Numerical Methods of Thermo-Fluid Dynamics I

Winter Semester 2017-2018

DELIVERABLE TASK I:
Numerical Solution of 2D Heat Equation

Given: Monday, 6/11/2017
Deadline: 27/11/2017

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Deliverable Task I

Consider the dimensionless 2D heat equation:

$$\frac{\partial w}{\partial t} = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}, \tag{1}$$

$$[x, y] \in [0, 1] \times [0, 1],\ t \in [0, 0.16] \tag{2}$$

The initial and boundary conditions are:

$$w(x, y, 0) = 0, \tag{3}$$

$$w(0, y, t) = 1 - y, \quad w(1, y, t) = 1 - y^2, \quad w(x, 0, t) = 1, \quad w(x, 1, t) = 0. \tag{4}$$

- Discretize the equation by a CDS scheme in space and the Crank-Nicolson method in time:
  - (10p) Show under what conditions the Crank-Nicolson scheme is stable.
  - (10p) Show that the numerical results will converge when the grid is refined.
- (35p) Write MATLAB/OCTAVE programs for the Crank-Nicolson and explicit Euler schemes to solve the heat equation. The programs should automatically generate all plots required.
- Use $h = 1/40$ in x and y directions and Δt with different values: 0.01, 0.001 and 0.0001.
  - (10p) Generate two plots (one for Explicit Euler, another one for Crank-Nicolson) by using the stable Δt, show the time evolution of the temperature at x=y=0.4, explain the results.
  - (10p) Use a stable Δt, generate two plots (one for Explicit Euler, another one for Crank-Nicolson) showing the vertical temperature profile at t=0.16 and x=0.4, explain the results.
  - (5p) Compare the performance of the two methods to achieve a given level of accuracy.
- (20p) Choose an appropriate scheme and time step and generate 5 plots of the numerical solution for the whole domain from t=0.01, 0.02, 0.04, 0.08, 0.16. Explain the physical phenomenon.

The Deliverable Task I contains 2 matlab code files as well as 1 report, all these must be submitted to `nan.chen@fau.de` before **November 27th, 2017, 23:59.**