$\boxed{\text{Assignment (01)}}$

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(a) Test for Consistency and solve.

(i) $2x-3y+7z=5$
$3x+y-3z=13$
$2x+19y-47z=32$

$$A=\begin{bmatrix}2 & -3 & 7\\ 3 & 1 & -3\\ 2 & 19 & -47\end{bmatrix},\quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix},\quad B=\begin{bmatrix}5\\ 13\\ 32\end{bmatrix}$$

$$[A:B]=\begin{bmatrix}2 & -3 & 7 & 5\\ 3 & -1 & -3 & 13\\ 2 & 19 & -47 & 32\end{bmatrix}\xrightarrow{R_1\to\frac{1}{2}R_1}=\begin{bmatrix}1 & -3/2 & 7/2 & 5/2\\ 3 & 1 & -3 & 13\\ 2 & 19 & -47 & 32\end{bmatrix}$$

$$\xrightarrow{R_2\to R_2-3R_1}\begin{bmatrix}1 & -\frac{3}{2} & \frac{7}{2} & \frac{5}{2}\\ 0 & 11/2 & -27/2 & 11/2\\ 2 & 19 & -47 & 32\end{bmatrix}\xrightarrow{R_3\to R_3-2R_1}\begin{bmatrix}1 & -3/2 & 7/2 & 5/2\\ 0 & 11/2 & -27/2 & 11/2\\ 0 & 22 & -54 & 27\end{bmatrix}$$

$$\xrightarrow{R_2\to R_2\times\frac{2}{11}}\begin{bmatrix}1 & -3/2 & 7/2 & 5/2\\ 0 & 1 & -27/2 & 1\\ 0 & 22 & 54 & 27\end{bmatrix}$$

$$\xrightarrow{R_3\to R_3-22R_2}\begin{bmatrix}1 & -3/2 & 7/2 & \frac{5}{2}\\ 0 & 1 & -27/11 & 1\\ 0 & 0 & 0 & 5\end{bmatrix}$$

as: $\boxed{\rho(A:B)=3}$ & $\boxed{\rho(A)=2}\neq$ no. of unk

$\rho(A)\neq\rho(A:B)$.

$\therefore$ Inconsistent & no solution

(ii) $2x-y+3z=8$
$-x+2y+z=4$
$3x+y-4z=0$

$$A=\begin{bmatrix}2 & -1 & 3\\ -1 & 2 & 1\\ 3 & 1 & -4\end{bmatrix},\quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix},\quad B=\begin{bmatrix}8\\ 4\\ 0\end{bmatrix}$$

$$[A:B]=\begin{bmatrix}2 & -1 & 3 & 8\\ -1 & 2 & 1 & 4\\ 3 & 1 & -4 & 0\end{bmatrix}\begin{matrix}R_1\to\frac{1}{2}R_1\\ R_2\to R_2+R_1\\ R_3\to R_3-3R_1\end{matrix}=\begin{bmatrix}1 & -\frac{1}{2} & \frac{3}{2} & 4\\ 0 & 3/2 & 5/2 & 8\\ 0 & 5/2 & -17/2 & -12\end{bmatrix}$$

$$\xrightarrow{\begin{matrix}R_2\to\frac{2}{3}R_2\\ R_3\to R_3-\frac{5}{2}R_2\end{matrix}}\begin{bmatrix}1 & -\frac{1}{2} & \frac{3}{2} & 4\\ 0 & 1 & 5/3 & 16/3\\ 0 & 0 & -38/3 & -16/3\end{bmatrix}$$

$\therefore \rho(A:B)=\rho(A)=3$

Consistent & unique solⁿ.

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$-\frac{38}{8} z = \frac{-76}{16} z$

$z = 2$

$y + 2\left(\frac{5}{3}\right) = \frac{16}{3}$

$y + \frac{10}{3} = \frac{16}{3}$

$y = \frac{16}{3} - \frac{10}{3} = 2$

$x - 4 + 3 = 4$ (sic)

$x + 2 = 4$

$x = 2$

$\therefore \underline{x + y + z = 2}$

(iii) $4x - y = 12$

$-x + 5y - 2z = 0$

$-2x + 4z = -8$

$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}$

$[A:B] = \begin{bmatrix} 4 & -1 & 0 & 12 \\ -1 & 5 & -2 & 0 \\ -2 & 0 & 4 & 8 \end{bmatrix}$ $R_1 \to \frac{R_1}{4}$, $R_2 \to R_2 + R_1$, $R_3 \to R_3 + 2R_1$ $= \begin{bmatrix} 1 & -\frac{1}{4} & 0 & 3 \\ 0 & 19/4 & -2 & 3 \\ 0 & -\frac{1}{2} & 4 & 14 \end{bmatrix}$

$R_2 \to \frac{4}{19} R_2$

$R_3 \to R_3 + \frac{1}{2} R_2$

$\therefore \rho(A:B) = \rho(A) = 3 = n.$

Consistent & Unique sol^n.

$\begin{bmatrix} 1 & -\frac{1}{4} & 0 & 3 \\ 0 & 1 & \frac{-8}{19} & 12/19 \\ 0 & 0 & \frac{72}{19} & \frac{272}{19} \end{bmatrix}$

$\therefore \frac{72}{19} z = \frac{272}{19}$

$\boxed{z = \frac{34}{9}}$

$y - \frac{8}{19}(z) = \frac{12}{19}$

$y - \frac{8}{19} \times \frac{34}{9} = \frac{12}{19}$

$\therefore \boxed{y = \frac{164}{171}}$

$x - \frac{1}{4}\left(\frac{164}{171}\right) = 3$

$x + \frac{164}{684} = 3$

$\boxed{x = \frac{1898}{684}}$

(b) for what value of λ & μ the given system of eq^n. has (i) ~~no~~ sol^n. (ii) a unique sol^n. (iii) infinite no. of sol^n.

$x + y + z = 6$

$x + 2y + 3z = 10$

$x + 2y + \lambda z = \mu$

$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$

$[A:B] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, $R_1 \to R_1 - R_2$ $= \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda - 1 & \mu - 6 \end{bmatrix}$

$R_3 \to R_3 - R_2$

$\therefore \rho(A:B) = \rho(A) = 3 =$ no. of unknowns.

Consistent & Unique sol^n.

$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda - 3 & \mu - 10 \end{bmatrix}$

(i) No. solⁿ :- if $\lambda = 3$, $\mu \neq 10$ rank $\rho(A) = 2$, $\rho(A:B) = 3$,
$\rho(A) \neq \rho(A:B)$
$\therefore$ inconsistent & no solⁿ.

(ii) Unique solⁿ :- if $\alpha \neq 3$. then $\rho(A) = \rho(A:B) = n$.
$\therefore$ Unique solⁿ.

(iii) Infinite solⁿ :- if $\lambda = 3$, $\mu = 10$ then $\rho(A) = \rho(A:B) = 2$
$\therefore$ infinite solⁿ.

(c) Find for what value of λ the given eqn. $x+y+z=1$, $x+2y+$ ... $=\lambda$, $x+4y+10z=\lambda^2$ have a solⁿ & solve them completely in each case.

$\Rightarrow$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & y-1 \\ 0 & 3 & 9 & y^2- \end{bmatrix}$$

$R_2 \leftrightarrow R_3$
$R_1 \to R_1 - \frac{1}{3}R_2$
$R_3 \to R_3 - \frac{1}{3}R_2$

$$\begin{bmatrix} 1 & 0 & -2 & 2 \\ 0 & 3 & 9 & y^3 - 1 \\ 0 & 0 & 0 & \frac{y^2 - 3y + 2}{3} \end{bmatrix}$$

$\therefore$ Here $\rho(A:B) = 3$, $\rho(A) = 2$
$\rho(A:B) \neq \rho(A)$
$\boxed{\text{Inconsistent}}$

(d) Find the solⁿ of the system of eqⁿ. $x+3y-2z=0$, $2x-y+4$ ..., $x-11y+14z$ ...

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\therefore [A:B] = \begin{bmatrix} 1 & 3 & -2 & 0 \\ 2 & -1 & 4 & 0 \\ 1 & -11 & 14 & 0 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - R_1 \end{matrix} = \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & -7 & 8 & 0 \\ 0 & -14 & 16 & 0 \end{bmatrix}$$

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$$\xrightarrow[\substack{R_2 \to \frac{R_2}{-7} \\ R_3 \to R_3 + 14R_2}]{} \begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & 1 & -\frac{8}{7} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\left.\begin{array}{l} \rho(A:B) = 2 \\ \rho(A) = 2 \end{array}\right\} \Rightarrow \rho(A:B) = \rho(A)$

$\boxed{\text{Consistent \& no. sol}^n}$

(e) Find for what values of λ the given eqn. $3x + y - \lambda z = 0$, $4x - 2y - 3z = 0$, $2\lambda x + 4y + \lambda z = 0$ may posses non-trivial solⁿ. & solve them completely in each case.

$\Rightarrow$ $A = \begin{bmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$[A:B] = \begin{bmatrix} 3 & 1 & -\lambda & 0 \\ 4 & -2 & -3 & 0 \\ 2\lambda & 4 & \lambda & 0 \end{bmatrix} \xrightarrow[\substack{R_1 \to \frac{1}{3}R_1 \\ R_2 \to R_2 - 2R_3 \\ R_3 \to R_3 - 2R_1}]{} = \begin{bmatrix} 1 & \frac{-1}{3} & -\lambda/3 \\ 0 & -10 & -3-2\lambda \\ 0 & -14/3 & 5\lambda/3 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 + \frac{7}{15}R_2} \begin{bmatrix} 1 & \frac{-1}{3} & -\lambda/3 \\ 0 & -10 & -3-2\lambda \\ 0 & 0 & \frac{11\lambda - 2}{15} \end{bmatrix}$$

Here, $\rho(A) =$ no. of unknowns $= 3$

Consistent & Unique solⁿ.

for system to possess non-trivial solⁿ (infinite solⁿ)

$\rho(A) <$ no. of unknown.

$\frac{11\lambda - 21}{15} = 0 \Rightarrow 11\lambda = 21 \therefore \boxed{\lambda = \frac{21}{11}}$

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Assignment (II)

There are the following sets of vectors linearly independent or Dependent?

1.) [1, 0, 0], [1 1 0], [1 1 1]

let $v_1 = [1, 0\ 0]$, $v_2 = [1\ 1\ 0]$, $v_3 = [1\ 1\ 1]$

$$\text{Matrix}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$\text{Det}(A) = 1(1-0) - 0 + 0 = 1 \neq 0$ so, it $\boxed{\text{Linearly Dependent}}$

2). $[7\ -3\ 11\ 6]$; $[-56\ 24\ -88\ 48]$

using eqn. & equating

$$[7c_1 - 56c_2 \quad -3c_1 + 24c_2 \quad 11c_1 - 88c_2 \quad -6c_1 + 48c_2] = [0\ 0\ 0\ 0]$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 8k \\ k \end{bmatrix} = k\begin{bmatrix} 8 \\ 1 \end{bmatrix}$$

$7c_1 = 56c_2$

$c_1 = 8c_2$

$c_1 = 8k$

$\boxed{\text{Linearly Dependent}}$

3) $[-1\ 5\ 0]$, $[16\ 8\ -3]$, $[-64\ 56\ 9]$

let, $v_1 = [-1\ 5\ 0]$, $v_2 = [16\ 8\ -3]$, $v_3 = [-64\ 56\ 9]$

$$\text{Mat}(A) = \begin{bmatrix} -1 & 5 & 0 \\ 16 & 8 & -3 \\ -64 & 56 & 9 \end{bmatrix}$$

$\text{Det}(A) = -1(72+168) - 16(45) - 6...$

$= -96 - 720 + 960$

$= 144 \neq 0$

$\boxed{\text{Linearly Dependent}}$

4) $[1\ -1\ 1]$, $[1\ 1\ -1]$, $[-1\ 1\ 1]$, $[0\ 1\ 0]$

$$[c_1 + c_2 - c_3 \quad -c_1 + c_2 + c_3 + c_4 \quad c_1 - c_2 + c_3] = [0\ 0\ 0]$$

$c_1 + c_2 - c_3 = 0$

$c_1 + c_2 = c_3$

$c_1 + c_1 + \cancel{c_3} = \cancel{c_3}$

$2c_1 = 0 \Rightarrow c_1 = 0$

$-c_1 + c_2 + c_3 + c_4 = 0$

$c_1 = c_2 + c_3 + c_4$

$c_4 = -2k$

$c_1 - c_2 + c_3 = 0$

$c_2 = c_1 + c_3$

$c_2 = c_3 = k$

$$\therefore v = k\begin{bmatrix} 0 \\ 1 \\ 1 \\ -2 \end{bmatrix}$$

$\boxed{\text{Linearly Dependent}}$

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5) $[2\ \ -4], [1\ \ 9], [3\ \ 5]$

$[2c_1+c_2+3c_3 \quad -4c_1+9c_2+5c_3] = [0\ \ 0]$

$\therefore 2c_1+c_2+3c_3=0$ $\quad 9c_2+5c_2+2c_2+6c_3=0$ $\quad \therefore 4c_1 = 9c_2+5c_3$

$\times 2 \quad 4c_1+2c_2+6c_3=0$ $\quad 11c_2+11c_3=0$ $\quad 4c_1 = 9k-5k$

$c_2 = -c_3 = k$ $\quad 4c_1 = 4k$

$c_1 = k.$

$$\begin{bmatrix} k \\ k \\ -k \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$ $\boxed{\text{Linearly Dependent}}$

6.) $[3\ \ -2\ \ 0\ \ 4], [5\ \ 0\ \ 0\ \ 1], [-6\ \ 1\ \ 0\ \ 1], [2\ \ 0\ \ 0\ \ 3]$

$$A = \begin{bmatrix} 3 & -2 & 0 & 4 \\ 5 & 0 & 0 & 1 \\ -6 & 1 & 0 & 1 \\ 2 & 0 & 0 & 3 \end{bmatrix}$$

$|A|=0$, because, all elements of c_3 are '0'

$\boxed{\text{Linearly independent}}$

7) $[3\ \ 4\ \ 7], [2\ \ 0\ \ 3], [8\ \ 2\ \ 3], [5\ \ 5\ \ 6]$

$$A = \begin{bmatrix} 3 & 2 & 8 & 5 \\ 4 & 0 & 2 & 5 \\ 7 & 3 & 3 & 6 \end{bmatrix}$$

$R_1 \to \frac{R_1}{3}$
$R_2 \to R_2 - 4R_1$
$R_3 \to R_3 - 7R_1$

$$= \begin{bmatrix} 1 & \frac{2}{3} & \frac{8}{3} & \frac{5}{3} \\ 0 & -8/3 & -\frac{26}{3} & -5/3 \\ 0 & -\frac{5}{3} & -\frac{47}{3} & -\frac{17}{3} \end{bmatrix}$$

$R_2 \to -\frac{3}{8}R_2$
$R_3 \to R_3 + \frac{5}{3}R_2$

$$\Rightarrow \begin{bmatrix} 1 & \frac{2}{3} & 8/3 & 5/3 \\ 0 & 1 & 13/4 & 5/8 \\ 0 & 0 & -41/4 & -37/4 \end{bmatrix}$$

Here by Rank Nullify theorem

$\rho(A) + N = 4$

$\Rightarrow 3 + N = 4 \quad \Rightarrow N = 1$

Here no. of unknown $= 4 = n$

but $\rho(A) = 3 \neq n$ $\to$ infinite solⁿ.

$\therefore$ $\boxed{\text{Linearly independent}}$

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8. [6 0 3 1 4 2], [0 -1 2 7 0 5], [12, 3 0 -19 8

Let, $c_1[6\ 0\ 3\ 1\ 4\ 2] + c_2[0\ -1\ 2\ 7\ 0\ 5] + c_3[12\ 3\ 0$

$6c_1 + 12c_3 = 0 \Rightarrow c_1 = -2c_3$

$-c_2 + 3c_3 = 0 \Rightarrow c_2 = 3c_3$

$3c_1 + 2c_2 = 0 \Rightarrow c_2 = \frac{-3c_1}{2}$

$c_1 + 7c_2 - 19c_3 = 0 \Rightarrow c_1 - \frac{21}{2}c_1 - 12c_3 = 0 \Rightarrow \frac{-19}{2}c_1 - 19c_3 = 0$

$\Rightarrow \frac{-19}{2}(-2c_3) - 19c_3 = 0$ $[\because c_1 = -2c_3]$

$\Rightarrow \frac{38}{2}c_3 - 19c_3 = 0 \Rightarrow 38c_3 - 38c_3 = 0 \Rightarrow \boxed{0=0}$

$4c_1 + 8c_3 = 0 \Rightarrow 4(-2c_3) + 8c_3 = 0 \Rightarrow 8c_3 + 8c_3 = 0$

$\boxed{0=0}$

$2c_1 + 5c_2 - 11c_3 = 0$

$\Rightarrow 2(-2c_3) + 5(3c_3) - 11c_3 = 0$

$\Rightarrow -4c_3 + 15c_3 - 11c_3 = 0$

$\Rightarrow -15c_3 + 15c_3 = 0$

$\Rightarrow \boxed{0=0}$

$c_3 = k$ then

$c_1 = -2k,\ c_2 = 3k$

$\therefore x = \begin{bmatrix} -2k \\ 3k \\ k \end{bmatrix} = k\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$ this is infinite solⁿ.

$\boxed{\text{Linearly dependent}}$

Assignment (III)   Jivan Jamdar

1) $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ $\therefore A - \lambda I = 0$

$$= \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} = (-2-\lambda)(-\lambda(1-\lambda)+6(-2)) - 2(-\lambda(2)+6(-1)) - 3(-4+1-\lambda)$$

$$= (-2-\lambda)(-\lambda+\lambda^2-12) + 2(+\lambda+6) + 3(+3+\lambda)$$

$$= +2\lambda - 2\lambda^2 + 24 + \lambda^2 - \lambda^3 + 12\lambda + 2\lambda + 12 + 9 + 3\lambda$$

$$= 19\lambda - \lambda^2 - \lambda^3 + 45$$

$$\Rightarrow \lambda^3 + \lambda^2 - 19\lambda - 45 = 0$$

2) $\begin{bmatrix} +4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \Rightarrow A - \lambda I = 0$

$$\therefore \begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} = (4-\lambda)(1-\lambda)(1-\lambda) + 1(+2(1-\lambda))$$

$$= (1-\lambda)((4-\lambda)(1-\lambda)+2)$$

$$= \cancel{(\lambda^2 - 5\lambda + 6)} \quad \lambda^3 - 6\lambda^2 + 9\lambda - 6 = 0$$

$$\Rightarrow (1+\lambda)(\lambda-2)(\lambda-3)$$

eigenvalue $\longrightarrow \lambda = -1, 2, 3$

for $\boxed{\lambda = -1}$

$$\Rightarrow \begin{bmatrix} 5 & 0 & 1 \\ -2 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\therefore$ $5x + z = 0 \quad \ldots (1)$

$-2x + 2y = 0 \quad \ldots (2)$

$-2x + 2z = 0 \quad \ldots (3)$

$\therefore$ from $2 \times (1)$ & $(3) \times 2$

$$10x + 2z = 0$$
$$\ominus \quad \underline{\overset{+}{-}2x \ \overset{-}{+}2z = 0}$$
$$12x = 0 \quad \therefore \boxed{x = 0} \quad \therefore z = 0, \ y = 0$$

$\therefore$ eigen vector $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

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for $\boxed{\lambda = 2}$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\therefore$ $2x + z = 0$ ... (1)

$-2x - y = 0$ ... (2)

$-2x - z = 0$ ... (3)

By solving them,

$x = K$

$y = -2K$

$z = 2K$

so, $\boxed{\text{eigenh vector} = K\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}}$

for $\boxed{\lambda = 3}$

$$\begin{bmatrix} +1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\therefore$ $x + z = 0$

$-2x - 2y = 0$

$-2x - 2z = 0$

By solving $\therefore$ $x = -z = -K$, $y = -x$

$z = K$ $y = K$

so, $\boxed{\text{eigen vector} = K\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}}$

8) $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

$\Rightarrow$ $A - \lambda I = \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$

$\therefore$ $(5-\lambda)(-\lambda)(3-\lambda) = 0$

$\boxed{\lambda = 0, 3, 5}$

so, for $\boxed{\lambda = 0}$

$\therefore$ $A = \begin{bmatrix} 5-0 & 0 & 0 \\ 0 & -0 & 0 \\ -1 & 0 & 3-0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

$\therefore$ $5x = 0 \Rightarrow x = 0$

$-x + 3z \Rightarrow z = 0$

$y = 0$

$\boxed{\text{vector} = K\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$

$$\boxed{\text{vector} = k\begin{bmatrix}0\\0\\1\end{bmatrix}}$$

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for $\boxed{\lambda = 3}$

$$\begin{bmatrix}2 & 0 & 0\\0 & -3 & 0\\-1 & 0 & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = 0$$

$\therefore$ $2x = 0 \Rightarrow x = 0$

$-3y = 0 \Rightarrow y = 0$

$-x = 0 \Rightarrow z = 0$

$\therefore$ so, ~~vecto~~ $\boxed{\text{eigenvector} = k\begin{bmatrix}0\\0\\1\end{bmatrix}}$

for $\boxed{\lambda = 5}$

$$\begin{bmatrix}0 & 0 & 0\\0 & -5 & 0\\1 & 0 & -2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = 0$$

$\therefore$ $-5y = 0 \Rightarrow y = 0$.

$x - 2z = 0$

$\therefore x = 2z = 2k$.

so, eigen vector $= \begin{bmatrix}-2k\\0\\k\end{bmatrix} = k\begin{bmatrix}-2\\0\\1\end{bmatrix}$

4) $A = \begin{bmatrix}0 & 0 & 0\\0 & 3 & 4\\0 & 0 & -2\end{bmatrix}$

$\Rightarrow$ $\therefore$ $A - \lambda I = \begin{bmatrix}0-\lambda & 0 & 0\\0 & 3-\lambda & 4\\0 & 0 & -2-\lambda\end{bmatrix} = 0$

$\therefore +\lambda(3-\lambda)(+2+\lambda) = 0$

$\boxed{\lambda = 0, 3, -2}$

$\therefore$ for $\boxed{\lambda = 0}$

$$\begin{bmatrix}0 & 0 & 0\\0 & 3 & 4\\0 & 0 & -2\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = 0$$

$3y = -4z$.

$y = \frac{-4}{3}z$

let, $z = k$

so,

$\therefore$ $3y + 4z = 0 \Rightarrow y = 0$.

$-2z = 0 \Rightarrow \cancel{z} \; \cancel{z} 0$

so, $\boxed{\text{eigen vector} = k\begin{bmatrix}0\\-4/3\\1\end{bmatrix}}$

for $\boxed{\lambda = 3}$:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\therefore 4z = 0 \Rightarrow z = 0$

$-5z = 0$

$x = 0, y = 0$

so, eigen vector $= k \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

for $\boxed{\lambda = -2}$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\therefore 5y + 4z = 0 \rightarrow y = -\frac{4}{5}z$

let, $z = k$, $y = -\frac{4}{5}k$

so, eigen vector $= k \begin{bmatrix} 0 \\ -\frac{4}{5} \\ 1 \end{bmatrix}$

Q.5). Our task is to find one eigenvalue of matrix without calculation so, we will tackle this by ~~2~~ 4 steps.

step 1: Indentify special properties of matrix:

as we see row [1, 2 3] is repeating which triggers the concept of determinant of matrix.

step 2: Relate the determinant to eigenvalues:

$\Rightarrow$ $\therefore$ determinant of a matrix is equal to the product of its eigen values.

step 3: Apply concept of determinant to find the eigenval

$\therefore$ If two rows of a matrix are identical, the determinant of matrix is 0,

since, determinant of given matrix is 0, we know that one of its eigenvalues must also be 0.

justification. $\Rightarrow$ as we derived determinant of matrix is 0 cause two of its rows are identical, as they are related to each other. so, in conclusion.

One of eigenvalue of given matrix is 0.