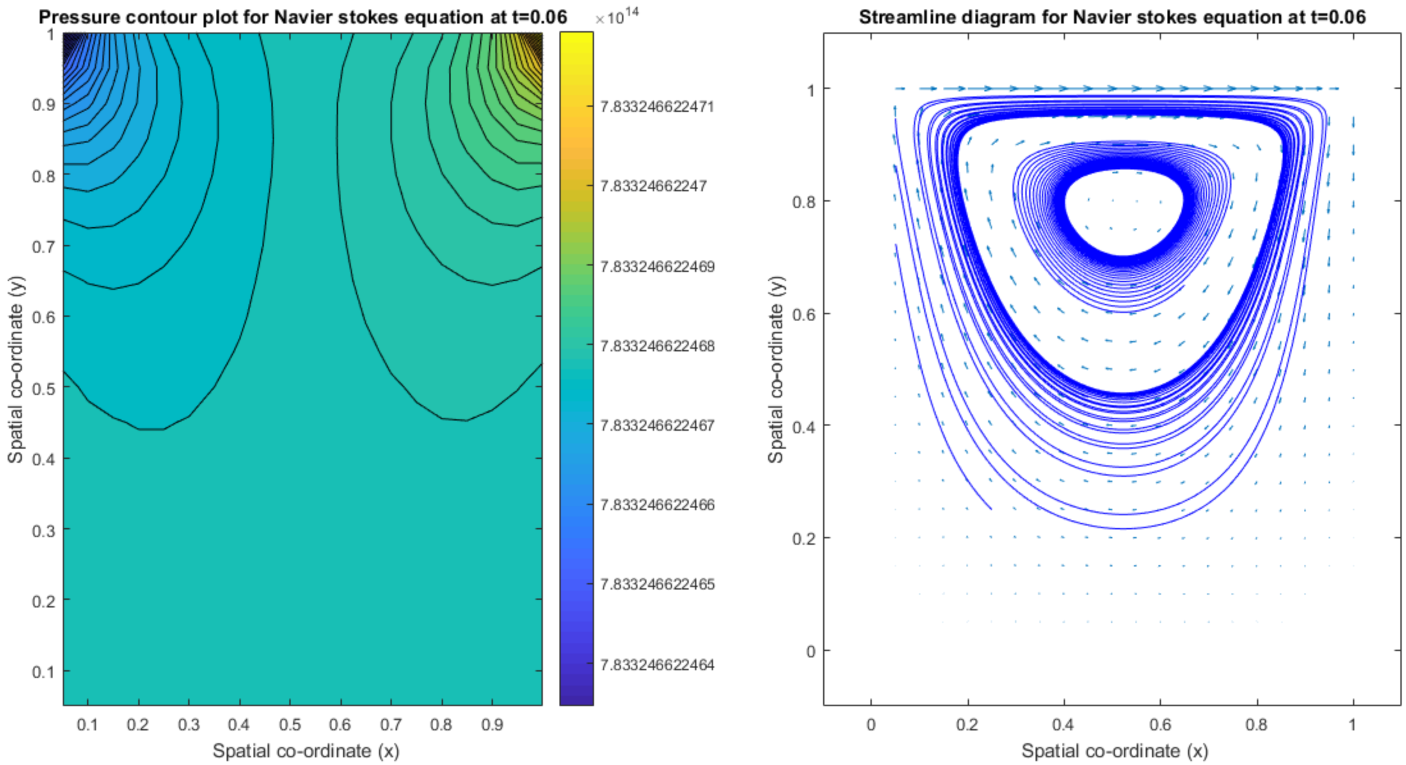
It is clear from the graphs that the swirl motion center is located approximately at $(x, y) = (0.5, 0.75)$ not $(x, y) = (0.5, 0.5)$ this is because the flow model is symmetric about y axis (as both left and right walls are stationary) but not x axis (as only the top lid is moving). It is intuitive to imagine that the center of swirl will be closer to the lid side and farther from the stationary sides.

It is also noticed that the magnitudes of velocities (sizes of arrows in the velocity vector field) are larger away from the swirl center and again smaller near the stationary walls. The largest magnitude will be for the first fluid layer at the lid.

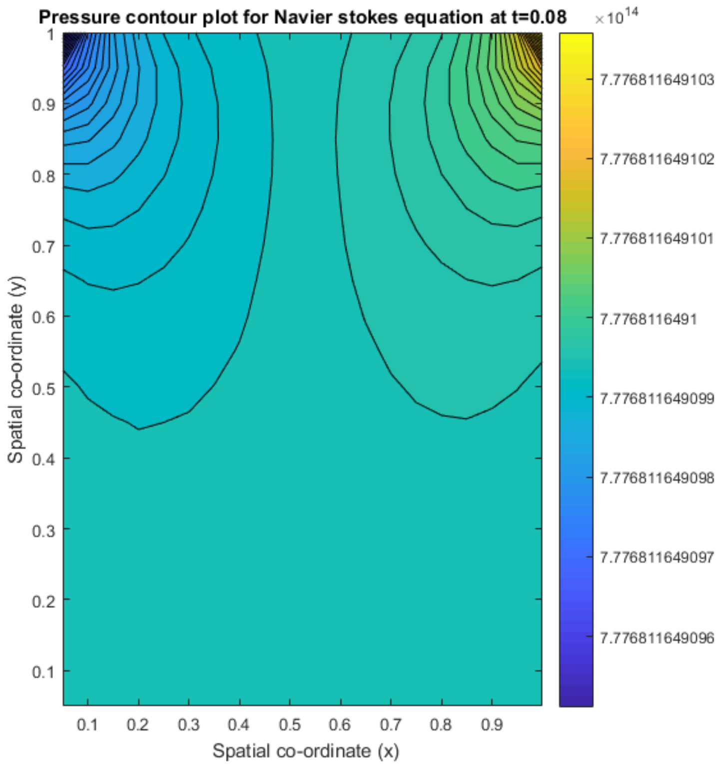

Pressure contour plot for Navier stokes equation at t=0.02
×10^14
7.883389087958
7.883389087957
7.883389087956
7.883389087955
7.883389087954
7.883389087953
7.883389087952
7.883389087951
7.88338908795
Spatial co-ordinate (y)
Spatial co-ordinate (x)
Streamline diagram for Navier stokes equation at t=0.02
Spatial co-ordinate (y)
Spatial co-ordinate (x)

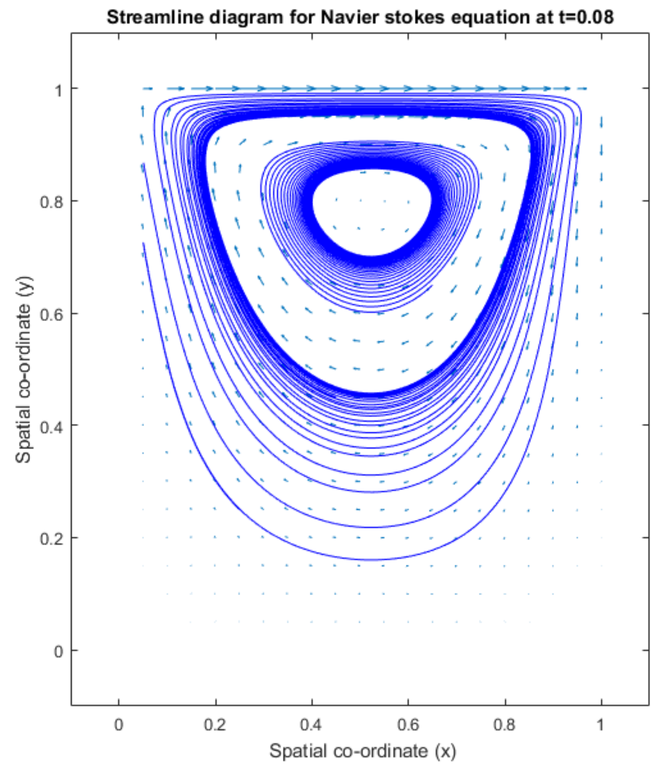


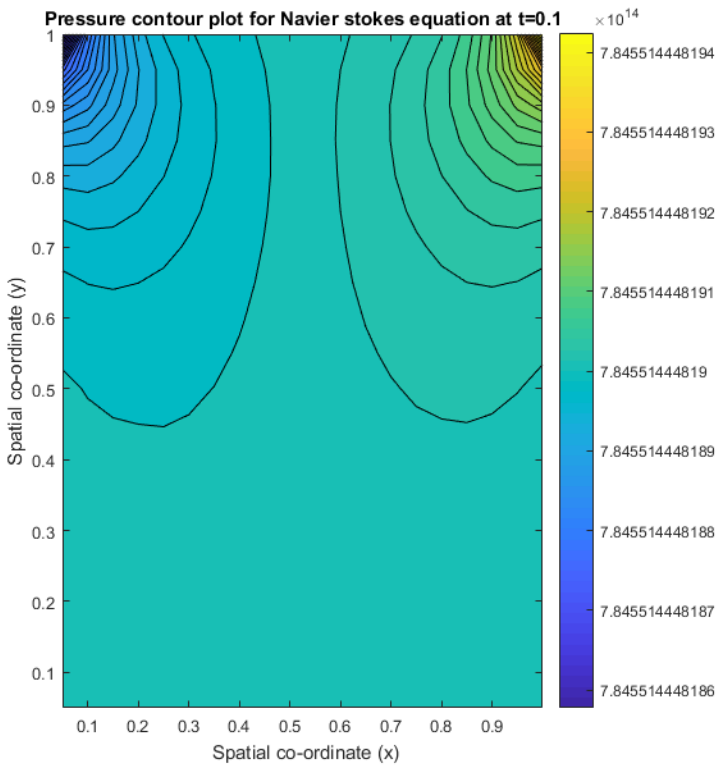
Pressure contour plot for Navier stokes equation at t=0.04
Spatial co-ordinate (x)
Spatial co-ordinate (y)
×10^14
7.83083160421
7.830831604209
7.830831604208
7.830831604207
7.830831604206
7.830831604205
7.830831604204
7.830831604203
Streamline diagram for Navier stokes equation at t=0.04
Spatial co-ordinate (x)
Spatial co-ordinate (y)

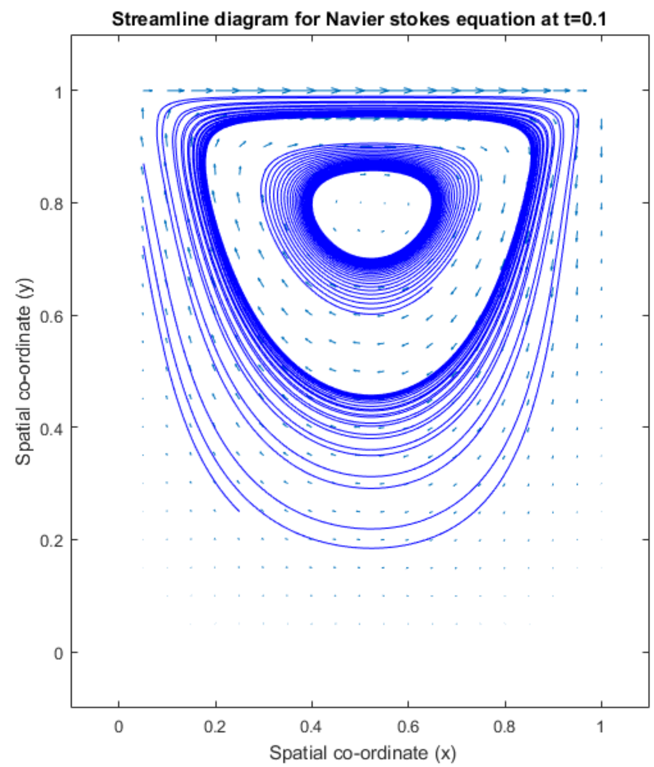


Pressure contour plot for Navier stokes equation at t=0.06
×10^14
7.833246622471
7.83324662247
7.833246622469
7.833246622468
7.833246622467
7.833246622466
7.833246622465
7.833246622464
Spatial co-ordinate (y)
Spatial co-ordinate (x)
Streamline diagram for Navier stokes equation at t=0.06
Spatial co-ordinate (y)
Spatial co-ordinate (x)

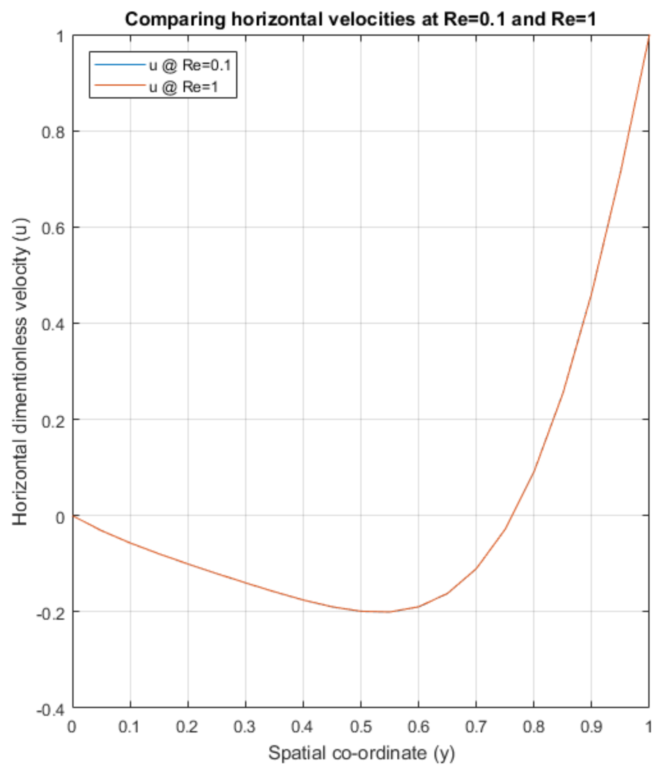

Pressure contour plot for Navier stokes equation at t=0.08
×10^14
7.776811649103
7.776811649102
7.776811649101
7.77681164910
7.776811649099
7.776811649098
7.776811649097
7.776811649096
Spatial co-ordinate (y)
Spatial co-ordinate (x)

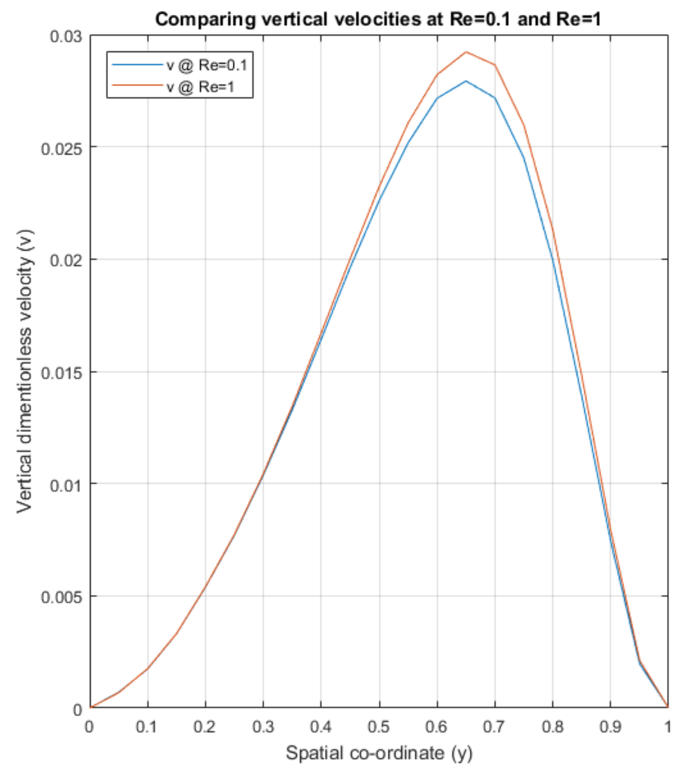

Streamline diagram for Navier stokes equation at t=0.08
Spatial co-ordinate (y)
Spatial co-ordinate (x)


Pressure contour plot for Navier stokes equation at t=0.1
×10^14
7.845514448194
7.845514448193
7.845514448192
7.845514448191
7.84551444819
7.845514448189
7.845514448188
7.845514448187
7.845514448186
Spatial co-ordinate (y)
Spatial co-ordinate (x)


Streamline diagram for Navier stokes equation at t=0.1
Spatial co-ordinate (y)
Spatial co-ordinate (x)

4. Comparing the flow at different Reynolds numbers:





The graph on the left shows that horizontal velocity profile at $Re = 0.1$and $Re = 1$, magnitudes of both the velocities are almost the similar. At the bottom of the cavity the horizontal velocity is zero which then gradually increases in the negative x direction and then reaches zero at swirl center $(y = 0.75)$ and continue to increase to its maximum velocity of `1 which is the velocity of the lid.

The graph of vertical velocity also remain almost similar with change in Reynolds number. The vertical velocity is 0 at the bottom wall and at the lid and attains its maximum in-between.

The reason why changing Reynolds number did not have great effect on the velocity profiles might be because different types of fluids having different viscosities will always reach a similar steady state. The only difference might be in how fast each fluid type will reach that steady state. So, it seems that in our case, the difference between the flow at $Re = 0.1$and $Re = 1$ might have been more significant at earlier time steps.