Lecture :- 6

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Normal Form :- It is of 2 types.

Conjunctive Normal Form :- It is a conjunction of 2 clauses where each clause is disjunction of 2 or more propositions or their negations.

AND of ORs

OR

It is a compound proposition obtained by ANDing together ORs of one or more propositions or their negations

eg :- p, $(p \vee q) \wedge r$, $(p \vee q) \wedge (q \wedge r) \wedge (\sim p)$, $(p \vee q) \wedge (p \vee \sim r) \wedge (\sim p \vee q \vee s \vee t \vee \sim u)$ are all in CNF.

But, $(p \vee q) \wedge (p \vee \sim r) \wedge (\sim p \wedge q)$,
$(p \vee q) \wedge (p \rightarrow r) \wedge (\sim p \vee q)$,
$(p \vee (q \wedge r)) \wedge (p \vee \sim r) \wedge (\sim p \vee q)$ are not in CNF.

* Using the equivalence $p \rightarrow q \equiv \sim p \vee q$, $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$ and Demorgan's laws we can convert/write a proposition to/in a CNF.

Construction to obtain CNF or DNF :-

1) Eliminate $\rightarrow$ + $\leftrightarrow$ using
$p \rightarrow q \equiv \sim p \vee q$ and $p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$
2) Use Demorgan's law to eliminate $\sim$ appearing before conjunction or disjunction.
3) Apply distributive laws repeatedly to eliminate conjunction of disjunctions or disjunction of conjunctions.

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(37) eg:- Write/obtain the CNF of

$(p \wedge q) \vee (\sim p \wedge q \wedge r)$

sol^n $(p \wedge q) \vee (\sim p \wedge q \wedge r)$

$\equiv (p \vee (\sim p \wedge q \wedge r)) \wedge (q \vee (\sim p \wedge q \wedge r))$

(By distributivity).

And of ORs

$\equiv [(p \vee \sim p) \wedge (p \vee q) \wedge (p \vee r)] \wedge [(q \vee \sim p) \wedge (q \vee q) \wedge (q \vee r)]$

$\equiv [T \wedge (p \vee q) \wedge (p \vee r)] \wedge [(q \vee \sim p) \wedge q \wedge (q \vee r)]$

$\equiv (p \vee q) \wedge (p \vee r) \wedge (q \vee \sim p) \wedge q \wedge (q \vee r)$

eg:- Write/obtain CNF of

$(\sim p \leftrightarrow r) \wedge (p \leftrightarrow q)$

sol^n $(\sim p \rightarrow r) \wedge (p \leftrightarrow q)$

And of ORs

$\equiv (\sim p \rightarrow r) \wedge [(p \rightarrow q) \wedge (q \rightarrow p)]$

$\equiv (p \vee r) \wedge [(\sim p \vee q) \wedge (\sim q \vee p)]$

$\equiv (p \vee r) \wedge (\sim p \vee q) \wedge (\sim q \vee p)$

<u>Disjunctive Normal Form</u>:- It is a disjunction of 2 clauses where each clause is a conjunction of 2 or more propositions or their negations.

OR of ANDs

eg:- $(p \wedge q) \vee (p \wedge \sim r) \vee (\sim p \wedge q)$ is in DNF.

$(p \wedge q) \vee \sim q$ is in DNF

* Just like CNF; any proposition can be written in its DNF.

Here also we use

$$\boxed{\begin{aligned} p \rightarrow q &\equiv \sim p \vee q \\ p \leftrightarrow q &\equiv (p \wedge q) \vee (\sim p \wedge \sim q) \end{aligned}}$$

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eg:- obtain the DNF of
$(p \to q) \wedge (\sim p \wedge q)$

OR of ANDs

Sol^n
$(p \to q) \wedge (\sim p \wedge q)$

$\equiv (\sim p \vee q) \wedge (\sim p \wedge q)$ } distributivity

$\equiv (\sim p \wedge \sim p \wedge q) \vee (q \wedge \sim p \wedge q)$

$\equiv (\sim p \wedge q) \vee (q \wedge \sim p)$

eg:- Write the following in its DNF
$(p \wedge \sim (q \wedge r)) \vee (p \to q)$

?

Doubt

Sol^n $(p \wedge \sim (q \wedge r)) \vee (p \to q)$

$\equiv (p \wedge (\sim q \vee \sim r)) \vee (\sim p \vee q)$

$\equiv (p \wedge \sim q) \vee (p \wedge \sim r) \vee (\sim p \vee q)$

$\equiv (p \wedge \sim q) \vee (p \wedge \sim r) \vee (\sim p$

Sometimes, it is easier to obtain CNF & DNF from Truth table.

eg:- Find DNF of $(\sim p \to r) \wedge (p \leftrightarrow q)$ by truth table

(34) Sol^n:-

	p	q	r	$\sim p$	$\sim p \to r$	$p \leftrightarrow q$	$(\sim p \to r) \wedge (p \leftrightarrow q) = X$	$\sim X$
1)	T	T	T	F	T	T	T	F
2)	T	T	F	F	T	T	T	F
	T	F	T	F	T	F	F	T
	T	F	F	F	T	F	F	T
	F	T	T	T	T	F	F	T
	F	T	F	T	F	F	F	T
3)	F	F	T	T	T	T	T	F
	F	F	F	T	F	T	F	T

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the required DNF is
$(p \wedge q \wedge r) \vee (p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge r)$

* For DNF we go for True Statements & for CNF " " " False " .

* CNF of X is negation of DNF of ~X.

DNF of ~X is
$(p \wedge \sim q \wedge r) \vee (p \wedge \sim q \wedge \sim r) \vee (\sim p \wedge q \wedge r) \vee (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge \sim q \wedge \sim r)$

⇒ CNF of X is
$(\sim p \vee q \vee \sim r) \wedge (\sim p \vee q \vee r) \wedge (p \vee \sim q \vee \sim r) \wedge (p \vee \sim q \vee r) \wedge (p \vee q \vee r)$

Remark:- $p, \sim p, p \vee q, p \wedge q, p \vee q \vee r, p \wedge q \wedge \sim r$ are CNF & DNF both.

35 eg:- Find the DNF of $(p \vee q) \rightarrow \sim r$.
sol^n $(p \vee q) \rightarrow \sim r$

$\equiv \sim(p \vee q) \vee \sim r$ $\quad (\because p \rightarrow q \equiv \sim p \vee q)$

OR of ANDs

$\equiv (\sim p \wedge \sim q) \vee \sim r$

This is the required DNF.

36 eg:- Write in CNF ; $\sim(p \rightarrow q) \vee (r \rightarrow p)$.
sol^n:- $\sim(p \rightarrow q) \vee (r \rightarrow p)$

AND of ORs

$\equiv \sim(\sim p \vee q) \vee (\sim r \vee p)$
$\equiv (p \wedge \sim q) \vee (\sim r \vee p)$ } distributive law
$\equiv (p \vee \sim r \vee p) \wedge (\sim q \vee \sim r \vee p)$

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38 eg:- Write the CNF of $(p \vee \sim q) \rightarrow q$.

sol^n $(p \vee \sim q) \rightarrow q$

$\equiv \sim (p \vee \sim q) \vee q$

$\equiv (\sim p \wedge q) \vee q$

AND of ORs

$\equiv (\sim p \vee q) \wedge (q \vee q)$

$\equiv (\sim p \vee q) \wedge q$

35 eg:- Write the DNF of $p \leftrightarrow (\sim p \vee \sim q)$.

sol^n:- $p \leftrightarrow (\sim p \vee \sim q)$

$\equiv (p \wedge (\sim p \vee \sim q)) \vee (\sim p \wedge \sim (\sim p \vee \sim q))$

$\left\{ \because p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \vee \sim q) \right.$

$\equiv ((p \wedge \sim p) \vee (p \wedge \sim q)) \vee (\sim p \wedge (p \wedge q))$

OR of Ands

$\equiv (p \wedge \sim p) \vee (p \wedge \sim q) \vee F$

$\equiv (p \wedge \sim p) \vee (p \wedge \sim q)$