MATH 301

Advanced Introduction to Probability

Spring 2021, Session 3

Dates / Synchronous meeting time:	Lectures:	Tuesdays and Thursdays 14:45—17:15
	Discussion:	Wednesdays 20:10—21:10
Academic credit:	4	
Hybrid course format:	video, lecture, discussion, presentation	

Instructor's information

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Office Hours:	Office: Wednesdays 14:30—16:30 or by appointment

What is this course about?

Probability is one of the most beautiful branches in mathematics that also applies to many areas. Different from MATH205, we shall introduce concept via rigorous proofs, for clarity and precision. Namely, a solid foundation shall be established, which applies to statistical methods, data science, information theory, etc.

We shall also study basic stochastic process, as simple random walks, branching processes, and very early stages of Brownian motions. Not only are the topics of great importance, the intuition on randomness with mathematical terms and the models for better understanding limit theorems are also essential for studying and understanding the reality, and for further application on the most edge-cutting projects.

What background knowledge do I need before taking this course?

Prerequisites: MATH 201, MATH 202 and MATH 205

What will I learn in this course?

We shall learn the main objectives for this course:

1. rigorous introduction on probability, including discrete and continuous distributions, random variables and limit theorems.
2. introductory stochastic process, including Markov chains and random walks.

Therefore, you will set up probabilistic reasoning in precise mathematical terms; will develop some fundamental results, tools and models for further applications; and will compute and analyze several basic random process models.

More precisely, you will be able to

- compute, for both discrete and continuous distributions, probabilities, expectations, variance, and moments
- classify and describe discrete and continuous random variables, and random vectors

- compute probabilities on joint, marginal and conditional distributions
- apply generating functions to compute e.g., independent random variables, random sum of i.i.d. random variables, etc.
- distinguish different convergence in probability
- demonstrate rigorous proofs of law of large numbers, central limit theorem, and Poisson limit theorems
- analyze and formulate random walks
- recognize and describe Markov chains
- compute the stationary distributions for random walks on finite sites

What will I do in this course?

The course will be comprised of (video) lectures, synchronous meetings dedicated to problem solving and lecturing, homework, and exams.

Quiz. In the middle of the first 12 lectures, there will be a quiz weighted 2% (i.e., 12 quizzes in total). Only the highest 10 quizzes will be counted.

Homework. There will be weekly homework 6 times in total and each is worth 5%.. Beginning from the Thursday of the first week and due the lecture of the next Thursday before the lecture. Each homework will be given on materials of the week.

Collaboration is encouraged, but you must write up your own solutions. Late homework will not be accepted, except as required by the University Policy.

Presentation. In the last lecture (or maybe last two lectures) you are required to give presentations for each individual (or maybe in groups if more students join before the Add/Drop session). Topics will be given in the middle of the course; and everyone is welcome to discuss and offer your own topic(s).

Exams. There will be a take-home final exam on Sakai. You will be given a 72-hour window for the final. It is an open book test, but communication on exam is prohibited.

We do not give make-up exams for any reasons if you miss an exam. An unexcused delay in taking any exam will be counted as a zero. Excuses may be accepted, at the discretion of the instructor, and any alternative arrangements must be made well in advance.

How can I prepare for the class sessions to be successful?

To succeed, you should be prepared to devote several hours to this course on a daily basis, more precisely, you are expected to spend approximately 24 hours of total in- and out-of-class time per week. It is also strongly encouraged to use the online tutoring resources of ARC, to work with classmates, and to contact instructors in a timely manner for additional help as needed.

What required texts, materials, and equipment will I need?

1. "A Natural Introduction to Probability Theory" by Ronald Meester.
   ISBN: 978-3-7643-8723-5
   Electronic Version Free from DKU/Duke Library: HERE

2. "Elementary Probability for Applications" (Chapter 4), by Richard Durrett.
   ISBN: 978-0-5218-6756-6.
   Electronic Version Free from DKU/Duke Library: HERE
3. "Essentials of Stochastic Processes" (Chapter 2) by Richard Durrett.
   ISBN: 978-3-319-45614-0
   Electronic Version Free from DKU/Duke Library: HERE
4. "Introduction to Probability" (Chapter 10), by Charles Grinstead and J. Laurie Snell.

   Open Textbook online free: HERE

What optional texts or resources might be helpful?

"Adventures in Stochastic Processes" by Sidney Resnick. ISBN: 0-8176-3591-2

How will my grade be determined?

1. Quizzes: 20%
2. Homework: 30%
3. Presentation: 20%
4. Final exam: 30%

Please refer to the following scale for your grading.

A+= 98% - 100% **A** = 93% - 97.99%; **A-** = 90% - 92.99%; **B+** = 87% - 89.99%; **B** = 83% - 86.99%; **B-** = 80% - 82.99%; **C+** = 77% - 79.99%; **C** = 73% - 76.99%; **C-** = 70% - 72.99%; **D+** = 67% - 69.99%; **D** = 63% - 66.99%; **D-** = 60% - 62.99% **F** = 59.99% and below

What are the course policies?

Academic Integrity:

As a student, you should abide by the academic honesty standard of the Duke Kunshan University. Its community Standard states: "Duke Kunshan University is a community comprised of individuals from diverse cultures and backgrounds. We are dedicated to scholarship, leadership, and service and to the principles of honesty, fairness, respect, and accountability. Members of this community commit to reflecting upon and upholding these principles in all academic and non-academic endeavors, and to protecting and promoting a culture of integrity and trust." For all graded work, students should pledge that they have neither given nor received any unacknowledged aid.

Please refer to http://undergrad.dukekunshan.edu.cn/undergraduate-bulletin-2020-2021/ and https://dukekunshan.edu.cn/en/academics/advising for DKU course policies and guidelines.

Duke Kunshan University is a community comprised of individuals from diverse cultures and backgrounds. We are dedicated to scholarship, leadership, and service and to the principles of honesty, fairness, respect, and accountability. Members of this community commit to reflecting upon and upholding these principles in all academic and non-academic endeavors, and to protecting and promoting a culture of integrity and trust. To uphold the Duke Kunshan Community Standard, each student is expected to pledge to hold

him/herself to the highest standards for honesty, integrity, fairness, and responsibility in his/her academic and non-academic endeavors, to respect other cultures and embrace all forms of diversity, and to uphold the standards if they are compromised.

Academic Policy & Procedures:
You are responsible for knowing and adhering to academic policy and procedures as published in University Bulletin and Student Handbook. Please note, an incident of behavioral infraction or academic dishonesty (cheating on a test, plagiarizing, etc.) will result in immediate action from me, in consultation with university administration (e.g., Dean of Undergraduate Studies, Student Conduct, Academic Advising). Please visit the Undergraduate Studies website for additional guidance related to academic policy and procedures. Academic integrity is everyone's responsibility.

Academic Disruptive Behavior and Community Standard:
Please avoid all forms of disruptive behavior, including but not limited to: verbal or physical threats, repeated obscenities, unreasonable interference with class discussion, making/receiving personal phone calls, text messages or pages during class, excessive tardiness, leaving and entering class frequently without notice of illness or other extenuating circumstances, and persisting in disruptive personal conversations with other class members. Please turn off phones, pagers, etc. during class unless instructed otherwise. If you choose not to adhere to these standards, I will take action in consultation with university administration (e.g., Dean of Undergraduate Studies, Student Conduct, Academic Advising).

Academic Accommodations:
If you need to request accommodation for a disability, you need a signed accommodation plan from Campus Health Services, and you need to provide a copy of that plan to me. Visit the Office of Student Affairs website for additional information and instruction related to accommodations.

What campus resources can help me during this course?

Academic Advising and Student Support
Please consult with me about appropriate course preparation and readiness strategies, as needed. Consult your academic advisors on course performance (i.e., poor grades) and academic decisions (e.g., course changes, incompletes, withdrawals) to ensure you stay on track with degree and graduation requirements. In addition to advisors, staff in the Academic Resource Center can provide recommendations on academic success strategies (e.g., tutoring, coaching, student learning preferences). All ARC services will continue to be provided online. Note, there is an ARC Sakai site for students and tutors. Please visit the Office of Undergraduate Advising website for additional information related to academic advising and student support services.

Writing and Language Studio

For additional help with academic writing—and more generally with language learning—you are welcome to make an appointment with the Writing and Language Studio (WLS). To accommodate students who are learning remotely as well as those who are on campus, writing and language coaching appointments are

available in person and online. You can register for an account, make an appointment, and learn more about WLS services, policies, and events on the WLS website. You can also find writing and language learning resources on the Writing & Language Studio Sakai site.

IT Support

If you are experiencing technical difficulties, please contact IT:

- China-based faculty/staff/students 400-816-7100, (+86) 0512- 3665-7100
- US-based faculty/staff/students (+1) 919-660-1810
- International-based faculty/staff/students can use either telephone option (recommend using tools like Skype calling)
- Live Chat: https://oit.duke.edu/help
- Email: service-desk@dukekunshan.edu.cn

What is the expected course schedule?

Week 0.5 **(Jan. 6—7)**	Review of Basic Discrete Distributions and Probability Space [1,Ch.1] ● Binomial, Multinomial, Geometric, Hypergeometric, Poisson ● Counting, Experiments ● Conditional Probability, Independence, and Bayes' Theorem	Homework 1, Due Jan. 14
Week 1.5 **(Jan. 11--14)**	Discrete Random Variables [1.Ch.2] ● Moments and Convolution ● Generating functions ● Compounding and Random Sum ● Limit Distribution ● The Law of Rare Events	Homework 2, Due Jan. 21
Week 2.5 **(Jan. 18--21)**	Markov Chain [2. Ch. 4] ● Construction and Examples ● Multistep transition probabilities ● Stationary distributions ● Limit behavior ● Gambler's ruin	Homework 3, Due Jan. 28

	• Absorbing chains	
Week 3.5 **(Jan. 25--28)**	Continuous Distributions [1.Ch. 5] • Experiments and Properties • Continuous Random Variables • Expectation & Variance • Random Vectors and Independence • Conditional Distributions and Expectations	Homework 4, Due Feb. 4
Week 4.5 **(Feb. 1—4)**	• Characteristic and Moment Generating Function • Functions of random variables • Joint distributions • Marginal and conditional distributions	Homework 5, Due Feb. 25
Week 5 **(Feb. 8—9)**	Limit Theorems [1, Ch. 8] • Weak Convergence • The Law of Large Numbers	
Week 6 **(Feb. 22--25)**	Limit Theorems (Cont'd) [1, Ch. 8] • The Central Limit Theorem Random Walk and Branching Processes [1, §3.1, §6.4, §6.5] • Random Walk and Counting • Branching Processes	Homework 6, Due Mar. 4
Week 7 **(Mar. 1--4)**	Poisson Processes [3, Ch. 2] • Exponential Distribution • Constructing the Poisson Process • More Realistic Models • Compound Poisson Processes • Transformations	

Final Exam Mar. 6—9