Tutorial 5

1) $H_0 : p = 0.7$
$H_1 : p \neq 0.7$

Level of Significance $= \alpha = 0.10$

Test Stat : Binomial val X with $p = 0.7$ $n = 15$

$x = 8$ and $np_0 = 15 \times 0.7 = 10.5$

$$\therefore p = 2P(X <= 8 \text{ when } p = 0.7)$$
$$= 2\left(\sum_{x=0}^{8} b(x_i, 15, 0.7)\right)$$
$$= 2 \times 0.1311$$
$$= 0.2622$$
(Via Binomial Prob Table)

$\because p > 0.1$ i.e. $p > \alpha$
There is no need to reject NULL Hypothesis H_0.
Insufficient reason to doubt builder's claim.

2) $H_0 : p = 0.6$ Given : $x = 70$, $n = 100$, $p = 06$
$H_1 : p > 0.6$

$$Z = \frac{x - np_0}{\sqrt{n \times p_0 \times q_0}} = \frac{70 - 100 \times 0.6}{\sqrt{100 \times 0.6 \times 0.4}} = 2.04$$

$p = P(Z > 2.04)$
$= \cancel{0.207}$
$= 0.0207$ (Via Table)

Since $p < \alpha$, reject the H_0 & conclude new drug is superior.

3) Let p_1 be the proportion of Mumbai voters.
p_2 be the proportion of surrounding area residents.

$p_1 = \frac{120}{200} = 0.6$, $p_2 = \frac{240}{500} = 0.48$,

$\hat{P}_p = \frac{120+240}{200+500} = 0.514$

$\alpha = 5\% = 0.05$

Hypothesis : $H_0 : p_1 <= p_2$

$H_1 : p_1 > p_2$

$$z = \frac{\hat{P}_1 - \hat{P}_2}{\sqrt{\hat{P}_p(1-\hat{P}_p)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.6-0.48}{\sqrt{(0.514)(1-0.514)\left(\frac{1}{200}+\frac{1}{500}\right)}}$$

$\therefore z = 2.869$

$p(z > 2.869) = 0.0044$

Since $p < \alpha$, we need to reject H_0 and conclude that proportion of Mumbai voters favouring the proposal is higher than proportion of surrounding area voters

4) a) $H_0: p = 0.2$ Critical Region is in Right Tail
$H_1: p > 0.2$

b) $H_0: \mu = 3$ Critical Region in Both ~~Sides~~ Tails
$H_1: \mu \neq 3$

c) $H_0: p = 0.15$ Critical Region is in left Tail.
$H_1: p < 0.15$

d) $H_0: \mu = 500$ Critical Region is in right tail.
$H_1: \mu > 500$

e) $H_0: \mu = 15$ Critical Region is in Both tails.
$H_1: \mu \neq 15$

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5) Let μ_1 = population mean "robustness" - laptops company A.

μ_2 = population mean "robustness" laptops company B.

$H_0: \mu_1 = \mu_2$
$H_1: \mu_1 \neq \mu_2$

$\alpha = 0.05$

$$\bar{x}_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} x_i = \frac{(9.3 + 8.8 + 6.8 + 8.7 + 8.5 + 6.7 + 8 + 6.5 + 9.2 + 7)}{10}$$

$\bar{x}_1 = 7.95$

$$\overline{X_2} = \frac{1}{n_2} \sum_{i=1}^{n_2} x_{2i} = \frac{(11.98 + 9.91 + 10.2 + 10.1 + 9.7 + 11 + 11.1 + 10.2 + 9.6)}{10}$$

$$\therefore \overline{X_2} = 10.26$$

$$s_1^2 = \frac{1}{n_1 - 1}\left(\sum_{i=1}^{n_1} (x_1)_i^2 - n_1 \bar{x}_1^2\right) = \frac{10.865}{9} = 1.207$$

$$s_2^2 = \frac{1}{n_2 - 1}\left(\sum_{i=1}^{n_2} (x_2)_i^2 - n_2 \bar{x}_2^2\right) = \frac{2.924}{9} = 0.325$$

As sample variances are very different, we cannot assume population variances equal, use the "unpooled t-test".

$$v = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1} \times \left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1} \times \left(\frac{s_2^2}{n_2}\right)^2} = 10.30$$

$$\therefore v \approx 10$$

Test statistic used to test hypo is $T = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

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which under null hypothesis follows approximately t-distribution. with $v = 10$ degrees of freedom.
Under null hypo, $(\mu_1 - \mu_2) = 0$

$$\therefore \text{Value of } T = \frac{7.95 - 10.26}{\sqrt{\frac{1.207}{10} + \frac{0.325}{10}}} = -5.90$$

As the test is two-sided, the value of test is doubled area under density curve of t-distribution with $(v = 10)$ right of the absolute value of tests.

$$|t| = |-5.9| = 5.9$$

$$p\text{-value} = 2P(T \geq |t|)$$
$$= 2P(T \geq 5.9)$$

$t_{0.0005}(10) = 4.587$ and since $|t| = 5.9$ is even greater than $P(T \geq 5.9) < 0.0005$ so ~~p-value is~~
p-value < 0.001 as $p < \alpha$ reject null Hypothesis. we conclude that "mean" "robustness" of the laptops is not same for both companies.

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