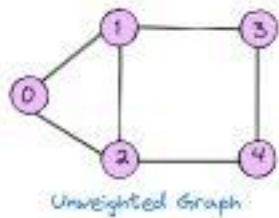


Graph Representation

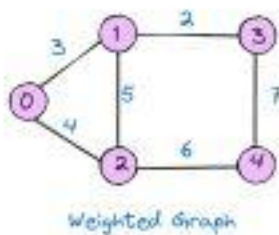
Adjacency Matrix



Vertex : {0,1,2,3,4} → 5
Edge : {(0,1),(1,2),(0,2),(2,4),(3,4),(1,3)}

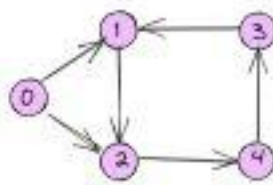
	0	1	2	3	4
0	0	1	1	0	0
1	1	0	1	1	0
2	1	1	0	0	1
3	0	1	0	0	1
4	0	0	1	1	0

2D Array



	0	1	2	3	4
0	0	3	4	0	0
1	3	0	5	2	0
2	4	5	0	0	6
3	0	2	0	0	7
4	0	0	6	7	0

2D Array



	0	1	2	3	4
0	0	1	1	0	0
1	0	0	1	0	0
2	0	0	0	0	1
3	0	1	0	0	0
4	0	0	0	1	0

2D Array

Similarly Weighted Directed Graph

CODE PART

TC - $O(V^2)$
SC - $O(V^2)$

```
1 #include<iostream>
2 #include<vector>
3 using namespace std;
4
5 // Adjacency Matrix
6 // Undirected Unweighted Graph
7 int main()
8 {
9     int vertex, edges;
10    cin >> vertex >> edges;
11
12    vector<vector<int>>> AdjMat(vertex, vector<int>(vertex,0));
13    int u,v;
14    for(int i=0;i<edges;i++)
15    {
16        cin >> u >> v;
17        AdjMat[u][v] = 1;
18        AdjMat[v][u] = 1;
19    }
20
21    for(int i=0;i<vertex;i++)
22    {
23        for(int j=0;j<vertex;j++)
24            cout<<AdjMat[i][j]<<" ";
25        cout<<endl;
26    }
27 }
```

Undirected Unweighted Graph

```
1 vector<vector<int>>> AdjMat(vertex, vector<int>(vertex,0));
2 int u,v;
3 for(int i=0;i<edges;i++)
4 {
5     cin >> u >> v;
6     AdjMat[u][v] = 1;
7     AdjMat[v][u] = 1;
8 }
```

Directed Unweighted Graph

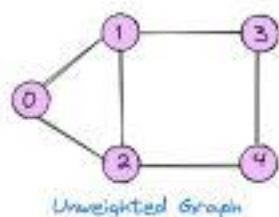
```
1 vector<vector<int>>> AdjMat(vertex, vector<int>(vertex,0));
2 int u,v,weight;
3 for(int i=0;i<edges;i++)
4 {
5     cin >> u >> v >> weight;
6     AdjMat[u][v] = weight;
7     AdjMat[v][u] = weight;
8 }
```

Undirected Weighted Graph

```
1 vector<vector<int>>> AdjMat(vertex, vector<int>(vertex,0));
2 int u,v,weight;
3 for(int i=0;i<edges;i++)
4 {
5     cin >> u >> v >> weight;
6     AdjMat[u][v] = weight;
7 }
```

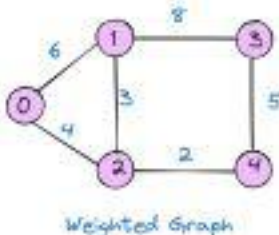
Directed Weighted Graph

Adjacency list



```
vector<int> AdjList[5]
AdjList[0].push_back(1)
AdjList[1].push_back(0)
```

	0	1	2	3	4
0	1 2				
1	0 3 2				
2	0 4 1				
3	1 4				
4	2 3				



	0	1	2	3	4
0	(1,6) (2,4)				
1	(0,6) (3,8) (2,3)				
2	(0,4) (4,2) (1,3)				
3	(1,8) (4,5)				
4	(2,2) (3,5)				

Which is Better

CODE PART

TC - $O(V+E)$
SC - $O(V+E)$

Worst Case

TC - $O(V^2)$
SC - $O(V^2)$

$E = V^2$ - Complete Graph

```
1 #include<iostream>
2 #include<vector>
3 using namespace std;
4
5 int main()
6 {
7     int vertex, edges;
8     cin >> vertex >> edges;
9     vector<pair<int,int>> AdjList[vertex];
10    int u,v,weight;
11    for(int i=0;i<edges;i++)
12    {
13        cin >> u >> v >> weight;
14        AdjList[u].push_back(make_pair(v,weight));
15        AdjList[v].push_back(make_pair(u,weight));
16    }
17
18    for(int i=0;i<vertex;i++)
19    {
20        cout<<"> ";
21        for(int j=0;j<AdjList[i].size();j++)
22            cout<<AdjList[i][j].first<<" "<<AdjList[i][j].second<<" ";
23        cout<<endl;
24    }
25 }
```

```
1 #include<iostream>
2 #include<vector>
3 using namespace std;
4
5 int main()
6 {
7     int vertex, edges;
8     cin >> vertex >> edges;
9     vector<list<pair<int,int>>> AdjList(vertex);
10    int u,v,weight;
11    for(int i=0;i<edges;i++)
12    {
13        cin >> u >> v >> weight;
14        AdjList[u].push_back(make_pair(v,weight));
15        AdjList[v].push_back(make_pair(u,weight));
16    }
17
18    for(int i=0;i<vertex;i++)
19    {
20        cout<<"> ";
21        for(int j=0;j<AdjList[i].size();j++)
22            cout<<AdjList[i][j].first<<" "<<AdjList[i][j].second<<" ";
23        cout<<endl;
24    }
25 }
```

Key Differences Between Graph and Tree

- Cycles:** Graphs can contain cycles, while trees cannot.
- Connectivity:** Graphs can be disconnected (i.e., have multiple components), while trees are always connected.
- Hierarchy:** Trees have a hierarchical structure, with one vertex designated as the root. Graphs do not have this hierarchical structure.
- Applications:** Graphs are used in a wide variety of applications, such as social networks, transportation networks, and computer science.

E.g. - Directed graph

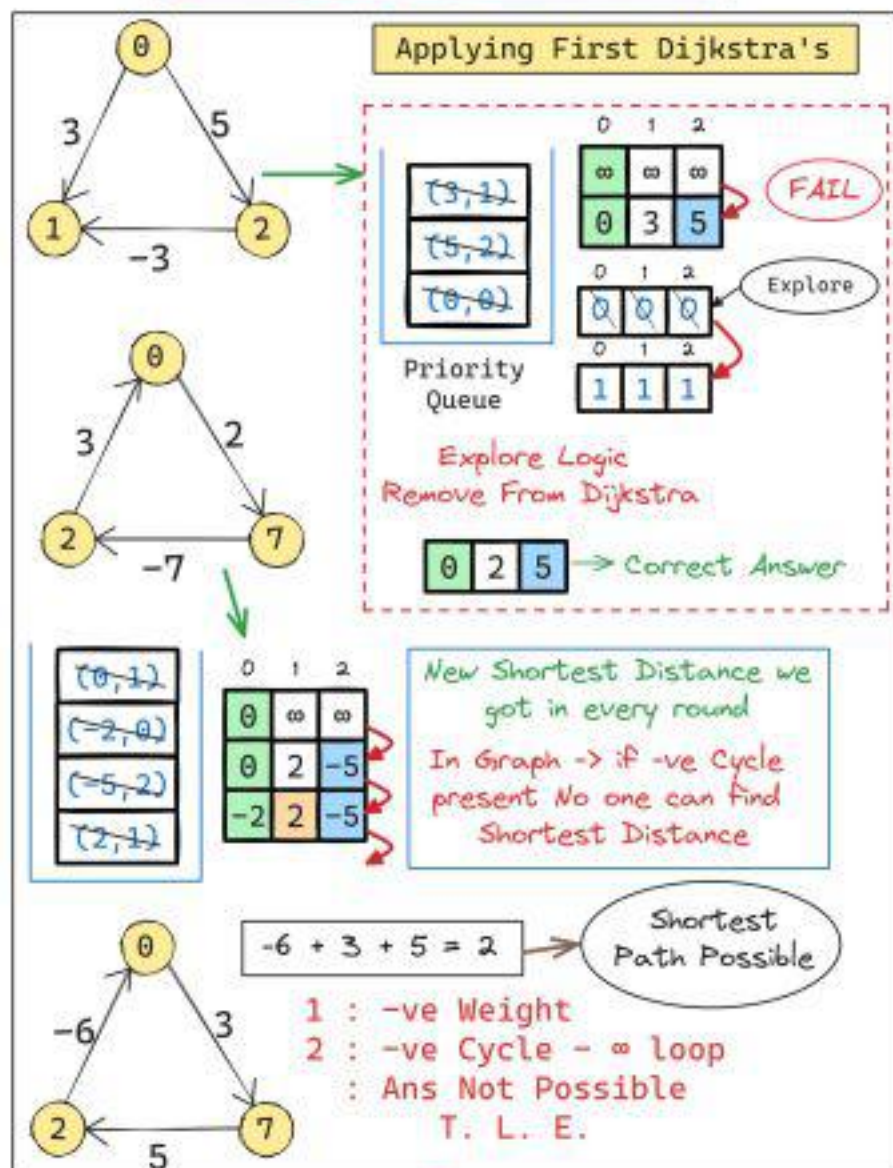
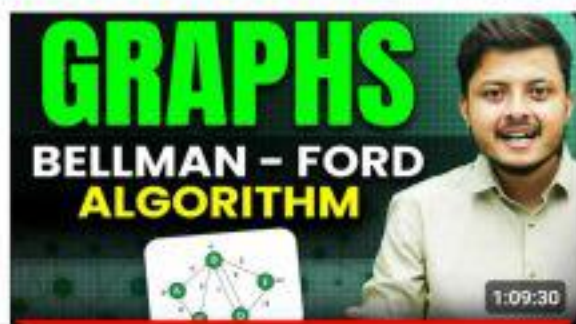


- Indegree(0) = 0
- Indegree(1) = 1
- Indegree(2) = 1
- Indegree(3) = 3
- Indegree(4) = 2
- Outdegree(0) = 3
- Outdegree(1) = 2
- Outdegree(2) = 1
- Outdegree(3) = 1
- Outdegree(4) = 0

Facebook - Sparse Graph → Adjacency List

#60daysofcode -> Day 14/60

Lecture - 145



```
vector<int> bellman_ford(int V,
vector<vector<int>>& edges, int S) {
    vector<int> dist(V, 1e8);
    // 1e8 10 power 8
    dist[S] = 0;
    int e = edges.size();

    for(int i=0; i<V-1; i++){
        // Relax all the edges
        bool flag = 0;
        for(int j=0; j<e; j++){
            int u = edges[j][0];
            int v = edges[j][1];
            int wt = edges[j][2];

            if(dist[u] == 1e8) continue;
            if(dist[u] + wt < dist[v]){
                flag = 1;
                dist[v] = dist[u] + wt;
            }
        }
        if(!flag) return dist;
    }

    // To deduct the cycle
    for(int j=0; j<e; j++){
        int u = edges[j][0];
        int v = edges[j][1];
        int wt = edges[j][2];

        if(dist[u] == 1e8) continue;
        if(dist[u] + wt < dist[v]){
            // cycle deducted
            vector<int> ans;
            ans.push_back(-1);
            return ans;
        }
    }
    return dist;
}
```

Optimized Code & Edge Cases Handle

Time Complexity :
Worst Case -> $V \times E$
Best Case -> E
Space Complexity :
Space -> V

Undirected Graph
-> Dijkstra ✓
-> Bellman ✗

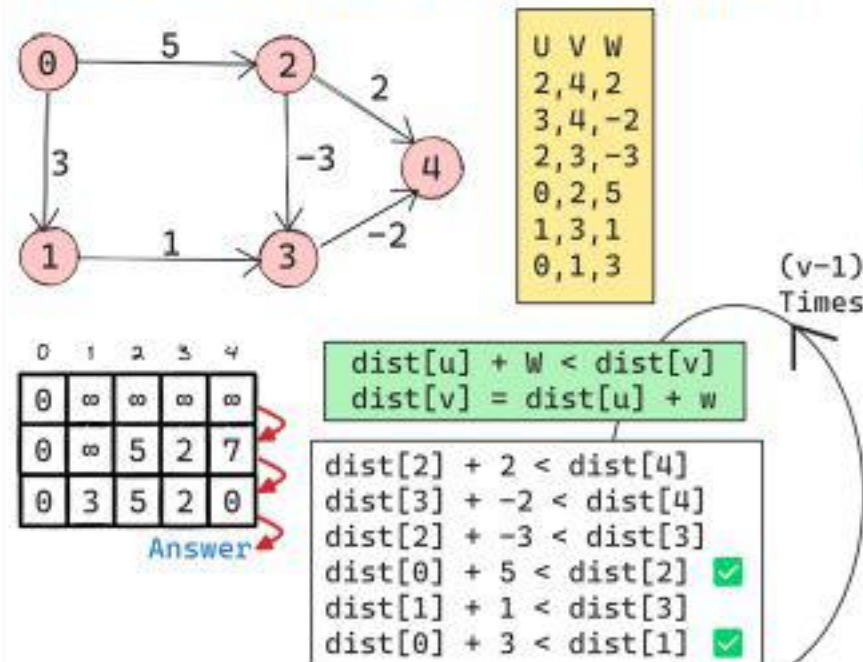
Undirected Graph with -ve Weight
-> Dijkstra ✗
-> Bellman ✗

Bellman Ford Algorithm

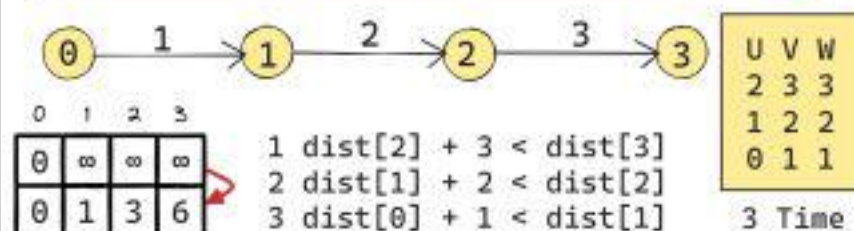
- Help In :
- Only For Directed Graph
 - If -ve Cycle present in Graph - Ans Not Found Else Ans Found

Working :

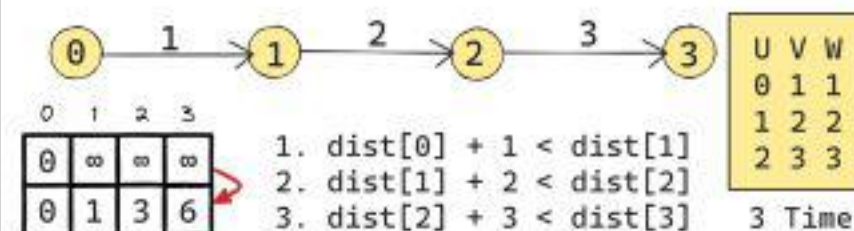
- (Relax your all edges) * (v-1) times
→ $dist[u] + wt < dist[v]$
→ $dist[v] = dist[u] + wt$
- Relax Your Edges one more to Detect Cycle



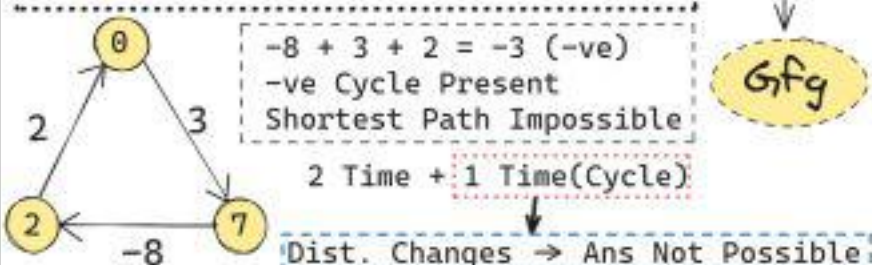
Why This Algorithm Working - Deep Dive



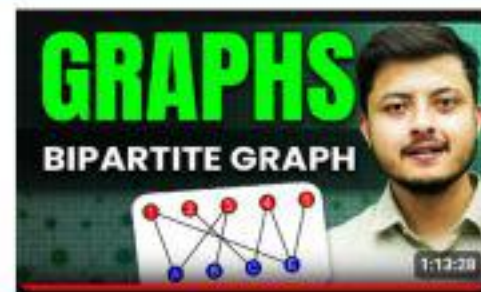
Vertex -> Dijkstra -> infinite
Edges -> Bellman -> Fixed



You have relaxed all the edges and if there is no change in distance Then This is Answer no need to move forward and relax edges

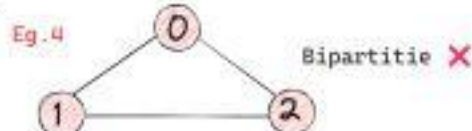
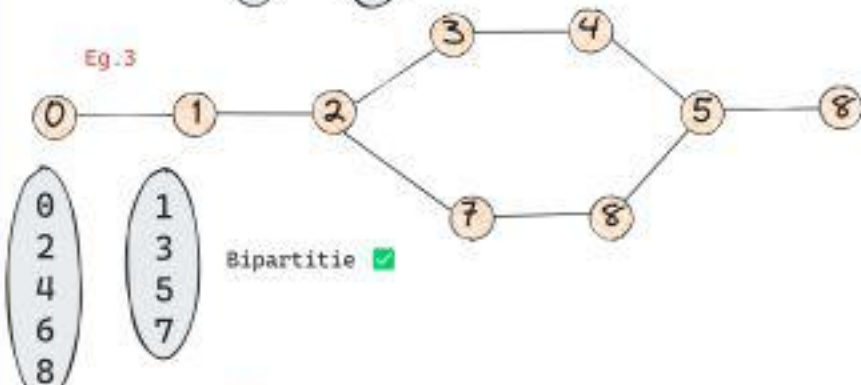
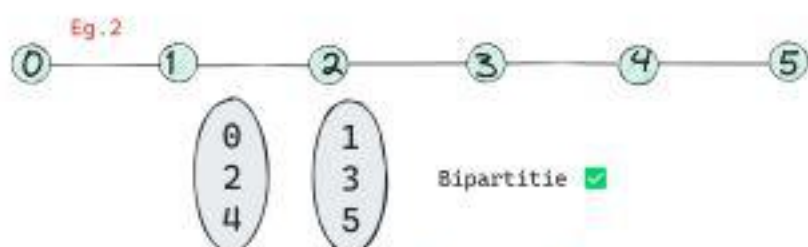
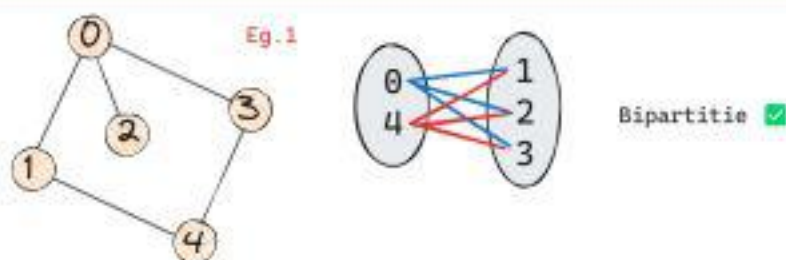


#60daysofcode Day 7 Lecture - 138



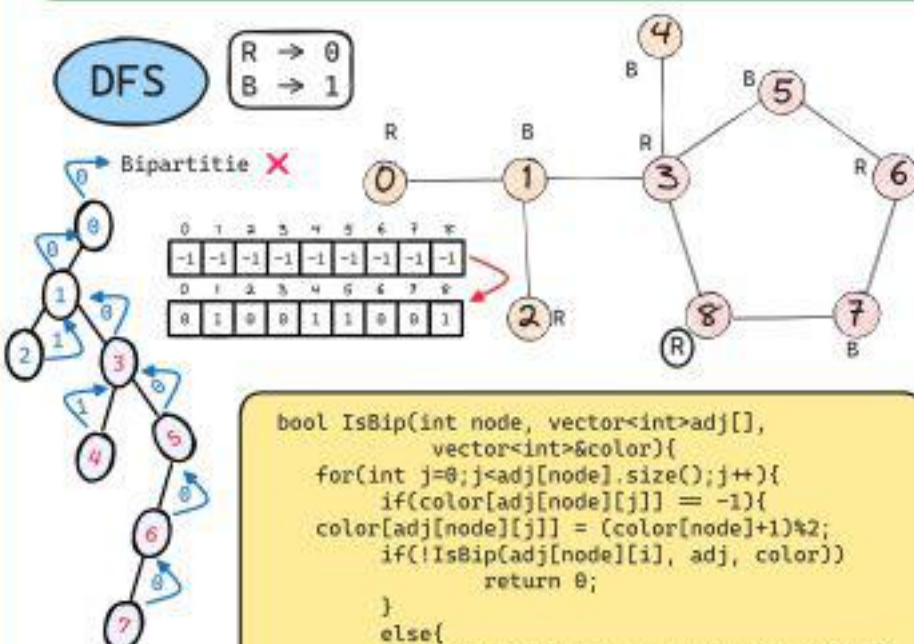
BIPARTITE GRAPH | BFS + DFS

It is a graph in which the vertices can be divided into two disjoint sets, such that no 2 vertices within the same set are adjacent. In other words, it is a graph in which every edge connects a vertex of one set to a vertex of other set.



DFS

R → 0
B → 1



```
bool IsBip(int node, vector<int>adj[],
vector<int>&color){
    for(int j=0;j<adj[node].size();j++){
        if(color[adj[node][j]] == -1){
            color[adj[node][j]] = (color[node]+1)%2;
            if(!IsBip(adj[node][j], adj, color))
                return 0;
        }
        else{
            if(color[node]==color[adj[node][j]])
                return 0;
        }
    }
    return 1;
}
```

TC - $O(V+E)$
SC - $O(V)$

BFS

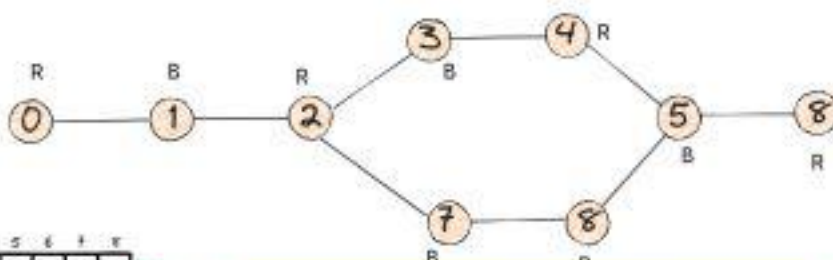
R → 0
B → 1

Queue

Color	0	1	2	3	4	5	6	7	8
0	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	0	1	0	0	1	1	0	0	1

CODE PART

TC - $O(V+E)$
SC - $O(V)$

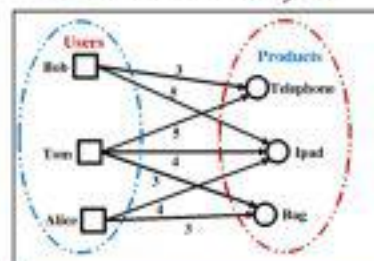


Steps

1. It will look at its all neighbour
 - If Color is not assigned, assign a color to them (Khud se opposite)
 - Push the neighbour into queue
 - Else → Color is already assigned
 - if neighbour color is same as present node we will declare it as not a bipartite graph.

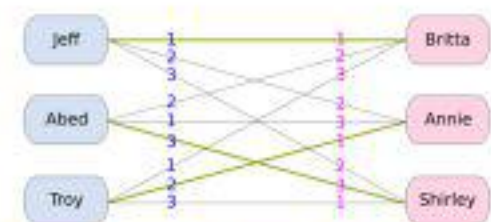
Real Life Example

1. Recommendation System



u1 → p1 p2 : p3 recommend
u2 → p1 p2 p3
u3 → p2 p3 : p1 recommend

2. Stable Marriage Problem

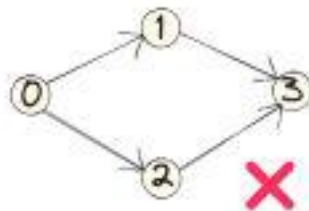
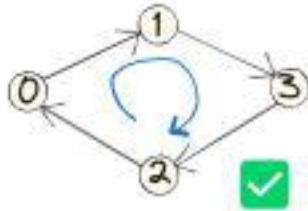
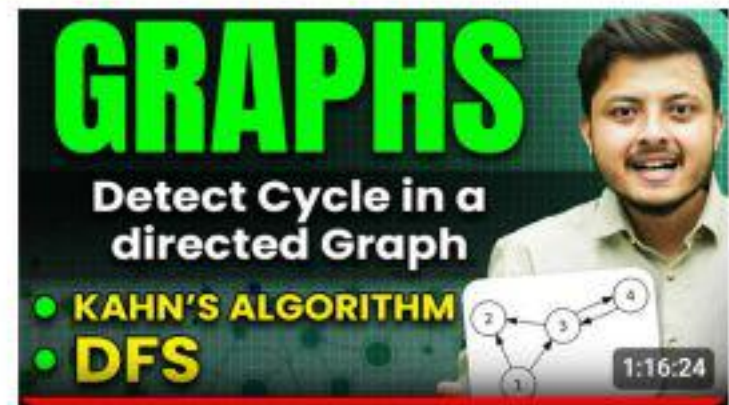


On the whole, bipartite graphs prove themselves in different fields, especially in matching problems, recommendation systems, social networks, and so on.

Bipartite graphs that capture relationships between various entities in an efficacious way become the essential tool for making decision and analysis, thus, they are essential in diverse real-life applications.

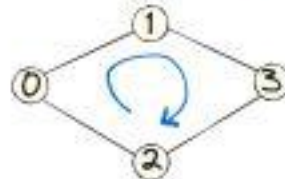
#60daysofcode
Day 6
Lecture - 137

Cycle Detection in an directed Graph (Bfs+Dfs)

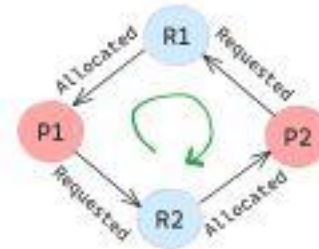


Revision of Undirected

if There is node, if its neighbour is already visited and it's not a parent. Cycle ✓



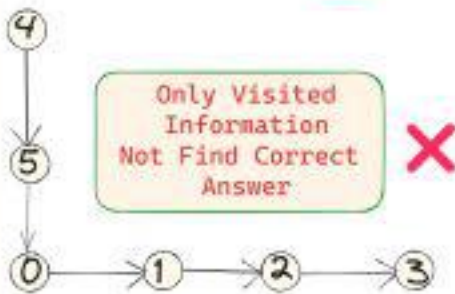
Real Life Example



Deadlock
→ Mutual Exclusion
→ Circular Wait
→ Hold And Wait
→ No Preemption

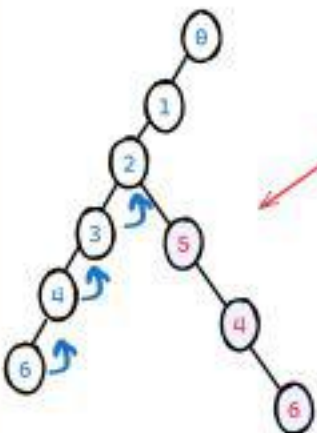
Banker's Algorithm Also Used to Detect Deadlock

Detect And Recover



Only Visited Information Not Find Correct Answer

0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	0	1	0



Visited Only Information

Cycle ✓

If a Node Appeared more than once in a path, then we will call it Cycle

Cycle X

CODE PART



if(visited[adj[node][j]]) continue;

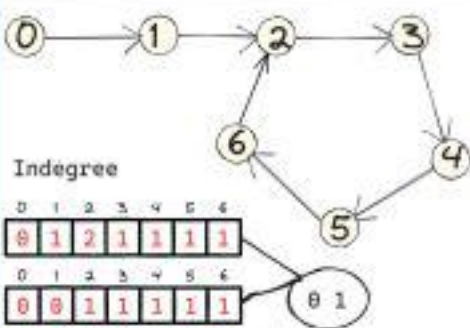
Optimized

TC - $O(V+E)$
SC - $O(V)$

```
bool DetectCycle(int Node, vector<int> adj[i],
vector<bool> &path, vector<bool> &visited ){
    path[node] = 1, visited[node] = 1;
    for(int j=0; j<adj[node].size(); j++){
        if(path[adj[node][j]])
            return 1;
        if(DetectCycle(adj[node][j], adj, path, visited))
            return 1;
    }
    path[node] = 0;
    return 0;
}
```

Not Optimized Code

BFs(Khan's Algorithm)



Indegree

0	1	2	3	4	5	6
0	1	2	1	1	1	1
0	0	1	1	1	1	1

DAG : Count == V
DCG : Count != V

HW : Topological Sort - Dfs ??

CODE PART

TC - $O(V+E)$
SC - $O(V)$

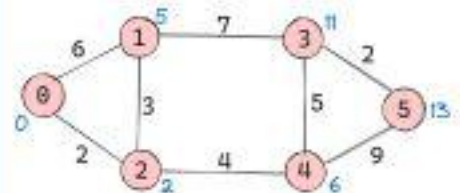


```
bool isCyclic(int V, vector<int> adj[]){
    vector<int> InDeg(V, 0);
    for(int i=0; i<V; i++){
        for(int j=0; j<adj[i].size(); j++){
            InDeg[adj[i][j]]++;
        }
    }
    queue<int> q;
    for(int i=0; i<V; i++){
        if(!InDeg[i])
            q.push(i);
    }
    int count = 0;
```

```
while(!q.empty()){
    int node = q.front();
    q.pop();
    count++;
    for(int j = 0; j<adj[node].size(); j++){
        InDeg[adj[node][j]]--;
        if(!InDeg[adj[node][j]])
            q.push(adj[node][j]);
    }
}
return count == V;
```


Dijkstra's Algorithm

Single Source Shortest Path Algorithm



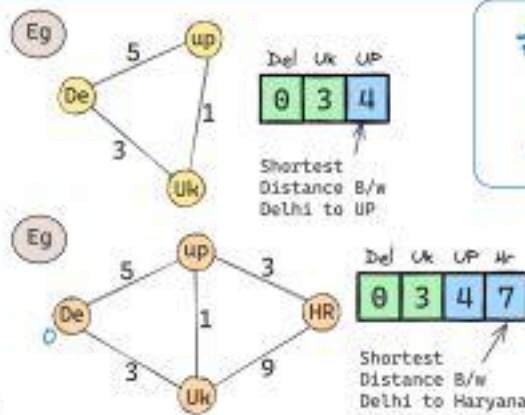
Vector = vector<int> adj[N];

	0	1	2	3	4	5
0	∞	6	2	∞	∞	∞
1	6	∞	3	7	∞	∞
2	2	3	∞	4	9	∞
3	∞	7	4	∞	5	2
4	∞	∞	9	5	∞	6
5	∞	∞	∞	2	6	∞

Explore

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	1	1	1	1	1	1

0 → {1,6}
 1 → {8,6}
 2 → {8,2}
 3 → {1,7}
 4 → {2,4}
 5 → {3,2}

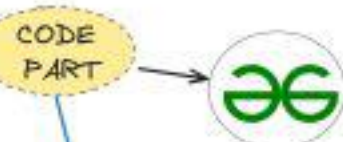


#60daysofcode
 Day 12/60
 Lecture - 143



M 1 Vs M 2	Method 1	Method 2
1	$T_c \rightarrow V^2$	$TC \rightarrow E \log v$
2	Dense Graph	Sparse Graph $E = V$
3	Minimum used	Maximum Time Used
4	Rarely Implement in Real Life	Real Life Applications Google Maps
Dijkstra's Not Works with -ve Weight		

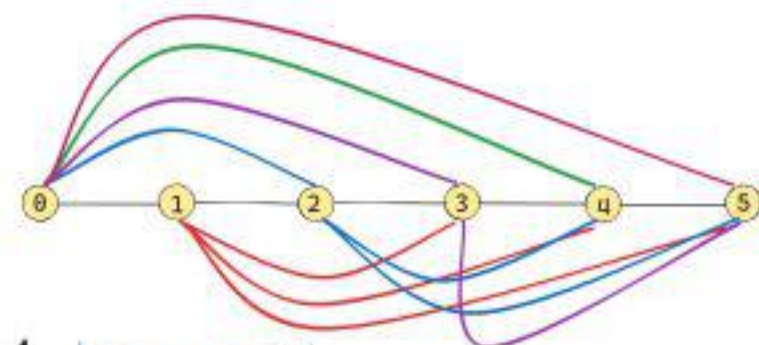
TC → $V + (V-1) = V * V = O(V^2)$
 SC → $O(V)$



OPTIMIZATION
 Decrease TC

Pseudo Code

1. Initialize Priority Queue with (0,5)
 dist[S] = 0;
 pair<int,int>
 2. While(priorityQueue is not empty)
 {
 Information of node
 if node is explored → Ignored
 else explore[node] = 1
 3. Relax the edges:
 → Look at your all the unexplored Neighbour
 → if(dist[node]+weight<dist[neighbour])
 dist[neighbour] = dist[node] + weight
 q.push(dist, neighbour);



5	5	5	5	5
4	4	4	4	4
3	3	3	3	3
2	2	2	2	2
1	1	1	1	1

Priority Queue
 MinHeap

$(n-2) + (n-3) + (n-4) + \dots$

$n(c,2) = \text{Edges}$

$$\frac{(n-2)(n-1)}{2} = \frac{(n)(n-1)}{2}$$

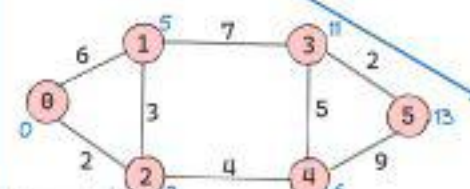
Maximum Size = Number of Edges

if Complete Graph TC → $E \log v$

TC → $E \log E$ (push) + $E \log E$ (pop) = $E \log E$
 SC → $V(E) + V(D) + \text{Maxsize of PQ}(E) = O(V+E)$

Check By Complete Graph

METHOD - 2



Priority Queue MinHeap

(7,2)
(7,5)
(11,2)
(7,5)
(8,4)
(8,2)
(5,2)
(2,2)

Used To Select Min Element

	0	1	2	3	4	5
0	∞	6	2	∞	∞	∞
1	6	∞	3	7	∞	∞
2	2	3	∞	4	9	∞
3	∞	7	4	∞	5	2
4	∞	∞	9	5	∞	6
5	∞	∞	∞	2	6	∞

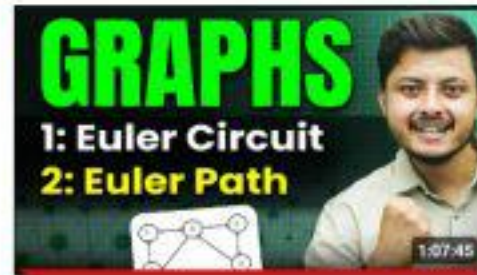
Explore

	0	1	2	3	4	5
0	0	0	0	0	0	0
1	1	1	1	1	1	1

Priority Queue
 MinHeap

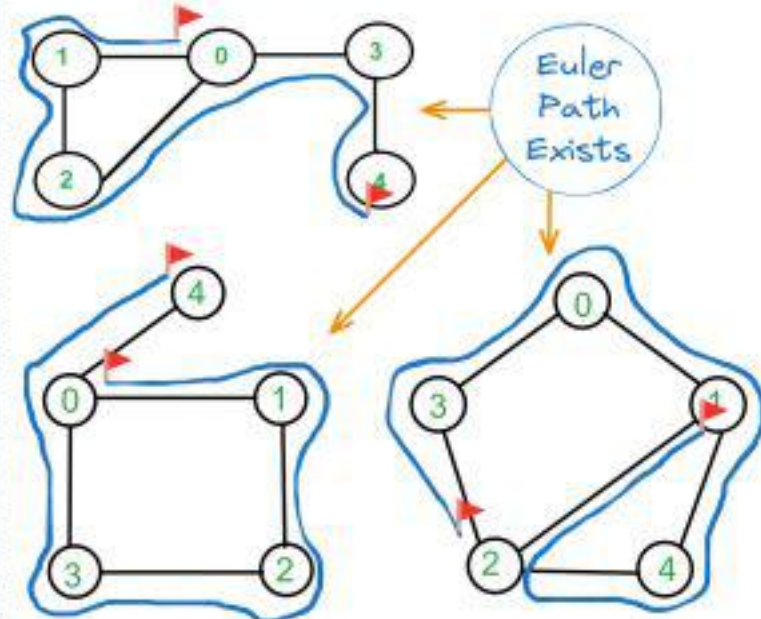
#60daysofcode -> Day 17/60

Lecture - 148



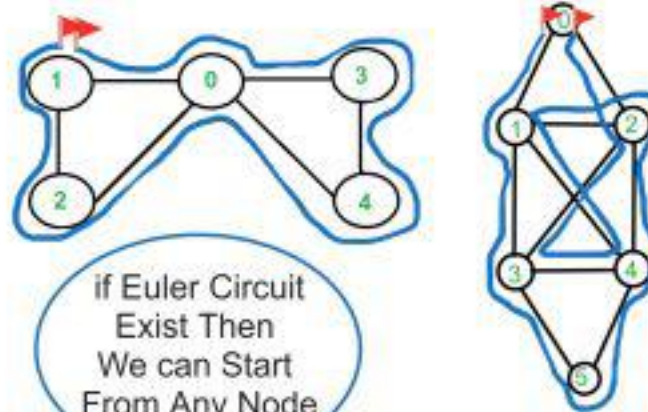
Euler Path And Euler Circuit

Eulerian Path is a path in a graph that visits every edge exactly once. Eulerian Circuit is an Eulerian Path that starts and ends on the same vertex.

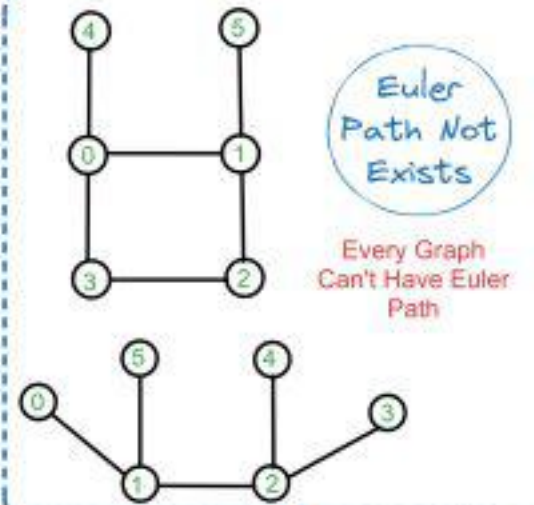


Euler Circuit

Starting & End Point Must be Same



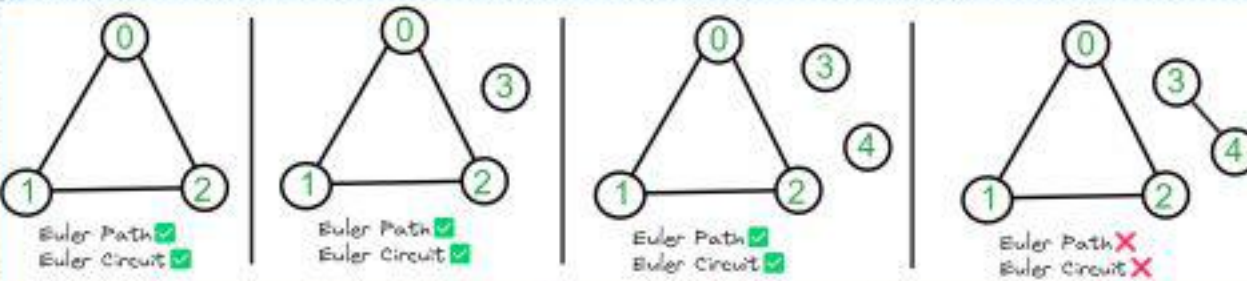
If Euler Circuit Exist Then We can Start From Any Node



Euler Path Not Exists

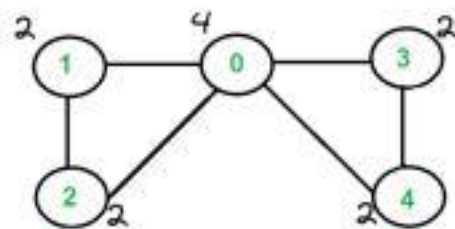
Every Graph Can't Have Euler Path

```
class Solution {
public:
    void DFS(int node, vector<int> adj[], vector<bool> &visited){
        visited[node] = 1;
        for(int j=0; j<adj[node].size(); j++){
            if(!visited[adj[node][j]])
                DFS(adj[node][j], adj, visited);
        }
    }
    int isEulerCircuit(int V, vector<int> adj[]){
        vector<int> Deg(V, 0);
        int Odd_Deg = 0;
        for(int i=0; i<V; i++){
            Deg[i] = adj[i].size();
            if(Deg[i] % 2)
                Odd_Deg++;
        }
        if(Odd_Deg != 0 && Odd_Deg != 2)
            return 0;
        return 1;
    }
    vector<bool> visited(V, 0);
    for(int i=0; i<V; i++){
        if(Deg[i] > 0){
            DFS(i, adj, visited);
            break;
        }
    }
    for(int i=0; i<V; i++){
        if(Deg[i] && !visited[i])
            return 0;
    }
    if(Odd_Deg == 0)
        return 2;
    else
        return 1;
    }
};
```



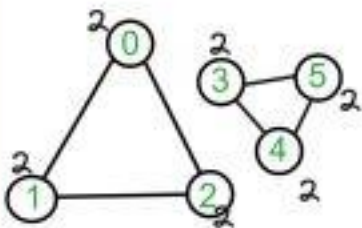
Whether Euler Circuit/Path Exist or Not

1. All Edges Should be visited exactly once,
2. start == end

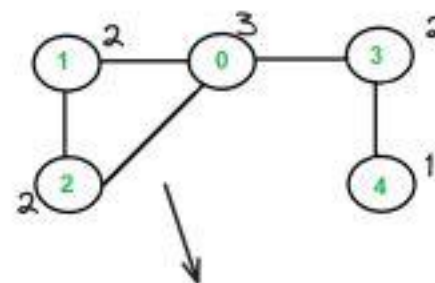


1. Every Node Degree -> Even

2. All Edges Should be part of 1 component



All Non Zero Degree Node Should be connected



1. Zero or two node can have odd Degree And Remaining Node Should Have Even Degree
2. All Non-Zero degree Node Should be Connected

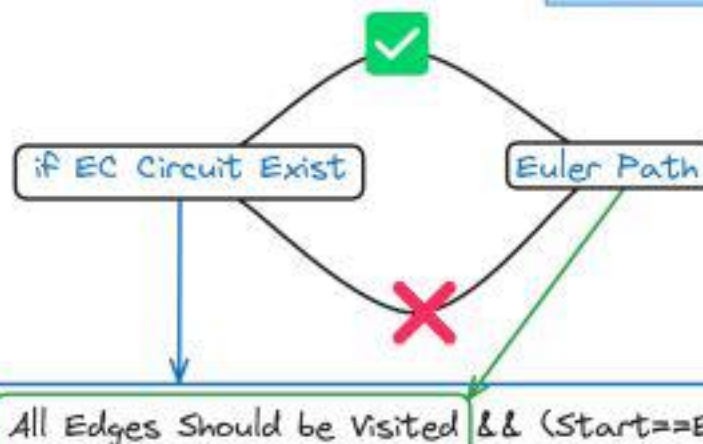
PSUEDO CODE

1. Find Degree of Each Node
2. If Deg of any Node is odd, Not a EC
3. Even Degree
4. Apply DFS, From Any Non-Zero Deg Node

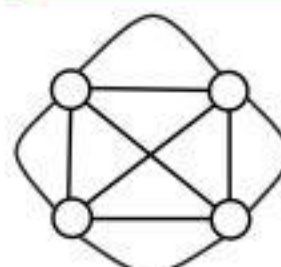
0	1	2	3	4	5
0	0	0	0	0	0
4	2	2	2	2	2

Deg Exists But Not Visited -> EC ✗
 Deg Exists and Visited -> EC ✓
 if Deg 0 -> Ignore (Don't Need to check Traverse)

Time Complexity : $O(V+E)$
 Space Complexity : $O(V)$



Complete Easy Implementation of Euler Path And Euler Circuit



We Can't Draw This Without lifting Pen -> Because it has 4 Odd Degree Nodes (5,5,5,5) -> Eulerian Path Not Possible

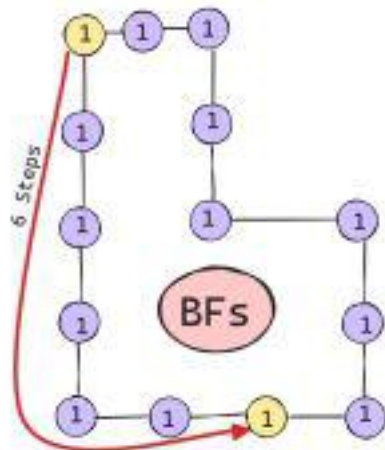
#60daysofcode -> Day 16/60

Lecture - 147

Shortest Source to destination Path

	0	1	2	3
0	1	1	1	0
1	1	0	1	0
2	1	0	1	1
3	1	0	0	1
4	1	1	1	1

Binary Matrix Given



6 Steps

BFs

	0	1	2	3
0	1	1	1	0
1	1	0	1	0
2	1	0	1	1
3	1	0	0	0
4	1	1	1	0

Visited Array initially 0

pair<int, pair<int, int>> p
(row, col, step)

Time Complexity : $O(n*m)$
Space Complexity : $\min(n, m)$

Knight Walk

	0	1	2	3	4	5	6	7
0								
1								
2								
3								
4								
5								
6								
7								

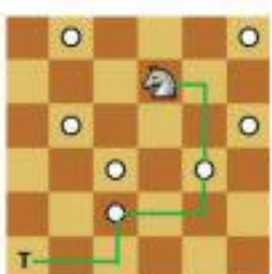
N = 8

Best Method

BFs

int row[8] = [2, 2, -2, -2, 1, -1, 1, -1]
int col[8] = [1, -1, 1, -1, 2, 2, -2, -2]

Yellow -> source
Blue -> 1 Step
Red -> 2 Step
Green -> 3 Step -> Answer



Complete
Optimized
Edge Cases
Handled Code
in Gfg

GRAPHS

1: Knight Walk

2: Shortest Source to Destination Path

3: Find whether path exist



1:11:50

Edges Cases Handled, Optimized, Without Using Visited Array Complete Code :

```
class Solution {
public:
    int row[4] = {1, -1, 0, 0};
    int col[4] = {0, 0, 1, -1};

    bool valid(int i, int j, int n, int m) {
        return i >= 0 && j >= 0 && i < n && j < m;
    }

    int shortestDistance(int N, int M, vector<vector<int>> A, int X, int Y) {
        // Edge Cases
        if(X == 0 && Y == 0)
            return 0;
        if(!A[0][0])
            return -1;

        // row, col, step
        queue<pair<int, int>> q;
        q.push({0, 0});
        int step = 0;
        A[0][0] = 0;

        while(!q.empty()) {
            int count = q.size();
            while(count-- > 0) {
                int i = q.front().first;
                int j = q.front().second;
                q.pop();
                // up down left right
                for(int k = 0; k < 4; k++) {
                    int new_i = i + row[k];
                    int new_j = j + col[k];
                    if(valid(new_i, new_j, N, M) && A[new_i][new_j]) {
                        if(new_i == X && new_j == Y)
                            // Check for the destination
                            return step + 1;

                        A[new_i][new_j] = 0;
                        q.push({new_i, new_j});
                    }
                }
            }
            step++;
        }
        return -1;
    }
};
```

Find Whether Path Exist

	0	1	2	3	4
0	3	0	3	3	0
1	3	0	2	0	3
2	3	3	3	3	3
3	0	3	0	0	3
4	0	3	1	0	3

1 -> Source
2 -> Destination
3 -> Blank
0 -> Wall

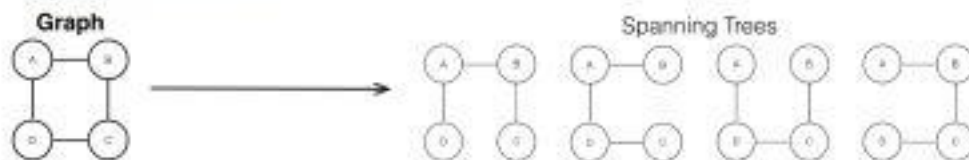
YES OR NO

BFs -> Homework
DFs -> Homework

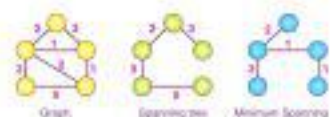
Traversed -> Make 0



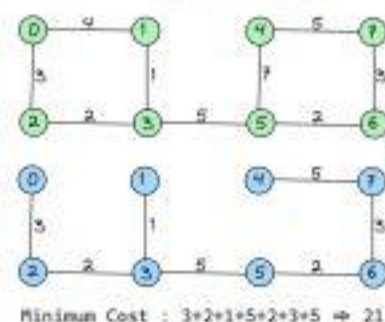
A spanning tree is a subset of Graph G , such that all the vertices are connected using minimum possible number of edges. Hence, a spanning tree does not have cycles and a graph may have more than one spanning tree.



A **minimum spanning tree (MST)** is defined as a spanning tree that has the minimum weight among all the possible spanning trees.



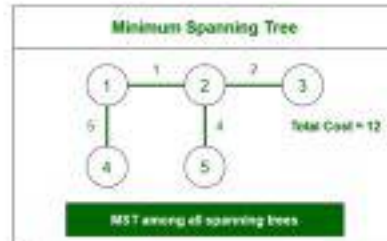
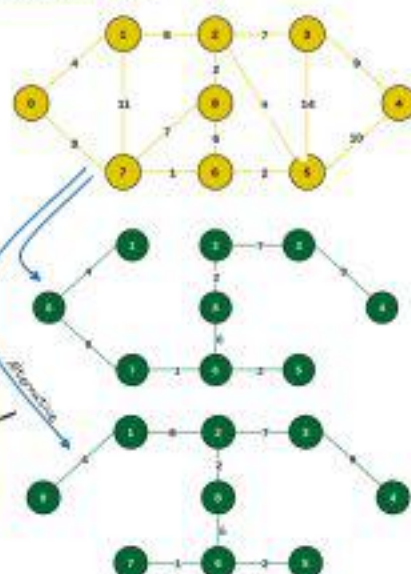
Prim's Algorithm
Or
Greedy Algorithm



Min Heap Priority Queue (Weight, node, parent)

IsMST	0	1	2	3	4	5	6	7	8
Initially 0	1	1	1	1	1	1	1	1	1
Parent	-1	0	1	2	3	6	8	6	2

Cost = 0 3 4 7 9 20 24 28 30
Draw MST = ?



COMPLETE CODE

```
class Solution{
public:
//Function to find sum of weights of edges of the Minimum Spanning Tree.
int spanningTree(int V, vector<vector<int>> adj[]){
    priority_queue<pair<int,pair<int,int>>,
    vector<pair<int,pair<int,int>>>,
    greater<pair<int,pair<int,int>>>> pq;
    vector<bool> IsMST(V,0);

    vector<int> parent(V); //Temporary
    int cost = 0;
    pq.push({0,{0,-1}});

    while(!pq.empty()){
        int wt = pq.top().first;
        int node = pq.top().second.first;
        int pqr = pq.top().second.second;
        pq.pop();

        if(!IsMST[node]){
            IsMST[node] = 1;
            cost += wt;
            parent[node] = pqr;

            for(int j=0;j<adj[node].size();j++){
                if(!IsMST[adj[node][j][0]]){
                    pq.push({adj[node][j][1], {adj[node][j][0], node}});
                }
            }
        }
    }
    return cost;
}
```

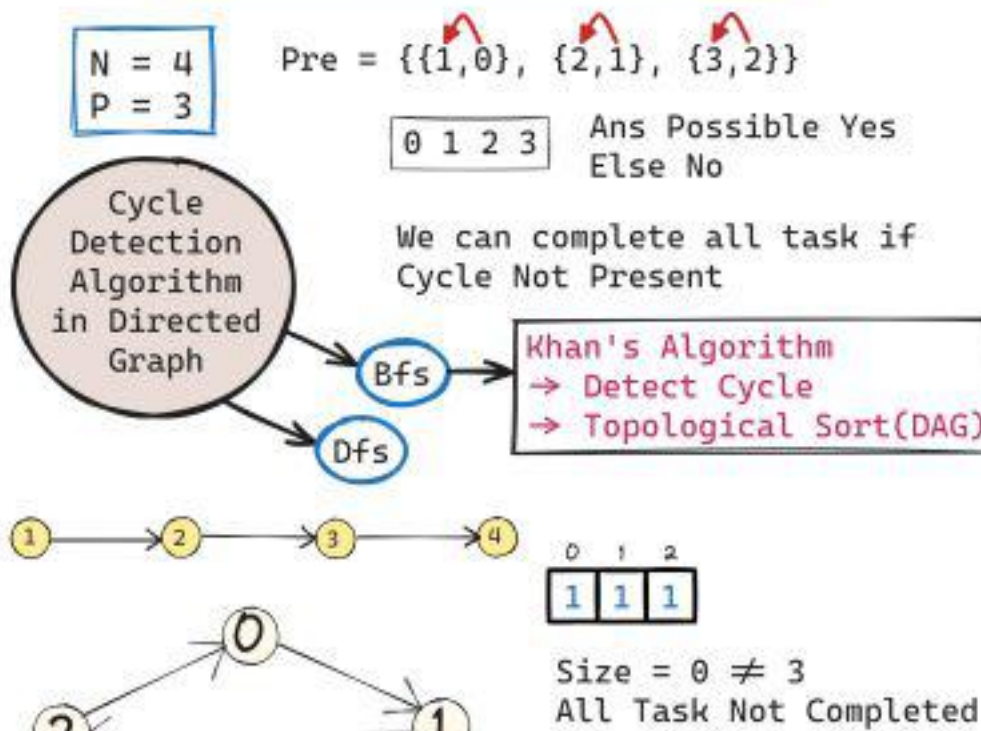
Time Complexity : $E \cdot \log E + E \log E$
 $\Rightarrow E \log E / E \log V$
Space Complexity : $V + V + E \Rightarrow V + E$

1. Prerequisite Tasks
2. Course Schedule
3. Parallel Courses 3
4. Alien Dictionary

#60daysofcode
Day 9/60
Lecture - 140



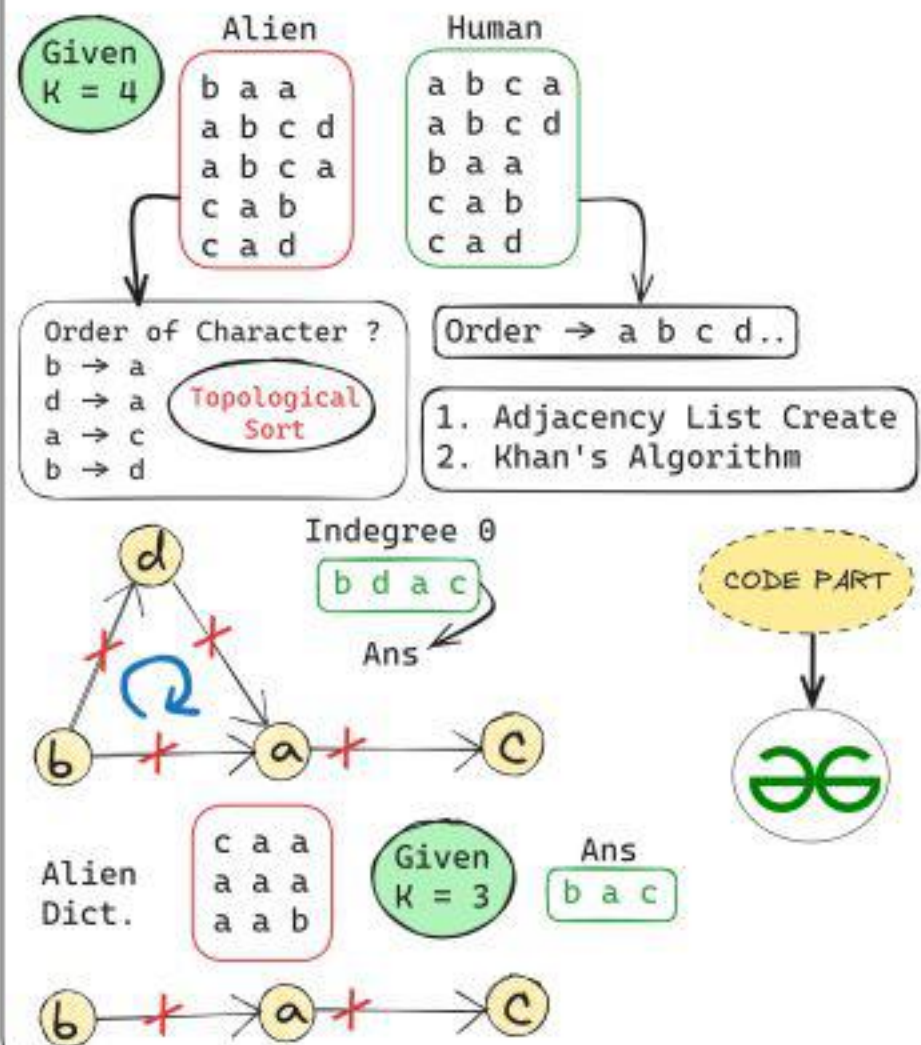
1 → Prerequisite Tasks



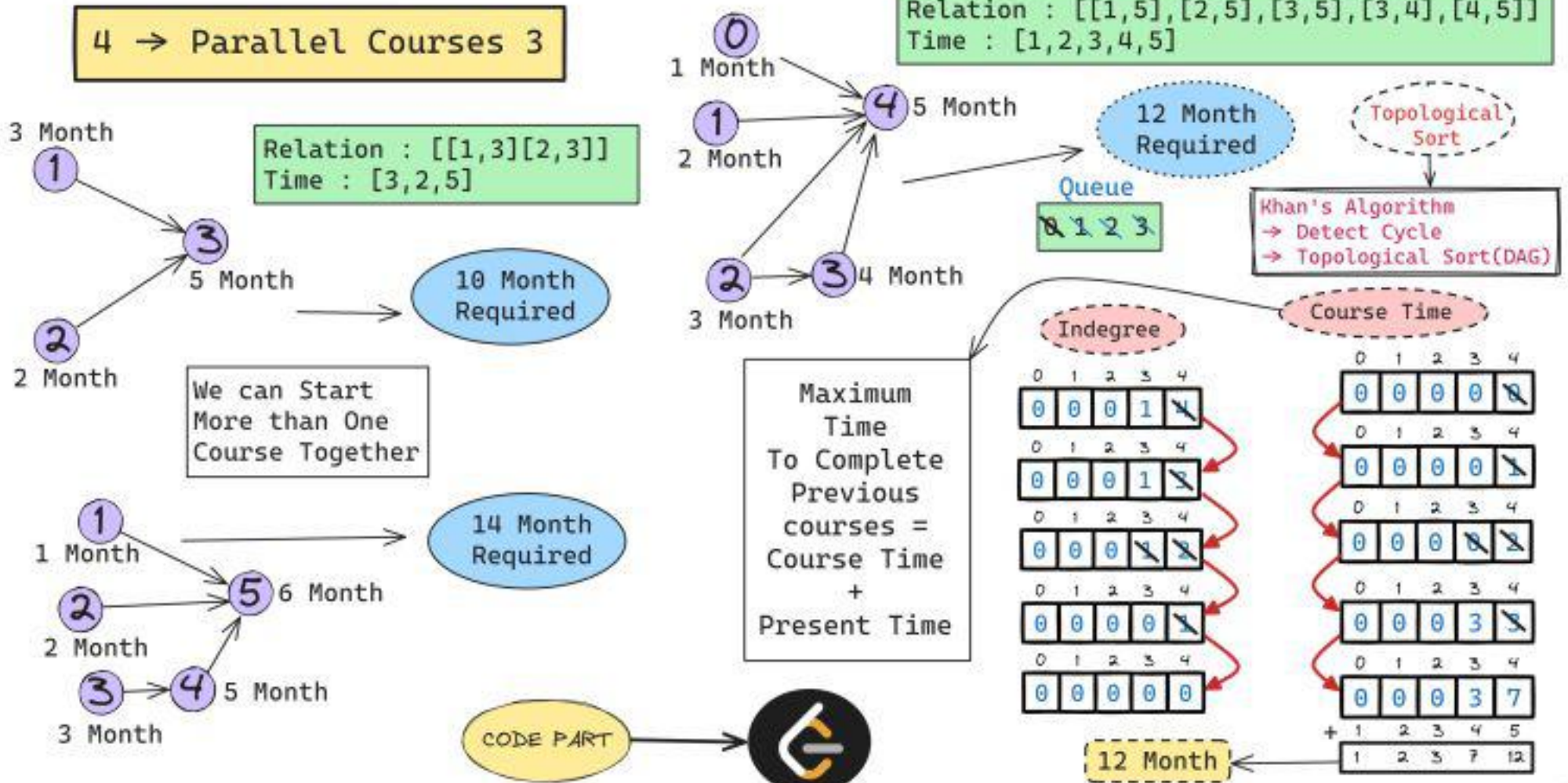
2 → Course Schedule



3 → Alien Dictionary



4 → Parallel Courses 3



1. Covid Spread
2. Find the number of Islands
3. Replace 0's with X's

#60daysofcode
Day 8
Lecture - 139

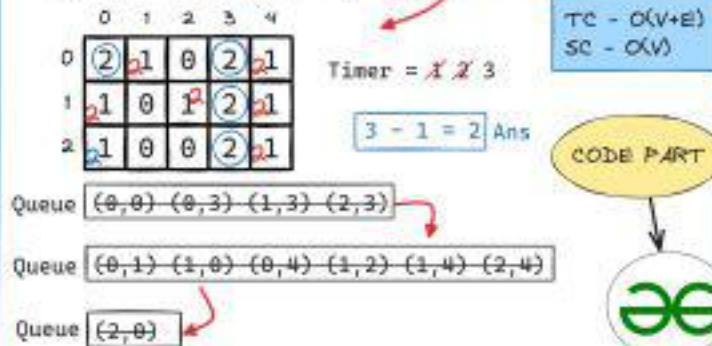
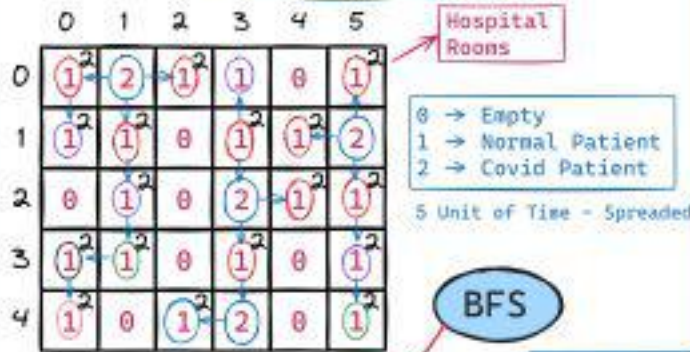
GRAPHS

- 1: COVID SPREAD
- 2: FIND THE NUMBERS OF ISLANDS
- 3: REPLACE 0's WITH X's

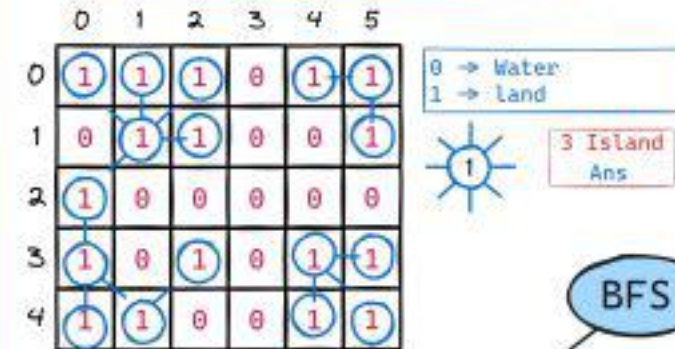
CODE + DRY RUN + H/W

1:49:42

Covid Spread



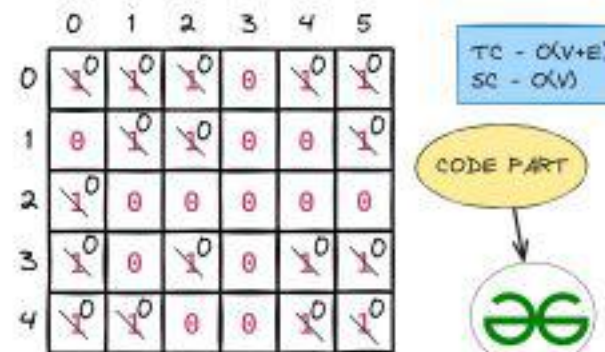
Find the Number of Island



Array for Visited

0	1	1	1	0	1
1	0	1	1	0	0
2	1	0	0	0	0
3	1	0	1	0	1
4	1	1	0	0	1

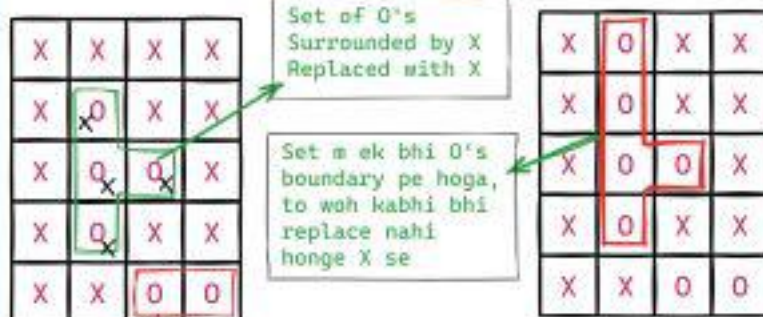
Solution Without Using Visited Array



1. Find 1
→ Count OF Island Increase
→ push in Queue (Row & Col)
→ Adjacent 1, Make them Zero



Replace 0's With X's

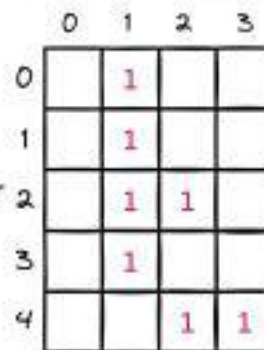


Boundary 0's can't be replaced With X

1. Traverse Starts from Boundary
→ If 0's Find - Put in Array
That not need to change
→ Apply BF's

TC - $O(V+E)$
SC - $O(V)$

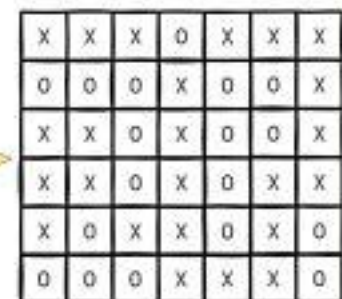
CODE PART



Without Using Extra Array



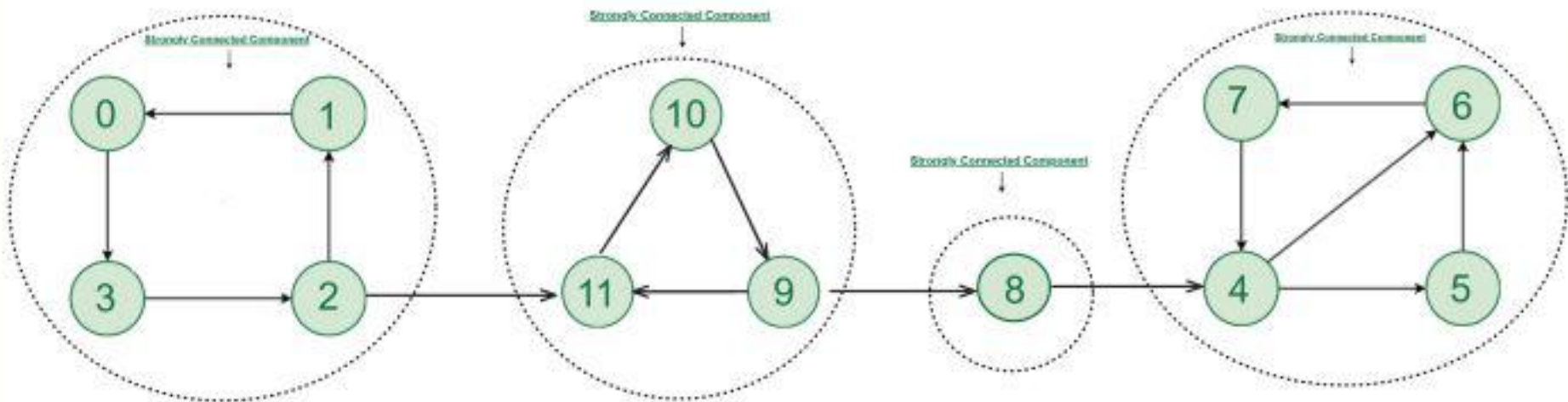
Boundary 0's → T
Other 0's → X



T → 0
Other 0's → X



In a directed graph, a Strongly Connected Component is subset of vertices where every vertex in the subset is reachable from every other vertex in the same subset by traversing the directed edges.

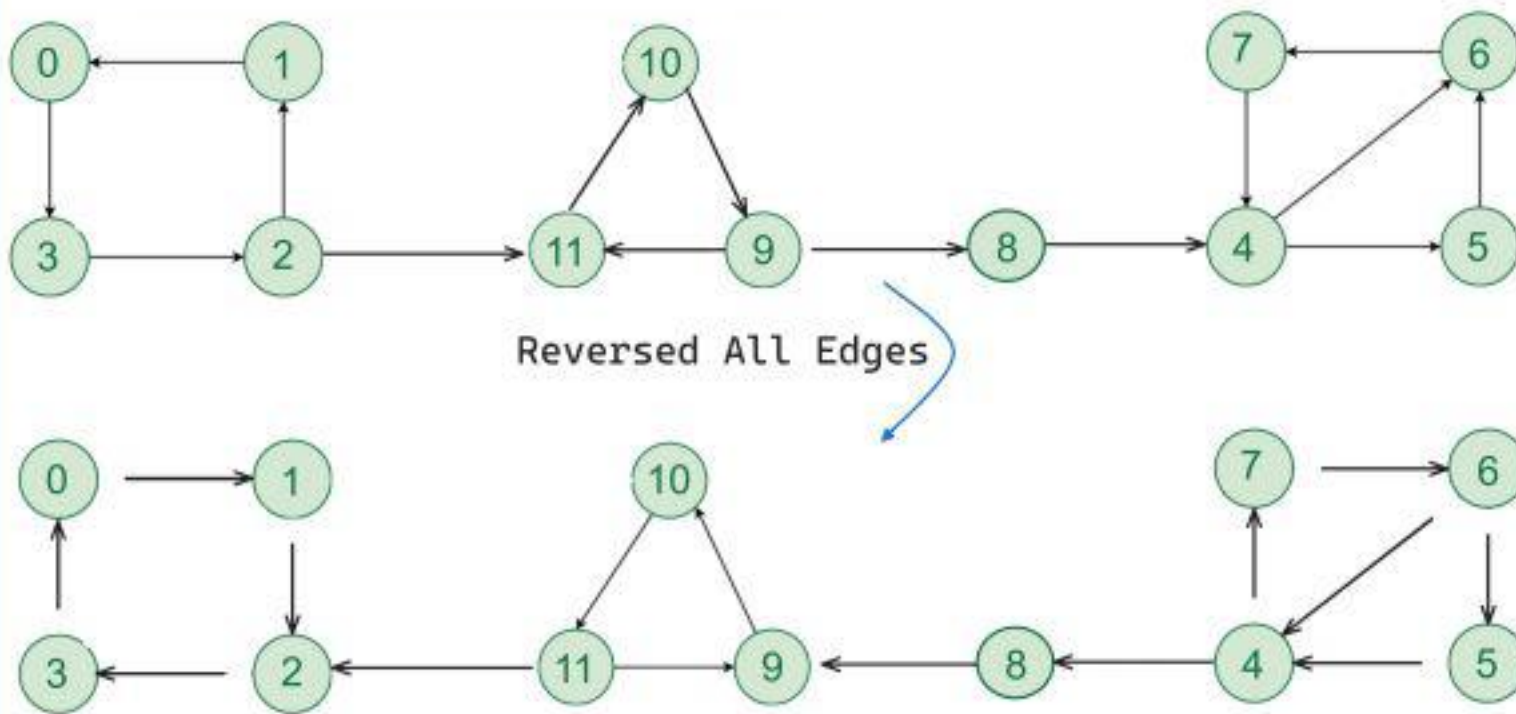


Answers

1. SCC -> 4
2. 0 1 2 3
10 11 9
8
5 4 6 7

How To Solve ?

Kosaraju's Algorithm



SCC 1 -> SCC 2 -> SCC 3 -> SCC 4

Pseudo Code

1. Kosaraju's Algorithm:

Kosaraju's Algorithm involves two main phases:

1. Performing Depth-First Search (DFS) on the Original Graph:
 - We first do a DFS on the original graph and record the finish times of nodes (i.e., the time at which the DFS finishes exploring a node completely).
2. Performing DFS on the Transposed Graph:
 - We then reverse the direction of all edges in the graph to create the transposed graph.
 - Next, we perform a DFS on the transposed graph, considering nodes in decreasing order of their finish times recorded in the first phase.
 - Each DFS traversal in this phase will give us one SCC.

Here's a simplified version of Kosaraju's Algorithm:

1. DFS on Original Graph: Record finish times.
2. Transpose the Graph: Reverse all edges.
3. DFS on Transposed Graph: Process nodes in order of decreasing finish times to find SCCs.

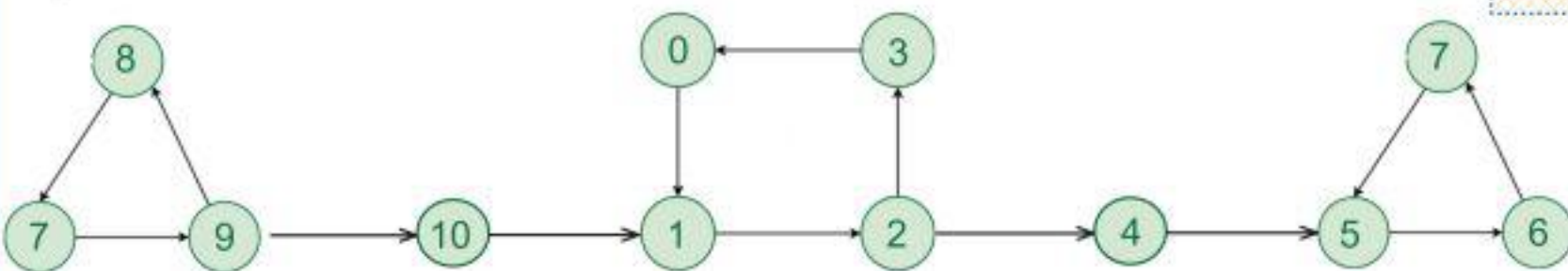
1. Topological Sort
 2. Reverse the Edge
 3. Pop Element from Stack One by One
- if Node is unvisited
-> Call the DFS
-> SCC++

Complete Code



TC -> V+E
SC -> V+E

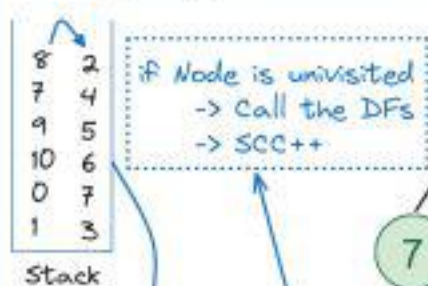
Eg.2



SCC 1 -> SCC 2 -> SCC 3 -> SCC 4 -> SCC 5

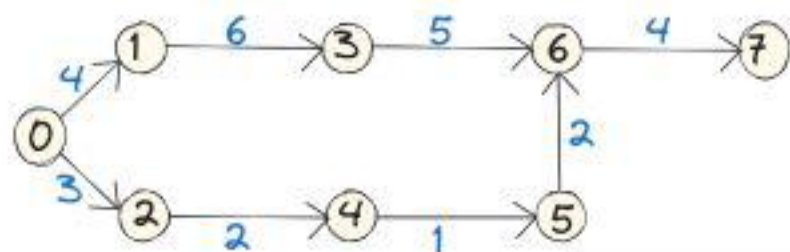
2. Reverse Direction

1. Topological Sort



3. Pop Elements

SCC 1 -> SCC 2 -> SCC 3 -> SCC 4 -> SCC 5



Adjacency List

```
0 → {1,4}, {2,3}
1 → {3,6}
2 → {4,2}
3 → {6,5}
4 → {5,1}
5 → {3,2}
6 → {7,4}
7 →
```

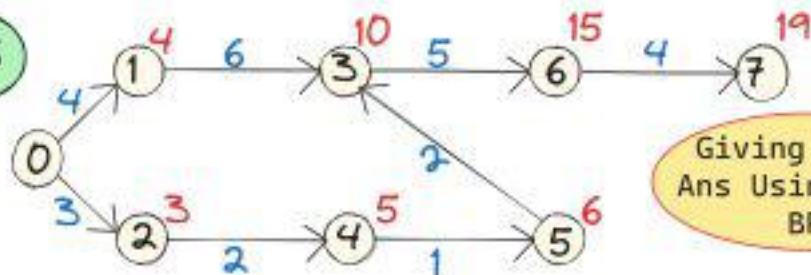
#60daysofcode
Day 11/60
Lecture - 142



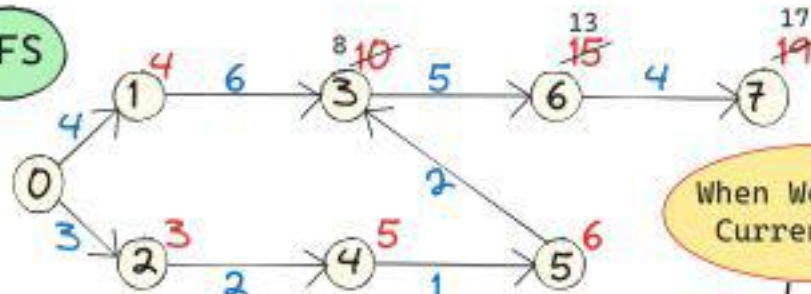
Distance
0 1 2 3 4 5 6 7
0 4 3 8 5 6 13 17

Required Array

BFS



DFS

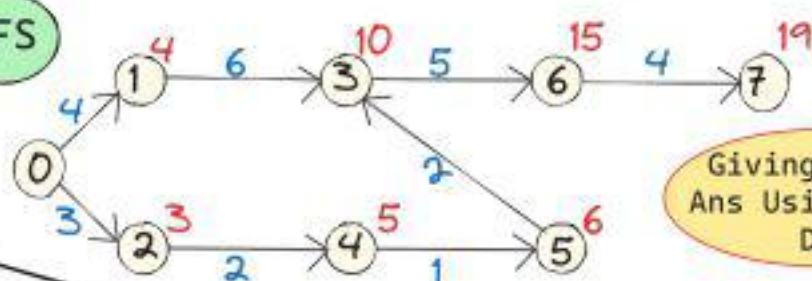


When We follow Current Path

We are Exploring a Path Multiple Times So that Time Complexity Very High

Decrease Time Complexity

DFS



DFS

Source

Topological Sort

Adjacency List

```
0 → {1,4}, {2,3}
1 → {3,6}
2 → {4,2}
3 → {6,5}
4 → {5,1}
5 → {3,2}
6 → {7,4}
7 →
```

TC → $O(V+E)$
SC → $O(V+E)$

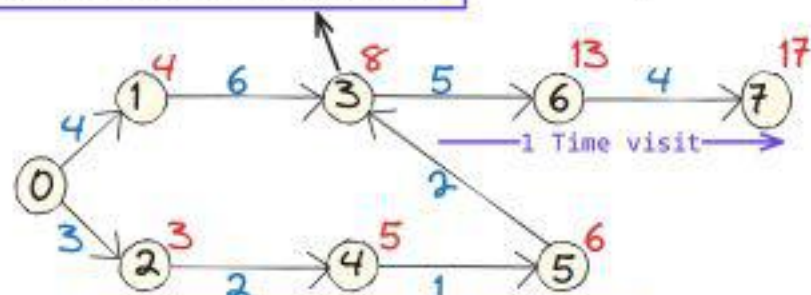
2 Step Solution
1. Find Topological Sort
2. Stack to empty karo ek ek karke

Stack

0	1	2	3	4	5	6	7	Dist
∞	∞	∞	∞	∞	∞	∞	∞	
0	4	3	∞	∞	∞	∞	∞	
0	4	3	∞	5	∞	∞	∞	
0	4	3	8	5	6	∞	∞	
0	4	3	∞	5	6	∞	∞	
0	4	3	8	5	6	13	∞	
0	4	3	8	5	6	13	17	

dist[neighbour] = min(dist[neighbour], weight(node,neighbour) + dist[node])

Minimum Distance from source Node



dist[3] = min(6+dist[1], 2+dist[5])

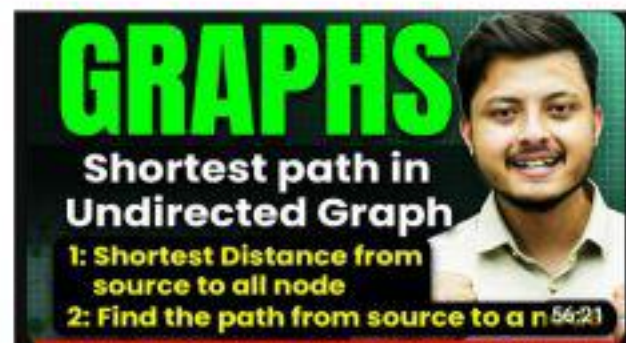
Topological Sort 0 2 4 1 5 3 6 7

Detailed Solution

CODE PART



#60daysofcode
Day 10/60
Lecture - 141


$$\begin{array}{lcl} 0 \rightarrow 1 & 2 \\ 1 \rightarrow 0 & 5 \\ 2 \rightarrow 0 & 4 & 3 \\ 3 \rightarrow 4 & 2 & 7 \\ 4 \rightarrow 2 & 3 & 8 \\ 5 \rightarrow 1 & 6 \\ 6 \rightarrow 5 & 8 \\ 7 \rightarrow 3 & 8 \\ 8 \rightarrow 7 & 6 \end{array}$$

```
unconnected Node = -1
```

CODE
PART



Figure 1 displays a sequence of 10 10x10 grids, each representing a snapshot of a 1D Ising spin chain at a different time step. The columns are indexed 0 to 9. The top row of each grid shows the column indices. The grids illustrate the evolution of the spin configuration over time, with red numbers indicating the state of the spin at the bottom row (index 9) for each column. The sequence shows a wave of spin flips moving from left to right.

[illegible]

If Apply Time Complexity Increases.
That's Why We Don't Use DFS

CODE
PART



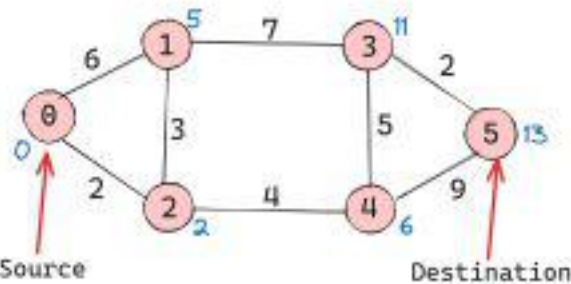
0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
0	1	2	3	4	5	6	7	8	9
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
0	1	2	3	4	5	6	7	8	9
-1	0	0	-1	-1	-1	-1	-1	-1	-1
0	1	2	3	4	5	6	7	8	9
-1	0	0	-1	-1	1	-1	-1	-1	-1
0	1	2	3	4	5	6	7	8	9
-1	0	0	2	2	1	-1	-1	-1	-1
0	1	2	3	4	5	6	7	8	9
-1	0	0	2	2	1	5	-1	-1	-1
0	1	2	3	4	5	6	7	8	9
-1	0	0	2	2	1	5	-1	4	-1
0	1	2	3	4	5	6	7	8	9
-1	0	0	2	2	1	5	3	4	-1

8 4 2 0
0 2 4 8 → Answer

Queue 0 1 2 5 4 3 6 8 7

[illegible]

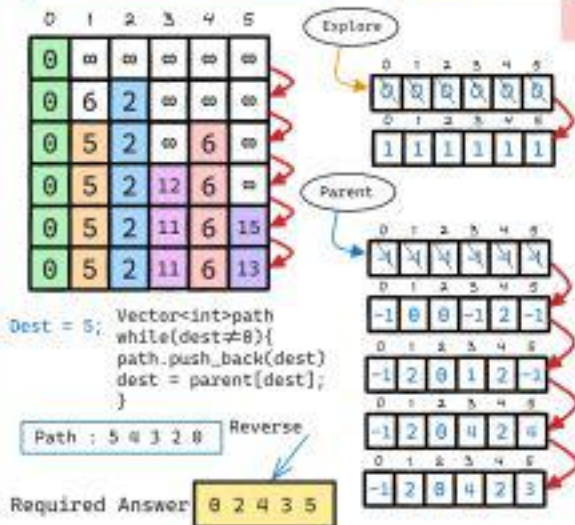
Shortest Path in Weighted Undirected Graph



Required Answer: 0 2 4 3 5

Applying Dijkstra

Tc - $E \log V + V$
Sc - $E + V$



Understanding With Real World Example



#60daysofcode
Day 13/60
Lecture - 144

Complete Code

```
vector<int> shortestPath(int V, int m, vector<vector<int>>& edges) {
    // adjacency list create
    // neighbour, weight
    vector<pair<int,int>>adj[V+1];
    for(int i=0;i<m;i++){
        int u = edges[i][0];
        int v = edges[i][1];
        int weight = edges[i][2];
        adj[u].push_back({v,weight});
        adj[v].push_back({u,weight});
    }

    // Dijkstra Algorithm
    vector<bool>Explored(V+1,0);
    vector<int>dist(V+1,INT_MAX);
    vector<int>parent(V+1,-1);
    priority_queue< pair<int,int>,vector< pair<int,int>>,greater< pair<int,int>>>p;
    p.push({0,1});
    dist[1]=0;
    while(!p.empty()){
        int node = p.top().second;
        p.pop();
        if(Explored[node])
            continue;
        Explored[node] = 1;
        for(int j=0;j<adj[node].size();j++){
            int neighbour = adj[node][j].first;
            int weight = adj[node][j].second;
            if(!Explored[neighbour]&&dist[node]+weight<dist[neighbour]){
                dist[neighbour] = dist[node]+weight;
                p.push({dist[neighbour],neighbour});
                parent[neighbour] = node; // line added } } }

        vector<int>path;
        // I can't reach my destination
        if(parent[V]==-1){
            path.push_back(-1);
            return path;
        }
        // I will reach my destination
        int dest = V;
        while(dest != -1){
            path.push_back(dest);
            dest = parent[dest];
        }

        path.push_back(dist[V]);
        reverse(path.begin(),path.end());
        return path;
    }
```

