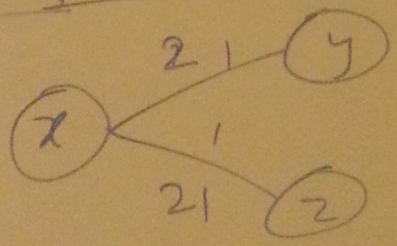
Lab 11_Q2. MST Proof:

Suppose that for every cut of G, there is unique light edge crossing the cut. Let us consider two MSTs, T & T', of G. We will show that T and T' are the same & hence there exists unique MST.

consider any edge $(u,v) \in T$. If we remove (u,v) from T, then it results in a cut $(S, V-S)$. The edge (u,v) is a light edge crossing the cut. Now, consider the edge $(x,y) \in T'$ that crosses $(S, V-S)$. It is also a light edge crossing the cut. Since the light edge crossing $(S, V-S)$ is unique these two edges are same Thus, $(u,v) \in T'$. And, it holds true for all edge in T, also in T'

Counter Example for the converse:



Here, the graph is its own MST, so the MST is unique. Consider the cut $(\{x\}, \{y,z\})$. Both of the edges (x,y) & (x,z) are edges crossing the cut, and they're both light edges, so the converse is not true.