I am an experimental mathematician working mainly in **number theory**, also intersecting with **combinatorics**, **probability**, and **differential geometry**. Computer software, programming languages, especially certain algorithms play an essential role in my research, by providing calculations, simulations, visualizations, and etc., for empirical evidence of conjectures. Primarily applying probabilistic and symbolic methods, I focus on certain *numbers, sequences, polynomials, special functions, combinatorial identities, geometric structures, algorithms*, and their properties that computer algorithms can (in-)directly guide or help to prove.

In the next sections, I will briefly describe current and future projects.

1 Probabilistic Approach

Large portion of my research topics, methods, and background involve probability: either by probabilistic approach, or some direct results in probability.

1.1 The Bernoulli and Euler Symbol

For all the special functions and polynomials that are either important, or have deep connections with other fields of mathematics, **Bernoulli and Euler polynomials** (e.g., [10, Chpt. 24]) are among my favorite ones. Some early results are summarized in the second part of my Ph. D thesis [Jiu16]. The main tools here are the *Bernoulli* and *Euler symbols*, denoted by $\mathcal{B}$ and $\mathcal{E}$, respectively. Each of the two symbols satisfies a simple evaluation rule: $\mathcal{B}^n = B_n$ and $\mathcal{E}^n = E_n/2$, where B_n and E_n are *Bernoulli* and *Euler numbers*. Originally, both $\mathcal{B}$ and $\mathcal{E}$ arise from the traditional umbral calculus (see, e.g., [12]). Meanwhile, the probabilistic interpretations (See, e.g., [4, Thm. 2.3] and [JMV14, Prop. 2.1]) view both symbols as random variables:

$$\mathcal{B} = iL_B - 1/2 \quad \text{and} \quad \mathcal{E} = iL_E - 1/2, \tag{1}$$

where $i^2 = -1$, and two random variables L_B and L_E have density functions p_B and p_E, respectively, as follows.

$$p_B(t) := \pi \operatorname{sech}^2(\pi t)/2 \quad \text{and} \quad p_E(t) := \operatorname{sech}(\pi t). \tag{2}$$

Note that, (1) implies the evaluation rules are exactly calculating the expectation. This probabilistic setup not only provides a rigorous background, also largely extends the application of symbolic computations.

1.1.1 Extensions

As random variables, we can consider the sum of independent and identically distributed (i.i.d.) sequence

$$\mathcal{B}^{(p)} := \mathcal{B}_1 + \cdots + \mathcal{B}_p \quad \text{and} \quad \mathcal{E}^{(p)} := \mathcal{E}_1 + \cdots + \mathcal{E}_p,$$

where $\mathcal{B}_i \sim \mathcal{B}$ and similarly, $\mathcal{E}_i \sim \mathcal{E}$. Then, extensions on higher-order polynomials and Bernoulli-Barnes polynomials also admit symbolic expressions, which do not appear in traditional umbral calculus.

- Bernoulli and Euler polynomials of order p, $B_n^{(p)}(x)$ and $E_n^{(p)}(x)$, are given by

$$B_n^{(p)}(x) = \mathbb{E}\left[\left(x + \mathcal{B}^{(p)}\right)^n\right] \quad \text{and} \quad E_n^{(p)}(x) = \mathbb{E}\left[\left(x + \mathcal{E}^{(p)}\right)^n\right]; \tag{3}$$

- and Bernoulli-Barnes polynomials $B_n(\mathbf{a}; x)$, with $\mathbf{a} = (a_1, \ldots, a_k)$ and $a_l \neq 0$ can be expressed as

$$B_n(\mathbf{a}; x) = \mathbb{E}\left[\frac{(x + a_1\mathcal{B}_1 + \cdots + a_k\mathcal{B}_k)^n}{a_1 a_2 \cdots a_k}\right], \tag{4}$$

  where $\boldsymbol{\mathcal{B}} = (\mathcal{B}_1, \ldots, \mathcal{B}_k)$ and $\mathbf{a} \cdot \boldsymbol{\mathcal{B}} = \sum_{l=1}^k a_l \mathcal{B}_l$.

Results based on symbolic expressions extends traditional ones. For instance, In [JMV16, Thm. 2.2], we extended a difference formula by Bayad and Beck [2, Thm. 5.1], symbolically, that for any polynomial P,

$$P(x - \mathbf{a} \cdot \boldsymbol{\mathcal{B}}) = \sum_{j=0}^{n} \sum_{|J|=j} |a|_{J^*} P^{(n-j)}(x + (\mathbf{a} \cdot \boldsymbol{\mathcal{B}})_J),$$

where $J \subset [n] := \{1, \ldots, n\}$ and $J^* = [n] \backslash J$.

While in [JMV14], not only did we apply the random variable interpretation to establish and prove identities, an expected observation (see [JMV14, Note 4.8]) triggers more probabilistic models involving objects in combinatorics and number theory, especially for finding and proving identities involving Bernoulli and Euler polynomials with their higher-order extensions. This will be stated in Subsection 1.2.1.

1.1.2 Orthogonal polynomials, Hankel determinants, and continued fractions

Admittedly, one does not need the probabilistic background, e.g., random variable, probability measure, moments, etc., to study orthogonal polynomials, Hankel determinants, continued fractions, weighted lattice path, generalized Motzkin number, and other related topics. Comprehensive introduction on basics and facts can be found, e.g., in [5, 7]. Personally, it IS the random variable interpretation of $E_n^{(p)}(x)$, Euler polynomials of higher-order defined in (3), leading me to study the corresponding orthogonal polynomials, denoted by $\Omega_n^{(p)}(y)$. In [JS19b], we recognize that $\Omega_n^{(p)}(y) = i^n n! P_n^{(p/2)}\left(-i\left(y - x + \frac{p}{2}\right); \frac{\pi}{2}\right)/2^n$, as the Meixner-Pollaczek polynomial $P_n^{(\lambda)}(y;\phi)$ (; see also [8, eq. 9.7.1]). Similarly, let $\varrho_n(y)$ be the the monic orthogonal polynomials with respect to the Bernoulli polynomial $B_n(x)$, then we identify that $\varrho_n(y) = n! p_n\left(y; \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)/(n+1)_n$, where $p_n(y;a,b,c,d)$ is the continuous Hahn polynomial [8, pp. 200–202]. This work opens the door for me to continue on Hankel determinants, especially of sequences related to Bernoulli and Euler polynomials, due to the following two reasons.

1. The classical formula for calculating the monic orthogonal polynomials of a given sequence contains the Hankel determinants (, see, e.g., [7, Eq. 2.1.6]);
2. Lemma 1 in [JS19b] actually provides an alternative proof, in the aspect of random variables and orthogonal polynomials, that Hankel determinants are invariant under binomial transforms [9, Item 445]. More precisely, the sequence c_n and its binomial transformed polynomial $c_n(x) := \sum_{k=0}^{n} \binom{n}{k} c_k x^{n-k}$ share the same Hankel determinants.

Therefore, in recent years, I accomplished a series studies on Hankel determinants [DJ21, DJ22, DJ23, JL, CJS, CJLW], involving mainly Bernoulli and Euler polynomials, with their extensions, in, e.g., Dirichlet characters and q-analogues. Besides major computations of new and important Hankel determinants, other highlights include the following.

- An incomplete table of Hankel determinant identities for numerous sequences containing Bernoulli and Euler numbers and polynomials was collected at the end of [DJ22].
- New general formulas of Hankel determinants of right-shifted sequence [DJ23, Lem. 2.5] and derivatives of Apell sequence [DJ22, Thm. 5.1] are proven.
- The Hankel determinants we studied in [JL] originally comes from statistically estimating the variance in non-parametric regression. Also the symbolic computation, or more precisely WZ method, was used and applied in the proofs.
- In the latest work [CJLW], we study the q-binomial transform sequence and its Hankel determinants, with partial result on the degree of polynomials and the leading coefficients. And some proofs make fully use of the Bernoulli and Euler symbols in (1).

1.2 Probabilistic Models

In [JMV14], based on the probabilistic interpretation, we obtained, for any positive integer N,

$$E_n(x) = \frac{1}{N^n} \sum_{l=N}^{\infty} p_l^{(N)} E_n^{(l)}\left(\frac{l-N}{2} + Nx\right),$$

where the coefficients $p_l^{(N)}$ appear in the series expansion of the reciprocal of the Nth Chebychev polynomial of the first kind T_N; while $p_\ell^{(N)}$ can also be viewed as transition probabilities in the context of a random walk over a finite number of sites [JMV14, Note 4.8], based on which, we continued to explore more of such probabilistic models.

1.2.1 Random walk, Brownian motion, and Bessel process

We consider both 1-dimensional reflected Brownian motion and 3-dimemsional Bessel process. More specifically, we decompose the hitting times for consecutive level sites. For instance, considering in the figure below: starting from site a_0, the hitting time to the site a_{m+1} can be decomposed, combinatorially, by the hitting times among sites $a_0, a_1, \ldots, a_m$ in between, where the back and forth walks between neighboring sites form loops, denoted by $L_1, L_2, \ldots, L_m$.

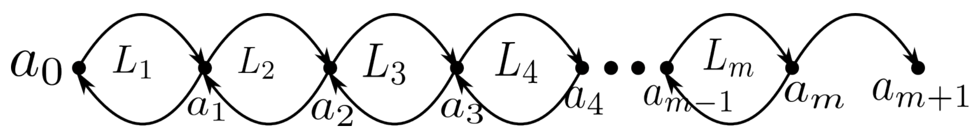

Figure 1: Loop Decomposition

With the combinatorial loop and hitting time decomposition, we can further derive non-trivial identities, involving $B_n^{(p)}(x)$ and $E_n^{(p)}(x)$. In the early work [JV19], results on only 1 or 2 loops, i.e. $m = 1$ or 2 in Figure 1, were obtained. Later in [JSY22], we successfully extended to the general n loops, by both induction and a combinatorial interpretation. Further identities are derived from equally distributed level sites. For instance, the 3-loop cases in 3-dim Bessel process implies

$$B_{n+1}\left(\frac{x+2}{5}\right) - B_{n+1}\left(\frac{x}{5}\right) = \frac{n+1}{5^n}\sum_{k=0}^{\infty}\frac{3^k}{2^{2k+3}}\sum_{\ell=0}^{k}\binom{k}{\ell}(-1)^\ell\frac{1}{12^\ell}E_n^{(2k+2\ell+3)}(k+\ell+x).$$

1.2.2 Shuffling model

The very recent work [CJS] studies the uniform shuffling on n cards, i.e., each result has probability $1/n!$. Then, merge the maximal consecutive integer subsequences and reduce the number of cards into the current remaining one. The model was introduced by Rao et al. [11] to model the number of times that catalysts must be added to n molecules to bond into a single lump. The molecules have a given hierarchical order which led to the above mathematical formulation of the process. They studied the number of permutations needed for the process to end, X_n; and obtained the asymptotic behavior of the mean as

$$n \le \mathbb{E}[X_n] \le n + \sqrt{n} \Rightarrow \mathbb{E}[X_n] \sim n.$$

Our work [CJS] not only improved it by

$$\mathbb{E}[X_n] = n + \left(1 + \frac{1}{2} + \cdots \frac{1}{n-1}\right) + \varepsilon_n, \quad \text{with} \quad 0 \le \varepsilon_n - \varepsilon_{n+1} \le \frac{1}{n^2}, \tag{5}$$

which also indicates that ε_n has a limit. More importantly, the simulation in [11], for $X_2, \ldots, X_{100}$ hinted that the asymptotic distribution of X_n would tend to be normal. While, we completely solve it in the affirmative, i.e., we proved that

$$\frac{X_n - n}{\sqrt{n}} \xrightarrow{w} \mathcal{N}(0,1),$$

by estimating all the central moments of X_n. As a by product, certain limits related to the sequence involve the Bell numbers.

1.3 Differential and Information Geometry

I studied differential and information geometry for my master's degree, which can be seen from some of my early work, e.g., [WJ06, JS07, PSJ07, ZSJP13, ZSJP14, LZJS16]. Switching to studying experimental mathematics in my Ph. D program does not prevent me from continuing my research in information geometry; in fact, I am gradually combining some work together.

- In both [TJKZ20] and [JK20], the Wishart distribution plays the essential rule, in defining the zonal polynomials and hence hypergeometric functions with matrix arguments. Alternatively, an equivalent definition of the zonal polynomials is the eigenfunction of the Laplace–Beltrami operator on the space of symmetric, positive definite matrices.
- I contributed to an awarded joint grant "Wuhan University—Duke Kunshan University—University of Minnesota, Twin Cities Joint Research Platform", together with my colleague Dr. Dongmian Zou.
- In the recent work [JP], we use the information geometry tools to study the exponential generalized beta of the second kind (EGB2) distribution or the beta-logistic distribution, with density function

$$f(x;\beta_1,\beta_2) = \frac{1}{B(\beta_1,\beta_2)} \cdot \frac{\exp(\beta_1 x)}{(1+\exp(x))^{\beta_1+\beta_2}},$$

where $\beta_1, \beta_2 > 0$ and $B(x, y)$ is the beta function. With some simplification, it is not hard to recognize that this family include both densities for Bernoulli and Euler symbols (2). Besides basic geometric structures, we also uncovers that the beta-logistic distribution admits an α-parallel prior for any real number α, that has the potential for application in geometric statistical inference.

1.4 Miscellaneous

There are also some work, inspired by probabilistic interpretations. For example,

- in [JS19a], we obtained a matrix representation of multiplicative nested sums, a special case of which is the stochastic transition matrix of a random walk on a finite number of sites;
- and in [JS22], some identities are obtained, based on the moments-cumulants relation, by applying Faà di Bruno's formula.

2 Symbolic Approach

In general, computer or machine proofs, as the major aim in symbolic computation, always contribute to mathematical proofs. For example, the WZ-method is applied in the proofs of [DJMV15, JK20, JL]. Meanwhile, Bernoulli and Euler symbols, mentioned above, can also be considered as generalized symbolic computation method.

2.1 Multiple zeta value at non-positive integers

The multiple zeta function

$$\zeta_r(n_1, \dots, n_r) = \sum_{k_1, \dots, k_r > 0} \frac{1}{k_1^{n_1}(k_1 + k_2)^{n_2} \cdots (k_1 + \cdots + k_r)^{n_r}}$$

has more than one analytic continuations at non-positive integers. For instance, Sadaoui [13, Thm. 1] used the Raabe's identity while Akiyama and Yanigawa [1, p. 350] considered the Euler-Maclaurin summation formula. Since both results involve Bernoulli number, applying Bernoulli symbol reveals, to our surprise, that both analytic continuations coincide. More precisely, for non-negative integers $n_1, \dots, n_k$, we have, symbolically,

$$\zeta_r(-n_1, \dots, -n_r) = \prod_{k=1}^{r} (-1)^{n_k} \mathcal{C}_{1,\dots,k}^{n_k+1} \tag{6}$$

where $\mathcal{C}_1^n = \mathcal{B}_1^n/n$ and recursively, $\mathcal{C}_{1,\dots,k+1} = (\mathcal{C}_{1,\dots,k} + \mathcal{B}_{k+1})^n/n$. This shows the two approaches, by Raabe's identity and Euler-Maclaurin summation formula that lead to analytic continuations of MZVs, coincide on non-positive integer values. Other results such as recurrence [JMV18, Thm. 3.1], contiguity identities [JMV18, Thm. 4.1], and generating functions [JMV18, Thm. 5.1] follow naturally from (6).

Also in [JVW20], we further symbolically expressed the r-fold harmonic sums at negative indices, which is similar to the multiple zeta value expression. For example [JVW20, Thm. 3.1], by defining the $\mathcal{H}$ symbol as

$$(\mathcal{H}(N))^n = H_{-n}(N) = 1^n + 2^n + \cdots + (N-1)^n,$$

the r-fold multiple power sums can be expressed as

$$H_{-n_1,\dots,-n_r}(N) := \sum_{N > i_1 > \cdots > i_r > 0} i_1^{n_1} \cdots i_r^{n_r} = \prod_{k=1}^{r} \mathcal{H}_{1,\dots,k}^{n_k},$$

where $\mathcal{H}_1 = \mathcal{H}(N)$ and recursively $\mathcal{H}_{1,\dots k} = \mathcal{H}\left(\mathcal{H}_{1,\dots,k-1}\right)$ for $k = 2, 3, \dots, r$. This is compatible with (6) by letting $N \to \infty$.

2.2 The method of brackets

The method of bracket is an efficient method for the evaluation of a large class of definite integrals on the half-line, i.e.,

$$\int_0^\infty f(x)dx,$$

with only 6 simple rules, with applications to evaluate certain Feymann integrals, which arise from Feynman diagrams. In theoretical physics, a Feynman diagram is a pictorial representation of the mathematical expressions describing the behavior and interaction of subatomic particles. It is also related to Dr. Moll, my Ph. D supervisor's long term project in proving all entries of [6]. Some other work of this project include [ADG$^+$16a]

Some early work and summary can be found in the first half of my Ph. D thesis [Jiu16]. Further generalization, novel evaluation techniques, and related discussion can be found, e.g., in [GJM16, GKJM17, GKJM18, GKJM20, BGJ$^+$23]. For instance, in [GJM16], we first observed several examples, all missing a common factor of 2. This is due to the analytic continuation of the Pochhammer symbol $(a)_k = \Gamma(a+k)/\Gamma(a)$, when both a and k are negative integers. In order to get the correct answer, the evaluation

$$(-km)_{-m} = \frac{k}{k+1}\frac{(-1)^m(km)!}{((k+1)m)!},$$

should be applied.

2.3 Zonal polynomials

Besides applying symbolic algorithms for proofs, it is also important to implement certain algorithms for calculation. In [JK20], a SageMath package[1] was built to compute zonal polynomials $C_\lambda(x_1,\dots,x_m)$ for any given integer partition λ. Based on computational results, some patterns are observed and proven. An independent Mathematica package[2] is built by my coauthor, Dr. Christoph Koutschan.

3 Remarks and some future plans

Here are some potential future research topics and plans, including both continuation of early work, and few completely new areas, with natural motivation.

1. A long-term project focuses on mutual connections among orthogonal polynomials, lattice paths, continued fractions and Hankel determinants; while current short-term aims at
   - algorithms in computing operations of continued fraction expressions;
   - labeled tree generating polynomials;
   - and Hankel determinant guessing and computing, in general.
2. My second concentration will continue to work on the random variable expressions and models in probability.
   - For instance, instead of only Bernoulli and Euler polynomials, the entire family of Sheffer sequences may be taken into consideration.
   - Some recent results, e.g., in [3], Budd studied the square lattice random walk related to Elliptic functions, can be a good start as a new model to explore new identities.
   - Those lattice and corresponding tree structures are also related to certain polynomials, especially with labeled trees.
   - Also, to continue the project on the shuffling model, we are now working on finding and proving the limit of ε_n in (5). And more general but similar shuffling models are also of our interests.
3. I would also involve more computational number theory research.
   - For example, continuing from the matrix representations of Bernoulli and Euler polynomials [JS19b], including some of their extensions to higher-order, we can consider other matrix representations of classic and new sequences of numbers, polynomials are of important interests.
   - Some q-identities, which have computational applications, either faster convergence, or more digits per term for computing certain constants, are also part of my future plans.
4. Finally, one important area is the normal numbers, which also admit probabilistic background. I will gradually study related topics, in theory, computations, constructions, and hopefully proofs.

Please find the publication list below, including some other work that does not fit into my major research projects. Among them, some are also suitable for undergraduate students, e.g., [BJMV14, JV16, JS22, JS].

[1] https://jiulin90.github.io/Packages/BNE.sage
[2] https://jiulin90.github.io/Packages/BNE.sage

List of Publications

[ADG+14] Tewodros Amdeberhan, Atul Dixit, Xiao Guan, Lin Jiu, and Victor H. Moll, *The unimodality of a polynomial coming from a rational integral. back to the original proof*, Journal of Mathematical Analysis and Applications **420** (2014), no. 5, 1154–1166.

[ADG+16a] Tewodros Amdeberhan, Atul Dixit, Xiao Guan, Lin Jiu, Alexey Kuznetsov, Victor H. Moll, and Christophe Vignat, *The integrals in gradshteyn and ryzhik. part 30: trigonometric functions*, Scientia Series A: Mathematical Sciences **27** (2016), 47–74.

[ADG+16b] Tewodros Amdeberhan, Atul Dixit, Xiao Guan, Lin Jiu, Victor H. Moll, and Christophe Vignat, *A series involving catalan numbers. proofs and demonstrations*, Elemente der Mathematik **71** (2016), no. 3, 109–121.

[BGJ+23] Zachary Bradshaw, Ivan Gonzalez, Lin Jiu, Victor H. Moll, and Christophe Vignat, *Compatibility of the method of brackets with classical integration rules*, Open Mathematics **21** (2023), no. 1, Article 20220581.

[BJMV14] Alyssa Byrnes, Lin Jiu, Victor H. Moll, and Christophe Vignat, *Recursion rules for the hypergeometric zeta functions*, International Journal of Number Theory **10** (2014), no. 7, 1761–1782.

[CJ24] Shane Chern and Lin Jiu, *Hankel determinants and jacobi continued fractions for q-euler number*, Comptes Rendus Mathématique **362** (2024), 203–216.

[CJLW] Shane Chern, Lin Jiu, Shuhan Li, and Liuquan Wang, *Leading coefficient in the hankel determinants related to binomial and q-binomial transforms*, under submission.

[CJS] Shane Chern, Lin Jiu, and Italo Simonelli, *A central limit theorem for a card shuffling problem*, under submission.

[DJ21] Karl Dicher and Lin Jiu, *Orthogonal polynomials and hankel determinants for certain bernoulli and euler polynomials*, Journal of Mathematical Analysis and Applications **497** (2021), no. 1, Article 124855.

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[GKJM17] Ivan Gonzalez, Karen Kohl, Lin Jiu, and Victor H. Moll, *An extension of the method of brackets. part 1*, Open Mathematics **15** (2017), no. 1, 1181–1211.

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