Engineering Mathematics-II Scilab

Lab-03: Linear dependence and independence

Linear span &
Linearly Independent and Dependent Vectors

- **Linear Span:** Let u,v,w be vectors in V(F) and a,b,c be scalars in the scalar field F, then the linear span of u,v,w is $au + bv + cw$.
- **Linearly dependent:** Let u,v,w be vectors in V(F) and there exist scalars a,b,c (not all zero) such that $au + bv + cw = 0$, then the set of vectors {u,v,w} is called linearly dependent vectors.
- **Linearly independent vectors:** Let u,v,w be vectors in V(F) and the linear span $au + bv + cw = 0$, only if a=b=c=0, then the set of vectors {u,v,w} are called linearly independent vectors.

Theory of Linear Independence

- What is the rank of a matrix?

ANS: Maximum No. of linearly independent Rows of the matrix.

- How do we find it?

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & -1 \\ 4 & 3 & 2 & 1 \end{bmatrix} \longrightarrow A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix} (\mathit{Echelon\ form})$$

r=No. of nonzero rows=2 (Scilab command : **rank(A)**

- What is LD/LI of vectors?

If at least one of the vectors can be expressed as a linear combination of others LD, if none LI

- How do we determine it?

Vectors v1=(1,1,1,1), v2=(2,1,0.-1), v3=(4,3,2,1), A=[v1;v2;v3]

If rank(A)=3(No. of vectors) , LI

Else LD

8.2. *if-else-end structure*

if-else-end in C

- Initialize or some input

```
if   (condition …)
   {
     statements(s);
     }
   else
     {
     statement(s)
      }
```

if-else-end in scilab

- Initialize or some input

```
if …    condition …  then
          statements(s)
     else
          statement(s)
      end
```

Algorithm for Linear Independence

1.Write clc and clear commands

2.Enter the vectors v1, v2,v3,…,vn etc

3.Form matrix A=[v1;v2;v3;…;vn]

4.Find rank of A: r=rank(A)

5. if r=n(no. of vectors),

   display the vectors are LI

   else LD

6. If L.I./L.D. is checking for two vectors $v_1, v_2 \in \mathbb{R}^2 or\ \mathbb{R}^3$, then plot the vectors as arrows.

Objective: (i) Write a script file to determine the Linear independence (LI) of vectors and plot these vectors in their respective space.

Exercise: Determine LI of the following vectors:

(i) (0,1), (1,0)

(ii) (2,4), (1,2)

(iii) (1,1), (1,3), (2,5)

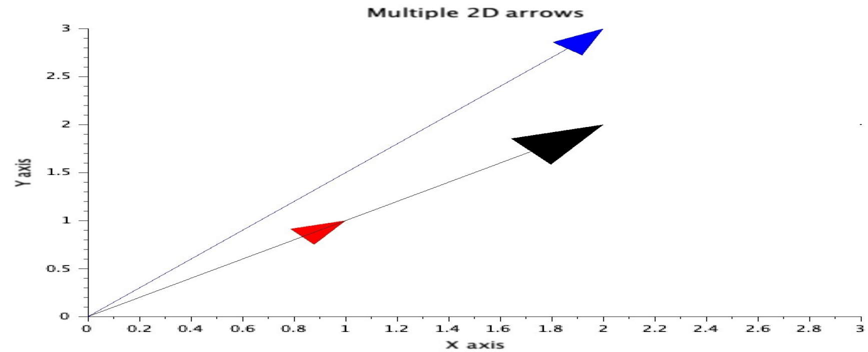
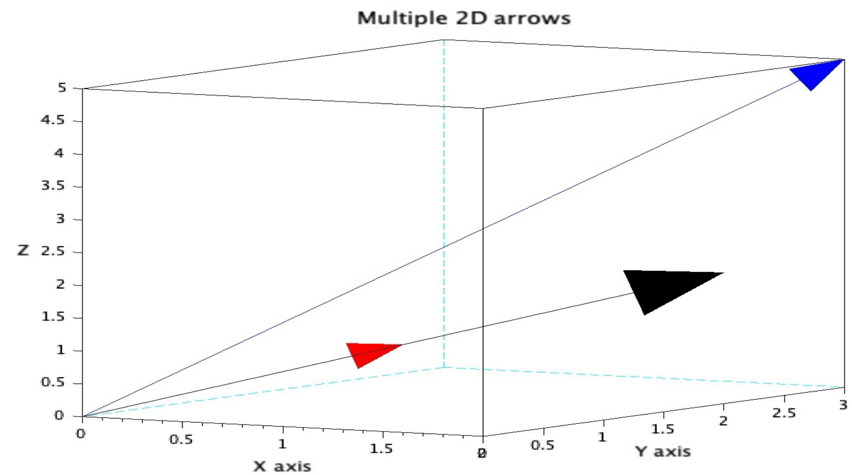
(iv) (1,2,3), (1,2,4)

(v) (1,1,0), (1,0,1), (0,1,1)

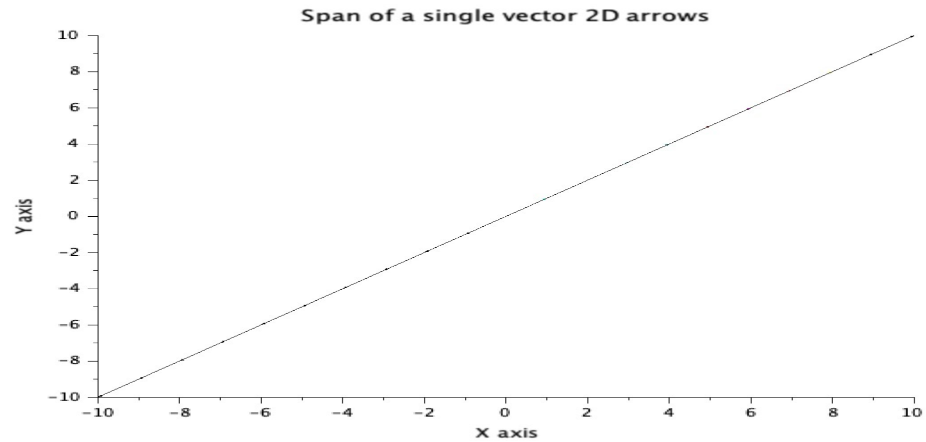
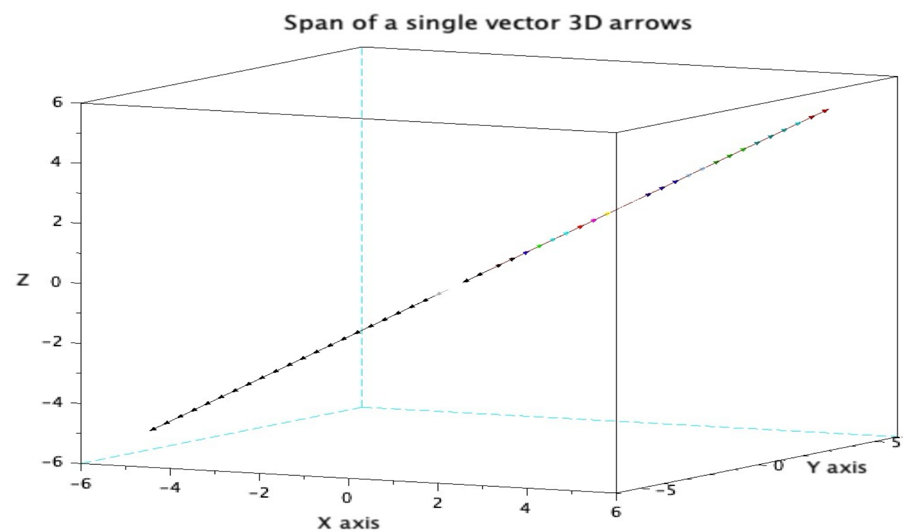
(vi) (2,2,1), (1,-1,1), (1,0,1)

(vii)(1,2,3,1),(2,1,-1,1),(4,5,5,3),(5,4,1,3)

Linearly independent and dependent vectors in 2D & 3D

Scilab Codes	Output
clf; A=[1,1]; B=[2,2]; plot2d([0,3],[0,2],[0,1]) xarrows([0,1],[0,1],4,5) *// Arrow for [1,1]* xarrows([0,2],[0,2]) *// arrow for [2,2]* *xarrows([0,2],[0,3],4,2)* xtitle('Multiple 2D arrows', 'X axis', 'Y axis');	
clf; A=[1,1]; B=[2,2]; plot3d([0,0,0],[0,0,0],[0,0,0]) xarrows([0,1],[0,1],[0,1],4,5) *// Arrow for [1,1]* xarrows([0,2],[0,2],[0,2]) *// arrow for [2,2]* xarrows([0,2],[0,3],[0,5],4,2) xtitle('Multiple 3D arrows', 'X axis', 'Y axis','Z-axis');	

Linear span of single vector in 2D and 3D

Scilab Codes	Output
// span of (1,1) clf plot2d([0,0],[0,0]) for i=-10:10 xarrows(i*[0,1],i*[0,1],2,4*i) end xtitle('Span of a single vector 2D arrows',"X axis","Y axis")	
// span of (1,1,1) clf plot3d([0,0,0],[0,0,0],[0,0,0]) for i=-5:.2:5 xarrows(i*[0,1],i*[0,1],i*[0,1],2, 4*i) end xtitle('Span of a single vector 2D arrows',"X axis","Y axis")	

Important Points

- Two vectors in $\mathbb{R}^2$ are l.d. if and only if they are multiple of each other.
- Three or more vectors in $\mathbb{R}^2$ are always l.d.
- Two vectors in $\mathbb{R}^3$ are l.d. if and only if they are multiple of each other.
- Three vectors in $\mathbb{R}^3$ are l.d. if and only if they are co-planner.