MATH 301

Advanced Introduction to Probability

昆山杜克大学
DUKE KUNSHAN UNIVERSITY

Fall 2023, Session 1

Dates / Synchronous meeting time:	**MoWe 14:45—17:15**	**ROOM: IB1010**.
Academic credit:	4	
Course format:	Lectures	

Instructor's Information

Dr. Lin Jiu — Assistant Professor of Mathematics
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Office: WDR 3004

Office Hours: We 10:00—11:30, TuTh 16:00—17:30, or **by appointment**

My main research area is number theory and combinatorics in mathematics. Some other projects involve computations, such as symbolic computation, i.e., computer proofs. Please check my personal website for more information: https://jiulin90.github.io/index.html

What is this course about?

Probability is one of the most beautiful branchs in mathematics that also applies to many areas. Different from MATH205, we shall introduce concept via rigorous proofs, for clarity and precision. Namely, a solid foundation shall be established, which applies to statistical methods, data science, information theory, etc.

What background knowledge do I need before taking this course?

Required: MATH 201, MATH 202 and MATH 205

What will I learn in this course?

We shall learn the main objectives for this course:

> rigorous introduction on probability, including discrete and continuous distributions, random variables and limit theorems.

Therefore, you will setup probabilistic reasoning in precise mathematical terms; will develop some fundamental results, tools and models for further applications.

More precisely, you will be able to

- compute, for both discrete and continuous distributions, probabilities, expectations, variance, and moments, including conditional expectation and variances.
- classify and discribe discrete and continuous random variables, and random vectors
- compute probabilities on joint, marginal and conditional distributions

- apply generating functions to compute e.g., independent random variables, random sum of i.i.d. random variables, etc.
- distinguish different convergence in probability
- demonstrate rigorous proofs of law of large numbers, central limit theorem, and Poisson limit theorems

What will I do in this course?

The course will be comprised of (video) lectures, synchronous meetings dedicated to problem solving and lecturing, homework, and exams.

Lectures. Lectures will be given in our regular classroom. If no remote student joins after the Add/Drop session, there will not be any recordings.

Homework. **(48%)** There will be weekly homework, 6 times in total and each is worth 8%. Except for the last week, each week, a homework assignment will be given before Friday noon, 12:00 and due the next Friday noon. All homework assignments will be released and submitted on Gradescope.
Collaboration is encouraged, but you must write up your own solutions. **Late homework will not be accepted**, except as required by the University Policy.

Presentation. **(20%)** There will be a individual presentation during the last lecture(s). The 20% points will be given based on the following items

- Topic 5%. The topic should be related to probability. The following situation should be avoided (otherwise, you will lose points):
  - The topic is too simple.
  - Your topic overlaps with someone else's for more than 20%.

  Both requirements are necessary for a conference/workshop/symposium talk; namely besides fitting the theme, your talk should have an adequate level and unique, comparing with the others'.
- Slides (typo, beamer, etc.) 5%. Due to the duration, your slides can have up to 10 typos; and the slides file should be a PDF file, generated by LaTeX beamer instead of PowerPoint.
- Articulation (English, explanation, Q&A, etc.) 5%,
- Peer review 5%.

Exams. **(32%)**. There will be a written final exam. Date and time shall be scheduled by the Registrar's Office, and will be announced once it is determined.

What required texts, materials, and equipment will I need?

"A Natural Introduction to Probability Theory" by Ronald Meester.
ISBN: 978-3-7643-8723-5
Electronic Version Free from DKU/Duke Library: HERE

How will my grade be determined?

1. Homework: 48%

2. Presentation: 20%
3. Final exam: 32%

Please refer to the following scale for your grading.

A+= 98.00% - 100% **A** = 93.00 - 97.99%%; **A-** = 90.00% - 92.99%;
B+ =87.00% - 89.99%; **B** = 83.00% - 86.99%; **B-** = 80.00% - 82.99%;
C+ = 77.00% - 79.99%; **C** = 73.00% - 76.99%; **C-** = 70.00% - 72.99%;
D+ = 67.00% - 69.99%; **D** = 63.00% - 66.99%; **D-** = 60.00% - 62.99%
F = 59.99% and below

What are the course policies?

Academic Integrity:

As a student, you should abide by the academic honesty standard of Duke Kunshan University. The DKU Community Standard states: "Duke Kunshan University is a community comprised of individuals from diverse cultures and backgrounds. We are dedicated to scholarship, leadership, and service and to the principles of honesty, fairness, respect, and accountability. Members of this community commit to reflecting upon and upholding these principles in all academic and non-academic endeavors, and to protecting and promoting a culture of integrity and trust." For all graded work, students should pledge that they have neither given nor received any unacknowledged aid.

Please also include an indication of your typical penalties for an academic integrity violation (such as resubmitting for a reduced grade, 0 on the problem or the assignment, etc.). While the actual penalty might depend on the details of the specific situation, including some indication will help students understand that DKU takes academic integrity seriously. Please also be clear about your policies regarding the use of any online resources, including language translation tools, problem-solving tools, **artificial intelligence**, etc.

Academic Policy & Procedures:

You are responsible for knowing and adhering to academic policy and procedures as published in the University Bulletin and Student Handbook. Please note, an incident of behavioral infraction or academic dishonesty (cheating on a test, plagiarizing, **unauthorized use of online tools**, etc.) will result in immediate action from me, in consultation with university administration (e.g., Dean or Associate Dean of Undergraduate Studies, Student Conduct, Academic Advising). Please visit the Undergraduate Studies website for additional guidance related to academic policy and procedures. Academic integrity is everyone's responsibility.

Academic Disruptive Behavior and Community Standard:

Please avoid all forms of disruptive behavior, including but not limited to: verbal or physical threats, repeated obscenities, unreasonable interference with class discussion, making/receiving personal phone calls, text messages or pages during class, excessive tardiness, leaving and entering class frequently without

notice of illness or other extenuating circumstances, and persisting in disruptive personal conversations with other class members. Please turn off phones, pagers, etc. during class unless instructed otherwise. Laptop computers may be used for class activities allowed by the instructor during synchronous sessions. If you choose not to adhere to these standards, I will take action in consultation with university administration (e.g., Dean of Undergraduate Studies, Student Conduct, Academic Advising).

Academic Accommodations:

Duke Kunshan University makes reasonable academic accommodations for qualified students with disabilities. All undergraduate accommodations must be approved through the Student Accommodation Services. Students requesting accommodation for this course should forward their official accommodation letter to the instructor and ask to schedule a time to meet and discuss the implementation of their accommodation(s). It is the student's responsibility to meet, discuss, and provide an electronic copy of the Instructor Accommodation Letter to each instructor. Accommodation will not be granted retroactively. Accommodations for test, quiz, or exam taking must be arranged with the professor at least a week before the date of the quiz, test, or exam, including finals.

What campus resources can help me during this course?

Academic Advising and Student Support

Please consult with me about appropriate course preparation and readiness strategies, as needed. Consult your academic advisors on course performance (i.e., poor grades) and academic decisions (e.g., course changes, incompletes, withdrawals) to ensure you stay on track with degree and graduation requirements. In addition to advisors, staff in the Academic Resource Center can provide recommendations on academic success strategies (e.g., tutoring, coaching, student learning preferences). Please visit the Office of Undergraduate Advising website for additional information related to academic advising and student support services.

Writing and Language Studio

For additional help with academic writing—and more generally with language learning—you are welcome to make an appointment with the Writing and Language Studio (WLS). You can register for an account, make an appointment, and learn more about WLS services, policies, and events on the WLS website. You can also find writing and language learning resources on the Writing & Language Studio Sakai site.

IT Support

If you are experiencing technical difficulties, please contact IT:

- China-based faculty/staff/students 400-816-7100, (+86) 0512- 3665-7100
- US-based faculty/staff/students (+1) 919-660-1810

- International-based faculty/staff/students can use either telephone option (recommend using tools like Skype calling)
- Live Chat: https://oit.duke.edu/help
- Email: service-desk@dukekunshan.edu.cn

What is the expected course schedule?

We will cover most of the following materials from the textbook (*Tentatively, may up to some perturbation*).

Lecture 1	Probability Space [Ch.1] ● Basic Set Theory ● Probability Measure: Definition, Continuity ● Properties of Probability Measures
Lecture 2	Probability Space (Cont'd) [Ch.1] ● Conditional Probabilities, Independence, and Bayes' Theorem General Probability Space [Ch. 9] ● General Probability Measures ● Sigma algebra, (sub)-additivity
Lecture 3	Discrete Random Variables [Ch.2] ● Discrete Random Variables: definition and examples ● Independent Random Variables ● Expectation
Lecture 4	Discrete Random Variables (Cont'd) [Ch.2] ● Variance ● Conditional Distribution and Expectation ● Generating Functions ● Compounding and Random Sum
Lecture 5	Random Walk [Personal Notes and Ch.3]

	• Simple Random Walk • Higher Dimensional Simple Random Walk
Lecture 6	Random Walk [Ch.3] • Dyth Paths, Arcsine Law Continuous Distributions [Ch. 5] • Review of continuous random distributions • Pointwise, uniform, and dominated convergence
Lecture 7	Continuous Distributions (Cont'd) [Ch. 5] • Random vectors and Independence • Expectation and Variance • Functions of random vectors • Characteristic and Moment Generating Functions
Lecture 8	Continuous Distributions (Cont'd) [Ch. 5] • Characteristic Function [Ch. 8] • Joint distributions • Marginal and conditional distributions
Lecture 9	Miscellaneous Distributions • Multivariate Normal Distribution
Lecture 10	• Mixed Example • Inequalities: Markov, Chebyshev, Hoelder, Minkovski [Ch. 2]
Lecture 11	Discrete Random Variables (Cont'd) [Ch.2] • Limit Distributions (Different Convergence: in probability, in distribution, almost surely (with probability 1), in rth mean.) • The Law of Rare Events

Lecture 12	Limit Theorems [Ch. 8] ● The Law of Large Numbers ● The Central Limit Theorem
Lecture 13	Miscellaneous Examples ● Buffon's Needle ● Shor's Algorithm/Factoring Problem ● Ramsey's Theorem
Lecture 14	Presentation

Final Exam
Oct 16th, 2023, IB1010, 13:00—15:00