DS Tutorial 4.

Ex-1

Given,

Scalar and statistically independent variables (random)

Mean = 0 and Variance = 1

and value of kurtosis - a to a, where $a > 0$

S_i is mixed ~~for~~ like,

$$x = \sum_i w_i s_i \quad \rightarrow (1)$$

$w_i \rightarrow$ constraint weight.

Q. Which constraints do you have to impose on the weights w_i to ~~guar~~ guarantee that the mixture has unit variance as well?

Hint: $\text{Var}(x) = \langle\langle(x - \hat{x})^2\rangle\rangle$
$= \langle x^2 \rangle - \langle x \rangle^2$

Continuing on the hint, substituting (1)

$$\text{var}(x) = \langle \left(\sum w_i s_i\right)^2 - \langle \sum w_i s_i \rangle^2$$

$$= \left\langle \left(\sum_i w_i s_i\right)^2 \right\rangle - \left(\sum_j w_j s_j\right)^2$$

$$= \left\langle \left(\sum_i w_i s_i \right) \left(\sum_j w_j s_j \right) \right\rangle - \left(\sum_i w_i \langle s_i \rangle \right) \left(\sum_j w_j \langle s_j \rangle \right)$$

$$= \left\langle \sum_{i,j} w_i w_j s_i s_j \right\rangle - \sum_{i,j} w_i w_j \langle s_i \rangle \langle s_j \rangle$$

$$= \sum_{i,j} w_i w_j \langle s_i s_j \rangle - \sum_{i,j} w_i w_j \langle s_i \rangle \langle s_j \rangle$$

$$= \sum_i w_i w_i \left(\langle s_i s_i \rangle - \langle s_i \rangle \langle s_i \rangle \right) +$$

$$\sum_{i,j \; i \neq j} w_i w_j \left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right)$$

$$= \sum_i w_i^2 \left(\langle s_i s_i \rangle - \langle s_i \rangle^2 \right)$$

$$= \mathrm{var}(s_i) = 1$$

$$+ \sum_{i,j \; i \neq j} w_i w_j \left(\langle s_i \rangle \langle s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right) = 0$$

$$= \sum_i w_i^2$$

$\therefore$ The constraint is $\sum w_i^2 = 1$

Ex-2

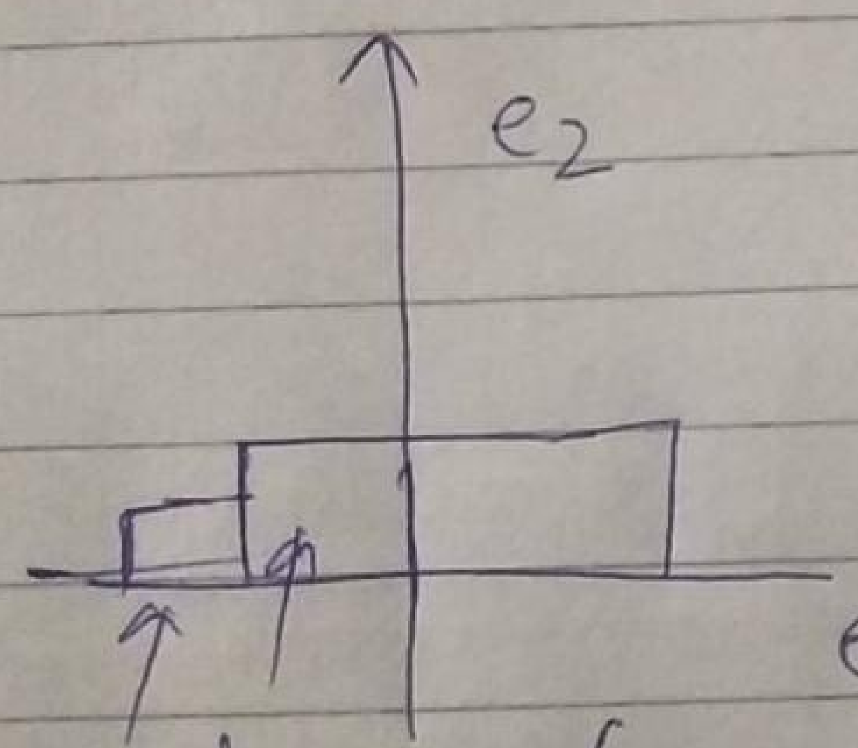
Need to guess independent components & Distribution

1) Decide whether the following distributions can be linearly separated into independent components if yes.

2) Sketch the axes into which the data must be ~~projected~~ projected to extract the independent components. Draw these axes also the marginal distributions of the corresponding component.

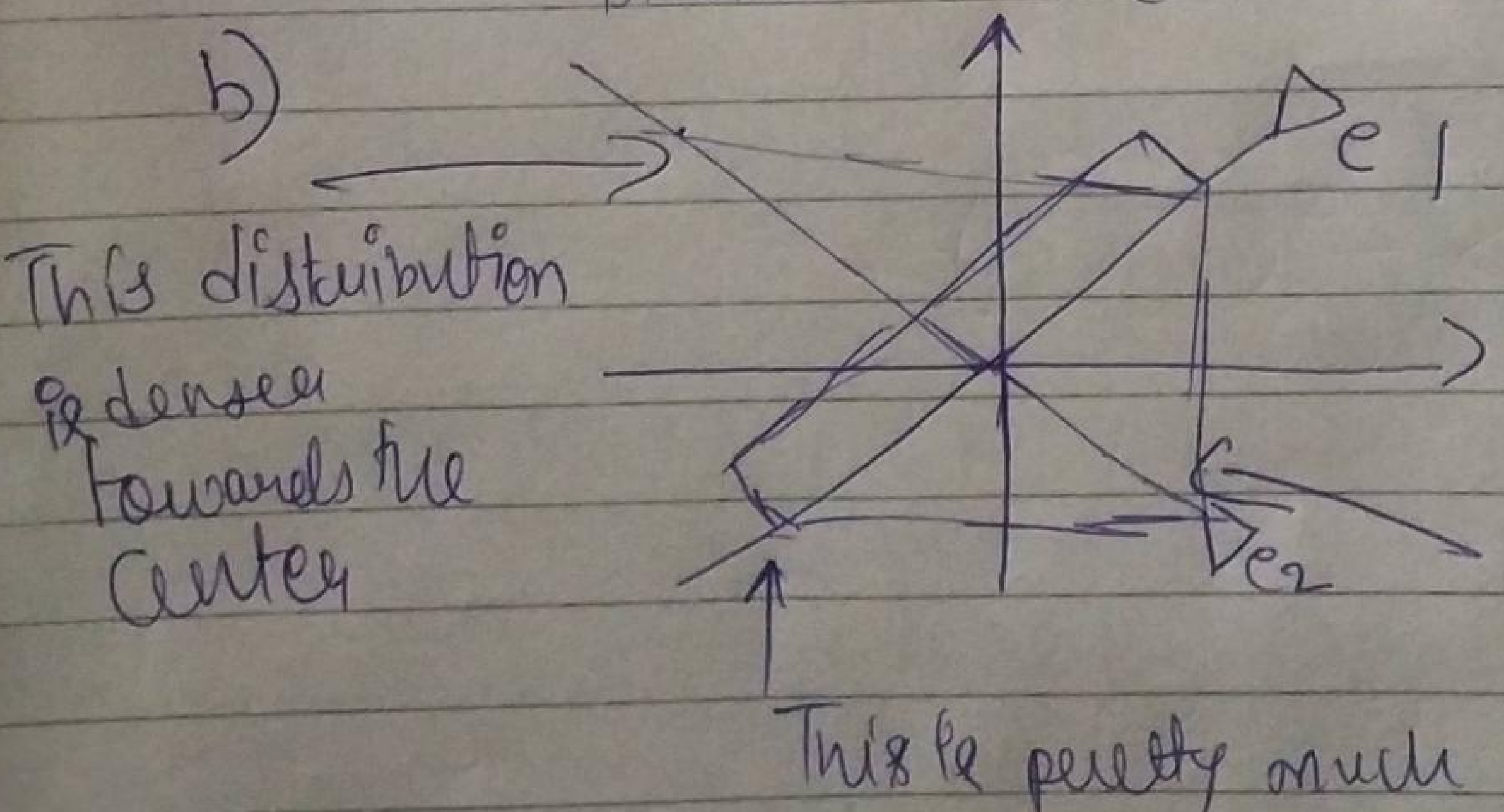
let us have e_1 & e_2 as vectors that help to extract the independent components.

a)



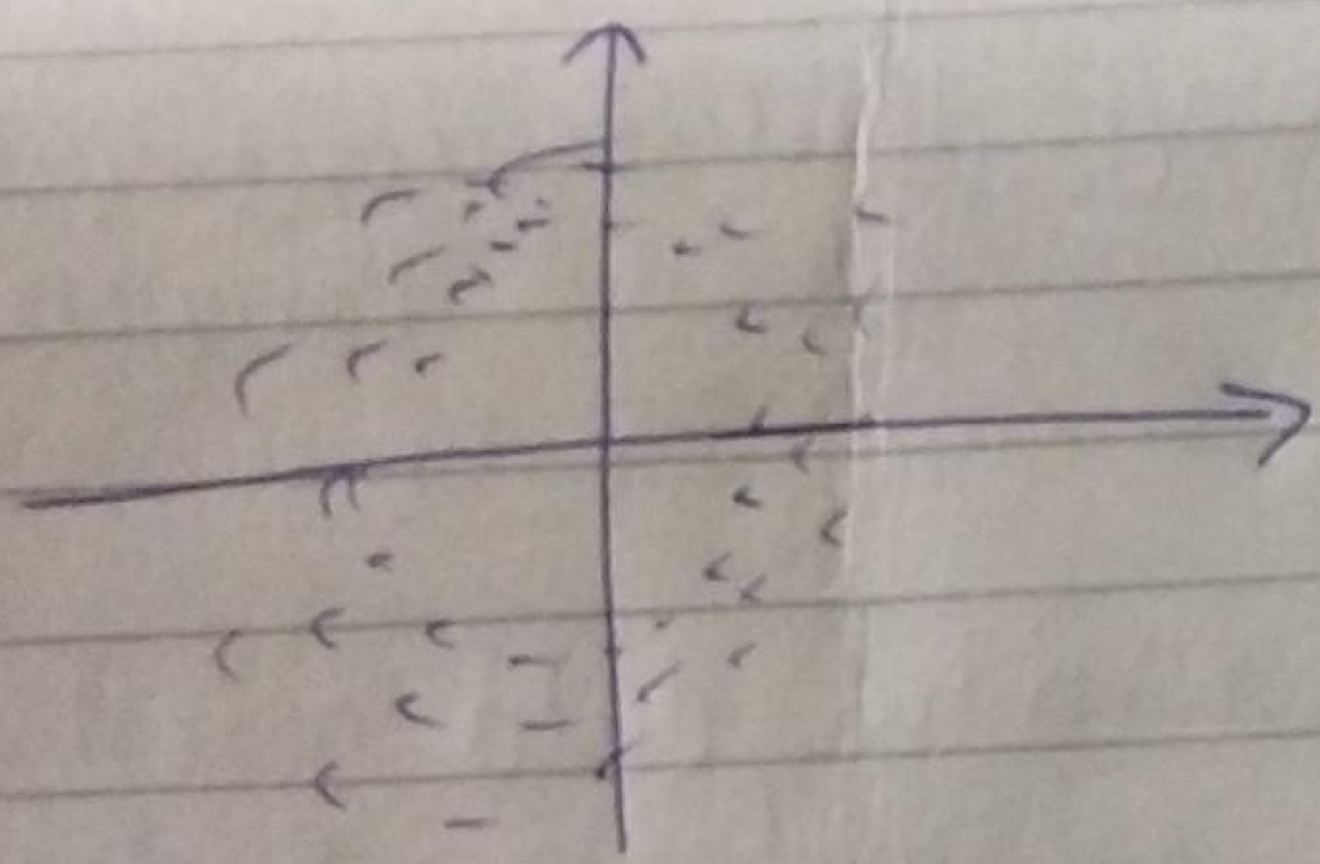
(As density is same almost in both the directions)

b)



This is pretty much even across.

c)



As the data is ~~pretty much~~ circular it is not possible to linearly separate it.