Lab-2 Group-2

problem 3

(A) $1+4n^2$ is $O(n^2)$

Now using limit facts:

Here $f(n) = 1+4n^2$

$g(n) = n^2$

If $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ = finite then $f(n) = O(g(n))$

So, $\lim_{n\to\infty} \frac{f(n)}{g(n)}$

$= \lim_{n\to\infty} \frac{1+4n^2}{n^2}$

using L-Hopital's rule

$= \lim_{n\to\infty} \frac{8\cancel{n}}{2\cancel{n}} = 4$ = finite

$\boxed{\text{So, } 1+4n^2 \text{ is } O(n^2)}$ proved

*Note:- Here $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ = finite (constant) means that $g(n)$ is upper bound of $f(n)$ & $f(n)$ grows no faster than $g(n)$ for all $(n \geq n_0)$

Lab-2 Group-2

problem 3

(8) $n^2-2n$ is not $O(n)$

~~We have to show~~

Here $f(n) = n^2-2n$

$g(n) = n$

We have, to show.

$\lim_{n\to\infty} \frac{f(n)}{g(n)}$ = infinite so, that $n^2-2n$ is not $O(m)$

Here $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{n^2-2n}{n}$

using L-Hopitals rule

$= \lim_{n\to\infty} \frac{2n-2}{1}$

$= \infty$

So, $f(n)$ is no $O(g(n))$

$\boxed{\text{i.e } n^2-2n \text{ is not } O(n)}$

proved

Lab-2 Group-2

Problem-3

(c) $\log(n)$ is $o(n)$

Soln Here $f(n) = \log(n)$

$g(n) = n$

We have to show that

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$$ so that $f(n) = o(g(n))$

Here

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{\log n}{n}$$

$=$ using L-Hopital's rule.

$$= \lim_{n\to\infty} \frac{1/n}{n}$$

$$= \lim_{n\to\infty} 1/n^2$$

$$= 1/\infty = 0$$

So, $f(n) = o(g(n))$

i.e $\boxed{\log n \text{ is } o(n)}$ proved

Lab-2 Group-2

Problem 3

(D) $n$ is not $o(n)$

Soln Here,

$f(n) = n$

$g(n) = n$

We have to show that

$\lim_{n \to \infty} \frac{f(n)}{g(n)} \neq 0$ So, that $f(n)$ is not $o(g(n))$

Here, $\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} n/n$

$= 1$ (~~finite~~) not equal to zero

So, $f(n)$ is not $o(g(n))$

$\boxed{\text{i.e } n \text{ is not } o(n)}$ proved

* Note: in this case, ~~$n = o(n)$~~

$n = O(n)$

but ~~n~~ $n \neq o(n)$