

1. Ans:

Given,

Total # of numbers to sort = 4

so, the total number of leaves in the decision tree =  $n! = 4!$

If the height of the tree is  $h$ , then  
(Since total no. of leaves  $\leq 2^h$ )

$$\Rightarrow 2^h \geq 4!$$

$$\begin{aligned}\Rightarrow h &\geq \log_2(4!) \\ &= \log_2(24) \\ &= 4.6 \\ &\approx 5\end{aligned}$$

$$\therefore h \geq 5$$

so, from this, we can conclude that the total number of comparisons to sort the 4 numbers should be at least 5.

$\Rightarrow$  In general, to sort  $n$  elements, comparison sort must make  $\Omega(n \log n)$  comparisons in the worst case.