Heat and Electricity Market Coordination: A Scalable Complementarity Approach - Online Appendix

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Abstract

In this Appendix we present the reformulation of the hierarchical heat market model introduced in Section 5.2 of the supporting paper into a single-level optimization problem. We then describe the steps of the proposed augmented regularized Benders algorithm introduced in Section 5.4. Finally, we provide the input data for the case study in Section 6 of the supporting paper.

Keywords: OR in energy Integrated energy system; Stochastic programming; Hierarchical optimization; Regularized Benders decomposition

1. Nomenclature

Sets and Indexes

$\mathcal{T}$ Set of time periods

$\mathcal{X}$ Set of day-ahead scenarios

$\mathcal{I}^{\mathbf{H}}$ Set of heat-only units

$\mathcal{I}^{\mathbf{HS}}$ Set of heat storage tanks

$\mathcal{I}^{\mathbf{CHP}}$ Set of CHPs

$\mathcal{I}^{\mathbf{HP}}$ Set of HPs

$\mathcal{I}^{\mathbf{E}}$ Set of electricity conventional generators and wind producers

$\Omega^{\mathbf{H}}$ Set of optimization variables of day-ahead sequential heat market

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$\Omega^{\mathbf{E}}_{\nu}$ Set of primal optimization variables of day-ahead electricity market for underlying scenario ν

$\Xi^{\mathbf{E}}_{\nu}$ Set of dual optimization variables of day-ahead electricity market for underlying scenario ν

$\Omega^{\mathbf{R}}_{\nu}$ Set of optimization variables of redispatch heat market for underlying scenario ν

$\Omega^{\mathbf{Int}}_{\nu}$ Set of optimization variables of integrated heat and electricity market for underlying scenario ν

$\Omega^{\mathbf{UL}}$ Set of optimization variables of upper-level problem in hierarchical heat market

Input Parameters

π_{ν} Probability of scenario ν

$\overline{Q}_j$ Maximum heat output of CHPs, HPs, heat-only, and heat storage tanks (MW)

$\rho^{\mathbf{H}}_j$,$\rho^{\mathbf{E}}_j$ Heat and electricity fuel efficiency of CHP

r_j Heat to electricity ratio of CHP

$\overline{F}_j$ Maximum fuel consumption of CHP (MW)

$\mathbf{COP}_j$ Coefficient of performance of heat pump

$\overline{S}_j$ Maximum heat stored in heat storage tanks (MWh)

$\underline{S}_j$ Minimum heat stored in heat storage tanks (MWh)

$S^{\mathbf{init}}_j$ Initial heat stored in heat storage tanks (MWh)

ρ^{-}_j,ρ^{+}_j Heat storage tanks charging and discharging efficiencies

l_j Heat storage losses (MWh)

$L^{\mathbf{E}}_{t\nu}$ Electricity load scenario (MW)

$L^{\mathbf{H}}_t$ Heat load (MW)

$\overline{P}_{jt\nu}$ Maximum power output of conventional generators and wind producers (MW)

$\underline{\alpha}$,$\overline{\alpha}$ Minimum and maximum price offer in the day-ahead electricity market; respectively -500€ /MWh and 3000€/MWh

α_j Marginal cost parameter of CHPs and heat-only units (€/MWh)

$\tilde{\alpha}^{\mathbf{E}}_{jt\nu}$ Marginal price offer of conventional generators and wind producers in day-ahead electricity market (€/MWh)

μ_{α_j} Mean value of marginal price offers' distribution (€/MWh)

σ_{α_j} Standard deviation of marginal price offers' distribution (€/MWh)

$\alpha_j^{\mathbf{E}}$ Electricity marginal cost of CHPs (€/MWh)

$\alpha_j^{\uparrow}$,$\alpha_j^{\downarrow}$ Up and down redispatch costs of CHPs, HPs, and heat-only units (€/MWh)

$\hat{\lambda}_t^{\mathbf{E}}$ Forecast electricity price (€/MWh)

Decision variables

Q_{jt} Day-ahead heat dispatch of CHPs, HPs, and heat-only units (MWh)

Q_{jt}^{-},Q_{jt}^{+} Charging and discharging of heat storage tanks in day-ahead market (MWh)

S_{jt} Heat stored in heat storage tanks in day-ahead market (MWh)

$Q_{jt\nu}^{\uparrow}$,$Q_{jt\nu}^{\downarrow}$ Upward and downward heat production adjustment of CHPs and heat-only units (MWh)

$Q_{jt\nu}$ Heat production of CHPs and heat-only units after redispatch (MWh)

$Q_{jt\nu}^{+}$,$Q_{jt\nu}^{-}$ Charging and discharging of heat storage tanks after redispatch (MWh)

$S_{jt\nu}$ Heat stored in heat storage tanks after redispatch (MWh)

$P_{jt\nu}$ Day-ahead electricity dispatch of conventional generators, wind producers, and CHPs (MWh)

$P_{jt\nu}^{0}$ Electricity production of CHPs below $\underline{P}_j(Q_{jt})$ (MWh)

$P_{jt\nu}^{+}$ Electricity production of CHPs over $\underline{P}_j(Q_{jt})$ (MWh)

$L_{jt\nu}^{\mathbf{HP}}$ Electricity consumption of HP (MWh)

$\underline{\mu}_{jt\nu}$,$\overline{\mu}_{jt\nu}$,$\underline{\mu}_{jt\nu}^{0}$,$\overline{\mu}_{jt\nu}^{0}$ Dual variables of the lower-level problems

$\lambda_{t\nu}^{\mathbf{E}}$ Day-ahead electricity prices

$\lambda_{t\nu}^{\mathbf{H}}$ Day-ahead heat prices

Functions

$\Gamma_j(.)$ Total production cost of CHPs, depending on electricity prices, heat and electricity production (€)

$\Gamma_j^{\mathbf{H}}(.)$ Expected heat cost of CHPs and HPs, depending on electricity prices, heat and electricity production (€)

$\alpha_j^{\mathbf{H}}(.)$ Expected heat marginal cost of CHPs and HPs, depending on electricity prices (€/MWh)

$\overline{P}_j(.)$ Maximum power output of CHPs depending on heat output (MW)

$\underline{P}_j(.)$ Minimum power output of CHPs depending on heat output (MW)

$\underline{L}_j^{\mathbf{HP}}(.)$ Minimum power output of CHPs depending on heat output (MW)

Benders algorithm indexes, variables and parameters

θ Iterations of Benders algorithm, in $\{1, ..., \theta^{\max}\}$

$\Omega^{\mathbf{MP}}$ Set of optimization variables of master problem

$\Omega_{\nu}^{\mathbf{SUB}}$ Set of optimization variables of subproblem SUB_{ν} for underlying scenario ν

ϵ Positive tolerance parameter

$\eta_{jt\nu}^{Q,(\theta)}$ Sensitivity of subproblem SUB_{ν} with respect to complicating variable Q_{jt} at iteration θ

$\eta_{jt\nu}^{S,(\theta)}$ Sensitivity of subproblem SUB_{ν} with respect to complicating variable S_{jt} at iteration θ

$\eta_{jt\nu}^{Q^+,(\theta)}$ Sensitivity of subproblem SUB_{ν} with respect to complicating variable Q_{jt}^+ at iteration θ

$\eta_{jt\nu}^{Q^-,(\theta)}$ Sensitivity of subproblem SUB_{ν} with respect to complicating variable Q_{jt}^- at iteration θ

$z_{\nu}^{\mathbf{SUB},(\theta)}$ Objective value of SUB_{ν} at iteration θ

$z^{\mathbf{MP},(\theta)}$ Objective value of master problem at iteration θ

$\mathbf{UB}^{(\theta)}$ Value of upper bound at iteration θ

$\mathbf{LB}^{(\theta)}$ Value of lower bound at iteration θ

$X^{(\mathbf{ref})}$ Reference point

$\tau^{(\mathbf{ref})}$ Penalization parameter

m Regularization parameter $(0 < m < 1)$

X Vector of complicating variables

β Decision variable of master problem

2. Single-Level Reformulation of the Hierarchical Market Model

The hierarchical heat market introduced in Section 5.2 of the supporting paper cannot be solved directly by traditional solvers. In this section we introduce various methods to reformulate this hierarchical (bi-level) optimization problem as single-level optimization problem.

2.1. Mathematical Problem with Equilibrium Constraints (MPEC) Formulation

The hierarchical optimization problem (17) in the supporting paper can be reformulated as a single-level optimization problem by replacing each linear lower-level optimization problem LL_ν by its equivalent Karush-Khun-Tucker (KKT) conditions:

$$\alpha_{jt\nu} + \overline{\mu}_{t\nu} - \underline{\mu}_{jt\nu} - \lambda^{\mathrm{E}}_{t\nu} = 0,\ \forall j \in \mathcal{I}^{\mathrm{E}}, t \in \mathcal{T} \tag{1a}$$

$$\underline{\alpha} + \overline{\mu}^{0}_{t\nu} - \underline{\mu}^{0}_{jt\nu} - \lambda^{\mathrm{E}}_{t\nu} = 0,\ \forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T} \tag{1b}$$

$$\alpha_{jt\nu}\rho^{\mathrm{E}}_{j} + \overline{\mu}_{t\nu} - \underline{\mu}_{jt\nu} - \lambda^{\mathrm{E}}_{t\nu} = 0,\ \forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T} \tag{1c}$$

$$-\overline{\alpha} + \overline{\mu}_{t\nu} - \underline{\mu}_{jt\nu} + \lambda^{\mathrm{E}}_{t\nu} = 0,\ \forall j \in \mathcal{I}^{\mathrm{HP}}, t \in \mathcal{T} \tag{1d}$$

$$L^{\mathrm{E}}_{t\nu} + \sum_{j \in \mathcal{I}^{\mathrm{HP}}} L^{\mathrm{HP}}_{jt\nu} = \sum_{j \in \mathcal{I}^{\mathrm{E}}} P_{jt\nu} + \sum_{j \in \mathcal{I}^{\mathrm{CHP}}} \left(P^{0}_{jt\nu} + P^{+}_{jt\nu}\right)\ : \lambda^{\mathrm{E}}_{t\nu},\ \forall t \in \mathcal{T} \tag{1e}$$

$$0 \leq \underline{\mu}_{jt\nu} \perp P_{jt\nu} \geq 0,\ \forall j \in \mathcal{I}^{\mathrm{E}}, t \in \mathcal{T} \tag{1f}$$

$$0 \leq \overline{\mu}_{jt\nu} \perp \left(\overline{P}_{jt\nu} - P_{jt\nu}\right) \geq 0,\ \forall j \in \mathcal{I}^{\mathrm{E}}, t \in \mathcal{T} \tag{1g}$$

$$0 \leq \underline{\mu}^{0}_{jt\nu} \perp P^{0}_{jt\nu} \geq 0,\ \forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T} \tag{1h}$$

$$0 \leq \overline{\mu}^{0}_{jt\nu} \perp \left(\underline{P}_{j}\left(Q_{jt}\right) - P^{0}_{jt\nu}\right) \geq 0,\ \forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T} \tag{1i}$$

$$0 \leq \underline{\mu}_{jt\nu} \perp P^{+}_{jt\nu} \geq 0,\ \forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T} \tag{1j}$$

$$0 \leq \overline{\mu}_{jt\nu} \perp \left(\overline{P}_{j}\left(Q_{jt}\right) - \underline{P}_{j}\left(Q_{jt}\right) - P^{+}_{jt\nu}\right) \geq 0,\ \forall j \in \mathcal{I}^{\mathrm{CHP}}, t \in \mathcal{T} \tag{1k}$$

$$0 \leq \underline{\mu}_{jt\nu} \perp L^{\mathrm{HP}}_{jt\nu} \geq 0,\ \forall j \in \mathcal{I}^{\mathrm{HP}}, t \in \mathcal{T} \tag{1l}$$

$$0 \leq \overline{\mu}_{jt\nu} \perp \left(\underline{L}^{\mathrm{HP}}_{j}\left(Q_{jt}\right) - L^{\mathrm{HP}}_{jt\nu}\right) \geq 0,\ \forall j \in \mathcal{I}^{\mathrm{HP}}, t \in \mathcal{T}. \tag{1m}$$

Equation (1a)-(1d) represent the stationarity conditions, and (1e)-(1m) represent the primal and dual constraints, and complementarity conditions. By replacing each lower-level problem LL_ν by its equivalent KKT conditions, the hierarchical optimization problem (17) in the supporting paper is recast as Mathematical Problem with Equilibrium Constraints (MPEC). Due to the non-convex complementarity conditions, MPECs are challenging to solve and few solvers (e.g. KNITRO, NLPEC) support them. Another approach is to reformulate the complementarity conditions by introducing SOS variables or auxiliary binary variables, as will be detailed in Section 3.

2.2. Primal-Dual Formulation

Additionally, as the lower-level problems LL_ν of the hierarchical optimization problem (17) in the supporting paper are linear in the continuous variables $\Omega^{\mathrm{E}}_{\nu}$, strong duality applies. Therefore, the hierarchical optimization problem can be reformulated as a single-level optimization problem by replacing each lower-level problem LL_ν by the following equivalent primal-dual formulation:

$$\begin{aligned}
&\sum_{j\in\mathcal{I}^{\mathrm{E}},t\in\mathcal{T}} \tilde{\alpha}^{\mathrm{E}}_{jt\nu}P_{jt\nu} + \sum_{j\in\mathcal{I}^{\mathrm{CHP}},t\in\mathcal{T}} \left(\underline{\alpha}P^{0}_{jt\nu} + \alpha^{\mathrm{E}}_{j}P^{+}_{jt\nu}\right) - \overline{\alpha}L^{\mathrm{HP}}_{jt\nu} \\
&= \sum_{t\in\mathcal{T}} \Bigg[- \sum_{j\in\mathcal{I}^{\mathrm{HP}}} \overline{\mu}_{jt\nu}\underline{L}^{\mathrm{HP}}_{j}\left(Q_{jt}\right) - \sum_{j\in\mathcal{I}^{\mathrm{CHP}}} \overline{\mu}^{0}_{jt\nu}\underline{P}_{j}\left(Q_{jt}\right) \\
&\quad - \sum_{j\in\mathcal{I}^{\mathrm{CHP}}} \overline{\mu}_{jt\nu}\left(\overline{P}_{j}\left(Q_{jt}\right) - \underline{P}_{j}\left(Q_{jt}\right)\right) - \sum_{j\in\mathcal{I}^{\mathrm{E}}} \overline{\mu}_{jt\nu}\overline{P}_{jt\nu} + \lambda^{\mathrm{E}}_{t\nu}L^{\mathrm{E}}_{t\nu}\Bigg] && \text{(2a)}\\
&\text{(1a)} - \text{(1d)} && \text{(2b)}\\
&L^{\mathrm{E}}_{t\nu} + \sum_{j\in\mathcal{I}^{\mathrm{HP}}} L^{\mathrm{HP}}_{jt\nu} = \sum_{j\in\mathcal{I}^{\mathrm{E}}\cup\mathcal{I}^{\mathrm{CHP}}} P_{jt\nu} \ : \lambda^{\mathrm{E}}_{t\nu},\ \forall t\in\mathcal{T} && \text{(2c)}\\
&0 \le P^{0}_{jt\nu} \le \underline{P}_{j}\left(Q_{jt}\right) \ : \underline{\mu}_{jt\nu}, \overline{\mu}_{jt\nu},\ \forall j\in\mathcal{I}^{\mathrm{CHP}}, t\in\mathcal{T} && \text{(2d)}\\
&0 \le P^{+}_{jt\nu} \le \overline{P}_{j}\left(Q_{jt}\right) - \underline{P}_{j}\left(Q_{jt}\right) \ : \underline{\mu}_{jt\nu}, \overline{\mu}_{jt\nu},\ \forall j\in\mathcal{I}^{\mathrm{CHP}}, t\in\mathcal{T} && \text{(2e)}\\
&0 \le P_{jt\nu} \le \overline{P}_{jt\nu} \ : \underline{\mu}_{jt\nu}, \overline{\mu}_{jt\nu},\ \forall j\in\mathcal{I}^{\mathrm{E}}, t\in\mathcal{T} && \text{(2f)}\\
&0 \le L^{\mathrm{HP}}_{jt\nu} \le \underline{L}^{\mathrm{HP}}_{j}\left(Q_{jt}\right) \ : \underline{\mu}_{jt\nu}, \overline{\mu}_{jt\nu},\ \forall j\in\mathcal{I}^{\mathrm{HP}}, t\in\mathcal{T} && \text{(2g)}\\
&\underline{\mu}_{jt\nu}, \underline{\mu}^{0}_{jt\nu}, \overline{\mu}_{jt\nu}, \overline{\mu}^{0}_{jt\nu} \ge 0,\ \forall j\in\mathcal{I}^{\mathrm{E}}\cup\mathcal{I}^{\mathrm{CHP}}\cup\mathcal{I}^{\mathrm{HP}}, t\in\mathcal{T}. && \text{(2h)}
\end{aligned}$$

Equation (2a) represents the strong duality theorem, (2b) stationarity conditions, (2c)-(2g) primal feasibility, and (2h) dual feasibility. The strong duality condition (2a) is non-convex due to the bilinear terms $\overline{\mu}^{0}_{jt\nu}\underline{P}_{j}\left(Q_{jt}\right)$, $\overline{\mu}_{jt\nu}\left(\overline{P}_{j}\left(Q_{jt}\right) - \underline{P}_{j}\left(Q_{jt}\right)\right)$, and $\overline{\mu}_{jt\nu}\underline{L}^{\mathrm{HP}}_{j}\left(Q_{jt}\right)$. As a result, this primal-dual formulation is challenging to solve directly using traditional solvers. To address this issue, we will introduce a solution method based on Benders decomposition in Section 4.

3. Mixed Integer Linear Problem (MILP) Formulation

The MPEC introduced in Appendix 2.1 can be reformulated as a Mixed Integer Linear Problem (MILP). First, the complementarity conditions (1f)-(1m) can be linearized using the well-known Fortuny-Amat linearization (Gabriel et al., 2012; Fortuny-Amat & McCarl, 1981). Additionally, the bilinear terms in the objective function (17a) in the supporting paper can be exactly linearized, using the complementarity conditions (1f)-(1m) and the strong duality theorem (2a) (Gabriel et al., 2012), such that:

$$\sum_{j\in\mathcal{I}^{\mathrm{HP}}} \lambda^{\mathrm{E}}_{t\nu}L^{\mathrm{HP}}_{jt\nu} - \sum_{j\in\mathcal{I}^{\mathrm{CHP}}} \lambda^{\mathrm{E}}_{t\nu}P_{jt\nu} = \sum_{j\in\mathcal{I}^{\mathrm{E}}} \left(\alpha_{jt\nu}P_{jt\nu} + \overline{\mu}_{jt\nu}\overline{P}_{jt\nu}\right) - \lambda^{\mathrm{E}}_{t\nu}L^{\mathrm{E}}_{t\nu},\ \forall t\in\mathcal{T}, \nu\in\mathcal{X}. \quad (3)$$

This MILP problem can be readily solved with traditional solvers. However, the number of auxiliary binary variables used to linearize the complementarity conditions increases proportionally to the number of scenarios considered, i.e. as $2\left|\mathcal{X}\right|\left|\mathcal{T}\right|\left|\mathcal{I}^{\mathrm{CHP}}\cup\mathcal{I}^{\mathrm{HP}}\cup\mathcal{I}^{\mathrm{E}}\right|$. In order to cope with the computational complexity, we will introduce a decomposition-based method below.

4. Augmented Regularized Benders Decomposition Algorithm

In this section we detail the steps of the proposed augmented regularized Benders algorithm, introduced in Section 5.4 of the supporting paper.

4.1. Benders Decomposition Structure

By using a Benders decomposition algorithm, the original stochastic hierarchical optimization problem can be decomposed into $|\mathcal{X}|$ subproblems, one per scenario $\nu \in \mathcal{X}$, by temporarily fixing the so-called complicating variables, i.e. the day-ahead heat dispatch variables $\Omega^{\mathrm{H}} = \left\{Q_{jt}, S_{jt}, Q^{+}_{jt}, Q^{-}_{jt}\right\}$. This method is an iterative process. At each iteration, the complicating variables are updated in the so-called master problem (step A), which only includes the constraints related to the complicating variables, optimality cuts, and auxiliary cuts derived from the subproblems at previous iterations. The objective value of the master problem provides a lower bound for the objective value of the original problem. The subproblems, one per scenario, are then solved independently with the fixed value of the complicating variables (step B). The hierarchical subproblems are reformulated as single-level problems using the primal-dual formulation introduced in Section 2.2 and solved in two steps (steps B1 and B2). If the decrease in the expected objective value of the subproblems is deemed sufficient, it provides a new upper bound on the objective value of the original problem, and a new reference point (step C). The master problem and the subproblems exchange information until the upper and lower bounds converge (step D).

4.2. Subproblems (Step B)

At iteration $\theta \geq 1$, each subproblem SUB_{ν}, one per scenario $\nu \in \mathcal{X}$, is a hierarchical optimization problem. The set of optimization variables $\Omega^{\mathrm{SUB}}_{\nu} = \Omega^{\mathrm{H}} \cup \Omega^{\mathrm{R}}_{\nu} \cup \Omega^{\mathrm{E}}_{\nu} \cup \Xi^{\mathrm{E}}_{\nu}$ includes the complicating variables, the redispatch variables, as well as the primal and dual lower-level variables for the corresponding scenario $\nu \in \mathcal{X}$. This hierarchical optimization problem is reformulated as a single-level optimization problem using a primal-dual formulation of the lower-level problem, such that:

$$\min_{\Omega_\nu^{\text{SUB}}} \pi_\nu \sum_{t\in\mathcal{T}} \Bigg[\sum_{j\in\mathcal{I}^{\text{CHP}}} \alpha_j \rho_j^{\text{E}} P_{jt\nu} + \sum_{j\in\mathcal{I}^{\text{E}}} \left(\alpha_{jt\nu} P_{jt\nu} + \overline{\mu}_{jt\nu}\overline{P}_{jt\nu}\right) - \lambda_{t\nu}^{\text{E}} L_{t\nu}^{\text{E}}$$

$$+ \sum_{j\in\mathcal{I}^{\text{CHP}}\cup\mathcal{I}^{\text{H}}} \left(\alpha_{jt\nu}^{\uparrow} Q_{jt\nu}^{\uparrow} - \alpha_{jt\nu}^{\downarrow} Q_{jt\nu}^{\downarrow}\right) \Bigg] \tag{4a}$$

$$\text{s.t.} \quad L_t^{\text{H}} = \sum_{j\in\mathcal{I}^{\text{H}}\cup\mathcal{I}^{\text{HP}}\cup\mathcal{I}^{\text{CHP}}} Q_{jt\nu} + \sum_{j\in\mathcal{I}^{HS}} \left(Q_{jt\nu}^{+} - Q_{jt\nu}^{-}\right), \ \forall t\in\mathcal{T} \tag{4b}$$

$$Q_{jt\nu} = Q_{jt} + Q_{jt\nu}^{\uparrow} - Q_{jt\nu}^{\downarrow}, \ \forall j\in\mathcal{I}^{\text{H}}\cup\mathcal{I}^{\text{HP}}\cup\mathcal{I}^{\text{CHP}}, t\in\mathcal{T} \tag{4c}$$

$$P_{jt\nu} = P_{jt\nu}^{0} + P_{jt\nu}^{+}, \ \forall j\in\mathcal{I}^{\text{CHP}}, t\in\mathcal{T} \tag{4d}$$

$$0 \le Q_{jt\nu}^{\uparrow}, \ 0 \le Q_{jt\nu}^{\downarrow}, \ \forall j\in\mathcal{I}^{\text{H}}\cup\mathcal{I}^{\text{HP}}\cup\mathcal{I}^{\text{CHP}}, t\in\mathcal{T} \tag{4e}$$

$$0 \le Q_{jt\nu} \le \overline{Q}_j, \ \forall j\in\mathcal{I}^{\text{CHP}}\cup\mathcal{I}^{\text{HP}}\cup\mathcal{I}^{\text{H}}, t\in\mathcal{T} \tag{4f}$$

$$P_{jt\nu} \ge r_j Q_{jt\nu}, \ \forall j\in\mathcal{I}^{\text{CHP}}, t\in\mathcal{T} \tag{4g}$$

$$\rho_j^{\text{H}} Q_{jt\nu} + \rho_j^{\text{E}} P_{jt\nu} \le \overline{F}_j, \ \forall j\in\mathcal{I}^{\text{CHP}}, t\in\mathcal{T} \tag{4h}$$

$$Q_{jt\nu} = \text{COP}_j L_{jt\nu}^{\text{HP}}, \ \forall j\in\mathcal{I}^{\text{HP}}, t\in\mathcal{T} \tag{4i}$$

$$S_{jt\nu} = S_{j(t-1)\nu} + \rho_j^{-} Q_{jt\nu}^{-} - \rho_j^{+} Q_{jt\nu}^{+} - l_j, \ \forall j\in\mathcal{I}^{HS}, t\in\mathcal{T} \tag{4j}$$

$$\underline{S}_j \le S_{jt\nu} \le \overline{S}_j, \ \forall j\in\mathcal{I}^{HS}, t\in\mathcal{T} \tag{4k}$$

$$0 \le Q_{jt\nu}^{-} \le \overline{Q}_{jt}, \ 0 \le Q_{jt\nu}^{+} \le \overline{Q}_{jt}, \ \forall j\in\mathcal{I}^{HS}, t\in\mathcal{T} \tag{4l}$$

$$S_{j(j(t=|\mathcal{T}|)\nu} \ge S_j^{init}, \ \forall j\in\mathcal{I}^{HS}, \tag{4m}$$

$$(2a) - (2h) \tag{4n}$$

$$Q_{jt} = Q_{jt}^{(\theta)} \ : \eta_{jt\nu}^{Q}, \ \forall j\in\mathcal{I}^{\text{CHP}}\cup\mathcal{I}^{\text{HP}}\cup\mathcal{I}^{\text{H}}, t\in\mathcal{T} \tag{4o}$$

$$S_{jt} = S_{jt}^{(\theta)} \ : \eta_{jt\nu}^{S}, \ \forall j\in\mathcal{I}^{HS}, t\in\mathcal{T} \tag{4p}$$

$$Q_{jt}^{+} = Q_{jt}^{+(\theta)} \ : \eta_{jt\nu}^{Q^{+}}, \ \forall j\in\mathcal{I}^{HS}, t\in\mathcal{T} \tag{4q}$$

$$Q_{jt}^{-} = Q_{jt}^{-(\theta)} \ : \eta_{jt\nu}^{Q^{-}}, \ \forall j\in\mathcal{I}^{HS}, t\in\mathcal{T}. \tag{4r}$$

The objective of the subproblem is to minimize the weighted cost (4a) for the underlying scenario $\nu \in \mathcal{X}$, subject to heat redispatch constraints (4b)-(4m), the primal-dual formulation of the lower-level problem LL_ν (4n), and the constraints fixing the complicating variables (4o)-(4r).

Due to the bilinear terms $\overline{\mu}_{jt\nu}^{0}\underline{P}_j(Q_{jt})$, $\overline{\mu}_{jt\nu}\left(\overline{P}_j(Q_{jt}) - \underline{P}_j(Q_{jt})\right)$, and $\overline{\mu}_{jt\nu}\underline{L}_j^{\text{HP}}(Q_{jt})$ in (2a), the subproblems are non-convex. Therefore we solve this subproblem in two steps. Each bilinear term is in fact a product of a complicating variable (say Q_{jt}) and a dual variable (say $\overline{\mu}_{jt\nu}$). At a given iteration (θ), in order to linearize these terms, each subproblem is solved in two steps. We first solve a linear auxiliary subproblem (aux-SUB_ν), in which the complicating variables Q_{jt} are treated as parameters and fixed to the values $Q_{jt}^{(\theta)}$ obtained in the master problem, and not as variables fixed to given values (step B1). These auxiliary subproblems provide the optimal values for dual variables $\overline{\mu}_{jt\nu}$, but do not give sensitivities required for generating cuts. In the second step, the dual variables $\overline{\mu}_{jt\nu}$ are treated as parameters and fixed to those values $\overline{\mu}_{jt\nu}^{(\theta)}$ obtained in Step B1, and subproblems SUB_ν treat complicating variables Q_{jt} as variables. However, the fixing constraints enforce the complicating variables Q_{jt} to take the values $Q_{jt}^{(\theta)}$ coming from the master problem. The dual variables of the fixing constraints (4o)-(4r), $\eta_{jt\nu}^{Q}$, $\eta_{jt\nu}^{S}$, $\eta_{jt\nu}^{Q^{+}}$, $\eta_{jt\nu}^{Q^{-}}$, provide sensitivities with respect to the complicating variables (step B2). These sensitivities are derived from the subproblems at each iteration in order to generate optimality cuts that further constraint in the master problem in the following iteration.

In order to initialize the Benders algorithm we propose to solve the original non-decomposed hierarchical optimization problem for a single scenario, representing the expected value of each uncertainty parameter. We use the solutions of this optimization problem to derive the initial values of the complicating variables $\Omega^{\mathrm{H}} = \left\{Q_{jt}, S_{jt}, Q_{jt}^{+}, Q_{jt}^{-}\right\}$. We then solve the subproblems at iteration $\theta = 1$ for this value of the complicating variables $Q_{jt}^{(\theta)}$, $S_{jt}^{(\theta)}$, $Q_{jt}^{+(\theta)}$, $Q_{jt}^{-(\theta)}$.

4.3. Augmented Regularized Master Problem (Step A)

At each iteration the master problem seeks to update the set of complicating variables Ω^{H}, using information from the previous iterations of subproblems. For notational simplicity, we introduce the vector of complicating variables $X = \left[Q_{jt}, S_{jt}, Q_{jt}^{+}, Q_{jt}^{-}\right]^{\mathrm{T}}$. The set of optimization variables $\Omega^{\mathrm{MP}} = \Omega^{\mathrm{H}} \cup \{\beta\}$ includes the complicating variables, and the auxiliary variable β representing the current linear estimate of the objective function of the subproblems. As a result, at iteration $(\theta + 1)$, the following master problem is solved to update the values of the complicating variables of the Benders algorithm:

$$\min_{\Omega^{\mathrm{MP}}} \sum_{\nu\in\mathcal{X},t\in\mathcal{T}} \left[\sum_{j\in\mathcal{I}^{\mathrm{H}}} \alpha_j Q_{jt} + \sum_{j\in\mathcal{I}^{\mathrm{CHP}}} \alpha_j \rho_j^{\mathrm{H}} Q_{jt} \right] + \beta + \frac{\left\| X - X^{(\mathrm{ref})} \right\|_2}{\tau^{(\mathrm{ref})}} \tag{5a}$$

$$\text{s.t.} \quad L_t^{\mathrm{H}} = \sum_{j\in\mathcal{I}^{\mathrm{HP}}\cup\mathcal{I}^{\mathrm{CHP}}\cup\mathcal{I}^{\mathrm{H}}} Q_{jt} + \sum_{j\in\mathcal{I}^{HS}} \left(Q_{jt}^{+} - Q_{jt}^{-} \right), \ \forall t\in\mathcal{T} \tag{5b}$$

$$0 \le Q_{jt} \le \overline{Q}_j, \ \forall j \in \mathcal{I}^{\mathrm{H}} \cup \mathcal{I}^{\mathrm{HP}} \cup \mathcal{I}^{\mathrm{CHP}}, t\in\mathcal{T} \tag{5c}$$

$$S_{jt} = S_{j(t-1)} + \rho_j^{-} Q_{jt}^{-} - \rho_j^{+} Q_{jt}^{+} - l_j, \ \forall j \in \mathcal{I}^{HS}, t\in\mathcal{T} \tag{5d}$$

$$0 \le Q_{jt}^{-} \le \overline{Q}_j, \ 0 \le Q_{jt}^{+} \le \overline{Q}_j, \ \forall j \in \mathcal{I}^{HS}, t\in\mathcal{T} \tag{5e}$$

$$\underline{S}_j \le S_{jt} \le \overline{S}_j, \ \forall j \in \mathcal{I}^{HS}, t\in\mathcal{T} \tag{5f}$$

$$S_{j(t=|\mathcal{T}|)} \ge S_j^{init}, \ \forall j \in \mathcal{I}^{HS} \tag{5g}$$

$$\begin{aligned} \beta \ge \sum_{\nu\in\mathcal{X}} \Bigg[z_\nu^{\mathrm{SUB},(k)} &+ \sum_{j\in\mathcal{I}^{\mathrm{H}}\cup\mathcal{I}^{\mathrm{CHP}}\cup\mathcal{I}^{\mathrm{HP}},t\in\mathcal{T}} \eta_{jt\nu}^{Q,(k)} \left(Q_{jt}^{(k)} - Q_{jt} \right) \\ &+ \sum_{j\in\mathcal{I}^{HS},t\in\mathcal{T}} \Bigg(\eta_{jt\nu}^{Q^{+},(k)} \left(Q_{jt}^{+,(k)} - Q_{jt}^{+} \right) + \eta_{jt\nu}^{Q^{-},(k)} \left(Q_{jt}^{-,(k)} - Q_{jt}^{-} \right) \\ &\qquad + \eta_{jt\nu}^{S,(k)} \left(S_{jt}^{(k)} - S_{jt} \right) \Bigg) \Bigg], \ \forall k \in [1, ..., \theta] \end{aligned} \tag{5h}$$

$$\beta \ge -M \tag{5i}$$

$$\sum_{\nu\in\mathcal{X}} \pi_\nu P_{jt\nu}^{(\theta)} \ge r_j \left(Q_{jt} + \sum_{\nu\in\mathcal{X}} \pi_\nu \left(Q_{jt\nu}^{\uparrow,(\theta)} - Q_{jt\nu}^{\downarrow,(\theta)} \right) \right), \ \forall j \in \mathcal{I}^{\mathrm{CHP}}, t\in\mathcal{T} \tag{5j}$$

$$\rho_j^{\mathrm{E}} \sum_{\nu\in\mathcal{X}} \pi_\nu P_{jt\nu}^{(\theta)} + \rho_j^{\mathrm{H}} \left(Q_{jt} + \sum_{\nu\in\mathcal{X}} \pi_\nu \left(Q_{jt\nu}^{\uparrow,(\theta)} - Q_{jt\nu}^{\downarrow,(\theta)} \right) \right) \le \overline{F}_j, \ \forall j \in \mathcal{I}^{\mathrm{CHP}}, t\in\mathcal{T} \tag{5k}$$

$$Q_{jt} + \sum_{\nu\in\mathcal{X}} \pi_\nu \left(Q_{jt\nu}^{\uparrow,(\theta)} - Q_{jt\nu}^{\downarrow,(\theta)} \right) \ge \mathrm{COP}_j \sum_{\nu\in\mathcal{X}} \pi_\nu L_{jt\nu}^{\mathrm{HP},(\theta)}, \ \forall j \in \mathcal{I}^{\mathrm{HP}}, t\in\mathcal{T}. \tag{5l}$$

The master problem aims at minimizing the day-ahead heat cost plus the current linear estimate of the objective function of the subproblems β, while the quadratic penalization term $\frac{\left\| X - X^{(\mathrm{ref})} \right\|_2}{\tau^{(\mathrm{ref})}}$ in (5a) ensures that the solutions remain close to the current reference point $X^{(\mathrm{ref})}$. The update of the reference point is detailed in Section 4.4. Efficient heuristic methods have been introduced to update the penalization parameter $\tau^{(\mathrm{ref})}$, that result in an accelerated convergence of the bundle method (Rey & Sagastizábal, 2002; Bonnans et al., 2006). The constraints of the master problem include all the constraints from the non-decomposed problem involving solely the complicating variables (5b)-(5g). Additionally, two sets of cuts are added to the master problem at each iteration. First, the optimality cuts (5h) are linear under-estimators of the objective function of the subproblems at previous iterations $k < \theta$. At each iteration a single optimality cut is added. Second, the set of auxiliary cuts (5j)-(5l) represent some of the primal constraints of the subproblems at previous iteration θ, and create an additional feedback from the subproblems.

4.4. Bounds and Reference Point Update (Step C)

At each iteration $(\theta+1)$, the solutions of the subproblems provide a new upper-bound of the objective value, and a reference point for the Benders algorithm. If the decrease in the expected objective value of the subproblems is significant, then the upper-bound is updated, in which case the step is called non-null. Otherwise it remains at the same value, in which case the step is called null. More precisely, for a given parameter $0 < m < 1$,

$$\mathrm{UB}^{(\theta+1)} = \begin{cases} \mathrm{UB}^{(\theta)}, \text{ if } z^{\mathrm{MP},(\theta+1)} + \sum_{\nu \in \mathcal{X}} z_{\nu}^{\mathrm{SUB},(\theta+1)} \geq \mathrm{ub}^{(\theta)} - m\left(\mathrm{UB}^{(\theta)} - \mathrm{LB}^{(\theta)}\right) \\ z^{\mathrm{MP},(\theta+1)} + \sum_{\nu} z_{\nu}^{\mathrm{SUB},(\theta+1)}, \text{ otherwise.} \end{cases} \tag{6}$$

Similarly, if the step is non-null we update the value of the current reference point of the Benders algorithm, such that:

$$X^{(\mathrm{ref})} = \left[Q_{jt}^{(\theta+1)}, S_{jt}^{(\theta+1)}, Q_{jt}^{+,(\theta+1)}, Q_{jt}^{-,(\theta+1)}\right]^{\mathrm{T}} \tag{7}$$

The choice of the parameter m influences the size of the Benders steps, and hence the frequency of the update of the upper-bound. However, for convex problems the bundle method has been proved to converge for any value $0 < m < 1$ (Rey & Sagastizábal, 2002; Bonnans et al., 2006). Additionally, the objective value of the master problem $z^{\mathrm{MP},(\theta+1)}$ provides a lower-bound for the solution of the Benders algorithm, such that $\mathrm{LB}^{(\theta+1)} = z^{\mathrm{MP},(\theta+1)}$.

4.5. Convergence Check (Step D)

Finally, we consider that the Benders algorithm has converged when the upper and lower bounds have converged.More precisely, for an arbitrarily small tolerance parameter $\epsilon > 0$, the algorithm has converged when $\left|\mathrm{UB}^{(\theta)} - \mathrm{LB}^{(\theta)}\right| \leq \epsilon$.

5. Case Study

5.1. Heat and Electricity Systems

A modified 24-bus IEEE Reliability Test System composes the integrated energy system. It consists of 7 thermal power plants, 6 wind farms, 3 CHPs, and 1 heat-only unit. Data for the power system is derived from the 24-bus IEEE Reliability Test System provided by Ordoudis et al. (2016). Data for the heat system and heat demand are derived from the greater Copenhagen area, and data presented by Madsen (2015) and Zugno et al. (2016). Tables 1-3 summarize the technical parameters, and marginal cost parameters of these units. Additionally, a heat-only peak unit,

Table 1: Electricity System: generation units' parameters

		G_1	G_2	G_3	G_4	G_5	G_6	G_7
$\overline{P}$	(MW)	76	76	175	30	200	200	1000
μ_{α}	(\$/MWh)	13.32	13.32	20.7	26.11	6.02	5.47	100
σ_{α}	(\$/MWh)	1.3	1.3	2.1	2.6	0.6 -	0.5	0

with a maximum capacity of $1,000$MW and a marginal cost of $100\$/\mathrm{MWh}$ is considered.

Table 2: Heat System: generation units' parameters

		CHP_1	CHP_2	CHP_3	CHP_4	HP_1	HP_2	HP_3
$\overline{Q}$	(MW)	300	300	300	400	250	250	250
$\overline{F}$	(MW)	600	600	600	600	-	-	-
COP	-	-	-	-	2.8	3.1	2.5	
r	0.5	0.5	0.5	0.5	-	-	-	
ρ^{E}	2.1	2.1	2.1	2.4	-	-	-	
ρ^{H}	0.25	0.21	0.25	0.21	-	-	-	
α	($/MWh)	5	7.5	10	12.5	-	-	-

Table 3: Heat storage tanks parameters

		HS_1	HS_2	HS_3
$\overline{S}$	(MWh)	150	150	150
$\overline{Q}$	(MW)	50	50	50
ρ^{+}	1.1	1.1	1.1	
ρ^{-}	0.9	0.9	0.9	
S^{init}	(MW)	100	100	100

5.2. Scenarios Generation

Scenarios of supply functions are generated from data in 1, assuming a normal distribution o fmean μ_{α_j} and standard deviation σ_{α_j} for each participants' offers, such that:

$$\tilde{\alpha}^{\mathrm{E}}_{jt\nu} \sim \mathcal{N}\left(\mu_{\alpha_j}, \sigma_{\alpha_j}\right) \tag{8}$$

Electricity demand scenarios are derived from market data from Nordpool for January 2018, available at Nordpool (2018). Wind power uncertainty is modeled by a set of scenarios with temporal and spatial correlation, which are available at Bukhsh. We first generate 15 independent scenarios per source of uncertainty, namely wind production, electricity loads and supply functions. We then use a scenario reduction technique to merge similar scenarios (Gabriel et al., 2009; Morales et al., 2009). Figure 1 illustrates the scenarios for wind production and electricity demand, before and after the scenario reduction.

Finally, we carry out an out-of-sample analysis to provide a more rigorous comparison of the performance of the three market frameworks against unseen scenarios. To this purpose, we generate 10 new scenarios per source of uncertainty, namely wind production, electricity loads and supply functions, from the same distributions that we generated in-sample scenarios. Figure 2 shows the out-of-sample scenarios for wind production, and electricity demand.

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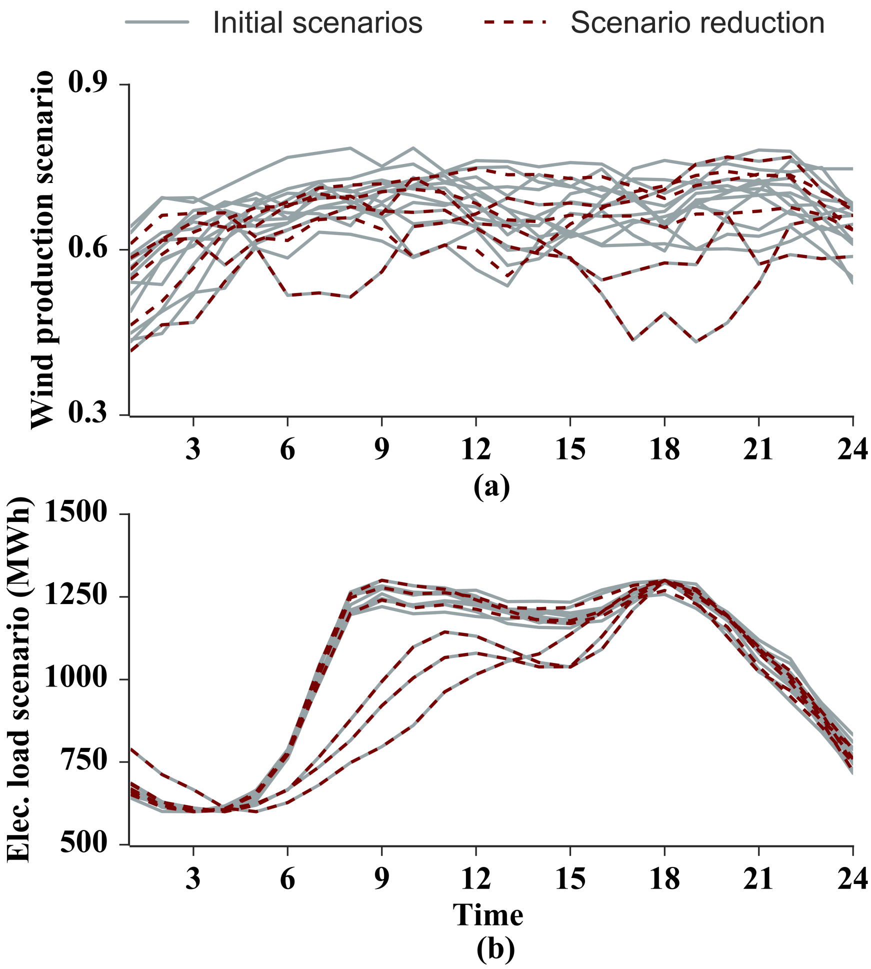

Figure 1: Original 15 scenarios and scenario reduction (6 scenarios) of (a) wind production (ratio of total installed wind capacity) and (b) electricity load (MWh)

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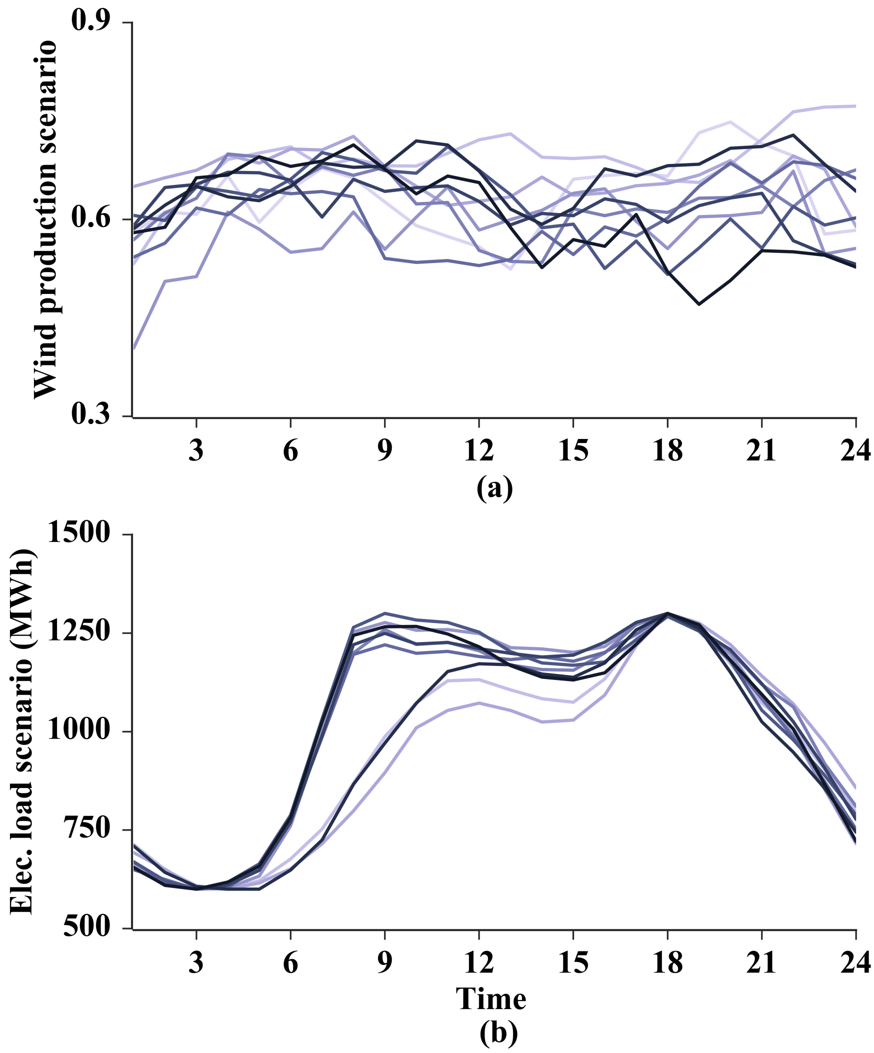

Figure 2: Out-of-sample scenarios of (a) wind production (ratio of total installed wind capacity) and (b) electricity load (MWh)

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