# Optimization for Engineers

## 4. Lab Exercise
25.06.2018
Summer Term 2018

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### Assignment 1: Projection, Active Index Set - 5 Credits

Complete the function *projectIntoBox.m*, which projects a point $x \in \mathbb{R}^n$ into a set of box constraints $\Omega_\square$ defined by lower bounds $a$ and upper bounds $b$:

a) Input: $x \in \mathbb{R}^n$, lower and upper bounds $a, b \in \mathbb{R}^n$ and $\varepsilon >= 0$.

b) Initialize $P \leftarrow x$ and $A \leftarrow \{\}$ and start a loop over $i = 1, ..., n$:

i) Set
$$P(x)_i \leftarrow \begin{cases} a_i & \text{if } x_i \leq a_i \\ x_i & \text{if } a_i < x_i < b_i \\ b_i & \text{if } x_i \geq b_i \end{cases} \text{ for } i = 1, \ldots, n$$

ii) Set $A(x) \leftarrow \{i \in \{1, \ldots, n\} | \; x_i \leq a_i + \varepsilon \text{ or } x_i \geq b_i - \varepsilon\}$.

c) Output: Projected point $P(x) \in \Omega_\square$ and $\varepsilon$-active set $A(x)$.

**Hints:**

a) Use **A=[ ]** to initialize an empty index set and **A=[A i]** to append index **i** to the index set.

b) Test the algorithm with the command **sheet04Script(1);**

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### Assignment 2: Projected Newton's Method - 5 Credits

Complete the projected Newton's method in the template *projectedNewton.m*, for minimizing $f : \mathbb{R}^n \to \mathbb{R}$ with projection $P$ into box constraints:

a) Input: $f \in \mathcal{C}^2$; $x_0 \in \mathbb{R}^n$; $P : \mathbb{R}^n \to \Omega_\square$; $\varepsilon > 0$.

b) Set $x_k \leftarrow P(x_0)$.

c) While $||x_k - P(x_k - \nabla f(x_k))|| > \varepsilon$ do:

i) Compute active index set $\mathcal{A}(x_k)$.

ii) Set $B_k = \nabla^2 f(x_k)$.

iii) For $i = \mathcal{A}(x_k)$ overwrite column $i$ and row $i$ of $B_k$ with column $i$ and row $i$ of the unit matrix.

iv) Solve $B_k d_k = -\nabla f(x_k)$ for $d_k$ with conjugate gradient.

v) If $d_k$ is not a descent direction set $d_k$ to the steepest descent direction.

vi) Compute $t_k$ by calling **projectedBacktracking.m** for $f$ at $x_k$ along $d_k$ respecting the projection $P$.

vii) Set $x_k \leftarrow P(x_k + t_k d_k)$.

d) Return $x_s \leftarrow x_k$.

**Hints:**

a) Use **getProjectedPoint(P_handle,x_k)** to get $P(x_k)$.

b) Use **getActiveIndexSet(P_handle,x_k)** to get $\mathcal{A}(x_k)$.

c) The syntax **for i = A** directly generates a loop over all indexes found in an index set (or row vector) **A**.

d) The syntax **B(:,i)=E(:,j)** overwrites column $i$ of **B** with column $j$ of **E**.

e) The syntax **B(i,:)=E(j,:)** overwrites row $i$ of **B** with row $j$ of **E**.

f) Test the algorithm with the command **sheet04Script(2);**

## Evaluation and Upload

Hand in the following files (unzipped) to StudOn using the **Exercises** object:

a) *projectIntoBox.m*

b) *projectedNewton.m*