MATH 306

Number Theory

Fall 2022 Session 1

Dates / Synchronous meeting time:	Tuesdayss and Thursdays 8:30—11:00
Room:	**IB 3039**
Academic credit:	4
Course format:	Lectures, presentations, tests

Instructor's information

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What is this course about?

Number Theory is one of the oldest subjects in mathematics, which begins with counting, literally; 1, 2, 3, ... As (probably) the only number theory course at DKU, we shall begin with elementary number theory, such as divisibility, congruence, continued fractions, and also with focus on prime number, including important conjectures such as Goldbach's conjecture, the twin primes conjecture, etc. Then, we shall switch to the introductory level of analytic number theory, on number-theoretic functions, L-functions, Riemann zeta-function and the statement of Riemann hypothesis. The rest of the class will be about several topics in number theory, involving integer partitions; p-adic numbers, algebraic number theory and computational number theory.

In short, the first 2/3 of the course will be standard elementary and analytic number theory class, with the concentration on basic objects; while the rest 1/3 will focus on various topics.

What background knowledge do I need before taking this course?

Required: MATH101/105 and MATH 205

Recommended: MATH307, MATH308, MATH401: If you haven't taken them, this will not affect your taking this course; but if you have taken them, you will have a different aspect viewing the materials.

What will I learn in this course?

By learning the materials step by step, students should be able to do the following.
1. determine the divisibility among several positive integers;
2. compute the least common divisor (LCD) via the Euclid algorithm;
3. present more than one proof on infinite many prime numbers;
4. compute the congruence classes and solve linear congruence equations;
5. state and prove theorems of Wilson, Fermat, and Euler;
6. state the properties of Jacobi and Legendre symbols and solve certain equations;
7. given a rational number, express it in terms of continued fractions, by Euclid algorithm;
8. given a positive integer:

(1) determine whether it can be written as the sum of two square; if so, present the sum;
(2) and express it in terms of four squares

9. state the following conjectures
   (1) Goldbach's
   (2) Twin prime
   (3) ABC
10. state Riemann hypothesis with at least two equivalent statements;
11. state the prime number theorem
12. state the definition of algebraic, transcendental, and normal numbers with examples;
13. state the definition of Bernoulli numbers, Euler numbers, Fibonacci numbers, Lucas numbers and their properties;
14. manipulate the integer partition via q-series:
   (1) prove the generating function by a combinatorial counting;
   (2) draw the Young diagram of a partition and vice versa;
   (3) state and prove Ramanujan's congruence theorems;
15. Illustrate the difference between p-adic distance and the normal distance between two real numbers;
16. calculate the p-adic valuation of a given rational number;
17. state the definition of Elliptic curves and the strategy of its application on Diophantine equations;
18. state the BBP formula and describe the significance of it;
19. define the normal numbers and Liouville numbers;

In addition, the presentation section can also be viewed as the training on

- finding interesting & important topics;
- preparation and presentation;
- basic LaTeX beamer usage;
- potential use of some computer packages in Mathematica or SageMath.

What required texts, materials, and equipment will I need?

We shall follow **my distributed notes** for this class, which is definitely adequate.

In addition, here are some materials for further reading:

1. G. E. Andrews, Number Theory, Dover Publications, New York, 1994.
2. B. Berndt, Number Theory in the Spirit of Ramanujan, AMS, 2006.
3. K. Broughan, Equivalents of the Riemann Hypothesis Volume One: Arithmetic Equivalents, Cambridge University Press, Cambridge, 2017.
4. K. Broughan, Equivalents of the Riemann Hypothesis Volume Two: Analytic Equivalents, Cambridge University Press, Cambridge, 2017
5. D. M. Burton, Elementary number theory, Third Edition, McGraw-Hill, New York, 1997.
6. H. M. Edgar, A first course in number theory, Wadsworth, 1988.
7. K. Ireland and M. Rosen, A classical introduction to modern number theory, Second Edition, Springer-Verlag, New York, 1990.
8. M. Kauers and P. Paule, The Concrete Tetrahedron: Symbolic Sums, 1Recurrence Equations, Generating Functions, Asymptotic Estimates, Springer-Verlag, Vienna, 2011.
9. T. Koshy, Fibonacci and Lucas Numbers with Applications, Wiley, Hoboken, New Jersey, 2019.
10. M. Petkověk, H S. Wilf, and D. Zeilberger, A=B, A . K. Peters/CRC Press 1997.
11. P. Ribenboim, The little book of big primes, Springer-Verlag, New York, 1991.

12. P. Ribenboim, The new book of prime number records, Springer-Verlag, New York, 1996.
13. P. Ribenboim, 13 lectures on Fermat's last theorem, Springer-Verlag, New York-Heidelberg, 1979.
14. P. Ribenboim, Fermat's last theorem for amateurs, Springer-Verlag, New York, 1999.

How will my grade be determined?

1. Homework: 50%
2. Presentation: 20%
3. Final exam: 30%

Please refer to the following scale for your grading.

A+= 98% - 100% **A** = 93% - 97.99%; **A-** = 90% - 92.99%; **B+** = 87% - 89.99%; **B** = 83% - 86.99%; **B-** = 80% - 82.99%; **C+** = 77% - 79.99%; **C** = 73% - 76.99%; **C-** = 70% - 72.99%; **D+** = 67% - 69.99%; **D** = 63% - 66.99%; **D-** = 60% - 62.99% **F** = 59.99% and below

Homework: Large percent of the final grade depends on the homework assignments. There will be FIVE assignments, each worth 10%. You are allowed to discuss or ask me for hints; but each of you should independently turn in individual work.

Presentation: There will be a presentation session during the last lecture time. Several potential topics shall be listed by the end of the 2nd week. In addition, you can also choose your own topic, related to number theory. Everyone should determine your title by the end of 5th week through a quick appointment with me; missing this deadline will let you lose 5% (by noting that the total percentage of Presentation is 20%, i.e., a quarter of it). Presentation will be graded based on the

- topic (5%),
- time control (5%),
- illustration (including language skill) (5%)
- slides typos and questions (5%).

Final Exam: There will be a standard 3-hour final exam. Instead of a closed-book test, you are allowed to bring one A4 size piece of formula sheet (double sided). The exact time slot and location will be arranged by the Registrar's Office and announced once it is determined.

What are the course policies?

Academic Integrity:

As a student, you should abide by the academic honesty standard of the Duke Kunshan University. Its community Standard states: "Duke Kunshan University is a community comprised of individuals from diverse cultures and backgrounds. We are dedicated to scholarship, leadership, and service and to the principles of honesty, fairness, respect, and accountability. Members of this community commit to reflecting upon and upholding these principles in all academic and non-academic endeavors, and to protecting and promoting a culture of integrity and trust." For all graded work, students should pledge that they have neither given nor received any unacknowledged aid.

Please refer to http://undergrad.dukekunshan.edu.cn/undergraduate-bulletin-2021-2022/ and

https://dukekunshan.edu.cn/en/academics/advising for DKU course policies and guidelines.

Duke Kunshan University is a community comprised of individuals from diverse cultures and backgrounds. We are dedicated to scholarship, leadership, and service and to the principles of honesty, fairness, respect, and accountability. Members of this community commit to reflecting upon and upholding these principles in all academic and non-academic endeavors, and to protecting and promoting a culture of integrity and trust. To uphold the Duke Kunshan Community Standard, each student is expected to pledge to hold him/herself to the highest standards for honesty, integrity, fairness, and responsibility in his/her academic and non-academic endeavors, to respect other cultures and embrace all forms of diversity, and to uphold the standards if they are compromised.

Academic Policy & Procedures:
You are responsible for knowing and adhering to academic policy and procedures as published in University Bulletin and Student Handbook. Please note, an incident of behavioral infraction or academic dishonesty (cheating on a test, plagiarizing, etc.) will result in immediate action from me, in consultation with university administration (e.g., Dean of Undergraduate Studies, Student Conduct, Academic Advising). Please visit the Undergraduate Studies website for additional guidance related to academic policy and procedures. Academic integrity is everyone's responsibility.

Academic Disruptive Behavior and Community Standard:
Please avoid all forms of disruptive behavior, including but not limited to: verbal or physical threats, repeated obscenities, unreasonable interference with class discussion, making/receiving personal phone calls, text messages or pages during class, excessive tardiness, leaving and entering class frequently without notice of illness or other extenuating circumstances, and persisting in disruptive personal conversations with other class members. Please turn off phones, pagers, etc. during class unless instructed otherwise. If you choose not to adhere to these standards, I will take action in consultation with university administration (e.g., Dean of Undergraduate Studies, Student Conduct, Academic Advising).

Academic Accommodations:
If you need to request accommodation for a disability, you need a signed accommodation plan from Campus Health Services, and you need to provide a copy of that plan to me. Visit the Office of Student Affairs website for additional information and instruction related to accommodations.

What campus resources can help me during this course?

Academic Advising and Student Support
Please consult with me about appropriate course preparation and readiness strategies, as needed. Consult your academic advisors on course performance (i.e., poor grades) and academic decisions (e.g., course changes, incompletes, withdrawals) to ensure you stay on track with degree and graduation requirements. In addition to advisors, staff in the Academic Resource Center can provide recommendations on academic success strategies (e.g., tutoring, coaching, student learning preferences). All ARC services will continue to be provided online. Note, there is an ARC Sakai site for students and tutors. Please visit the Office of

Undergraduate Advising website for additional information related to academic advising and student support services.

Writing and Language Studio

For additional help with academic writing—and more generally with language learning—you are welcome to make an appointment with the Writing and Language Studio (WLS). To accommodate students who are learning remotely as well as those who are on campus, writing and language coaching appointments are available in person and online. You can register for an account, make an appointment, and learn more about WLS services, policies, and events on the WLS website. You can also find writing and language learning resources on the Writing & Language Studio Sakai site.

IT Support

If you are experiencing technical difficulties, please contact IT:

- China-based faculty/staff/students 400-816-7100, (+86) 0512- 3665-7100
- US-based faculty/staff/students (+1) 919-660-1810
- International-based faculty/staff/students can use either telephone option (recommend using tools like Skype calling)
- Live Chat: https://oit.duke.edu/help
- Email: service-desk@dukekunshan.edu.cn

What is the expected course schedule? (*Tentatively, may up to some perturbation)*

Aug. 23	representation of an integer, divisibility, GCD**,** Euclid algorithm
	LCM, linear Diophantine equation
	Pell's equation
Aug. 25	unique factorization, infinitely many primes
	twin prime and Goldbach's conjectures. ABC conjecture
	congruence, residue classes, Euler's phi function
Aug. 30	linear congruences, Chinese Remainder theorem
	theorems of Wilson, Fermat and Euler, Primitive Roots
	Primitive Roots
Sept. 1	quadratic residues, Legendre symbol,
	quadratic reciprocity
	Jacobi symbol,

Sept. 6	Multiplicative functions
	The Möbius inversion formula, gamma function
	The Riemann zeta-function, Euler's infinite product formula
Sept. 8	Analytic continuation of Riemann zeta-function
	Riemann Hypothesis
	Asymptotic densities: square-free integers, prime number theory
	Homework I---Due Day
Sept. 13	Dirichlet characters and L-functions
	algebraic and transcendental numbers: existence and examples, Liouville numbers
	Irrationality of e and pi, further transcendence results.
Sept. 15	generating functions, Bernoulli polynomials and numbers
	Euler numbers, Fibonacci numbers, and Lucas numbers
	Catalan numbers
	Homework II---Due Day
Sept. 20	integer partitions, q-series.
	Ramanujan's congruence theorems
	Sum of squares, Young diagram
Sept. 22	metric and normed spaces, non-Archimedean metric/norm, p-adic valuation
	p-adic norm, Ostrowski's Theorem, Field of p-adic numbers
	Sequences and series of p-adic numbers
	Homework III---Due Day
Sept. 27	The cubic Diophantine equation.
	Basic concept of elliptic curves, addition
	Fermat's Last Theorem
Sept. 29	examples in computational number theory, BBP formula of Pi
	Normal number, Borwein integrals, Symmetric cryptography
	The RSA-cryptosystem, Digital signatures

	Homework IV---Due
Oct. 11	WZ Method
	Holonomic sequences
	Mathematica package demonstration
Oct. 13	Presentation Day
	Homework V---Due Day

- ***Final Exam: 10/18/2021, 16:00—18:00, IB 1056***