# Lecture :- 5

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**Argument** :- A sequence of statements that end with a conclusion.
OR A sequence of propositions.

**Premise & Conclusion** :- All but the final propositions in an argument are called premises and the final proposition is called conclusion.

**Valid Argument** :- An argument is valid if the truth of all its premises implies that conclusion is true.

* To show that an argument is valid, we can use truth table, laws of logic but if we have large no. of propositions then laws of inference come to our rescue to prove the validity of arguments.

## Laws of Inference :-

1) **Modus tollens / Law of Contraposition** :-

$\sim q,\ p \to q \quad \therefore \sim p$ (ie) $(\sim q \wedge (p \to q)) \Rightarrow \sim p$

eg:- "If it is raining then i wear my coat"
$\Rightarrow$ It means If I don't wear my coat then it is not raining.

2) **Modus ponens / Law of Detachment** :-

$p \to q,\ p \quad \therefore q$ (ie) $((p \to q) \wedge p) \Rightarrow q$

eg. if 2 triangles are similar then their corresponding sides are proportional.

3) **Hypothetical Syllogism** :-

$p \to q,\ q \to r \quad \therefore p \to r$ (ie) $((p \to q) \wedge (q \to r)) \Rightarrow (p \to r)$

4) **Disjunctive Syllogism** :-

$p \vee q,\ \sim p \quad \therefore q$ (ie) $((p \vee q) \wedge \sim p) \Rightarrow q$

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disjunctive syllogism

eg:- 1) Either it is below freezing or raining now
2) It is not below freezing
1) + 2) conclude that It is raining now.

hypothetical syllogism

eg:- 1) If weather is good then we will go for an outing
2) If we go for an outing, we will play a game
1) + 2) $\Rightarrow$ If weather is good then we will play a game

5) Law of Addition:-

$p \quad \therefore \; p \vee q$ (ie) $p \Rightarrow p \vee q$

eg:- It is below freezing now : $p$
then clearly $p$ concludes that
"Either it is below freezing now or it is raining.

6) Law of Simplification:-

$p \wedge q \quad \therefore \; p$ (ie) $p \wedge q \Rightarrow p$

eg:- ~~If~~ "It is below freezing and it is raining now"
concludes that "It is raining now".

7) Law of Conjunction:-

$p, q \quad \therefore \; p \wedge q$ (ie) $(p) \wedge (q) \Rightarrow (p \wedge q)$

eg:- "It is raining now" + "It is below freezing now"
Concludes that
"It is raining and below freezing now".

8) Law of Resolution:-

$p \vee q, \; \sim p \vee r \quad \therefore \; q \vee r$ (ie) $((p \vee q) \wedge (\sim p \vee r)) \Rightarrow (q \vee r)$

eg:- "It is raining or I will make tea" and
"It is not raining or I will read a book"
conclude that
"I will make tea or I will read book".

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eg:- Show that the premises

1) "It is not sunny this afternoon and it is colder than yesterday"

2) "We will go swimming only if it is sunny"

3) "If we do not go swimming, then we will take a canoe trip"

4) "If we take a canoe trip, then we will be home by sunset"

leads to the conclusion

" We will be home by sunset".

sol^n:- Let

$p$: It is sunny this afternoon

$q$: It is colder than yesterday

$r$: We will go swimming

$s$: we will take a canoe trip.

$t$: we will be home by sunset.

So, we need to prove that premises 1, 2, 3 & 4 leads to the conclusion $t$.

clearly 1), 2), 3), 4) are $(\sim p \wedge q)$, $r \to p$, $\sim r \to s$, $s \to t$ respectively

Now to prove conclusion $t$, we construct an argument as follows

| | | | |
|---|---|---|---|
| Step:-1 | Premise 1 | $\sim p \wedge q$ | |
| Step:- 2 | Law of Simplication | $\sim p$ | using step 1) |
| Step:- 3 | Premise 2 | $r \to p$ | |
| Step:- 4 | Modus tollens / Contraposition | $\sim r$ | using step 2) & 3) |
| Step:- 5 | Premise 3 | $\sim r \to s$ | |
| Step:- 6 | Modus ponnens | $s$ | using step 4) & 5) |
| Step:- 7 | Premise 4 | $s \to t$ | |
| Step:- 8 | Modus Ponens | $t$ | using step 6) & 7). |

thus, we arrive at ~~contradiction~~ conclusion $t$.

If we do it by truth table then because we are working with 5 variables (propositional) then we will be dealing with 32 rows.

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**eg:-** Show that the premises $(p \wedge q) \vee r$ and $r \to s$ implies the conclusion $p \vee s$.

**Sol^n:-**

Step:-1 We can replace $(p \wedge q) \vee r$ as 2 clauses $(p \vee r)$ & $(q \vee r)$ (by distributivity)

Step:-2 $r \to s$ can be replaced by $\sim r \vee s$

Step:-3 $(p \vee r) \wedge (\sim r \vee s) \Rightarrow p \vee s$ Law of Resolution.

41 **eg:-** Show (using Laws of Inference) that the premises
1) "Randy works hard"
2) "If Randy works hard, then he is a dull boy" and
3) "If Randy is a dull boy, then he will not get the Job" imply the conclusion
"Randy will not get the Job".

**Sol^n:-** Let $p$: Randy works hard
$q$: Randy is a dull boy.
$r$: Randy will get the Job

So we need to prove that the premises 1), 2), 3) imply the conclusion $\sim r$.
Clearly, we see that premises 1), 2), 3) are $p$, $p \to q$, $q \to \sim r$.

We construct the argument as follows:

| | | |
|---|---|---|
| Step:-1 | $p \to q$ | Premise (2) |
| Step:-2 | $p$ | Premise (1) |
| Step:-3 | $q$ | Modus ponents using step 1) & 2) |
| Step 4 | $q \to \sim r$ | Premise (3) |
| Step 5 | $\sim r$ | Modus ponens using step 3) & 4) |

Thus, we arrive at conclusion.

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42 eg.- For each of these arguments, explain which rules of inference are used for each step :
" Danish, a student in this class, knows how to write program in JAVA".
Everyone who knows how to write programe in JAVA can get a high paying Job. Therefore
Someone in this class can get a high paying Job".

sol^n $P(x)$ : $x$ is in this class.
$Q(x)$ : $x$ knows how to write a program in JAVA
$R(x)$ : $x$ can get a high paying Job.

thus the premises are

$$P(\text{Danish}), \; Q(\text{Danish}), \; \forall x(Q(x) \to R(x))$$

| | | |
|---|---|---|
| Step:- 1 | $Q(\text{Danish})$ | Premise 2 |
| Step:- 2 | $Q(\text{Danish}) \to R(\text{Danish})$ | Using universal instantiation & Premise 3 |
| Step:- 3 | $R(\text{Danish})$ | Modus Ponens |
| step:- 4 | $P(\text{Danish})$ | Premise 1 |
| Step:- 5 | $P(\text{Danish}) \wedge R(\text{Danish})$ | Law of conjunction |
| Step:- 6 | $\exists x\,(Q(x), R(x))$ | Existential generalisation |

## # Universal Specification $\forall x\, P(x) \Rightarrow P(a)$

Universal Generalisation $P(a) \Rightarrow \forall x P(x)$ for arbitrary a

Existential Generalisation $P(a) \Rightarrow \exists x\, P(x)$ for some a

Existential Specification $\exists x, P(x) \Rightarrow P(a)$ for some a

'Specification' & 'Instantiation' are same.

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eg: Consider the argument :

42 "If you invest in stock market, then you will get rich"

"If you get rich, then you will be happy"

Therefore.

"If you invest in stock market, then you will be happy".

Check whether argument is valid.

sol$^n$: Let

$p$: You invest in stock market

$q$: you get rich

$r$: you will be happy.

Thus premises here are $p \to r$, $q \to r$

Conclusion is $p \to r$

Here, clearly premises implies conclusion because of Hypothetical Syllogism.

Thus, the argument is valid.

eg: Test the validity of following argument:

43 "If philosopher is not money-minded and some money-minded persons are not clever, then there are some persons who are neither philosopher nor clever".

sol$^n$:- $P(x)$: $x$ is philosopher

$Q(x)$: $x$ is money minded

$R(x)$: $x$ is clever.

thus premises are $P(x) \to \sim Q(x)$, (1)

(2) $\exists x\, (Q(x) \wedge \sim R(x))$

& the conclusion is

$\exists x\, (\sim P(x) \wedge \sim R(x))$

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By existential specification and premise (2) $\exists$ some c st $Q(c) \wedge \sim R(c)$

Now, By law of simplication
$Q(c), \sim R(c)$

Now $Q(c)$, Premise (1) $\Rightarrow \sim P(c)$ ($\because$ Modus tollens)
$\Rightarrow \sim P(c), \sim R(c) \Rightarrow \sim P(c) \wedge \sim R(c)$ (Law of Conjunction)
$\Rightarrow$ By existential generalization
$\exists x (\sim P(x) \wedge \sim R(x))$ for some c.

Thus premise imply conclusion,
hence, argument is valid.

eg:- Show that the premises
" A student in this class has not read the book",
" Everyone in this class passed the first exam" imply the conclusion
" Someone who passed the first exam has not read the book".

sol$^n$ let $P(x)$: $x$ is in this class
$Q(x)$: $x$ has read the book.
$R(x)$: $x$ passed the first exam.

The premises here are $\exists x (P(x) \wedge \sim Q(x))$ ← $P_1$ and
$\forall x (P(x) \rightarrow R(x))$. ← $P_2$

The conclusion here is $\exists x (R(x) \wedge \sim Q(x))$.

Step:- 1) $\exists x (P(x) \wedge \sim Q(x))$ Premise $P_1$
Step 2) $P(a) \wedge \sim Q(a)$ Existential Specification from $P_1$
Step 3) $P(a)$ Law of Simplification
Step 4) $\forall x (P(x) \rightarrow R(x))$ Premise $P_2$
Step 5) $P(a) \rightarrow R(a)$ Universal Specification
Step 6) $R(a)$ Modus ponens
Step 7) $\sim Q(a)$ Law of Simplification from $P_1$
Step 8) $R(a) \wedge \sim Q(a)$ Law of Conjunction.
Step 9) $\exists x (R(x) \wedge \sim Q(x))$. Conclusion.

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