

---

**MATH 301**

# Advanced Introduction to Probability



## Spring 2022, Session 3

---

Dates / Synchronous meeting time:	TBA
Academic credit:	4
Hybrid course format:	video, lecture, recitation, discussion

---

### Instructor's information

---

Dr. Lin Jiu	Lecturer of Mathematics, Duke Kunshan University Assistant Professor of the Practice, Duke University
Email:	<a href="mailto:lin.jiu@dukekunshan.edu.cn">lin.jiu@dukekunshan.edu.cn</a>
Office:	CC2057
Office Hours:	Tuesdays 14:00—15:00
Zoom ID:	999 1505 2538      PASSCODE: MATH301

### What is this course about?

---

Probability is one of the most beautiful branches in mathematics that also applies to many areas. Different from MATH205, we shall introduce concept via rigorous proofs, for clarity and precision. Namely, a solid foundation shall be established, which applies to statistical methods, data science, information theory, etc.

### What background knowledge do I need before taking this course?

---

Required:      MATH 201, MATH 202 and MATH 205

### What will I learn in this course?

---

We shall learn the main objectives for this course:

rigorous introduction on probability, including discrete and continuous distributions, random variables and limit theorems.

Therefore, you will setup probabilistic reasoning in precise mathematical terms; will develop some fundamental results, tools and models for further applications.

More precisely, you will be able to

- compute, for both discrete and continuous distributions, probabilities, expectations, variance, and moments, including conditional expectation and variances.
- classify and describe discrete and continuous random variables, and random vectors
- compute probabilities on joint, marginal and conditional distributions
- apply generating functions to compute e.g., independent random variables, random sum of i.i.d. random variables, etc.
- distinguish different convergence in probability
- demonstrate rigorous proofs of law of large numbers, central limit theorem, and Poisson limit theorems

## What will I do in this course?

---

The course will be comprised of (video) lectures, synchronous meetings dedicated to problem solving and lecturing, homework, and exams.

**Lectures.** Lectures will be given in our regular classroom. If no remote student joins after the Add/Drop session, there will not be any recordings.

**Homework. (48%)** There will be weekly homework, 6 times in total and each is worth 8%. Except for the last week, each week, a homework assignment will be given before Friday noon, 12:00 and due the next Friday noon. All homework assignments will be released and submitted on Gradescope. Collaboration is encouraged, but you must write up your own solutions. **Late homework will not be accepted**, except as required by the University Policy.

**Presentation. (20%)** There will be a individual presentation during the last lecture(s). The 20% points will be given based on the following items

- Topic 5%. The topic should be related to probability. The following situation should be avoided (otherwise, you will lose points):
  - The topic is too simple.
  - Your topic overlaps with someone else's for more than 20%.Both requirements are necessary for a conference/workshop/symposium talk; namely besides fitting the theme, your talk should have an adequate level and unique, comparing with the others'.
- Slides (typo, beamer, etc.) 5%. Due to the duration, your slides can have up to 10 typos; and the slides file should be a PDF file, generated by LaTeX beamer instead of PowerPoint.
- Articulation (English, explanation, Q&A, etc.) 5%,
- Peer review 5%.

**Exams. (32%).** There will be a written final exam. Date and time shall be scheduled by the Registrar's Office, and will be announced once it is determined.

## How can I prepare for the class sessions to be successful?

---

To succeed, you should be prepared to devote several hours to this course on a daily basis, more precisely, you are expected to spend approximately 24 hours of total in- and out-of-class time per week. It is also strongly encouraged to use the online tutoring resources of ARC, to work with classmates, and to contact instructors in a timely manner for additional help as needed.

## What required texts, materials, and equipment will I need?

---

"A Natural Introduction to Probability Theory" by Ronald Meester.

ISBN: 978-3-7643-8723-5

Electronic Version Free from DKU/Duke Library: [HERE](#)

## How will my grade be determined?

---

- |                  |     |
|------------------|-----|
| 1. Homework:     | 48% |
| 2. Presentation: | 20% |
| 3. Final exam:   | 32% |

Please refer to the following scale for your grading.

**A+** = 98.00% - 100% **A** = 93.00 - 97.99%; **A-** = 90.00% - 92.99%;  
**B+** = 87.00% - 89.99%; **B** = 83.00% - 86.99%; **B-** = 80.00% - 82.99%;  
**C+** = 77.00% - 79.99%; **C** = 73.00% - 76.99%; **C-** = 70.00% - 72.99%;  
**D+** = 67.00% - 69.99%; **D** = 63.00% - 66.99%; **D-** = 60.00% - 62.99%  
**F** = 59.99% and below

## What are the course policies?

---

### Academic Integrity:

As a student, you should abide by the academic honesty standard of the Duke Kunshan University. Its community Standard states: "Duke Kunshan University is a community comprised of individuals from diverse cultures and backgrounds. We are dedicated to scholarship, leadership, and service and to the principles of honesty, fairness, respect, and accountability. Members of this community commit to reflecting upon and upholding these principles in all academic and non-academic endeavors, and to protecting and promoting a culture of integrity and trust." For all graded work, students should pledge that they have neither given nor received any unacknowledged aid.

Please refer to <http://undergrad.dukekunshan.edu.cn/undergraduate-bulletin-2020-2021/> and <https://dukekunshan.edu.cn/en/academics/advising> for DKU course policies and guidelines.

Duke Kunshan University is a community comprised of individuals from diverse cultures and backgrounds. We are dedicated to scholarship, leadership, and service and to the principles of honesty, fairness, respect, and accountability. Members of this community commit to reflecting upon and upholding these principles in all academic and non-academic endeavors, and to protecting and promoting a culture of integrity and trust. To uphold the Duke Kunshan Community Standard, each student is expected to pledge to hold him/herself to the highest standards for honesty, integrity, fairness, and responsibility in his/her academic and non-academic endeavors, to respect other cultures and embrace all forms of diversity, and to uphold the standards if they are compromised.

### Academic Policy & Procedures:

You are responsible for knowing and adhering to academic policy and procedures as published in University Bulletin and Student Handbook. Please note, an incident of behavioral infraction or academic dishonesty (cheating on a test, plagiarizing, etc.) will result in immediate action from me, in consultation with university administration (e.g., Dean of Undergraduate Studies, Student Conduct, Academic Advising). Please visit the Undergraduate Studies website for additional guidance related to academic policy and procedures. Academic integrity is everyone's responsibility.

### Academic Disruptive Behavior and Community Standard:

Please avoid all forms of disruptive behavior, including but not limited to: verbal or physical threats, repeated obscenities, unreasonable interference with class discussion, making/receiving personal phone

calls, text messages or pages during class, excessive tardiness, leaving and entering class frequently without notice of illness or other extenuating circumstances, and persisting in disruptive personal conversations with other class members. Please turn off phones, pagers, etc. during class unless instructed otherwise. If you choose not to adhere to these standards, I will take action in consultation with university administration (e.g., Dean of Undergraduate Studies, Student Conduct, Academic Advising).

### **Academic Accommodations:**

If you need to request accommodation for a disability, you need a signed accommodation plan from Campus Health Services, and you need to provide a copy of that plan to me. Visit the Office of Student Affairs website for additional information and instruction related to accommodations.

## **What campus resources can help me during this course?**

---

### **Academic Advising and Student Support**

Please consult with me about appropriate course preparation and readiness strategies, as needed. Consult your academic advisors on course performance (i.e., poor grades) and academic decisions (e.g., course changes, incompletes, withdrawals) to ensure you stay on track with degree and graduation requirements. In addition to advisors, staff in the Academic Resource Center can provide recommendations on academic success strategies (e.g., tutoring, coaching, student learning preferences). All ARC services will continue to be provided online. Note, there is an ARC Sakai site for students and tutors. Please visit the [Office of Undergraduate Advising website](#) for additional information related to academic advising and student support services.

### **Writing and Language Studio**

For additional help with academic writing—and more generally with language learning—you are welcome to make an appointment with the Writing and Language Studio (WLS). To accommodate students who are learning remotely as well as those who are on campus, writing and language coaching appointments are available in person and online. You can register for an account, make an appointment, and learn more about WLS services, policies, and events on the [WLS website](#). You can also find writing and language learning resources on the [Writing & Language Studio Sakai site](#).

### **IT Support**

---

If you are experiencing technical difficulties, please contact IT:

- China-based faculty/staff/students **400-816-7100**, (+86) 0512- 3665-7100
- US-based faculty/staff/students (+1) 919-660-1810
- International-based faculty/staff/students can use either telephone option (recommend using tools like Skype calling)
- Live Chat: <https://oit.duke.edu/help>
- Email: [service-desk@dukekunshan.edu.cn](mailto:service-desk@dukekunshan.edu.cn)

## What is the expected course schedule?

---

Tentative:

<b>Lecture 1</b>	Probability Space [Ch.1] <ul style="list-style-type: none"><li>● Basic Set Theory</li><li>● Probability Measure: Definition, Continuity</li><li>● Properties of Probability Measures</li></ul>
<b>Lecture 2</b>	Probability Space (Cont'd) [Ch.1] <ul style="list-style-type: none"><li>● Conditional Probabilities, Independence, and Bayes' Theorem</li></ul> General Probability Space [Ch. 9] <ul style="list-style-type: none"><li>● General Probability Measures</li><li>● Sigma algebra, (sub)-additivity</li></ul>
<b>Lecture 3</b>	Discrete Random Variables [Ch.2] <ul style="list-style-type: none"><li>● Discrete Random Variables: definition and examples</li><li>● Independent Random Variables</li><li>● Expectation</li></ul>
<b>Lecture 4</b>	Discrete Random Variables (Cont'd) [Ch.2] <ul style="list-style-type: none"><li>● Variance</li><li>● Inequalities: Markov, Chebyshev, Hoelder, Minkovski</li><li>● Conditional Distribution and Expectation</li></ul>
<b>Lecture 5</b>	Discrete Random Variables (Cont'd) [Ch.2] <ul style="list-style-type: none"><li>● Generating Functions</li><li>● Compounding and Random Sum</li><li>● Limit Distributions (Different Convergence: in probability, in distribution, almost surely (with probability 1), in rth mean.)</li><li>● The Law of Rare Events</li></ul>
<b>Lecture 6</b>	Random Walk [Personal Notes and Ch.3] <ul style="list-style-type: none"><li>● Simple Random Walk</li></ul>

	<ul style="list-style-type: none"> <li>● Higher Dimensional Simple Random Walk</li> <li>● Dyth Paths, Arcsine Law</li> </ul>
<b>Lecture 7</b>	<p>Continuous Distributions (Cont'd) [Ch. 5]</p> <ul style="list-style-type: none"> <li>● Random Vectors and Independence</li> <li>● Conditional Distributions and Expectations</li> <li>● Moment Generating Function</li> </ul>
<b>Lecture 8</b>	<p>Continuous Distributions (Cont'd) [Ch. 5]</p> <ul style="list-style-type: none"> <li>● Functions of random variables</li> <li>● Joint distributions</li> <li>● Marginal and conditional distributions</li> </ul>
<b>Lecture 9</b>	<p>Continuous Distributions (Cont'd) [Ch. 5]</p> <ul style="list-style-type: none"> <li>● Higher dimensional joint distributions</li> <li>● n-dimensional Gaussian Distribution</li> <li>● Characteristic Function [Ch. 8]</li> </ul>
<b>Lecture 10</b>	<p>Miscellaneous Distributions</p> <ul style="list-style-type: none"> <li>● F- distribution,</li> <li>● t- distributions,</li> <li>● Chi-square distribution</li> </ul>
<b>Lecture 11</b>	<p>Limit Theorems [Ch. 8]</p> <ul style="list-style-type: none"> <li>● Weak Convergence</li> <li>● The Law of Large Numbers</li> <li>● The Central Limit Theorem</li> </ul>
<b>Lecture 12</b>	<p>Software packages demonstrations</p> <ul style="list-style-type: none"> <li>● Mathematica</li> <li>● SageMath</li> </ul>
<b>Lecture 13</b>	<p>Applications</p> <ul style="list-style-type: none"> <li>● Probability method in number theory and combinatorics</li> </ul>

	● 2-dimensional Dirichlet problem
Lecture 14	Presentation

***Final Exam***  
***March 7<sup>th</sup>, 2022,***  
***IB1050, 13:00—15:00***