Basic Probability

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Sample Space

- A Sample Space is a LIST of all possible outcomes.

- A Sample Space Diagram is a TABLE which shows all the possible outcomes of a scenario.

X	1	2	3	4	5	6
P(X)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Table 1: The probability matrix of the outcomes of a single role of a die.

Independent Events

Events are ***independent*** if the outcome of each event is not affected by any other event outcomes.

Example

A bag contains 2 blue, 3 green and 5 red balls.

If I draw one ball at random, the probability of a blue ball …

$$P(\text{Blue}) = 2/10$$

If I draw another ball the probability of it being Blue depends on what colour ball was drawn first –

not independent.

Probability - Notation

The probability of an event A happening can be written as P(A)

$$P(A) = \frac{\text{number of ways event } A \text{ can happen}}{\text{total number of possible outcomes (including } A\text{)}}$$

X	1	2	3	4	5	6
P(X)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Table 1: The probability matrix of the outcomes of a single role of a die.

Example

The probability of getting an "Ace" by drawing one card from a pack of 52 cards:

$$P(\text{Ace}) = \frac{\text{number of ways of getting an ace}}{\text{total number of possible cards picked}} = \frac{4}{52}$$

Probability of Event NOT happening

$P(A')$ is probability of event A NOT happening.

$$P(A') = 1 - P(A)$$

e.g.

$$P(\text{not an ace}) = 1 - \frac{4}{52} = \frac{48}{52}$$

Probability of Independent Events

If two events A and B are independent then the probability of them both happening is:

$$P(A \textit{ and } B) = P(A) \times P(B)$$

Example

A dice and a coin are thrown. What is the probability of a HEAD and a 4?

Answer:

$$P(H \text{ and } 4) = P(H) \times P(4)$$
$$= 1/2 \times 1/6$$
$$= 1/12$$

Mutually Exclusive

Events are mutually exclusive if they can't happen at the same time

e.g. If coin is tossed it cannot land as HEAD and TAIL

Events "showing a head" or "showing a tail" are mutually exclusive.

Probability of Mutually Exclusive

If two events *A* and B are mutually exclusive, the probability of either event *A* or event B occurring …

$P(A \text{ or } B) = P(A) + P(B)$

Example

If a dice is thrown what is the probability of an even number or a "3"

Since three is not an even number (!)

$$P(\text{Even or "3"}) = P(\text{Even}) + P(3)$$
$$= 3/6 + 1/6$$
$$= 4/6$$