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Linear Algebra Assignment (10 Qns.)

Q.1) Sol^n:

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$R_4 \to R_4 - 4R_1$

$R_4 \to R_4 - R_3$

$R_4 \to R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

no. of non-zero rows, $\boxed{\text{Rank} = 3}$

Q.2) Sol^n: ~~T: W~~ $T : W \to P_2$

$$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$$

so, task is to find rank & nullity of T.

$$\Rightarrow T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a-b) + (b-c)x + (c-a)x^2$$

let $X = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ then

$$T(X) = (a-b) + (b-c)x + (c-a)x^2$$
$$= a - bx + c(x^2 - x + 1)$$

$\therefore$ The image of T is the set of all polynomials of degree at most 2, denoted as P_2.

Rank of T:

The rank of T is the dimension of its image since P_2 has a dimension of 3 (coefficients for x^0, x^1 & x^2) the rank of T is 3.

The null space of symmetric matrix

$$T(X) = 0$$

this leads to the system of equations.

$a - b = 0$ $\quad b - c = 0$ $\quad c - a = 0$

$\therefore a = b = c$

$\therefore$ T is the set of symmetric matrices of the form

$\begin{bmatrix} t & t \\ t & t \end{bmatrix}$ where t is any scaler.

$\therefore$ Dimension $= 1$ (using only t)

$$\boxed{\begin{array}{l} \text{rank T is 3.} \\ \text{the nullity of T is 1} \end{array}}$$

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Q.5) Sol^n:

$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{4-1}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

$|A - \lambda I|$ $= \begin{bmatrix} \frac{2}{3}-\lambda & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3}-\lambda \end{bmatrix} = \left(\frac{2}{3}-\lambda\right)^2 - \left(\frac{1}{3}\right)^2 = 0$

eigen equation

$\Rightarrow \left(\frac{1}{3}-\lambda\right)(1-\lambda) = 0$

.... $((a^2-b^2) = (a+b)(a-b))$

let, $\boxed{\lambda = 1, \frac{1}{3}}$ ← eigenvalues

Now, $\boxed{\lambda = 1} \Rightarrow$

$\begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

$-x\left(\frac{1}{3}\right) + y\left(\frac{1}{3}\right) = 0$

$\boxed{\text{eigen vector } \underline{k}\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$

so, $x = y = k$

$\boxed{\lambda = \frac{1}{3}} \Rightarrow$

$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

$x = -y$ eigen vector $= \begin{bmatrix} k \\ -k \end{bmatrix} = k\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Now, so, for A^{-1} ~~eig and~~ eigen values & vectors are $\lambda = 1, \frac{1}{3}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ & $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ respectively.

Now,

$z = A + 4I = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$

$\Rightarrow |z - \lambda I| = 0$

$= \begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix} = (6-\lambda)^2 - (1)^2$

using..$(a^2-b^2 = (a+b)(a-b))$

$= (6-\lambda-1)(6-\lambda+1)$

$= (5-\lambda)(7-\lambda) = 0$

$\therefore \boxed{\text{eigenvalues} = 5, 7}$

Now, $\lambda = 5$ $\boxed{\text{eigenvector} = k\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$

$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$ $\therefore x - y = 0$ $\boxed{x = y}$

$\lambda = 7$

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$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0 \Rightarrow x = y \Rightarrow k\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$\hookrightarrow \boxed{\text{eigen vector} = k\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$

Q.4) Sol^n:

$3x - 0.1y - 0.2z = 7.85$ — (1)

$0.1x + 7y - 0.3z = -19.3$ — (2)

$0.3x - 0.2y + 10z = 71.4$ — (3)

given

$\Rightarrow$ $x = \frac{1}{3}[7.85 + 0.1y - 0.2z]$

$y = \frac{1}{7}[-19.3 - 0.1x + 0.3z]$

$z = \frac{1}{10}[71.4 - 0.3x + 0.2y]$

Iteration-1: let, ($z = y = 0$)

$x = \frac{7.85}{3} = 2.61\bar{6} \cong 2.62$

Now, $z = 0$:.

$y = \frac{1}{7}\left(-19.3 - 0.1\left(\frac{7.85}{3}\right)\right) = 2.79$

$y(1) = 2.79$

$z = \frac{1}{10}(71.4 - 0.3(2.61) + 0.2(2.79))$

$z = \frac{1}{10}(71.4 - 0.783 + 0.558)$

$z = 7.1175$

Iteration-2:

$x_{(2)} = \frac{7.85 - 0.1(2.6167) - 0.2(7.1408)}{3}$

$= 2.9255$

$y_{(2)} = \frac{19.3 - 0.1(2.9255) - 0.3(7.1408)}{7}$

$= 3.0123$

$z_{(2)} = \frac{71.4 - 0.3(2.9255) - 0.2(3.0123)}{10} = 7.0132$

Iteration-3:

$x_{(3)} = \frac{7.85 \times 0.1(2.9255) - 0.2(7.0132)}{3} = 3.0032$

$y_{(3)} = \frac{19.3 - 0.1(3.0032) - 0.3(7.0132)}{7} = 3.0001$

$z_{(3)} = \frac{71.4 - 0.3(3.0032) - 0.2(3.0001)}{10} = 7.00$

$\boxed{x = 3.0032, \quad y = 3.0001, \quad z = 7.000}$

Q.5) sol^n.

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given eq^n: $x + 3y + 2z = 0,$

$2x - y + 3z = 0,$

$3x - 5y + 4z = 0$

$x + 17y + 4z = 0$

$\Rightarrow X = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}$ after performing row reduction, we obtained echlon form.

$X = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix}$ $R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - 3R_1$, $R_4 \rightarrow R_4 - R_1$

$\Rightarrow X = \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ By $R_3 \rightarrow R_3 - 2R_2$

$X = \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ This Corresponds to the system

$x + 3y + 2z = 0$

$-7y - z = 0$

Now, lets express the variables in term of parameters. let $y = t$

$\therefore x = -3t, \; z = -7t.$

So, the system has infinity many sol^n. give by

$x = -3t, \; y = t, \; z = 7t$

$\therefore$ The system is Consistent and dependent.

Q.6) sol^n.

given: $T: P_2 \rightarrow P_2$ is linear transformation.

$T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$

is a linear transformation, we had to check two properties.

1) Additivity: $T(u + v) = T(u) + T(v)$

2) homogenity of degree:

$T(ku) = kT(u)$ for all u in the domain of T and all scalars k.

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i) $T(u+v) = T\left((a_1+b_1x+c_1)+(a_2+b_2x+c_2)\right)$
$= T\left((a_1+a_2)+(b_1+b_2)x+(c_1+c_2)\right)$
$= (a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2$
$= (a_1+1)+(b_1+1)x+(c_1+1)x^2+(a_2+1)+(b_2+1)x+(c_2+1)x^2$
$T(a_1+b_1x+c_1)+T(a_2+b_2x+c_2)$

So, solution is additive.

$\therefore$ Homogenity of Degree 1.

$T(ku) = T\left(k(a+bx+c)\right)$
$= T(ka+kbx+kc) = (ka+1)+(kb+1)x+(kc+1)$
$= k(a+1)+k(b+1)x+k(c+1)x^2$
$= kT(a+bx+c)$

so, the function is homogeneous of degree 1.
$\therefore$ it indeed linear Transformal.

Q.7) given: $S = \{(1,2,3),(3,1,0),(-2,1,3)\}$ is a basis of $V_3(R)$
In cases S is not a basis, we have to determine subspace spanned by S.

$\Rightarrow S = \{(1,2,3),(3,1,0),(-2,1,3)\}$
can be arranged as a matrix

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$$

Now, lets perform row reduction to obtained the echelon form.

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$ & $R_3 \leftarrow R_3 - 3R$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix}$$

$R_3 \to R_3 + \frac{9}{5}R_2 \Rightarrow A = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore$ Third row of zeros indicates that the vectors in S are linearly dependent for basis of the subspace spanned by S.

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \end{bmatrix}$$

$(1,3,2)$ & $(0,-5,5)$ these vectors form a basis for the subspace spanned by S.

∴ Dimension of subspace spanned by $s = 2$.

∴ set s is not a basis of R^3 because the row-reduced form has a row of zeroes.

∴ the basis for the subspace spanned by s is $\{(1, 3, -2), (0,$

∴ The dimension of s's subspace is 2.

Q.8). Given : equations: $3x - 6y + 2z = 23$,

$-4x + y - z = -15$,

$x - 3y + 7z = 16$

initial values. $x_0 = 1, y_0 = 1, z_0 = 1$

⟹ iteration - I):

$$x_{(1)} = \frac{23 + 6y_{(0)} - 2(z_{(0)})}{3} \simeq 9.0$$

$$y_{(1)} = \frac{-15 + 4(x_0) + z_{(0)}}{1} \simeq -9.0$$

$$z_{(1)} = \frac{16 - x_{(1)} - 3(y_{(1)})}{7} = 2.0$$

Iteration - II):

$$x_{(2)} = \frac{23 + 6(y_{(1)}) - 2(z_{(1)})}{3} = 5.0$$

$$y_{(2)} = \frac{-15 + 4x_{(1)} + z_{(1)}}{1} = -5.0$$

$$z_{(2)} = \frac{16 - x_{(1)} + 3y_{(1)}}{7} \simeq 3.0$$

Iteration - III):

$$x_{(3)} = \frac{23 + 6y_{(2)} - 2z_{(2)}}{3} = 6.0$$

$$y_{(3)} = \frac{-15 + 4x_{(2)} + z_{(2)}}{1} \simeq -6.0$$

$$z_{(3)} = \frac{16 - x_{(2)} + 3y_{(2)}}{7} \simeq 2.0$$

Q.9) Matrix operations in image processing is applying filters to an image. Filters are used for various ~~appl~~ tasks like blurring, sharpening and edge detection.

Lets explain using matrix operation how we blur an image.
* imagine a small 3x3 matrix representing a blurring filter:

$$\begin{vmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{vmatrix}$$

* Each element (1/9 in this case) represents the weight given to a surrounding pixel in the image.
* To blur the image, this filter matrix applied using an operation called Convolution. In convolution, the filter slides across image, element-wise multiplication is performed between the filter and the corresponding pixels in the image, and product are then summed to get a new pixel value for the output image.

* This blurring effect is achieved because the filter averages the values of surrounding pixels, replacing each pixel with a slightly blended version of its neighborhood.

By using different filter matrices with varying weights, you can achieve various image processing effects like sharpening, edge detection, and noise reduction.

Q-10) ~~Lets understand it by~~ Brief Description of LT for Computer Vision
⇒ for rotating 2D image.
⇒ LT's are mathematical operations that map one vector space to another. In Computer vision, they are used to manipulate images, such as rotating, scaling, translating them.
For rotating a 2D image, a rotation matrix is used. This matrix is multiplied by a vector representing the image co-ordinates, which results in a new vector representing the rotated co-ordinates.
Heres a breakdown of the rotation matrix for a 2D image with Counter clockwise rotation of θ:
* Rotation Matrix: $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

Multiplying this matrix with a vector (x, y) representing an image co-ordinate will give you a new vector (x', y') representing the rotated co-ordinate after a counter clockwise rotation of θ°.