Differentiation

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September 2020

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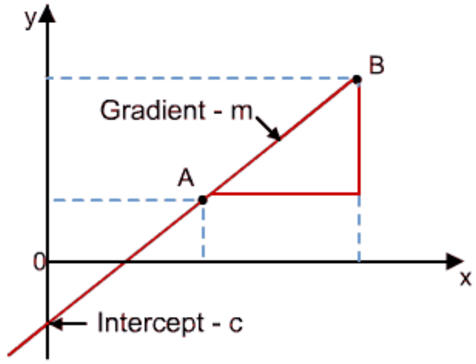
Motivation

- We often need to compute how the output of a function changes, when we alter a parameter by a small amount
- This process is called computing the gradient (also called the derivative) of a function
  - Also is known as differentiation.

Differentiation is all about measuring change!
Measuring change in a linear function:



$$y = c + mx$$

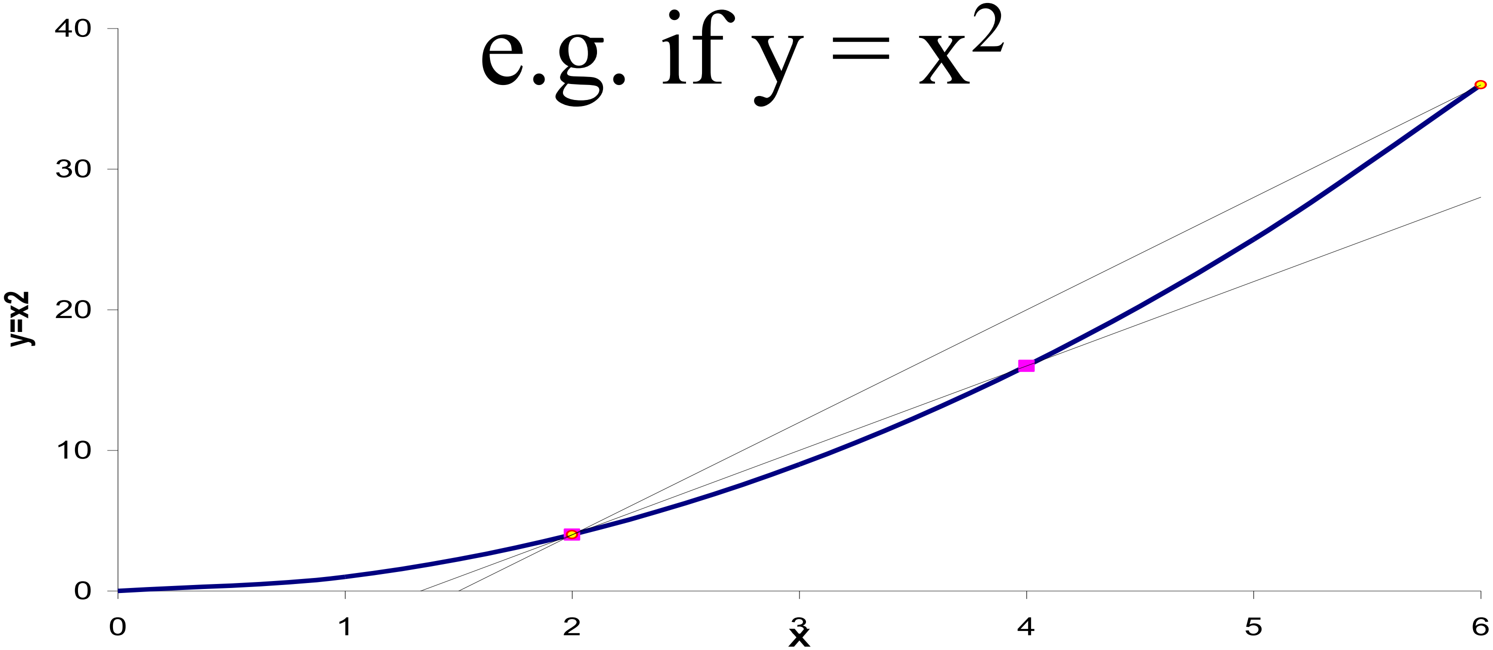
c = intercept

m = constant slope i.e. the impact of a unit change in x on the level of y

$$\mathbf{m} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

If the function is non-linear:

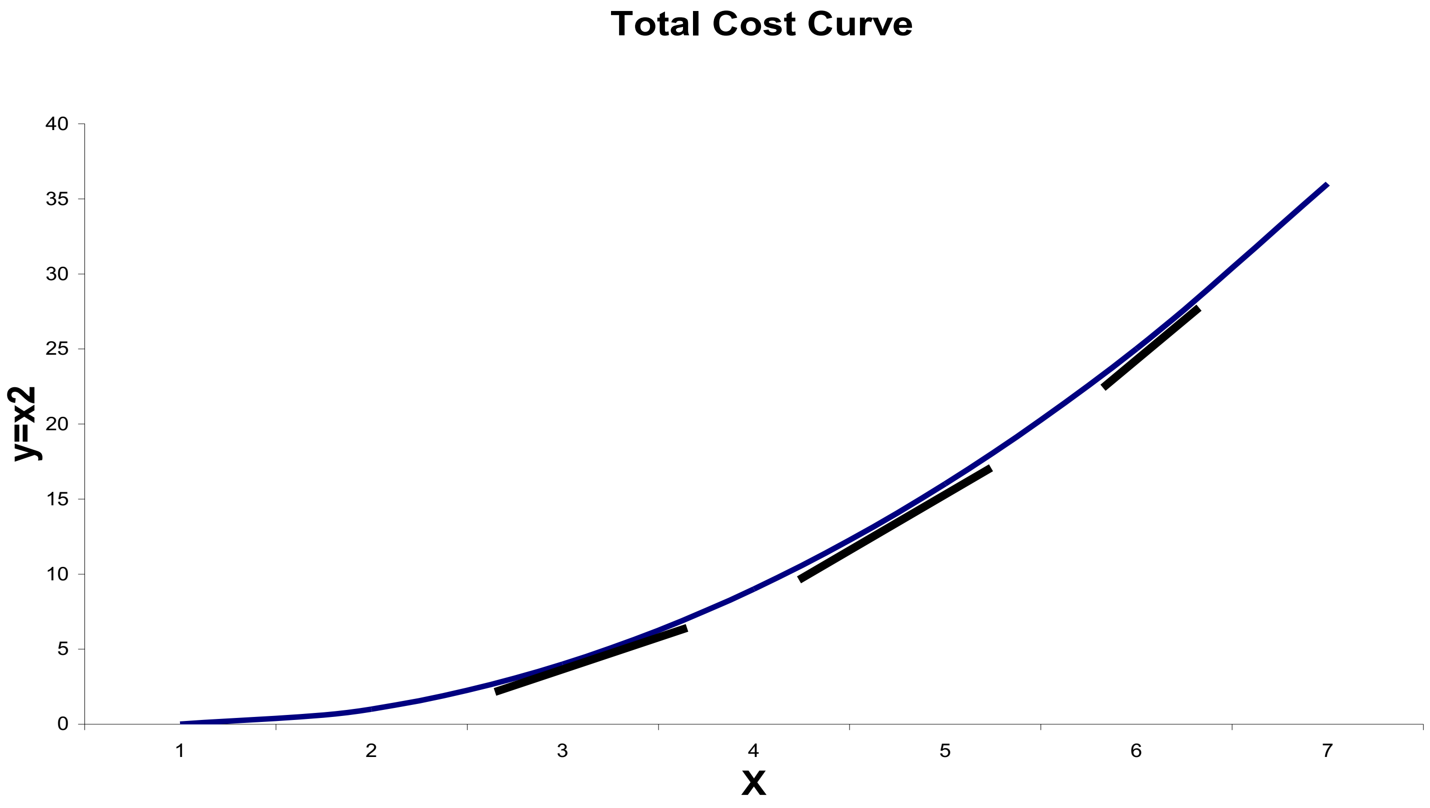
e.g. if $y = x^2$



$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ gives slope of the *line* connecting 2 points (x_1, y_1) and (x_2, y_2) on a curve

- (2,4) to (4,16): slope = $^{(16-4)}/_{(4-2)} = 6$
- (2,4) to (6,36): slope = $^{(36-4)}/_{(6-2)} = 8$

The slope of a curve is equal to the slope of the line (or tangent) that touches the curve at that point



which is different for different values of x

The slope of the graph of a function is called the derivative of the function

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

- The process of differentiation involves letting the change in x become arbitrarily small, i.e. letting $\Delta x \to 0$

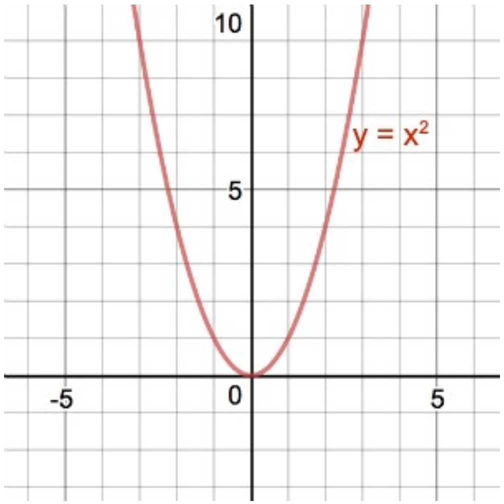
the slope of the non-linear function $Y = X^2$ is $2X$

- the slope tells us the change in y that results from a very small change in X
- We see the slope varies with X

e.g. the curve at $X = 2$ has a slope $= 4$

and the curve at $X = 4$ has a slope $= 8$

- In this example, the slope is steeper at higher values of X



Rules for Differentiation

1. The Constant Rule

If y = c where c is a constant,

$$\frac{dy}{dx} = 0$$

e.g. y = 10 then $\frac{dy}{dx} = 0$

2. The Linear Function Rule

If $y = a + bx$

$$\frac{dy}{dx} = b$$

e.g. $y = 10 + 6x$ then $\frac{dy}{dx} = 6$

3. The Power Function Rule

If $y = ax^n$, where a and n are constants

$$\frac{dy}{dx} = n.a.x^{n-1}$$

i) y = 4x => $\frac{dy}{dx} = 4x^0 = 4$

ii) y = $4x^2$ => $\frac{dy}{dx} = 8x$

iii) y = $4x^{-2}$ => $\frac{dy}{dx} = -8x^{-3}$

4. The Sum-Difference Rule

If $y = f(x) \pm g(x)$

$$\frac{dy}{dx} = \frac{d[f(x)]}{dx} \pm \frac{d[g(x)]}{dx}$$

If y is the sum/difference of two or more functions of x:

differentiate the 2 (or more) terms separately, then add/subtract

(i) $y = 2x^2 + 3x$ then $\dfrac{dy}{dx} = 4x + 3$

(ii) $y = 5x + 4$ then $\dfrac{dy}{dx} = 5$

5. The Product Rule

If y = $u.v$ where u and v are functions of x,
($u = f(x)$ *and* $v = g(x)$) Then

$$\frac{dy}{dx} = u\,\frac{dv}{dx} + v\,\frac{du}{dx}$$