# Chapter 13
# Spectral Analysis of Signals

So far in the exploration of signals measured by an oscilloscope, the signals have been measured, displayed and analyzed in the time domain. While this is the real world and is often the domain in which to extract important figures of merit, sometimes further insight can be gained about the behavior or signature of signals by looking at their frequency components in the frequency domain. This is achieved by converting their time behavior into the frequency domain using spectral analysis.

## 13.1 A Spectrum

In the frequency domain, the only waveforms we are allowed to consider are sine waves. Unique combinations of sine waves in the frequency domain can describe any time domain waveforms.

However, this property is not the only reason sine waves were singled out to use in spectra analysis. There are other special waveforms, combinations of which can describe any time-domain waveform, such as Hermite polynomials, Laguerre polynomials, Jacobi polynomials, Legendre polynomials, or even wavelets. These functions are in the general class of complete, orthonormal basis functions or eigenfunctions.

The reason we single out sine waves for a frequency domain description, is that sine waves are solutions to second order, linear, differential equations, the equations found so often in electrical circuits involving resistor, capacitor and inductor elements. This means signals that arise or have interacted with RLC circuits are described more simply when using combinations of sine waves than any other function because sine waves naturally occur.

The only reason we would ever leave the real world of the time domain to enter any other domain is to get to an answer or insight faster. Sometimes, a complex waveform in the time domain can be more simply analyzed and understood by looking at its frequency domain description composed of combinations of sine waves.

An ideal sine wave is described by only three figures or merit or parameters: its frequency, amplitude and phase. A sine wave waveform, measured by an oscilloscope with one million voltage-time, V(t), data points in the acquisition buffer is described by only three numbers in the frequency domain.

This is a dramatic simplification. This comparison between the time domain measurement of a sine wave and its frequency domain description is shown in **Figure 13.1**.

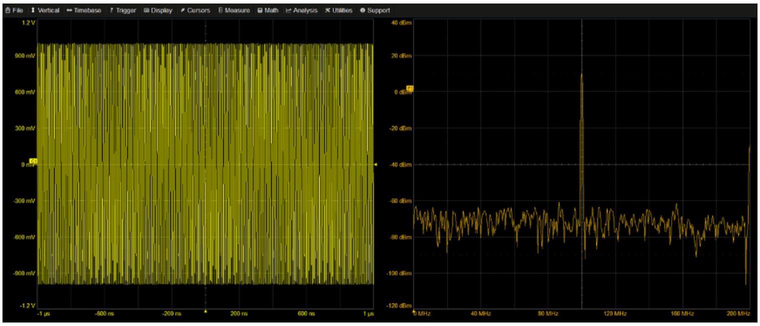

***Figure 13.1*** *A 100 MHz sine wave in the time domain and its spectrum in the frequency domain showing the one peak at 100 MHz.*

In principle, any waveform in the time domain over some time interval, can be described by a unique combination of sine waves, each with its own frequency, amplitude and phase. This combination of sine wave components is called its spectrum. It is a unique fingerprint describing the time domain waveform.

The DC or average value of any ideal sine wave frequency component is 0. The DC offset of the time domain waveform is stored as the 0 Hz frequency component of the signal. It is treated as just another frequency component.

The most common application of spectral analysis is to identify the frequency components in a time domain waveform as a fingerprint to help identify the root cause of a problem.

What appears as a random, noisy waveform in the time domain may reveal specific frequency components in the frequency domain. For example, the measured near field emissions from a digital system is a complex waveform in the time domain. The frequency domain decomposition into its spectrum, reveals a pattern specific frequency peaks, as shown in **Figure 13.2**.

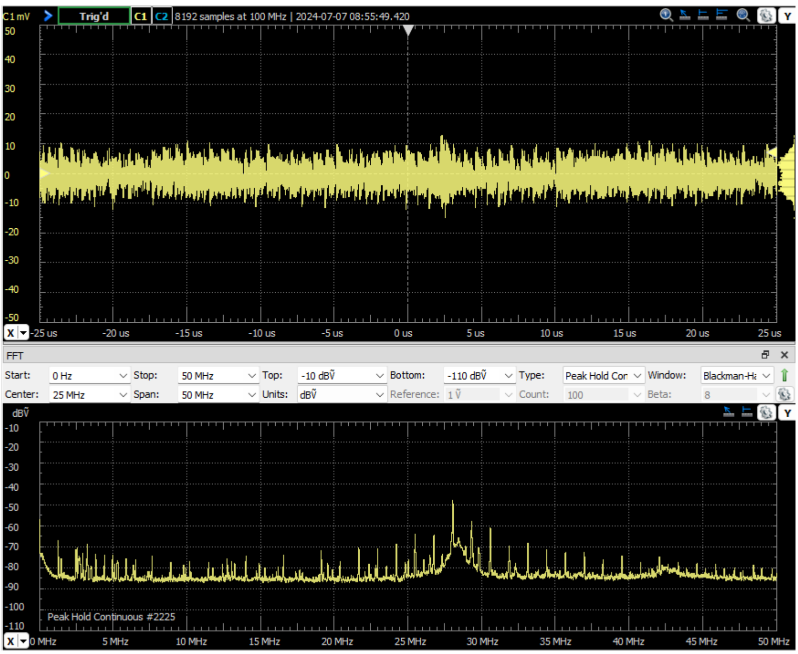


***Figure 13.2*** *Top: time domain waveform of the near field emissions from a digital device, Bottom: the spectrum of this waveform, showing distinct peaks starting at 1 MHz with a broad peak at 28 MHz.*

This spectrum shows the three common features found in most spectra:

- ✓ *Sharp peaks, suggesting repetitive signals, such as clocks, carrier frequencies or loops in the code running on the digital device.*
- ✓ *Broad peaks, suggesting periodic transient signals or modulated carrier frequencies*
- ✓ *Flat regions, suggesting broadband, random white noise.*

The spectrum reveals a fingerprint identification of a possible source of noise or interference. In this example, searching in the performance of the DUT for clock frequencies that are 1 MHz or multiples, or repetitive operations at 28 MHz, might reveal the originating source of these features.

A second question is how the source of the noise then interacts with interconnects inside the DUT to create the near field emissions. This is the start of the debug process.

Not all problems can be solved more quickly in the frequency domain, but when signals or noise are periodic and have specific frequency components, spectral analysis might be applied to the problem to gain insight faster.

## 13.2 Spectrum analyzer or real-time spectrum analyzer

Traditionally, the instrument used to measure the spectral content of a signal in the time domain is a spectrum analyzer. As a standalone instrument, a spectrum analyzer will scan across the spectrum with a tuned, narrow band filter, measuring the voltage amplitude that gets through the narrow band filter. This is a direct measurement of the power in the signal within the bandwidth of the narrow band filter.

The voltage through the filter, as the center frequency of the narrow band filter is swept, is displayed on the front screen. There is no phase information extracted from the signal. This is why these instruments are sometimes referred to as **scalar spectrum analyzers**.

If the center frequency of the narrow band filter is scanned quickly, it may appear like a continuous display of the spectrum on the screen, but in fact, the amplitude of the signal in each frequency band is really a snapshot measurement of the spectral content at the moment the filter happened to be centered at that frequency value.

If the spectrum of the signal is dynamically changing, such as in a spread spectrum clock, the dynamic nature of the spectrum of the signal may not be captured, or it would be an averaged response. Sometimes this is enough from which to interpret the behavior of the signal source. If the source frequency changes more quickly compared to the sweep rate, information about the signal may be missed or misinterpreted.

A scope displays the spectral response of a signal in a fundamentally different way than a spectrum analyzer. A scope measures the signal in real time and records the V(t) buffer of measurements into a file. It then performs a calculation based on the Fast Fourier Transform (FFT) to convert the real time voltage data into the spectrum. For this reason, the scope’s spectrum is sometimes referred to as a **real time spectrum analyzer**.

Using either hardware acceleration or an efficient software algorithm, the calculation of the FFT can be performed in a short enough time to appear to be instantaneous. Even though the time domain signal and the spectral response are shown continuously on the screen, there is some deadtime between successive time domain buffer measurements, FFT calculation, and display as the spectral response. Some scopes allow multiple buffers to stream into memory and the FFT calculated after the total acquisition. This reduces the deadtime between consecutive buffer acquisitions.

The process of converting an arbitrary signal in the time domain into its spectral components in the frequency domain is based on five fundamental principles, outlined in the next section.

# 13.3 Principles of FFT Analysis

Every scope calculates the spectrum of a signal recorded in the time domain using five principles. By understanding these principles, we can identify and reduce potential artifacts created by the FFT process. In addition, the FFT process introduces some limitations in the features displayed in the spectrum.

Generally, all five principles are applied "under the hood" by oscilloscopes with an FFT function. While it is not necessary to understand these five principles, understanding these principles will eliminate the black box nature of the spectrum, and you will get the most out of your real time spectrum analyzer.

Every DSO available from the < $100 units to the >$1M units have spectral analysis built in. It is just as important a tool in analyzing signals and gaining insight into the root cause of problems as the real time voltage measurements in the time domain. If you wish to master the scope and the analysis of signals, you should master spectral analysis as well.

### *13.3.1 Create a periodic waveform*

When we take a waveform in the time domain and transform it into the frequency domain, we end up with a collection of sine waves, each with a frequency value, an amplitude, and a phase. Each frequency is separated into frequency bins. Alternatively, we can describe the waveform in the time domain as a collection of both sine waves and cosine waves, each with a frequency and amplitude value. The phase is contained in the relative sine and cosine amplitudes.

The amplitude of each frequency component bin is selected so that the combination of all the spectral component bins recreates the original waveform:

$$V(t) = \sum_n \left( a_n \sin(n2\pi f_0 t) + b_n \cos(n2\pi f_0 t) \right)$$

In this expansion, the $a_n$ and $b_n$ coefficients are the amplitudes of the sine and cosine components at each frequency bin, $nf_0$. In this formalism, $f_0$ is the initial, lowest repeat frequency bin in the acquisition buffer and n is the harmonic number of each frequency bin in the spectrum. The lowest frequency is related to the total acquisition time. The highest value of n is related to the Nyquist frequency of the sample rate.

The coefficients, and each frequency bin, define the spectrum. They are calculated from the measured V(t) waveform using the Discrete Fourier Transform (DFT). This calculation is implemented with a special matrix math algorithm referred to as the Fast Fourier Transform (FFT) algorithm.

In the time domain, the measurements are taken and stored in an acquisition buffer with a total acquisition time, $T_0$, and a time interval between samples, $\Delta T$. When we describe the same waveform in the frequency domain, we refer to the collection of all the sine and cosine wave components, each with a frequency and amplitude, as the spectrum.

Unfortunately, we can only use the Discrete Fourier Transform (DFT) on a V(t) waveform that is periodic. If it is not periodic, we must artificially make it periodic. The trick we use to turn any arbitrary acquisition buffer of measured data into a periodic waveform is to take the acquisition buffer of total time, $T_0$, and concatenate identical, repeated acquisition buffers to the front and the back.

This creates an artificially repetitive waveform that repeats with a period equal to the acquisition buffer time, forever in the past and forever in the future. This is illustrated in **Figure 13.3**.

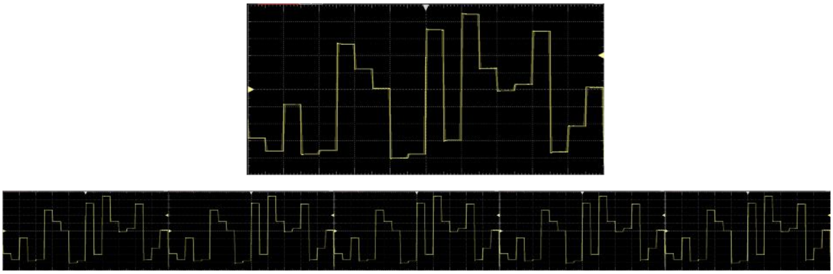

***Figure 13.3*** *Top: a single measured acquisition buffer of some arbitrary waveform is turned into a period waveform by, bottom: repeating the acquisition buffer forever in the past and forever in the future.*

When we have this artificially repetitive waveform, we can apply the power of the DFT to mathematically calculate each frequency component in the spectrum. Each frequency bin in the DFT are multiples of the fundamental, $f_0 = 1/T_0$.

The amplitude and phase of each frequency bin are calculated using:

$$a_n = \frac{1}{T_0}\int_0^{T_0} V(t)\sin\left(\frac{2\pi}{T_0}nt\right)dt$$

$$b_n = \frac{1}{T_0}\int_0^{T_0} V(t)\cos\left(\frac{2\pi}{T_0}nt\right)dt$$

The amplitide of each frequency component bin is given by

$$\text{Amplitude}_n = \sqrt{a_n^2 + b_n^2}$$

The spectrum calculated using the DFT is literally the collection of each frequency bin with its amplitude of each sine and cosine wave.

### *13.3.2 The Lowest Frequency*

In the calculated spectrum, only discrete frequency values appear. The lowest frequency component bin is called the *fundamental*. It is the lowest frequency sine wave that will fit in the acquisition buffer time. The period of this lowest frequency sine wave, is the total acquisition time, $T_0$.

$$f_0 = \frac{1}{T_0}$$

Each frequency component in the spectrum is a frequency that is an integer multiple of the fundamental, $f_n = n \times f_0$.

Multiples of the fundamental are the only frequency components that appear as bins in the spectrum. This means, the frequency spacing between each frequency component bin, or the **resolution**, is the fundamental frequency.

If you want a higher resolution to distinguish narrower frequency features in the spectrum, you need to use a longer acquisition time in the oscilloscope. This means a longer full-scale time interval.

The average value of an ideal sine wave is always 0. This means that when we use a collection of sine waves to describe a real waveform, the average value of the recreated time-domain waveform is always 0.

But real waveforms have an average value, or DC offset. To account for this, we store the DC component in the 0 Hz frequency component, which is 0 x the fundamental frequency. When calculating the DFT, the sine term for n = 0 is zero, but the cosine term is 1. This means the $b_0$ coefficient is literally the average value of the signal over the time interval, which is an approximation of the DC component.

In most oscilloscopes, you can suppress plotting the $0^{th}$ frequency component to increase the dynamic range of the display.

### *13.3.3 The Highest Frequency*

The highest frequency component in the spectrum is related to the sample rate of the measurements. At a minimum, two sampled points are required during one cycle to measure the amplitude of a frequency component. This means that the highest frequency component that can be extracted from sampled measurements is ½ the sample rate. This is the basis of the Nyquist theorem. The Nyquist frequency of sampled data is ½ the sample rate. The highest sine wave frequency that can be extracted is the Nyquist frequency.

$$F_{\text{max}} = F_{\text{Nyquist}} = \frac{1}{2} \times F_{\text{sample rate}}$$

If the sample rate is 10 GS/s, the Nyquist frequency is 5 GHz. The highest frequency we can calculate in the spectrum is 5 GHz.

Do not confuse the digitizing sample rate to record a signal with acceptable fidelity, with the sample rate to identify a specific frequency component of a sine wave component.

When the signal is a stable frequency sine wave and the acquisition trigger is synchronous with the signal phase, the measured waveform will have any arbitrary amplitude from the full value, to nearly 0, depending on the relative phase between the signal and the trigger. This is illustrated in **Figure 13.4**.

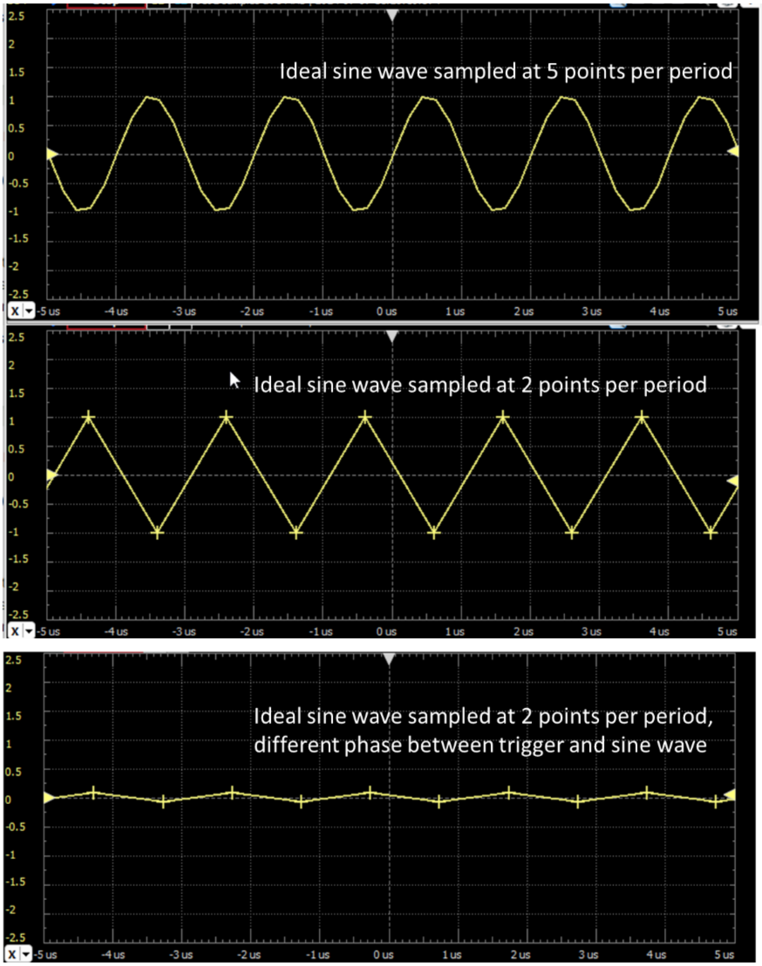


***Figure 13.4***. *Impact of sample rate and measured signal for a sine wave. Top: acquisition rate = 5 x frequency. Middle and bottom, acquisition rate = 2 x sine wave frequency = Nyquist, but different phases.*

If the frequency components of the signal are higher than the Nyquist frequency, the waveform is under sampled, the resulting recorded waveform is some arbitrary collection of points with no relationship to the original signal. Undersampling results in an artifact called aliasing. A signal will be recorded, and a spectrum created that is not an accurate representation of the original waveform.

As an example, **Figure 13.5** shows a 10 MHz sine wave signal, sampled at 1.37 MS/s. This means there is a voltage acquisition roughly every 7.299 cycles. Sometimes, the sampled point will catch a peak, sometimes a valley, and sometimes, somewhere in between. When the phase of the sine wave and trigger are both stable, their relative phases will be locked, and the sampled waveform will appear to be periodic. This is an artifact, termed aliasing, that will create artifacts in the spectrum at frequencies that are below the Nyquist frequency.

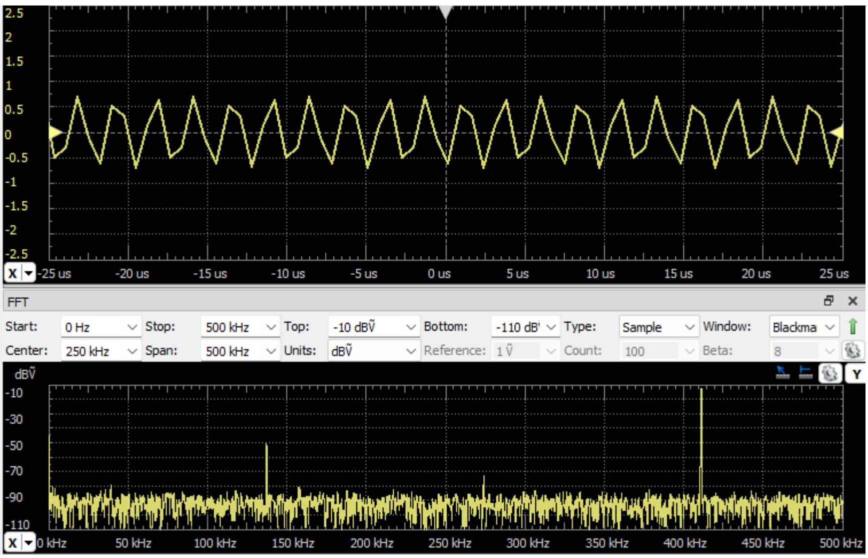


***Figure 13.5*** *A 1 V amplitude, 10 MHz sine wave sampled at 1.37 MS/s. The sine wave and trigger are stable. The sine wave is under sampled. The resulting waveform and its spectrum is not representative to the 10 MHz waveform. Spectral components in the signal appear at 140 kHz, 275 kHz and 410 kHz. These are aliasing artifacts.*

This artifact is most apparent when the signal is composed of stable sine waves, which is common when measuring rf communications signals. It is less apparent when measuring analog or digital signals.

To avoid this artifact, two steps are important. First, the input signal measured should be filtered so that frequency components above the Nyquist are removed. This eliminates the possibility of higher frequency components aliasing down to appear at lower frequencies. This is performed with an anti-aliasing filter. Sometimes this is available and built into the scope.

Second, the signal should be over sampled. At the very least, the Nyquist should be 2.5 x the frequency of the highest frequency component to measure. To preserve some fidelity of the signal features, the sample rate should be at least 5 x the highest frequency component in the signal. This will allow at least 5 samples over one period.

This is why, in most scopes, the highest sample rate achievable on all channels simultaneously is usually about 2.5 x the scope bandwidth. This is a cost-performance balance between the bandwidth of the scope, the ADC sample rate, the streaming rate for data into the memory buffer and the impact from aliasing due to under sampling the signal.

With all channels recording, frequency components above the Nyquist will be filtered by the scope amplifier's bandwidth. If the scope is used at a lower sample rate than the scope's bandwidth, such as when the time base is reduced, aliasing artifacts can arise. This is why being aware of aliasing and adding a hardware filter is so important.

In some scopes which use one ADC for all four channels, the sample rate is 10x the scope bandwidth. When one channel is measured, it is over sampled by 10x. But when all four channels are measured, each channel is over sampled by 2.5x.

### *13.3.4 Number of Points in the FFT Calculation*

Calculating the DFT of a time domain waveform requires an integral over all the measurement points at each frequency bin value. With one million data points in the acquisition buffer, about one trillion calculations are required to create one spectrum. This may take too long and would not appear in real time.

To get around this problem, all scopes use a much faster version of the DFT called the Fast Fourier Transform (FFT). It calculates the same integrals as the DFT, but it applies matrix math to perform the calculations. The FFT matrix math can only operate on a total number of points that is a power of 2. If there are one million points in the buffer, the highest number of points that could be included in the FFT calculation would be 2^19 = 524,288 points. The scope throws out almost half the measured data to gain incredibly fast computation time.

The first step in performing an FFT is to define the region of the acquisition buffer that contains the 2^n points that will be computed. Most oscilloscopes allow you to pick either the central region of the time-domain screen or a count from the left edge. **Figure 13.6** shows the region of the acquisition buffer that will be included in the FFT calculation between the dashed lines on the screen.

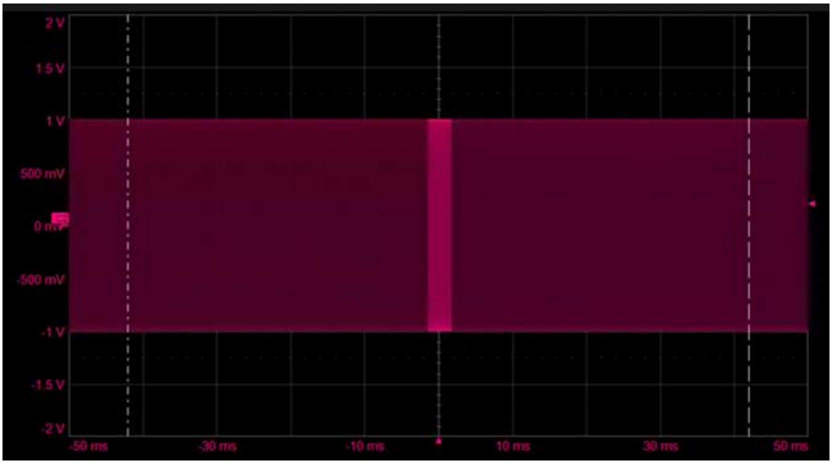


***Figure 13.6*** *Between the vertical dashed lines is the region of the acquisition buffer that contains the 2^n points which will be used in the FFT.*

When the acquisition buffer time is 1 µs, and we have one million points, we expect the fundamental frequency to be 1 MHz. In the spectrum, the FFT acquisition buffer is smaller than this, which means the actual resolution is slightly larger than 1 MHz. But these estimates are still a good rule of thumb to use when thinking about the features of the spectrum.

### *13.3.5 Windowing functions*

To create a periodic waveform, we repeat the acquisition buffer indefinitely in the future and in the past. When using the FFT function, we further truncated the acquisition buffer and repeat the truncated buffer indefinitely. This means that at the boundaries of each appended acquisition buffer, there may be a discontinuity in the waveform corresponding to the end of one buffer and the beginning of the next one. This is illustrated in **Figure 13.7**.

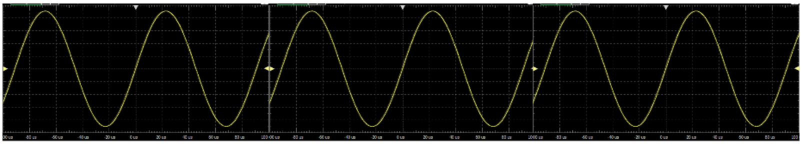

***Figure 13.7*** *Example of a repeated buffer that ends at a different value than it starts. When concatenated, there is a discontinuity at each boundary in this infinitely repeated waveform.*

In principle, the spectrum of a sine wave should be a single peak at the sine wave's frequency. If there are an integral number of cycles in the acquisition buffer the end of one cycle matches up with the beginning of the next buffer and there is no discontinuity. The spectrum will have a sharp peak and no other components in the spectrum.

However, if there is not an integral number of cycles in the buffer then there is a discontinuity at the boundaries of the beginning and end of the buffer. This means there will be a distortion in the spectrum. The discontinuity spreads spectral information from one frequency component into nearby components. This is called **spectral leakage**.

An example of the spectrum of a measured sine wave signal with an integral number of cycles and a slightly shifted frequency that does not have an integral number of cycles is shown in **Figure 13.8**. In this example, the sample rate was 100 MS/s which is about 10 samples per cycle. The time base was 50 usec full scale so the frequency resolution was 1/50 usec = 20 kHz. The frequency scale for the spectrum was 2 MHz/div. This is 100 frequency bins per division.

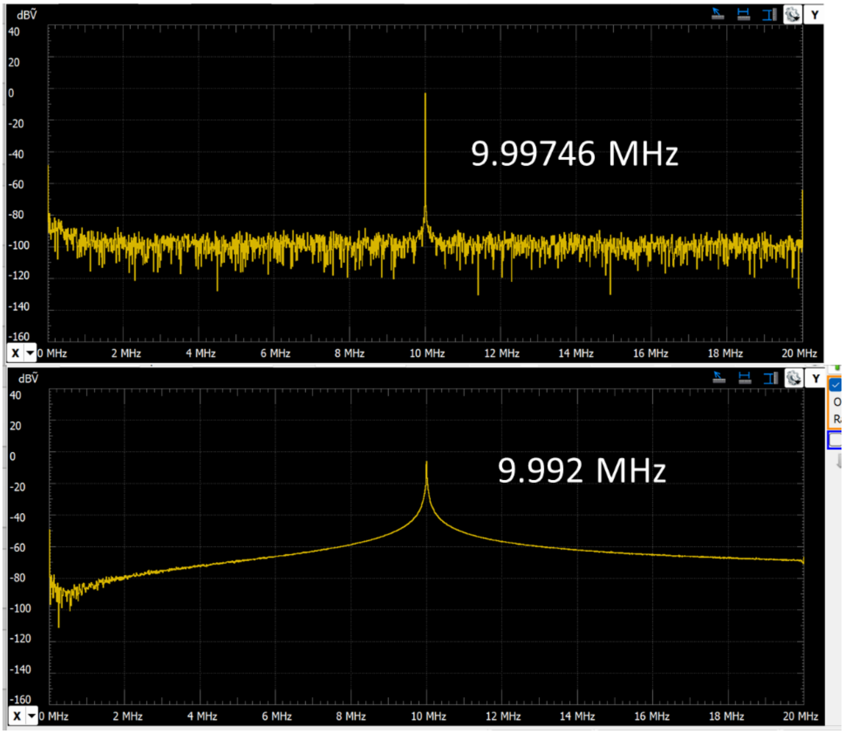


***Figure 13.8*** *Examples of the spectrum of measured sine waves, at about 10 MHz, sampled at 100 MS/s with no windowing function. Top: with an integral number of cycles per data acquisition window. Bottom: with a non-integral number of sine waves per acquisition buffer. Note the large spectral leakage.*

Spectral leakage is an artifact of the discontinuity at the boundaries of the buffers due to the first voltage value not being the same as the last voltage value. The way to reduce this artifact is to artificially reduce the discontinuity at the ends of the buffer by multiplying the entire acquisition buffer by a window function. This gradually forces the voltage value at the ends of the acquisition buffer to be 0, guaranteeing that the end of one buffer is continuous with the beginning of the next buffer.

There are a number of windowing functions commonly used. They differ in how much spectral leakage they allow and the resulting resolution. Unless you have a strong compelling reason otherwise, the Blackman-Harris function is a good compromise between resolution and spectral leakage.

Examples of four common window functions are shown in **Figure 13.9**.

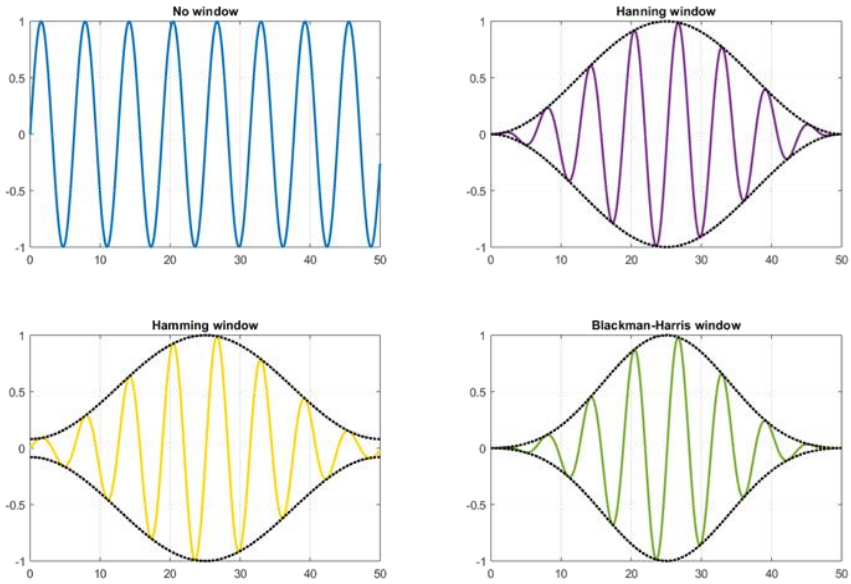


***Figure 13.9*** *Examples of four common windowing functions, courtesy of National Instruments.*

An example of a measured sine wave with a non-integral number of cycles in the acquisition buffer using these four windowing functions is shown in **Figure 13.10**. The resolution of the spectrum, based on the 50 usec acquisition buffer size is 20 kHz. This is 5 frequency bins per division on this scale. This is the width of the spectral peak of the sine wave with an integral number of cycles in the acquisition window.

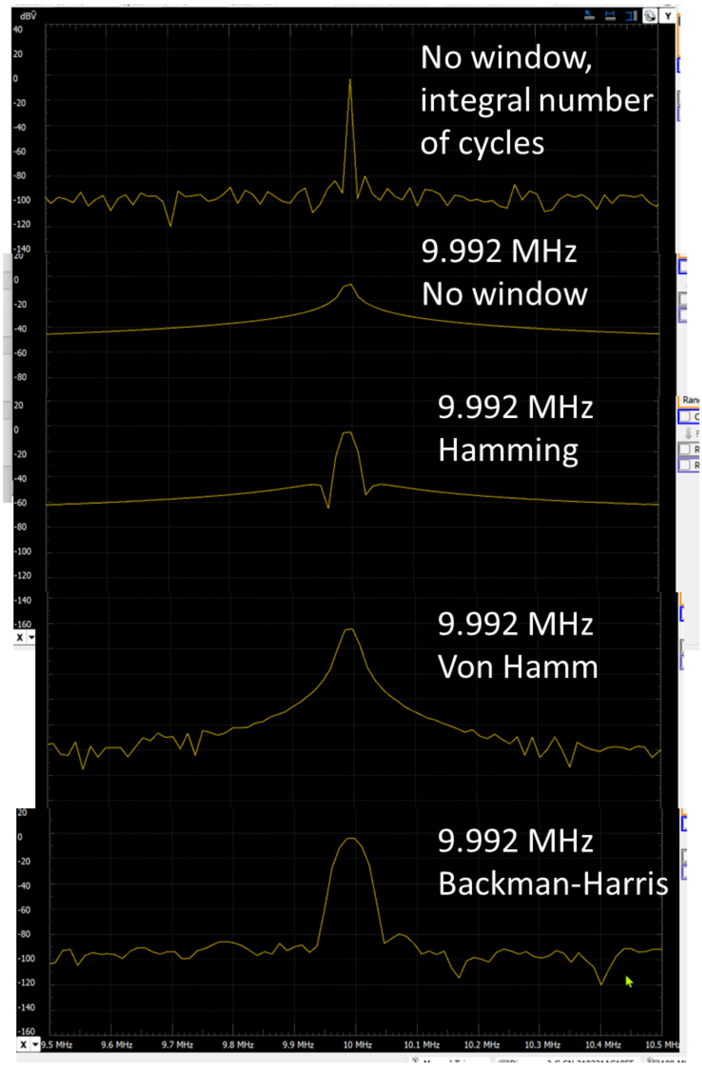


***Figure 13.10*** *Examples of the same sine wave with different windowing functions. The resolution is 20 kHz.*

Each windowing function is a different balance between frequency resolution and distribution of the spectral leakage. For general applications, the Blackman-Harris window is a good compromise between resolution and reduction in spectra leakage. If this window is available on your scope, it should be selected. Otherwise, the von Hann window is a second choice.

### *13.3.6 Summary of FFT principles*

The DC component is the amplitude of the 0 Hz frequency component. This is the average value over the acquisition time window.

The second frequency bin, and the frequency resolution of the spectrum, is 1/ the total acquisition time window. To enable a higher resolution, use a longer acquisition time window.

The highest frequency in the spectrum is the Nyquist frequency, ½ the sample rate. If you want to see a higher frequency component in the spectrum, you have to increase the sample rate.

Aliasing occurs when the signal has frequency components higher than the Nyquist. This will appear as anaomolus frequency components at lower than the Nyquist. Reduce this artifact by adding an anti-aliasing filter to limit the measured bandwidth below the Nyquist.

To implement the FFT, a number of points in the form of 2^n are sampled. Most scopes will take the largest number of points in the form of 2^n from the center of the acquisition buffer and use these measurements when calculating the FFT.

To minimize spectral leakage, use a windowing function such as the Blackman-Harris. This will reduce spectral leakage while maintaining acceptable resolution.

This suggests that to see the highest frequency in the spectrum, use the highest sampling rate to which the scope is capable. To get the highest resolution, use the longest acquisition window practical. This combination results in a large number of points in the acquisition buffer:

$$\#\text{points} = \text{acquisition time x sample rate}$$

Or

$$\text{acquistion buffer size} = \frac{\#\text{points}}{\text{sample rate}}$$

As a rough rule of thumb, depending on the speed of the scope, a reasonable number of samples in the acquisition buffer that takes less than 1 second to calculate the FFT is about 1-3 Million points.

A practical algorithm to follow is:

The scope should be set for the highest sample rate based on the highest frequency required in the spectra.

The time base is selected so that there are less than about 1 million points in the buffer.

Then the FFT is calculated in real time using a Blackman-Harris windowing function.

For example, if the scope is capable of 10 GSps, the highest calculated frequency in the spectrum is 5 GHz. For 1 million samples in the acquisition buffer, this is a total acquisition time interval of 1 million/10 GSps = 100 usec. With typically 10 divisions full scale for the time base, this is a time scale of 10 usec/div.

## 13.4Units for amplitudes

The amplitude of a sine wave signal is fundamentally measured in volts. However, other units are also commonly used. It is important to always be aware of the units used and be able to transform between them.

### *13.4.1 Amplitude and RMS in v*

The fundamental figure of merit of a sine wave is its amplitude. This is the value the signal reaches above and below the average value.

Real sine waves often have a DC offset. This means it can be misleading using the maximum value of a sine wave as an intrinsic measure of the sine wave. The peak to peak value is not sensitive to the DC component since the average value is common to both the minimum and maximum values.

The peak to peak value of a sine wave is 2 x the amplitude.

Another common unit to measure the magnitude of a sine wave is the root mean square (RMS) value. For an ideal sine wave, the RMS value of the wave is related to its amplitude, V, as

$$\mathrm{RMS} = \sqrt{\frac{1}{T_0}\int_0^{T_0} \sin^2\left(\frac{2\pi}{T_0}t\right)dt} = \sqrt{\frac{1}{2}V^2} = \frac{1}{\sqrt{2}}V = 0.707 \times V$$

The RMS value of a sine wave is linearly proportional to the amplitude. Why then is it necessary to consider the RMS value? This is related to the power associated with the voltage signal.

The power associated with a voltage signal is ambiguous. By itself, the voltage in a circuit is not a measure of the power associated with the signal. If this signal were to appear across a 1 ohm resistor or a 100 ohm resistor, very different powers would be consumed by the resistor, with the same voltage amplitude.

By agreed convention, unless otherwise specified, the power associated with a voltage signal always corresponds to the power that would be consumed if that voltage were to be applied across a 50 ohm resistor. This power, based on the signal's amplitude, V, is,

$$P = \frac{1}{2}\frac{V^2}{50\,\Omega} = \frac{V_{RMS}^2}{50\Omega}$$

The factor of ½ comes from the fact it is a sine wave. This is why the RMS value has significance in addition to the sine wave's amplitude. The power associated with a sine wave is the square of the RMS value divided by the 50 ohm resistance. This power level is often measured in units of dBm.

### *13.4.2 Units of dB, dBW and dBm*

The dB unit is used throughout engineering and is often a source of confusion. It is based on the unit of Bels, named for Alexander Grahm Bell. Note that when spelling the bel, one L is lost.

The bel is ALWAYS the log of the ratio of two powers. This is written as:

$$P[\text{bels}] = \log\left(\frac{P_1}{P_{ref}}\right)$$

For example, if the reference power is 1 watt, then the value of 100 watts in bels is log(100) = 2 bels. Likewise, the value of 0.01 watts in units of bels, is log(0.01) = -2.

The range of the bel is not very large, given how much change there is in the power level. In 1924, the Bell Telephone Company introduced the unit of decibels as a more sensitive measure, instead of the bel. A deci is 1/10$^{th}$, just as the centi is 1/100$^{th}$.

This makes the value of a ratio of two powers, in decibels, abbreviated dB, as

$$P[\text{dB}] = 10 \times \log\left(\frac{P_1}{P_{ref}}\right)$$

and

$$P_1 = P_{ref} \times 10^{\frac{P_1[dB]}{10}}$$

In principle, the dB is a ratio of any two powers. A power with a value of 10 dB means it is 10x the reference power. A power value of 20 dB means the power is 100x the reference value, and a value of -20 dB means the power is 0.01x the reference power. In this way, a power level can be compared to a reference power level using units of dB.

Without knowing the reference power value used in the definition of the dB, it is ambiguous what the absolute power measured is when it is referred to units of dB. It depends on the value of the reference power.

To avoid confusion from the value of the reference power, the convention is to use 1 watt as the reference power, and to make note of this by adding a W at the end of the dB, written as units of dBW. This is the conventional way of describing a power in units of dBW:

$$P[dBW] = 10 \times \log\left(\frac{P_1}{1watt}\right)$$

and

$$P_1[watt] = 1watt \times 10^{\frac{P_1[dBW]}{10}}$$

A power of 1 watt is always 10 x log(1/1) = 0 dBW. A power of 30 watts is 14.8 dBW.

Unfortunately, this convention is not followed often enough. Instead, the W is often dropped when describing a power in dB.

There is ambiguity in the use of the units of just the dB. In principle, it refers to a change in value from an initial value to a final value. If the power level in a reactor's output goes from 1 Mwatt to 100 Mwatts, this is a change of 100x in the power level, or a 20 dB change.

The absolute power went from 60 dBW to 80 dBW. When it is written as 60 dB or 80 dB, this is also an increase of 20 dB in the output power, but ambiguity is introduced. To avoid confusion in the use of the units of dB, the units should be written as dBW when referring to an absolute power level.

If we deal with small power levels, it is common to use a reference power level of 1 mwatt instead of 1 watt. In this case, to avoid confusion, the units are changed from dBW to dBm to signify that the reference power level is really mwatts. Likewise, if the reference power level were 1 uwatt, the units would be dBu.

This means that the conversion between power and dBm is,

$$P[\mathrm{dBm}] = 10 \times \log\left(\frac{P_1}{1\mathrm{mwatt}}\right)$$

and

$$P_1[\mathrm{mwatt}] = 1\mathrm{mwatt} \times 10^{\frac{P_1[\mathrm{dBm}]}{10}}$$

A power of 0 dBm is 1 mwatt. A power of 20 dBm is 10^(2) = 100 mwatt.

### *13.4.3 Using the dBm to describe voltage amplitude*

The dBm units can also be used to describe the power in a voltage, which would be consumed when this voltage amplitude appears across a 50 ohm resistor.

A 1 v amplitude sine wave, which has an RMS value of 0.707 v, would produce a power level of

$$P = \frac{1}{2}\frac{V^2}{50\,\Omega} = \frac{1}{2}\frac{1^2}{50\,\Omega} = 10\text{mwatt} = 10\text{dBm}$$

A voltage amplitude which would create a power of 1 mwatt across a 50 ohm resistor is,

$$V = \sqrt{P \times 100} = \sqrt{0.001 \times 100} = 0.316\text{ v}$$

When the dB scale is used to describe a voltage amplitude or RMS value, it is the ratio of the powers associated with two voltages, which is:

$$V[\text{dB}] = 10 \times \log\left(\frac{V^2}{V_{\text{ref}}^2}\right) = 20 \times \log\left(\frac{V}{V_{\text{ref}}}\right)$$

The factor of 20 comes from the fact that we are using the ratio of voltages inside the log operation. When the dBm scale is used, the reference voltage level is the voltage in a signal that generates 1 mwatt of power across a 50 ohm resistor, or $V_{ref}$ = 0.316v.

This results in conversions as,

$$V[\text{dBm}] = 20 \times \log\left(\frac{V[v]}{0.316v}\right) = 20 \times \log\left(V[v]\right)\text{dBm} + 10\text{ dBm}$$

And

## 13.4.3 Using the dBm to describe voltage amplitude



$$V[v] = 0.316\ v \times 10^{\frac{V[dBm]}{20}}$$

For example, a 1 v amplitude sine wave would be described as +10 dBm. A 1 mV signal would be described as -60 dBm + 10 dBm = -50 dBm.

Many scales in spectral displays use dBm as the vertical axis. This means a sine wave with an amplitude of 1 V would appear as a frequency component of + 10 dBm. **Figure 13.11** shows the example of a measured sine wave with amplitude of 1 v and frequency of 10 MHz. The spectrum shows a peak at 10 MHz with an amplitude of +10 dBm.

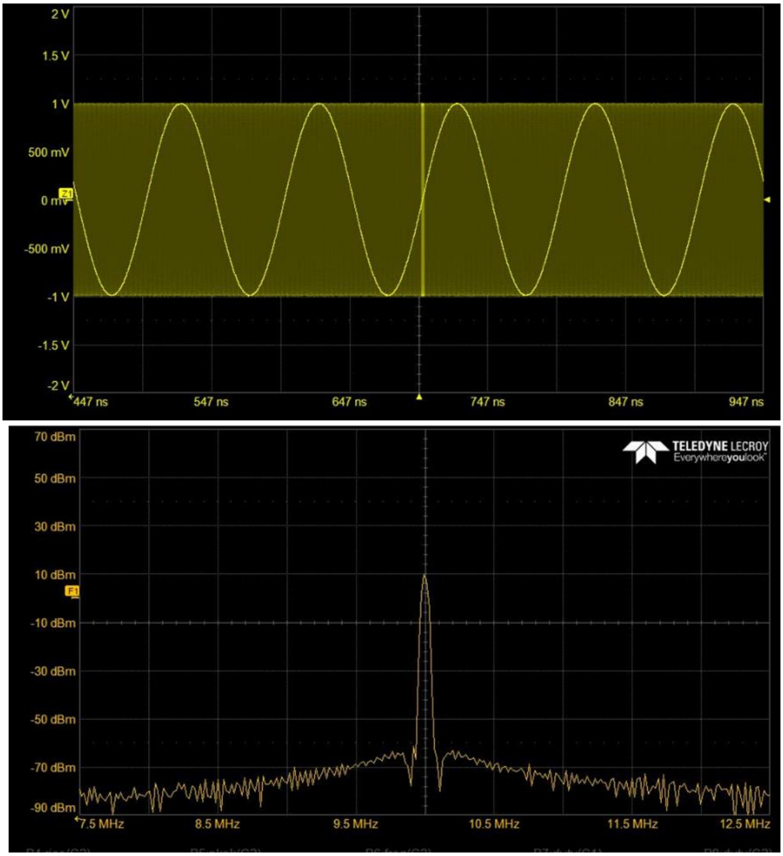


***Figure 13.11*** *An example of the measured sine wave with 1 v amplitude displayed as 10 dBm amplitude in the spectrum.*

It is convenient to remember that +10 dBm is 1 v and 0 dBm is 0.316. A 20 dBm change is a factor of 10 change in voltage. This means that 30 dBm is 10 x 1 V = 10 v. A value of -10 dBm is 0.1 v and a value of -30 dBm is 0.01 v. And a value of -20 dBm is 0.316 v x 0.1 = 0. 316 v.

### *13.4.4 Scales of dBV*

Alternatively, it is also common to use a scale in dBV. This means the reference voltage to which other voltages are measured is a 1 V amplitude. In general, the connection between voltage amplitude and dBV is

$$V[\mathrm{dBV}] = 20 \times \log\left(\frac{V[v]}{1v}\right) = 20 \times \log\left(V[v]\right)\mathrm{dBV} + 0\mathrm{dBV}$$

And

$$V[v] = 1\,v \times 10^{\frac{V[\mathrm{dBV}]}{20}}$$

A value of 0 dBV is a voltage amplitude of 1 V. A value of -40 dBV is a voltage amplitude of 0.01 V.

## 13.5 Quick Start Guide using the FFT function in a free sound card scope

If you do not have access to a scope, the spectral analysis features compared with time-domain signals can be explored using either the free MAUI Studio scope emulator and the function generator source or the PC sound card using Waveforms software and audio signals picked up by your computer's microphone.

In the free version of Digilent Waveforms, using your PC sound card as the scope front end, on the top of the toolbar, under the Scope window tab, is the button labeled as FFT. This is shown in **Figure 13.12**. Click this button, and an FFT window will open up below the real-time scope window.

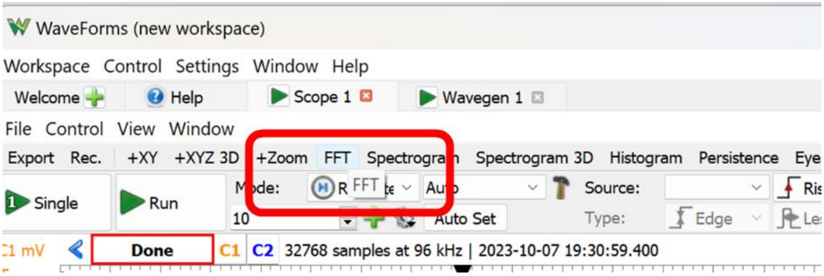


***Figure 13.12***. *Select the FFT button to open up a live spectral display of the real time voltage waveform measured by the scope.*

You can experiment with the horizontal scale of the frequency and the vertical scale for the amplitude of each frequency component. An example of a real time spectrum of sounds in the room is shown in **Figure 13.13**.

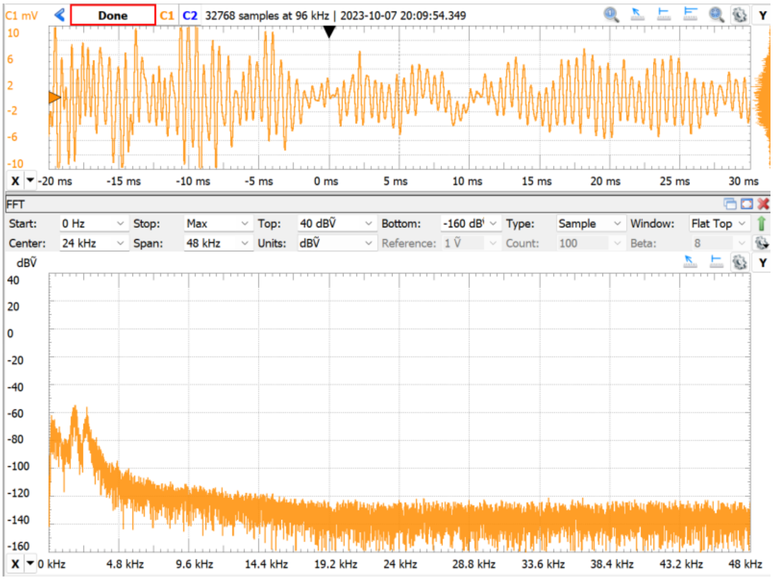


***Figure 13.13*** *Example of the spectral response of a measured voltage waveform displayed using the FFT function. This identifies frequency peaks at 2.4 kHz and 3 kHz and most of the spectral components below about 4.8 kHz.*

In this example, the acquisition sample rate is fixed by the sound card's ADC at 96 kSps. This means the highest frequency to display is 48 kHz. This is seen on the right edge of the spectral scale.

The total acquisition window is about 300 msec. This is 32k samples/96 kSps. This makes the lowest frequency measurable about 3 Hz and the resolution is 3 Hz. The time base is set up to display only 60 msec of the 300 msec measured acquisition buffer.

In this example, the vertical scale is in dBV. This means a value of -60 dBV is an amplitude of 1 mV.

On this linear scale, the amplifier sensitivity drops off above about 10 kHz. The noise floor corresponds to voltage amplitudes on the order of -120 dBV, which is 1 uV amplitude.

The Digilent Waveforms software tool is a great way of exploring many of the features of advanced scopes, with no additional components required, runs on your PC, and is free.

## 13.6 Spectra of Common Signals

There are a few commonly seen patterns in spectra which relate to features in the time domain signals. While the correlation between the time domain features and the frequency domain features are well established from the mathematical transform from the time to the frequency domain, a few measurement examples will help build engineering judgement.

With a suitable function generator, many of these waveforms can be synthesized and anyone can practice using the FFT function to observe these patterns.

Look for these patterns when observing spectra of signals and they will hint back to features in the time domain signal from which they might arise.

### *13.6.1 Setting up the scope*

The following examples are using a Teledyne LeCroy HDO6104B scope. It has a maximum sample rate of 10 GS/s at 12 bit vertical resolution.

While the acquisition buffer size is well in excess of 100 M samples, the buffer size was set to a max of 1 M sample to keep the display appearing as real time with no perceptible delay. At 10 GSps, it takes 1 MS/10 GSps = 100 usec to fill an acquisition buffer. This means the time per division should be limited to about 10 usec/div. Unless there is a strong compelling reason, this is a reasonable set up to observe signals which vary on time frames shorter than 100 usec. These settings take full advantage of the scope's features. Otherwise, the scale can be adjusted accordingly.

To see the details of any signals that vary on time frames faster than can be displayed at 10 sec/div, the measured acquisition buffer can be zoomed in to show the faster-changing signals. The data acquisition rate is at the scope limit and will not change as the time base is zoomed in.

On this time base of 100 usec full scale, the frequency resolution is 1/100 usec = 10 kHz. The highest sample rate of 10 GSps means the highest displayed frequency component in the spectrum is 5 GHz. The scope bandwidth of 1 GHz will act as an anti-aliasing filter.

This means the spectrum displayed will span 0 Hz to 5 GHz with a resolution of 10 kHz. This is 500,000 frequency bins available. To explore this frequency range, regions can be zoomed in, either in live mode or after an acquisition has been recorded.

Before looking at the spectra of measured signals, it is useful to get a feel for the system level noise and its spectrum. On the highest voltage resolution scale of 1 mV/div, the noise floor of the scope's amplifier can be measured, and the background spurious frequency components identified.

**Figure 13.14** shows the time base and spectrum scale with the most sensitive voltage scale of 1 mV/div. This identifies three features in the spectrum.

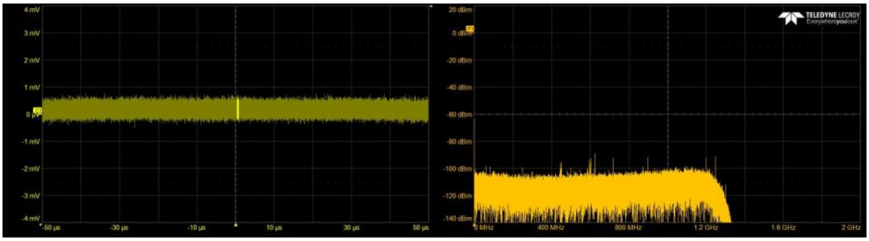


***Figure 13.14*** *The measured noise floor of the scope with the input set to gnd.*

First is the drop off in the spectral response at about 1.2 GHz. This is from the DSP filter in the scope that introduces a sharp drop in response above the rated scope bandwidth of 1 GHz.

Second is the baseline noise floor limiting the peak detectable values to about -100 dBm. This is a factor of 100,000 down from the 0 dBm value of 0.316 V, or a noise floor corresponding to an amplitude level of 3.156 uV detectable above the scope noise.

There are also spurious frequency components extending to about -90 dBm or to a voltage amplitude of 10 uV. These are due to the interleaved ADC sampling harmonics. Any peaks in a measured spectrum above this spurious peak level of -90 dBm or 10 uV amplitude are probably real.

There is a $0^{th}$ harmonic value at about -60 dBm. This corresponds to a DC value of about 0.316 mV. This matches the observation in the time domain signal of the DC offset of about 0.3 mV.

### *13.6.2 A sine wave*

When using a new instrument or new features of an instrument for the first time, the best practice is to always measure something for which you know the answer. This way, you can apply Rule #9 and verify you are measuring what you expect to see.

A good initial signal to measure is a sine wave. In this first example, an external function generator was used to generate a 10 MHz sine wave with amplitude of 1 V. We expect the spectrum to contain one peak with a value of 1 V amplitude, which is +10 dBm.

**Figure 13.15** shows the measured signal in the time domain and the FFT response. While there is a peak at about 1 MHz with an amplitude of +10 dBm, there are also other peaks, some near 620 MHz but down in amplitude to -60 dBm. This corresponds to a 0.001 x 0.316 v or 0.316 mV amplitude signal amplitude. Are these real or part of the scope's spurious signals? One way of determining this is to measure the spectral response with the function generator unplugged.

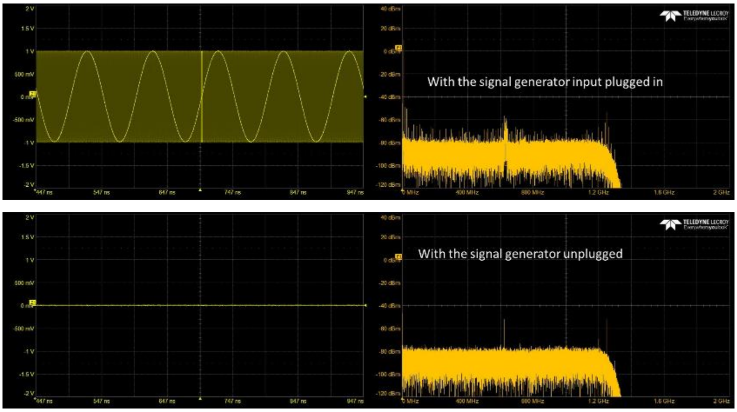


***Figure 13.15*** *Measured 10 MHz sine wave from the function generator shown in the time domain on the left and the spectrum on the right. Note the response with the function generator unplugged.*

The spurious peaks are present only with the function generator plugged in and are clearly from the function generator. They are at the roughly -70 dBm level corresponding to a voltage amplitude of about -80 dB down from 10 dBm or 1 v. This is an amplitude of about 100 uV. This is a measure of the spectral contamination from the function generator.

On this frequency range, the features of the 10 MHz sine wave are difficult to resolve. Without changing the acquisition settings, the spectrum can be zoomed to show the features of the 10 MHz frequency components. This is shown in **Figure 13.16**.

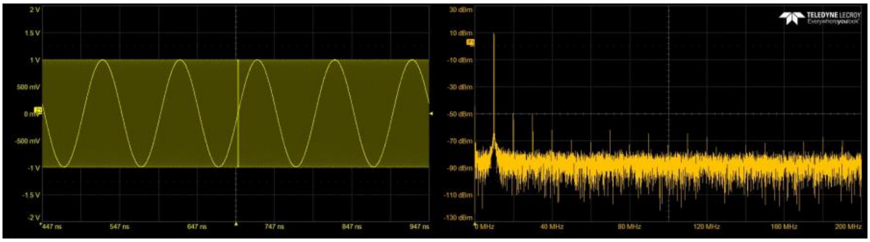


***Figure 13.16*** *Measured spectrum of the 10 MHz sine wave from the function generator. Note the higher harmonics from the sine wave.*

The amplitude of the first harmonic is 10 dBm, or 1 v amplitude. The second harmonic at 20 MHz, is down to -50 dBm. This is a drop of -60 dB or an amplitude of 0.1% of the first harmonic. This is a very low level of total harmonic distortion. The 3$^{rd}$ harmonic is also of this order, but the higher harmonics drop off quickly, exactly as expected for a high-quality function generator.

The time base is 100 usec full scale so the frequency resolution of each bin is 1/100 usec = 10 kHz. **Figure 13.17** shows the 10 MHz peak on a zoomed scale, highlighting the width of the peak as about 1/10$^{th}$ a division. The frequency scale is 500 kHz per division. This is a width of about 50 kHz, higher than the expected 10 kHz resolution. On this scale, there are 50 frequency bins per division.

However, the peak width is defined as the full width at half maximum (FWHM). This is the peak width when the value drops by -3 dB from its peak value. On this scale of 20 dB/div, this is nearly at the very tip of the peak. The FWHM width is closer to the expected frequency resolution of the FFT of 10 kHz.

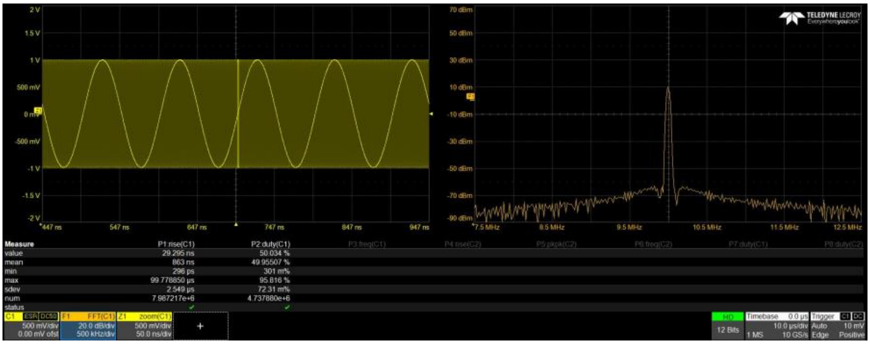

***Figure 13.17*** *A zoomed frequency scale of the spectrum of the sine wave showing the width of the peak as about 50 kHz. The FWHM width, at the very tip of the peak, is closer to the expected 10 kHz.*

### *13.6.3 A square wave*

An ideal square wave is an example of a waveform that is periodic, with a repeat frequency. In any periodic waveform, there will be frequency components in the spectrum multiples of the repeat frequency. Each frequency component is called a harmonic.

When you see a spectrum with evenly spaced frequency components, they are due to a repetitive waveform in the time domain.

In an ideal square wave with a short rise time and 50% duty cycle, the frequency components will be odd multiples of the repeat frequency. Even if the rise time is 0 psec, the amplitudes of each component will drop off with 1/f. An example of a square wave and its spectrum is shown in **Figure 13.18**.

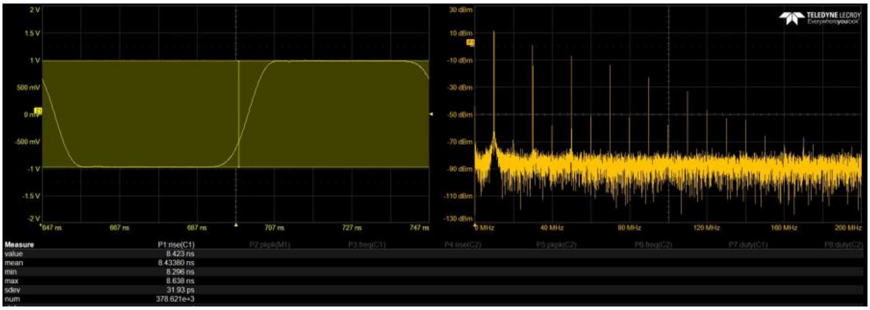


***Figure 13.18*** *The spectrum of a square wave generated by the function generator. The frequency scale has been expanded to 20 MHz/div.*

The drop off in amplitude of the harmonics shows the 1/f dependency. The rise time is measured as 8.4 nsec. This would result in a bandwidth of about 0.35/8.4 nsec = 42 MHz. This is the frequency at which the amplitudes of the harmonics would drop off faster than 20 dB/decade. This is roughly after the 5$^{th}$ harmonic.

In principle, in a waveform anti-symmetric about its center, so that f(t) = -f(t-1/2 T), as is a square wave with a 50% duty cycle, the amplitude of the even harmonics will be zero. The duty cycle of this synthesized square wave was adjusted to minimize the even harmonic amplitudes.

In the best case, the 2$^{nd}$ harmonic was reduced -80 dB from the 1$^{st}$ harmonic. With the noise floor below -80 dBm, even harmonics with amplitude on the order of -60 dBm are clearly visible. This is an example of how a sensitive measurement can reveal imperfections in waveforms, compared to their ideal behavior.

The magnitude of the even harmonics in a repetitive waveform are an indication of the asymmetry in the waveform.

By increasing the duty cycle from 50% to just 50.22%, the magnitude of the even harmonics are increased from -70 dBm to -30 dBm, a change of 40 dB or a factor of 100 in amplitude. The spectrum of the square wave with this slight increase in duty cycle and large increase in even harmonic amplitudes is shown in **Figure 13.19**.

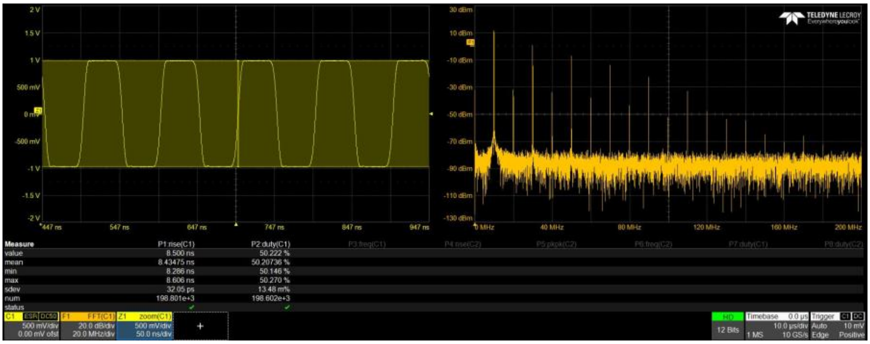


***Figure 13.19*** *Increasing the duty cycle slightly to 50.22% dramatically increases the amplitude of the even harmonics.*

### *13.6.4 A Modulated Sine wave*

A single-frequency sine wave has a single peak in the spectrum. When this frequency is modulated, as in FM modulation, the frequency component is spread out with a width equal to the frequency span of the modulation.

When the frequency is modulated at a slow rate compared with the total acquisition time, the spacetime appears as a peak that moves back and forth. When the frequency excursions are modulated in a time short compared to the acquisition time, the peak is broadened.

**Figure 13.20** shows the spectrum of a 10 MHz sine wave modulated by a 4 MHz FM modulation amplitude, with a sweep frequency 0f 50 kHz. This is a period of 20 uses, short compared to the 100 uses total equation down. This means that the acquisition window has all the frequencies in the modulated waveform. The spectrum will be a broadened peak with a width of 8 MHz.

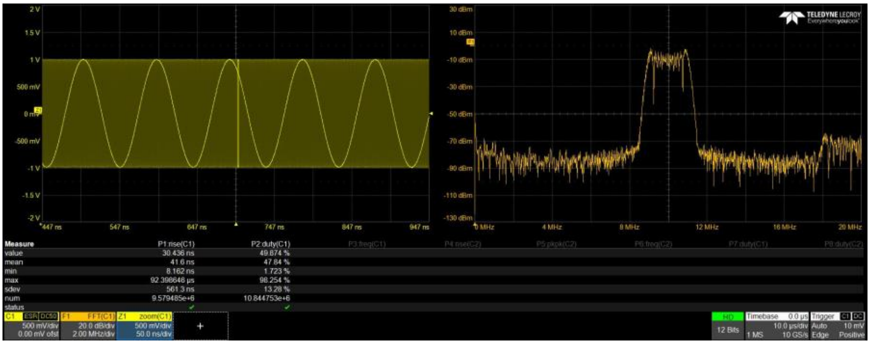


***Figure 13.20*** *The spectrum of a 10 MHz sine wave with a frequency modulation amplitude of 4 MHz. Note the signature of a flat, broad peak.*

This is an important signature of a broad spectral peak with a nearly flat top. The modulation waveform, either sinusoidal, triangular, or other pattern, will change the shape of the peak's top, not its width.

### *13.6.5 Narrow pulses*

As the pulse width of a repetitive signal gets shorter, it approaches an impulse response. This means the amplitude of the spectral components at the repeat frequency will approach a flat response. The spectrum of a pulse is shown in **Figure 13.21**. The amplitude of the first few harmonics is nearly constant. This is the signature of a narrow, reactive pulse.

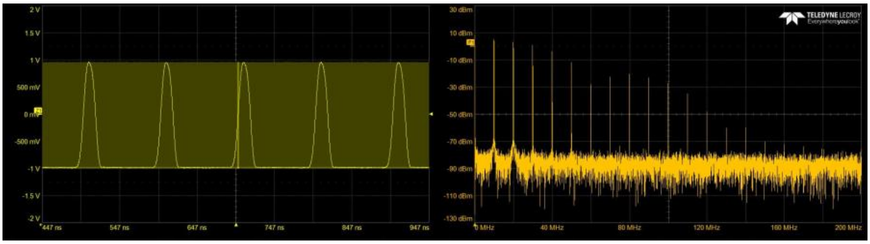


***Figure 13.21*** *The spectrum of a 10MHz repetitive pulse shows a flat spectrum of harmonics at the repeat frequency.*

## 13.7RF Pickup Noise

We live in a noisy electromagnetic environment. If we could see with RF-sensitive eyes, we would be surrounded by a dense fog. Every frequency range has some common sources.

Noise with frequency components in the 1 kHz to 100 kHz band might be from switch mode power supplies. This noise radiates from interconnect cables or circuit board traces.

Components in the 40 kHz band are usually from the inverter circuits used to power LED lights.

AM broadcast radio is in the 535 kHz to 1.7 MHz band.

The shortwave radio is in the 5.95 MHz to 26.10 MHz band.

Free television stations are in the 54 MHz to 88 MHz and 174 MHz to 220 MHz bands.

Garage door openers are around 40 MHz.

Cordless phones use bands from 43 MHz to 50 MHz, 900 MHz, 1.9 MHz, 2.4 GHz and 5.8 GHz.

Many electronic devices use the Instrumentation, Scientific, and Medical (ISM) bands. These are unlicensed frequency bands in which the FCC allows radiated emissions. There are three bands:

- ✓ *902-928 MHz*
- ✓ *2.4 to 2.483 GHz*

- ✓ *5 GHz*

A short wire loop is a magnetic dipole antenna. A voltage is induced across the ends of the loop, which can be measured by a scope. A simple version is just shorting the ends of a mini grabber at the end of a coax cable.

Using the standard settings of 10 GS/s and 100 uses full scale will show frequency components from 10 kHz to 1.2 GHz, limited by the scope bandwidth. It can be used as a quick and simple probe to sniff the local electromagnetic field background. **Figure 13.22**shows the measured spectrum of the scope with no input and then with the mini grabber attached. There are distinctive peaks present.

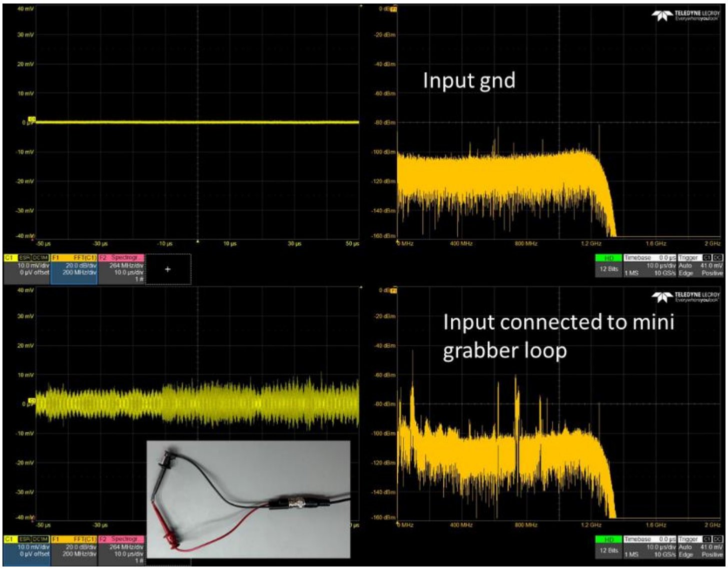


***Figure 13.22*** *The measured noise spectrum was picked up by a simple magnetic dipole loop showing peaks. Top: scope basic noise level, bottom: spectrum of rf pick up from the small coil seen in inset.*

The measured spectrum has four distinct frequency bands: 40 MHz, 600 MHz, 750 MHz, and 900 MHz. These are common communication bands. For example, the band around 750 MHz is expanded in **Figure 13.23**to reveal a signature of flat bands in narrow channels. This is the signature of FM modulated signals probably from communications systems.

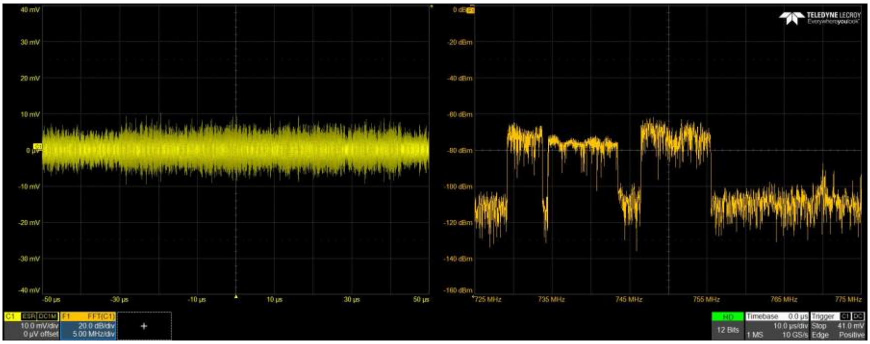


***Figure 13.23*** *The measured noise spectrum expanded in the 750 MHz range, showing a flat top signature similar to FM-modulated communications signals.*

## 13.8 The Bottom Line