Discrete Maths

Argument → A sequence of statements or propositions that end with a conclusion is called argument.

Premise and Conclusion → All but the final propositions in an argument are called premises and the final proposition is called conclusion.

Valid Argument → An argument is valid if the truth of all its premises implies that the conclusion is truth.

or

An argument $P_1, P_2, ---, P_n \vdash Q$ is said to be valid if Q is true whenever all the premises $P_1, P_2, ---, P_n$ are true.

An argument which is not valid is called fallacy.

The argument $P_1, P_2, ---, P_n \vdash Q$ is valid iff the proposition $(P_1 \wedge P_2 \wedge --- \wedge P_n) \rightarrow Q$ is a tautology.

$p \rightarrow q$ or $\sim p \vee q$

Converse : $q \rightarrow p$

Inverse : $\sim p \rightarrow \sim q$

Contrapositive : $\sim q \rightarrow \sim p$

$p \rightarrow q$

- if p, then q
- if p, q
- p is sufficient for q
- q if p
- q when p
- a necessary condition for p is q
- q unless $\neg p$
- p implies q
- p only if q
- a sufficient condition for q is p
- q whenever p
- q is necessary for p
- q follows from p

Q: How can this English sentence be translated into a logical expression?

~~Sol: Let p:~~

"You can access the Internet from campus only if you are a computer science major or you are not a freshman".

Sol: Let p : You can access the Internet from campus

q : You are a computer science major

r : You are a freshman

$$\Rightarrow p \rightarrow (q \vee \sim r)$$

Tautology : A compound proposition that is always true

Contradiction : A compound proposition that is always false.

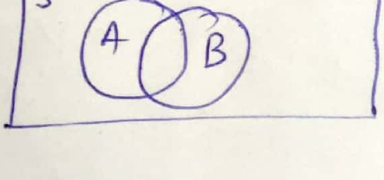
Contingency : A compound proposition that is neither a tautology nor a contradiction.

For eg. : $p \vee \sim p$ is always true, it is a tautology

$p \wedge \sim p$ is always false, it is a contradiction.

Logical Equivalent : The compound propositions p and q are called logical equivalent if $p \leftrightarrow q$ is a tautology.

Symbol : $\equiv$



Laws →

$p \vee (\sim p) \equiv T$	$A \cup A^c = S$
$p \wedge (\sim p) \equiv F$	$A \cap A^c = \phi$
$p \vee T \equiv T$	$A \cup S = S$
$p \vee F \equiv p$	$A \cup \phi = A$
$p \wedge T \equiv p$	$A \cap S = A$
$p \wedge F \equiv F$	$A \cap \phi = \phi$

Rules of Inference →

(1) Modus Ponens. (Law of Detachment)

$p, p \to q \vdash q$

(2) Modus Tollens (Law of Contrapositive)

$\sim q, p \to q, \vdash \sim p$

(3) Hypothetical Syllogism

$p \to q, q \to r \vdash p \to r$

(4) Disjunctive Syllogism

$p \vee q, \sim p \vdash q$

(5) Addition

$p \vdash p \vee q$

(6) Simplification

$p \wedge q \vdash p$

(7) Conjunction

$p, q \vdash p \wedge q$

(8) Resolution

$p \vee q, \sim p \vee r \vdash q \vee r$

Ex → • If weather is pleasant then we'll go for outing.
If we'll go for outing then we'll play a game.
If it is raining.

• ~~Either~~ it is freezing below or it is raining.
I'll make a tea

• It's raining ~~or~~ I'll read a book.
It's not raining or I'll

Q: Show that the premises

- It's not sunny this afternoon and it is colder than yesterday
- We'll go swimming only if it is sunny
- If we don't go swimming then we'll take a trip.
- If we take a trip then we'll be home by sunset.

leads to the conclusion
'We'll be home by sunset'

Sol: p : It's sunny this afternoon
q : It is colder than yesterday
r : We'll go swimming
s : We'll take a trip
t : We'll be home by sunset

i.e.; S_1 : $\sim p \wedge q$
S_2 : $r \rightarrow p$
S_3 : $\sim r \rightarrow s$
S_4 : $s \rightarrow t$

T.S. : t is true

Step 1 : S_1 : $\sim p \wedge q$
Step 2 : Law of Simplification, $\sim p$
Step 3 : S_2 : $r \rightarrow p$
Step 4 : Modus Tollens, $\sim r$
Step 5 : S_3 : $\sim r \rightarrow s$
Step 6 : Modus ponens, s
Step 7 : S_4 : $s \rightarrow t$
Step 8 : Modus ponens, t

Thus, we arrive our conclusion.

Q: Show that the premises "If you send me an e-mail message, then I'll finish writing the program," "If you don't send me an e-mail message then I'll go to sleep early" "If I go to sleep early then I'll wake up feeling refreshed" lead to the conclusion "If I don't finish writing the program, then I'll wake up feeling refreshed".

Sol: Let p : You send me an e-mail message

q : I'll finish writing the program

r : I'll go to sleep early

s : I'll wake up feeling refreshed

$S_1 : p \to q$

$S_2 : \sim p \to r$

$S_3 : r \to s$

$S : \sim q \to s$

Step 1 : $p \to q$ $(\because S_1)$

Step 2 : $\sim q \to \sim p$ $(\because$ Law of Tollens$)$

Step 3 : $\sim p \to r$ $(\because S_2)$

Step 4 : $\sim q \to r$ $(\because$ Hypothetical Syllogism$)$

Step 5 : $r \to s$ $(\because S_3)$

Step 6 : $\sim q \to s$ $(\because$ Hypothetical Syllogism$)$

Q: Show that $(p \wedge q) \vee r$ and $r \to s$ imply the conclusion $p \vee s$.

Sol: $(p \wedge q) \vee r \equiv (p \vee r) \wedge (q \vee r)$

$r \to s \equiv (\sim r \vee s)$

$\Rightarrow (p \vee r)$, $(\sim r \vee s)$, $(q \vee r)$ are true

Law of Resolution, $(p \vee s)$ is true.

Q: Test the validity of the foll. argument:

If I study then I'll not fail Maths.

If I don't play basketball, then I'll study

But I failed maths.

$\therefore$, I must have played basketball.

Sol: p: I study

q: I fail Maths

r: I play basketball

S_1: $p \rightarrow \sim q$

S_2: $\sim r \rightarrow p$

S_3: q

S: r

Steps
1. $\sim r \rightarrow p$ $(\because S_2)$
2. $p \rightarrow \sim q$ $(\because S_1)$
3. $\sim r \rightarrow \sim q$ $(\because$ Hypothetical Syllogism$)$
4. $q \rightarrow r$ $(\because$ ~~Laws~~ Modus Tollens$)$

Q: Use the laws of Inference, show that the premises

- "~~Rajay~~ Ram works hard"
- "If Ram works hard then he is a dull boy"
- "If Ram is a dull boy, then he will not get the job"

imply the conclusion "Ram will ~~not~~ not get the job".

Sol: Let p: Ram works hard

q: Ram is a dull boy

r: Ram will get the job

S_1: p T.S. $\sim r$ is true.

S_2: $p \rightarrow q$

S_3: $q \rightarrow \sim r$

Steps 1. $p \to q$ $(\because S_2)$

2. p $(\because S_1)$

3. q $(\because$ Modus ponens$)$

4. $q \to \sim r$ $(\because S_3)$

5. $\sim r$ $(\because$ Modus ponens using step 3 & 4$)$

Thus, we arrive our conclusion.

Rules of Inference for Quantified Statements

1. Universal Instantiation / Specification

$\forall x,\ P(x)$ is true $\vdash$ $P(c)$ is true.

2. Universal Generalization

$P(c)$ is true for an arbitrary c $\vdash$ $P(x)$ is true $\forall x$

3. Existential Instantiation/Specification

$\exists x,\ P(x)$ is true $\vdash$ $P(c)$ is true for some element c

4. Existential Generalization

$P(c)$ is true for some element c $\vdash$ $\exists x,\ P(x)$ is true.

Q: Show that the premises "Everyone in this discrete maths class has taken a course in computer science" and "Marla is a student in this class" imply the conclusion "Marla has taken a course in computer science"

Sol → Let $D(x)$: x is in this discrete maths class

$C(x)$: x has taken a course in computer science.

Then S_1 : $\forall x,\ D(x) \longrightarrow C(x)$

S_2 : $D(\text{Marla})$

S : $C(\text{Marla})$

Steps ① $\forall x, D(x) \to C(x)$ ($\because S_1$)

② $D(\text{Marla}) \to C(\text{Marla})$ ($\because$ Universal Instantiation)

③ $D(\text{Marla})$ ($\because S_2$)

④ $C(\text{Marla})$ ($\because$ Modus ponens from ② + ③)

Q: Show that the premises "A student in this class has not read the book", and "Everyone in this class passed the Ist exam" imply the conclusion "Someone who passed the Ist exam has not read the book".

Sol: Let $C(x)$: x is in this class

$B(x)$: ~~x~~ has read the book

$P(x)$: x passed the Ist exam.

S_1 : $\exists x, (C(x) \land \sim B(x))$

S_2 : $\forall x, C(x) \to P(x)$

S : $\exists x, P(x) \land \sim B(x)$

Steps ① $\exists x, C(x) \land \sim B(x)$ ($\because S_1$)

② $C(a) \land \sim B(a)$ ($\because$ Existential Instantiation from ①)

③ $C(a)$ ($\because$ Law of Simplification from ②)

④ $\forall x, C(x) \to P(x)$ ($\because S_2$)

⑤ $C(a) \to P(a)$ ($\because$ Universal Instantiation from ④)

⑥ $P(a)$ ($\because$ Modus ponens from ③ + ⑤)

⑦ $\sim B(a)$ ($\because$ Simplification from ②)

⑧ $P(a) \land \sim B(a)$ ($\because$ Conjunction from ⑥ & ⑦)

⑨ $\therefore \exists x, P(x) \land \sim B(x)$. ($\because$ Existential generalization from ⑧)

Q: "Danish, a student in the class, knows how to write program in Java"
"Everyone who knows how to write programs in Java can get a high paying job"
Therefore, "Someone in this class can get a high paying job".

Sol → $P(x)$: x is in this class
$Q(x)$: x knows how to write programs in Java.
$R(x)$: x can get a high paying job

S_1 : $P(\text{Danish})$
S_2 : $Q(\text{Danish})$
S_3 : $\forall x,\ Q(x) \rightarrow R(x)$

S : $\exists x,\ P(x) \wedge R(x)$

Steps
1. $P(\text{Danish})$ $(\because S_1)$
2. $Q(\text{Danish})$ $(\because S_2)$
3. $\forall x,\ (Q(x) \rightarrow R(x))$ $(\because S_3)$
4. $Q(\text{Danish}) \rightarrow R(\text{Danish})$ ($\because$ Universal Instantiation ~~from~~ from 2 & 3)
5. $R(\text{Danish})$ ($\because$ Modus ponens from 2 & 4)
6. $P(\text{Danish}) \wedge R(\text{Danish})$ ($\because$ Conjunction from 1 & 5)
7. $\exists x,\ (P(x) \wedge R(x))$ ($\because$ Existential generalization of 6)

Q: If philosopher is not money-minded and some money-minded persons are not clever, then there are some persons who are neither philosopher nor clever.

Sol: $P(x)$: x is a philosopher

$Q(x)$: x is money-minded

$R(x)$: x is clever

S_1 : $P(x) \rightarrow \sim Q(x)$

S_2 : $\exists x,\ Q(x) \wedge \sim R(x)$

S : $\exists x,\ \sim P(x) \wedge \sim R(x)$

Steps
1. $\exists x,\ Q(x) \wedge \sim R(x)$ $(\because S_2)$
2. $Q(c) \wedge \sim R(c)$ $(\because$ Existential Instantiation of 1$)$
3. $Q(c)$ $(\because$ Simplification from 2$)$
4. ~~$\sim P(c)$ $(\because$ Modus Tollens from~~
4. $P(x) \rightarrow \sim Q(x)$ $(\because S_1)$
5. $\sim P(c)$ $(\because$ Modus Tollens from 3 & 4$)$
6. $\sim R(c)$ $(\because$ ~~Existential~~ Simplification from 2$)$
7. $\sim P(c) \wedge \sim R(c)$ $(\because$ Law of Conjunction from 5 & 6$)$
8. $\exists x,\ \sim P(x) \wedge \sim R(x)$ $(\because$ Existential Generalization of 7$)$

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Normal Form → It is of 2 types:

① Conjunctive Normal Form / ~~Sum of Products~~ (CNF)
[Products of Sums / Ands of ORs]

② Disjunctive Normal Form (DNF)
[Sum of Products / OR of Ands]

① Conjunctive Normal Form → It is a conjunction of two clauses where each clause is disjunction of two or more propositions or their negations.

Example: p, $(p \vee q) \wedge r$, $(p \vee q) \wedge (p \vee \sim r) \wedge (\sim p \vee q \vee s \vee t \vee \sim u)$, $(p \vee q) \wedge p \wedge (q \vee r) \wedge (\sim p)$ are all in CNF.

But, $(p \vee q) \wedge (p \vee \sim r) \wedge (\sim p \wedge q)$, $(p \vee q) \wedge (p \to r) \wedge (\sim p \vee q)$, $(p \vee q) \wedge (p \vee \sim r) \wedge (\sim p \vee q)$, $(p \vee (q \wedge r)) \wedge (p \vee \sim r)$ are not in CNF.

Note →

$$p \to q \equiv \sim p \vee q$$
$$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$$

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

Construction to obtain CNF or DNF →

① Eliminate '→' and '↔' using
$p \to q \equiv \sim p \vee q$, $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

② Use Demorgan's law to eliminate $\sim$ attaining before conjunction or disjunction

③ Apply disjunctive laws repeatedly to eliminate conjunction of disjunctions or disjunction of ~~cg~~ conjunctions.

Q: Write the CNF of $(p \wedge q) \vee (\sim p \wedge q \wedge r)$

Sol → $(p \wedge q) \vee (\sim p \wedge q \wedge r)$

$\equiv [p \vee (\sim p \wedge q \wedge r)] \wedge [q \vee (\sim p \wedge q \wedge r)]$ (∵ Distributivity)

$\equiv [(p \vee \sim p) \wedge (p \vee q) \wedge (p \vee r)] \wedge [(q \vee \sim p) \wedge (q \vee q) \wedge (q \vee r)]$

$\equiv [T \wedge (p \vee q) \wedge (p \vee r)] \wedge [(q \vee \sim p) \wedge q \wedge (q \vee r)]$ --- (∵ $p \vee \sim p \equiv T$)

$\equiv (p \vee q) \wedge (p \vee r) \wedge (q \vee \sim p) \wedge q \wedge (q \vee r)$

Q: $(\sim p \to r) \wedge (p \leftrightarrow q)$

$\equiv [\sim(\sim p) \vee r] \wedge [(p \to q) \wedge (q \to p)]$

$\equiv [p \vee r] \wedge [(\sim p \vee q) \wedge (\sim q \vee p)]$

$= (p \vee r) \wedge (\sim p \vee q) \wedge (\sim q \vee p)$

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(12) **Disjunctive Normal Form** → It is a disjunctive of two clauses where each clause is a conjunction of two or more propositions or their negations.

Examples: $(p \wedge q) \vee (p \wedge \sim r) \vee (\sim p \wedge q)$ is in DNF.

$(p \wedge q) \vee \sim q$ is in DNF.

Q: Write DNF of $(p \to q) \wedge (\sim p \wedge q)$

Sol: $(p \to q) \wedge (\sim p \wedge q) \equiv (\sim p \vee q) \wedge (\sim p \wedge q)$

$= (\sim p \wedge \sim p \wedge q) \vee (q \wedge \sim p \wedge q)$ (∵ Distributivity)

$= (\sim p \wedge q) \vee (q \wedge \sim p)$

Q: Find DNF of $(p \vee q) \to \sim r$

Sol: $(p \vee q) \to \sim r$

$\equiv \sim(p \vee q) \vee \sim r$ $\quad (\because p \to q \equiv \sim p \vee q)$

$\equiv (\sim p \wedge \sim q) \vee \sim r$

Q: Write CNF of $\sim(p \to q) \vee (r \to p)$.

Sol: $\sim(p \to q) \vee (r \to p)$

$\equiv \sim(\sim p \vee q) \vee (\sim r \vee p)$

$\equiv (p \wedge \sim q) \vee (\sim r \vee p)$

$\equiv (p \vee \sim r \vee p) \wedge (\sim q \vee \sim r \vee p)$ $\quad$ ($\because$ distributivity law)

Q: Write CNF of $(p \vee \sim q) \to q$

Sol: $(p \vee \sim q) \to q$

$\equiv \sim(p \vee \sim q) \vee q$

$\equiv (\sim p \wedge q) \vee q$

$\equiv (\sim p \vee q) \wedge (q \vee q)$ $\quad$ ($\because$ Distributivity law)

$(\because q \vee q \equiv q)$

$\equiv (\sim p \vee q) \wedge q$

Q: Write DNF of $p \leftrightarrow (\sim p \vee \sim q)$

Sol: $p \leftrightarrow (\sim p \vee \sim q)$

$\equiv [p \wedge (\sim p \vee \sim q)] \vee [\sim p \wedge \sim(\sim p \vee \sim q)]$

$\equiv [(p \wedge \sim p) \vee (p \wedge \sim q)] \vee [\sim p \wedge p \wedge \sim q]$ $\quad$ ($\because$ Distributivity law)

$\equiv [F \vee (p \wedge \sim q)] \vee [F \wedge \sim q]$ --- $(\because p \wedge \sim p \equiv F)$

$\equiv (p \wedge \sim q) \vee F$ --- $[\because F \wedge \sim q \equiv F]$

$\equiv (p \wedge \sim q)$

Q: Write DNF

Sol: $(p \wedge \sim(q \wedge r)) \vee (p \to q)$

$\equiv (p \wedge (\sim q \vee \sim r)) \vee (\sim p \vee q)$

$\equiv (p \wedge \sim q) \vee (p \wedge \sim r) \vee (\sim p \vee q)$

$\equiv (p \wedge \sim q) \vee (p \wedge \sim r) \vee$

CNF of X $= (\text{DNF of } \sim X)^c$

Q: Find DNF & CNF of $(\sim p \to r) \wedge (p \leftrightarrow q)$ by Truth Table

Sol:

p	q	r	$\sim p$	$\sim q$	$\sim r$	$\sim p \to r$	$p \leftrightarrow q$	$X = (\sim p \to r) \wedge (p \leftrightarrow q)$	$\sim X$
T	T	T	F	F	F	T	T	T	F
T	T	F	F	F	T	T	T	T	F
T	F	T	F	T	F	T	F	F	T
T	F	F	F	T	T	T	F	F	T
F	T	T	T	F	F	T	F	F	T
F	T	F	T	F	T	F	F	F	T
F	F	T	T	T	F	T	T	T	F
F	F	F	T	T	T	F	T	F	T

Then, the reqd. DNF is $(p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r)$

Note → ① For DNF, we go for True statements & ~~CNF is~~ for CNF, we go for False statements.

② CNF of X is negation of DNF of $\sim X$.

So, DNF of $\sim X$ is $(p \wedge \sim q \wedge r) \vee (p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r)$

& CNF of X is $(\sim p \vee q \vee \sim r) \wedge (\sim p \vee q \vee r) \wedge (p \vee \sim q \vee \sim r) \wedge (p \vee \sim q \vee r) \wedge (p \vee q \vee r)$

Introduction to Proofs →

(1) Direct Proofs $p \to q$

If p is true then q is true.

Example: Give a direct proof of the theorem. "If n is an odd integer then n^2 is odd".

Sol: $\forall n$, $P(n) \longrightarrow Q(n)$ where $P(n)$: n is odd integer
$Q(n)$: n^2 is odd

If n is odd integer then $n = 2k+1$ f.s. $k \in \mathbb{Z}$.

~~T.S. $n^2 = (2k+1)^2 = 4k^2+4k+1$~~

~~To show: $n^2 = (2k+1)^2 = 4k^2+4k+1 = 2(2k^2+2k)+1$ is odd~~

Consider, $n^2 = (2k+1)^2$
$= 4k^2+4k+1$
$= 2(2k^2+2k)+1$, which is an odd integer by the defn of odd integer.

Thus, n^2 is odd.

The integer n is even if $\exists$ an integer k s.t. $n = 2k$
& n is odd if $\exists$ " " " s.t. $n = 2k+1$

Two integers have the same parity when both are even or both are odd; they have opposite parity when one is even and other is odd.

An integer a is a perfect square if $\exists$ an integer b s.t. $a = b^2$.

Q: Give a direct proof that if m and n are both perfect squares, then nm is also a perfect square.

Sol: If m and n are both perfect squares then $\exists$ integers m_1, n_1 s.t. $m = m_1^2$, $n = n_1^2$

Consider, $mn = m_1^2 n_1^2 = (m_1 m_1)(n_1 n_1)$ [$\because$ Commutativity & Associativity of multiplication]

$= (m_1 n_1)(m_1 n_1)$

$= (m_1 n_1)^2$ where $m_1 n_1 \in \mathbb{Z}$

Thus, mn is a perfect square.

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② Proof by Contraposition →

$$p \to q \equiv \sim q \to \sim p$$

Q: Prove that if n is an integer and $3n+2$ is odd then n is odd.

Sol: Ist we attempt a direct proof. To construct a direct proof, we first assume that $3n+2$ is an odd integer.

$3n+2 = 2k+1$ f.s. $k \in \mathbb{Z}$

$\Rightarrow 3n+2$

$\Rightarrow 3n+1 = 2k$

but there doesn't seem to be any direct way to conclude that n is odd. Because our attempt at a direct proof failed, we next try a proof by contraposition.

Now, we'll do it by contraposition.

i.e, ~~if~~ Assume that n is even. Then, by def^n of an even integer, $n = 2k$ f.s. $k \in \mathbb{Z}$.

Consider, $3n+2 = 3(2k)+2 = 6k+2 = 2(3k+1)$.

This tells us that $3n+2$ is even, and therefore not odd.

(Hence proved)

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Q: Prove that if $n = ab$ where $a, b \in \mathbb{Z}^+$ then $a \le \sqrt{n}$ or $b \le \sqrt{n}$.

Sol: We'll do it by contraposition.

Assume, $(a \le \sqrt{n}) \vee (b \le \sqrt{n})$ is false.

$\Rightarrow$ ~~$a \ne \sqrt{n}$~~ $(a > \sqrt{n})$ and $(b > \sqrt{n})$ are true.

$\Rightarrow ab > \sqrt{n} \cdot \sqrt{n} = n$ $\quad [\because$ if $0 < s < t$, $0 < u < v$ then $su < tv]$

~~ab~~ This shows $ab \ne n$.

(Hence proved)

Q Prove that if n is an integer & n^2 is odd, then n is odd.

Sol: If we do it by direct way.

say if n^2 is odd then $n^2 = 2k+1$ f.s. $k \in \mathbb{Z}$

$\Rightarrow n = \pm\sqrt{2k+1}$ which we can't say about that n is odd.

So, we'll do it by contrapositive.

If n is even then $n = 2k$ f.s. $k \in \mathbb{Z}$.

$\Rightarrow n^2 = 4k^2 = 2(2k^2)$ is an even. (Hence proved)

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③ Proof by Contradiction →

$\sim p \rightarrow (r \wedge \sim r)$ is true.

Q: Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

Sol: Let p: $\sqrt{2}$ is irrational.

If $\sqrt{2}$ is rational then $\exists$ $a, b \in \mathbb{Z}$ s.t. $\sqrt{2} = \frac{a}{b}$,

where $b \neq 0$ and $\gcd(a,b) = 1$

$\Rightarrow 2 = \frac{a^2}{b^2}$

$\Rightarrow a^2 = 2b^2$

By the def^n of even integers it follows that a^2 is even.

If a^2 is even then a must also be an ~~int~~ even.

$\Rightarrow a = 2c$ f.s. $c \in \mathbb{Z}$.

$\Rightarrow 4c^2 = 2b^2$

$\Rightarrow b^2 = 2c^2$

By the def^n of even, b^2 is even $\Rightarrow$ b is even. ~~which is a contradiction~~

So, $\gcd(a,b) = 2$ which is a contradiction

i.e. $2 \mid a$ and $2 \mid b$

~~to~~ the fact that $\gcd(a,b) = 1$.

Thus, $\sqrt{2}$ is irrational.

Q: By contradiction,
if $3n+2$ is odd then n is odd.

Sol: Let p: $3n+2$ is odd
q: n is odd

Assume that n is not odd
i.e. n is even $\Rightarrow n = 2k$ f.s. $k \in \mathbb{Z}$
$\Rightarrow 3n+2 = 3 \cdot 2k + 2 = 2(3k+1)$

So, $3n+2$ is an even integer which is a contradiction to $3n+2$ is odd.

Thus, n is odd.

Q: If n is integer then n is odd iff n^2 is odd.

Sol: Let p: n is odd,
q: n^2 is odd.

T.S.: $p \leftrightarrow q$ i.e. $p \to q$ and $q \to p$.

We have shown $p \to q$ and $q \to p$ in previous examples.
— (Hence proved)