Q.3 TSP is NP-Complete - Proof

We first show that TSP is NP problem. Consider we have an instance of the problem and a sequence of n vertices as the certificate. The verification algorithm can be defined in polynomial time so that it will check for the distinct vertices, sums of the edge cost and compare the cost with the value k.

Now, we show that HAM-CYCLE $\xrightarrow{\text{Poly}}$ TSP. Let $G = (V, E)$ is an instance of HAM-CYCLE. Let's say H is a complete graph formed from G by adding the remaining edges. We define cost as follows:

$$c(u,v) = \begin{cases} 0, & \text{if } (u,v) \in G \\ 1, & \text{if } (u,v) \notin G \end{cases}$$

Now, Suppose G has HAM-CYCLE h. Each edge in h has cost 0. Thus, h is a tour in H with cost 0, i.e, $TSP < H, c, 0>$. Conversely, Suppose H has a tour with cost at most 0. For this to happen, every edge in H should have cost 0, i.e, each edge belongs to G. From this, we conclude that the hamiltonian cycle is in G. So, the TSP is NP-complete problem.