Tutorial 6

1) a) $P(H) = \lambda$
$P(T) = 1 - \lambda$

$$P(\text{first head at } k+1) = (1-\lambda)^{k} \cdot \lambda$$

b) Let M be number of tosses required to get first head

Let $S = E[M]$

Since Tosses are Independent and equation is Additive

$\therefore$ ~~S~~ =

$$S = \lambda \times 1 + (1-\lambda)(S+1)$$
$$= \lambda + S + 1 - \lambda S - \lambda$$
$$S = S + 1 - \lambda S$$
$$\therefore \lambda S = 1$$

$$\therefore \boxed{S = \frac{1}{\lambda}}$$

2) $X \rightarrow$ Random Variable

a) $Var(\cancel{X}) = E[X - E(X)^2]$

To Prove : $Var(X) = E[X^2] - (E[X])^2$

so now we have,

$$Var(X) = E[(X - E[X])^2]$$

$$= E[X^2 - 2XE[X] + (E[X])^2]$$

$$= E[X^2] - 2E[XE[X]] + E[\cancel{X}]^2$$

$$= E[X^2] - (E[X])^2 \quad \text{Hence Proved.}$$

b) $E[X] = 0, E[X^2] = 1$

i) $Var(X) = E[X^2] - (E[X])^2$
$= 1 - 0^2$
$= 1$

ii) $Y = a + bX$
$E[Y^2] = E[(a+bX)^2]$

$= E[a^2 + 2abX + b^2X^2]$
$= E[X] \cdot 2ab + a^2 + b^2 E[X^2]$
$= 0 + a^2 + b^2$

$E[Y] = E[a+bX] = a + bE[X] = a + b(0) = a$

$\therefore Var(Y) = E[X]^2 + E[X^2]$
$= a^2 + b^2 - a^2 = b^2$

3) A → Aku predicts given horse is winning horse.
~A → Aku predicts given horse is not winning horse.

B → Event that given horse wins.
~B → Event that given horse does not win.

a) Given a horse probability, it wins.

$P(B) = P(B, A) + P(B \cap A)$
$= P(B/A) \cdot P(A) + P(B/{\sim A}) \cdot P(\sim A)$
$= P(B/A) \cdot P(A) + P(B/{\sim A}) \cdot P(\sim A)$
$= 0.99 \times 10^{-5} + (1 - 0.99999)(1 - 10^{-5})$

$\boxed{\therefore P(B) = 1.99 \times 10^{-5}}$

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b) Probability that Aku predicts black beauty is winning.

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A/B)\,P(A)}{P(B)} = \frac{0.99 \times 10^{-5}}{1.99 \times 10^{-5}}$$

$$\boxed{P(A/B) = 0.497}$$