(2)

(a) 4^n is $O(2^n)$

Here, $f(n) = 4^n$

$g(n) = 2^n$

If $\lim_{n \to \infty} \frac{f(n)}{g(n)}$ = finite, then $f(n) = O(g(n))$

So,

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{4^n}{2^n}$$

$$= \lim_{n \to \infty} \left(\frac{4}{2}\right)^n$$

$$= \lim_{n \to \infty} 2^n$$

$$= \infty$$

Since, $f(n)/g(n)$ is not finite, 4^n is not $O(2^n)$

$$\boxed{4^n \text{ is } O(2^n) \text{ is false}}$$

(a)

$\cancel{\log_b x} = \cancel{\frac{\log_a x}{\log_a b}}$

(b) $\log n$ is $\Theta(\log_3 n)$

Here, $f(n) = \log n$

$g(n) = \log_3 n$

Now,

If $\lim_{n \to \infty} \frac{f(n)}{g(n)}$ ~~or~~ and $\lim_{n \to \infty} \frac{g(n)}{f(n)}$ is finite,

$f(n)$ is $\Theta(g(n))$.

So,

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \lim_{n \to \infty} \frac{\log n}{\log_3 n}$$

$$= \lim_{n \to \infty} \frac{\log n}{\frac{\log_2 n}{\log_2 3}}$$

$$= \lim_{n \to \infty} \frac{\log n}{\log_2 n} \times \log_2 3$$

$$= \lim_{n \to \infty} \log_2 3 \, \frac{\log n}{\log n}$$

$$= \cancel{\log_2 3} \lim_{n \to \infty} \log_2 3 \, \frac{1/n}{1/n}$$

$$= \lim_{n \to \infty} \log_2 3 \, \frac{1}{n} \times n$$

$$= \log_2 3$$

Similarly, $\lim_{n \to \infty} \frac{g(n)}{f(n)} = \frac{1}{\log_2 3}$

Since, ~~lim~~ $\lim_{n \to \infty} \frac{f(n)}{g(n)}$ and $\lim_{n \to \infty} \frac{g(n)}{f(n)}$ is both finite

$\boxed{\log n \text{ is } \Theta(\log_3 n)}$ is True

(c) $(n/2)\log(n/2)$ is Θ to $(n\log n)$

Here, $f(n) = (n/2)\log(n/2)$

$g(n) = n\log n$

If $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ and $\lim_{n\to\infty} \frac{g(n)}{f(n)}$ is both finite, $f(n)$ is $\Theta(g(n))$

So, $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{(n/2)\log(n/2)}{n\log n}$

$$= \lim_{n\to\infty} \frac{\frac{d}{dn}\left(n/2 \log(n/2)\right)}{\frac{d(n\log n)}{dn}}$$

$$= \lim_{n\to\infty} \frac{\frac{d(n/2)}{dn}\log(n/2) + (n/2)\frac{d(\log(n/2))}{d(n/2)} \times \frac{d(n/2)}{dn}}{\frac{dn}{dn}\log n + n\frac{d\log n}{dn}}$$

$$= \lim_{n\to\infty} \frac{1/2 \log(n/2) + n/2 \cdot \frac{1}{n/2} \times \frac{1}{2}}{\log n + n \cdot \frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{1/2 \log(n/2) + \cancel{\frac{n}{2} \times \frac{2}{n}}\ 1/2}{\log n + 1}$$

$$= \frac{\infty}{\infty} = \infty$$

Similarly, $\lim_{n\to\infty} \frac{g(n)}{f(n)} = \infty$

Since, both $\lim_{n\to\infty} \frac{f(n)}{g(n)}$ and $\lim_{n\to\infty} \frac{g(n)}{f(n)}$ is both infinite

$\boxed{n/2 \log(n/2) \text{ is } \Theta(n\log n) \text{ is false}}$