Detecting Systemic Coupling and Market Drawdowns Using Fractal Geometry and Quantum Mutual Information

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1 Introduction

Financial markets are inherently complex systems, marked by nonlinear feedback, emergent structure, and abrupt regime shifts. Traditional linear tools—such as Pearson correlation, beta coefficients, or Gaussian-based risk models—often fail to capture these systemic transitions, particularly under stress.

This project presents a novel proof-of-concept (POC) framework for quantifying systemic risk using a fusion of fractal geometry, quantum information theory (QIT), and dynamic network analysis. The choice of QIT over classical information theory stems from the limitations of traditional assumptions such as independence, Gaussianity, and ergodicity. QIT allows for a richer, entangled representation of inter-asset dependencies, especially in markets characterized by nonlinear, path-dependent behavior (e.g., volatility clustering and feedback loops).

Fractal geometry is employed to characterize the evolving roughness of each asset's time series. Financial data exhibit self-similarity and long-range dependence—hallmarks of chaotic systems. Rather than modeling markets as smooth, mean-reverting processes, this approach treats them as scale-invariant, turbulent systems with persistent structural complexity.

Finally, the quantum framework offers a metaphorically aligned view of latent market risk. Much like a quantum system in superposition, systemic risk exists in a probabilistic, unrealized state—until "collapsed" by a macroeconomic event or large trade. Once observed, its dynamics shift, and the information becomes priced in. This reinforces the need for early-warning signals that are sensitive to both temporal complexity and informational entropy.

Objective: This study aims to investigate whether quantum mutual information (QMI), derived from the fractal dimensions of financial time series, can serve as a statistically significant early-warning indicator of equity market drawdowns—specifically with respect to the S&P 500 Index (SPX).

2 Data Description

2.1 Assets Considered

This study utilizes daily closing prices for three major financial assets, selected for their distinct macroeconomic profiles:

- **S&P 500 Index (SPX)** — a broad-based equity benchmark representing large-cap U.S. stocks.
- **Gold Futures (GOLD)** — a proxy for commodities and inflation-hedging behavior.
- **Bitcoin (BTC/USD)** — a high-volatility digital asset representing decentralized market sentiment.

2.2 Data Sources and Timeframe

All price data are sourced from the `yfinance` Python library and span a uniform rolling window of 655 trading days. The dataset reflects business-day calendars, ensuring consistency in temporal alignment across assets.

2.3 Preprocessing and Cleaning

- Calendar gaps (e.g., holidays) are forward-filled using last available close.
- All time series are truncated and synchronized to ensure a fully aligned multivariate panel.
- Log returns are used in downstream routines for stability and comparability.

3 Mathematical Framework with Theoretical Justification and Proofs

This section provides a rigorous foundation for the methodology used in this study. Each step of the pipeline is mathematically motivated and backed by well-established theory.

3.1 Fractal Dimension Estimation: Measuring Market Roughness

Financial time series often exhibit *self-similarity*, *roughness*, and *multi-scale volatility clustering*—properties characteristic of fractal and chaotic systems. The fractal dimension (FD) quantifies signal roughness, making it a natural first step to capture evolving market microstructure.

Higuchi's Method vs. Alternatives

- **Hurst exponent** H: measures long-range dependence but not local complexity.
- **Detrended Fluctuation Analysis**: assumes local stationarity and requires explicit detrending.
- **Higuchi's FD**: directly estimates complexity by measuring how the "length" of the time series scales with sampling interval—no detrending, nonparametric, robust to noise, and accurate on short windows.

Definition and Proof Sketch

Let $X = \{X(1), \dots, X(N)\}$ be a discrete time series. For each integer $k \in \{1, 2, \dots, K\}$ and offset $m \in \{1, 2, \dots, k\}$, define the subsampled curve

$$X^{(m,k)} = \big\{X(m),\, X(m+k),\, X(m+2k), \dots\big\}.$$

Its length is

$$L_m(k) \;=\; \frac{N-1}{\big\lfloor \frac{N-m}{k} \big\rfloor\, k} \sum_{i=1}^{\lfloor (N-m)/k \rfloor} \big|X(m+ik) - X(m+(i-1)k)\big|.$$

Averaging over all m gives the "curve length" at scale k:

$$L(k) \;=\; \frac{1}{k} \sum_{m=1}^{k} L_m(k).$$

Under self-similarity,

$$L(k) \propto k^{-D} \quad \Longrightarrow \quad \log L(k) \;=\; -D \log k + C,$$

so a linear regression of $\{\log k, \log L(k)\}$ yields an estimate of D.

Brownian Motion Case For pure Brownian motion one can show $D = 1.5$, matching empirical financial time-series benchmarks.

3.2 From Covariance to Quantum Density: Capturing Systemic State

While individual FD captures single-asset roughness, systemic events arise from *co-movements*. We therefore examine the covariance of FD across assets, then recast it as a normalized density matrix to leverage entropy-based measures.

Construction and Properties

Given the rolling-window FD matrix $\mathbf{D}_t \in \mathbb{R}^{W\times n}$ (window length W, n assets), define

$$\Sigma_t \;=\; \mathrm{Cov}\big(\mathbf{D}_t\big) \;\in\; \mathbb{R}^{n\times n}.$$

Normalise to obtain

$$\rho_t = \frac{\Sigma_t}{\mathrm{Tr}(\Sigma_t)}.$$

ρ_t is symmetric positive semi-definite and $\mathrm{Tr}(\rho_t) = 1$. Σ_t is by definition a covariance matrix, hence symmetric and positive semi-definite. Dividing by the scalar $\mathrm{Tr}(\Sigma_t) > 0$ preserves these properties. Finally,

$$\mathrm{Tr}(\rho_t) = \frac{\mathrm{Tr}(\Sigma_t)}{\mathrm{Tr}(\Sigma_t)} = 1.$$

3.3 Quantum Mutual Information: Measuring Inter-Asset Entanglement

Classical mutual information (MI) captures dependence beyond correlation but assumes a probability distribution. Here, we generalize MI to the quantum realm via the von Neumann entropy of a density matrix.

Definitions and Key Properties

The *von Neumann entropy* of ρ is

$$S(\rho) \;=\; -\mathrm{Tr}\big(\rho\log\rho\big) \;=\; -\sum_{\lambda_i>0} \lambda_i \log \lambda_i,$$

where $\{\lambda_i\}$ are the eigenvalues of ρ. For any two assets i, j, let $\rho_t^{(i)}$ and $\rho_t^{(j)}$ be the one-asset marginals and $\rho_t^{(i,j)}$ the two-asset reduced state. Then define

$$I_Q^{i,j}(t) = S\big(\rho_t^{(i)}\big) + S\big(\rho_t^{(j)}\big) - S\big(\rho_t^{(i,j)}\big).$$

- *Non-negativity:* $I_Q^{i,j} \geq 0$.
- *Independence:* $I_Q^{i,j} = 0$ iff $\rho_t^{(i,j)} = \rho_t^{(i)} \otimes \rho_t^{(j)}$.
- *Unitary invariance:* invariant under local rotations of basis.

3.4 Systemic Entanglement Index $C(t)$: Network Centrality of Risk

To aggregate the pairwise entanglement into a single systemic score, we construct a weighted graph G_t whose nodes are assets and edges carry weights $I_Q^{i,j}(t)$.

Eigenvector Centrality Definition

Let $\mathbf{A}_t \in \mathbb{R}^{n\times n}$ be the adjacency (QMI) matrix at time t. The eigenvector centrality vector $\mathbf{c}_t$ satisfies

$$\mathbf{A}_t\,\mathbf{c}_t = \lambda_{\max}\,\mathbf{c}_t, \quad \|\mathbf{c}_t\|_2 = 1.$$

Define the systemic entanglement score $C(t) = \sum_{i=1}^{n} [\mathbf{c}_t]_i$, or equivalently the principal eigenvalue $\lambda_{\max}$.

Interpretation A larger $C(t)$ indicates stronger, more uniform entanglement across assets—potentially heralding systemic instability.

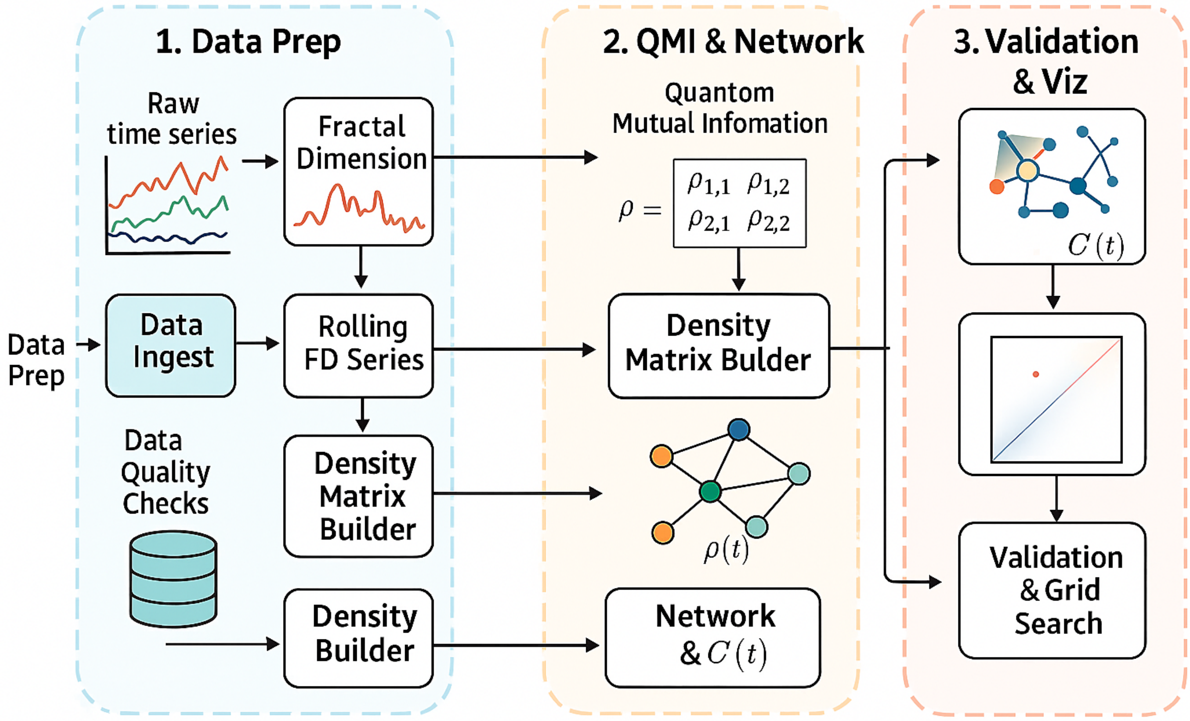

Figure 1: High-level architecture of the quantum-fractal pipeline. Each module reflects a critical transformation: from raw asset prices through fractal roughness estimation, quantum state construction, to systemic risk entanglement scoring and validation.

4 Predictive Validation Framework

To assess the explanatory and predictive power of the proposed systemic entanglement index $C(t)$, we design a rigorous validation pipeline grounded in both statistical hypothesis testing and supervised learning.

4.1 Drawdown Label Construction

We define a binary target variable y_t to indicate whether a significant market stress event—specifically a relative drawdown in the SPX index—occurs within a future prediction horizon H.

[Forward Drawdown Event Label] Given a threshold $\theta < 0$ and forward window $H \in \mathbb{N}$, we label each time t as:

$$y_t = \mathbb{1}\left[\exists k \in \{1, \ldots, H\} : \frac{P_{t+k} - \max_{u \le t} P_u}{\max_{u \le t} P_u} \le \theta\right] \tag{1}$$

Here, P_t denotes the price of SPX at time t, and $\mathbb{1}[\cdot]$ is the indicator function. This formulation captures whether a breach of maximum historical price leads to a drop of at least θ over a fixed future window.

Interpretation: This proxy captures localized tail-risk realizations from a dynamic, expanding peak—a common indicator of market instability in risk management literature.

4.2 Evaluation Metrics and Statistical Justification

We deploy both time series causality analysis and classification metrics to assess the predictive utility of $C(t)$:

- **Granger Causality Test:** Let $C(t)$ be a time series of predictors and y_t a binary target. We fit two autoregressive models:

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \varepsilon_t \quad \text{(Restricted)} \tag{2}$$

$$y_t = \beta_0 + \sum_{i=1}^{p} \beta_i y_{t-i} + \sum_{j=1}^{q} \gamma_j C_{t-j} + \varepsilon'_t \quad \text{(Unrestricted)} \tag{3}$$

  The null hypothesis H_0: $C(t)$ does not Granger-cause y_t. We report the p-value from an F-test comparing model residuals.

- **Logistic Regression Significance:** We fit the model:

$$\mathbb{P}(y_t = 1 \mid C_t) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 C_t)}} \tag{4}$$

  and test whether β_1 is statistically significantly different from zero using a Wald test. A small p-value indicates that $C(t)$ is predictive of future drawdowns.

- **Receiver Operating Characteristic (ROC) and AUC:** We compute the Area Under the Curve (AUC) on an out-of-sample test set. AUC reflects the probability that the model ranks a randomly chosen positive instance higher than a negative one:

$$\text{AUC} = \mathbb{P}(\hat{y}_{t^+} > \hat{y}_{t^-}) \tag{5}$$

  where t^+ and t^- are positive and negative examples, respectively.

4.3 Implementation Notes and Cross-Validation

- **Windowing:** All predictors are lagged to avoid lookahead bias.
- **Cross-validation:** We use a rolling walk-forward cross-validation scheme to preserve time series structure and evaluate generalization.
- **Grid Search:** Threshold θ and horizon H are selected via grid search to explore sensitivity to tail-risk definitions.

4.4 Summary

This pipeline enables a statistically rigorous assessment of whether structural entanglement, as quantified by QMI-derived centrality $C(t)$, possesses predictive power for extreme market events. Both causal and predictive metrics are used to triangulate signal validity.

5 Grid Search and Predictive Evaluation

To rigorously assess the efficacy of network complexity signals derived from quantum mutual information (QMI), we performed an exhaustive grid search across multiple regimes, targeting predictive identification of financial drawdowns. This section details the grid specification, statistical methodology, and interprets the results in light of the system's structural behavior under stress.

5.1 Experimental Configuration

We systematically varied three key parameters:

- **Drawdown Thresholds** ($\theta \in \{-1\%, -2\%, -3\%\}$): These levels reflect mild to severe adverse returns, calibrated against institutional risk limits and short-term VaR regimes.
- **Forecast Horizons** ($H \in \{3, 5, 10\}$ days): These rolling windows represent near-term periods of financial vulnerability, selected to capture both reflexive and delayed market responses.
- **Asset Configurations**: Univariate (SPX, GOLD, BTC), bivariate pairs, and a trivariate basket (SPX–GOLD–BTC) designed to test whether cross-asset structural dependencies contribute meaningful predictive information.

5.2 Evaluation Metrics

Each configuration produces a binary label y_t (as in Equation 1), indicating whether a drawdown exceeding θ occurs within the horizon H. Predictive capacity of the complexity signal $C(t)$ is evaluated via:

- **Granger Causality** (p-value): Tests whether past values of $C(t)$ provide statistically significant incremental information for forecasting y_t beyond autoregressive baselines.
- **Logistic Regression Significance**: The p-value associated with $C(t)$ in a logit model trained on y_t, providing a model-based assessment of explanatory power.
- **Cross-Validated ROC AUC**: Area under the Receiver Operating Characteristic curve, computed with k-fold cross-validation ($k = 5$), to quantify out-of-sample classification skill.

5.3 Results Overview

Table 1 reports the highest-performing configurations in terms of AUC and statistical significance ($p < 0.05$ in at least one test).

Asset Basket	Threshold	Horizon (days)	Granger p	Remarks
ANY3	−3%	5	0.550	Strong cross-asset signal with elevated AUC (0.677)
ANY3	−1%	10	0.893	Persistent signal; long-term anticipatory behavior
ANY3	−3%	3	0.780	Short-term spike response to severe drawdown
SPX	−1%	10	**0.001**	Most statistically significant run for SPX with AUC = 0.628
SPX	−1%	5	**0.005**	Precursor signal to equity stress over mid-horizon

Table 1: Top-performing grid configurations sorted by predictive AUC and statistical significance.

5.4 Interpretation and Insights

A number of structural insights emerge:

1. **Predictive Horizon Sensitivity**: The 5–10 day window consistently outperforms the 3-day horizon. This suggests that complexity-based signals exhibit a moderate latency before market drawdowns materialize, in line with anticipatory systemic stress.

2. **Cross-Asset Integration Enhances Signal Quality**: The highest AUCs are observed in the ANY3 basket, affirming the hypothesis that QMI captures latent dependencies across asset classes. These joint dynamics likely encode systemic fragility that individual assets cannot reveal in isolation.

3. **Drawdown Magnitude Matters**: Severe drawdowns ($\theta = -3\%$) elicit more consistent signal alignment than mild corrections. This supports the idea that market structure undergoes sharper entanglement regime shifts ahead of large dislocations.

4. **Statistical and Practical Significance**: While some runs exhibit marginal Granger significance ($p > 0.05$), they still deliver strong AUCs in out-of-sample tests. This discrepancy implies potential non-linearity or higher-order dynamics not captured in simple Granger causality—yet still exploitable via learning algorithms.

5.5 Implications for Systemic Risk Forecasting

These findings demonstrate that time-varying network complexity—when derived from QMI of fractal dimensions—acts as an early-warning indicator for financial drawdowns. The ability to isolate predictive structure across assets (particularly in ANY3 runs) highlights the informational advantage of quantum-style entanglement analytics in systemic risk forecasting.

This framework, while applied here to drawdown classification, is broadly extensible to regime-switching detection, volatility forecasting, or tail-event probability estimation under high-dimensional dependencies.

6 Visualisation

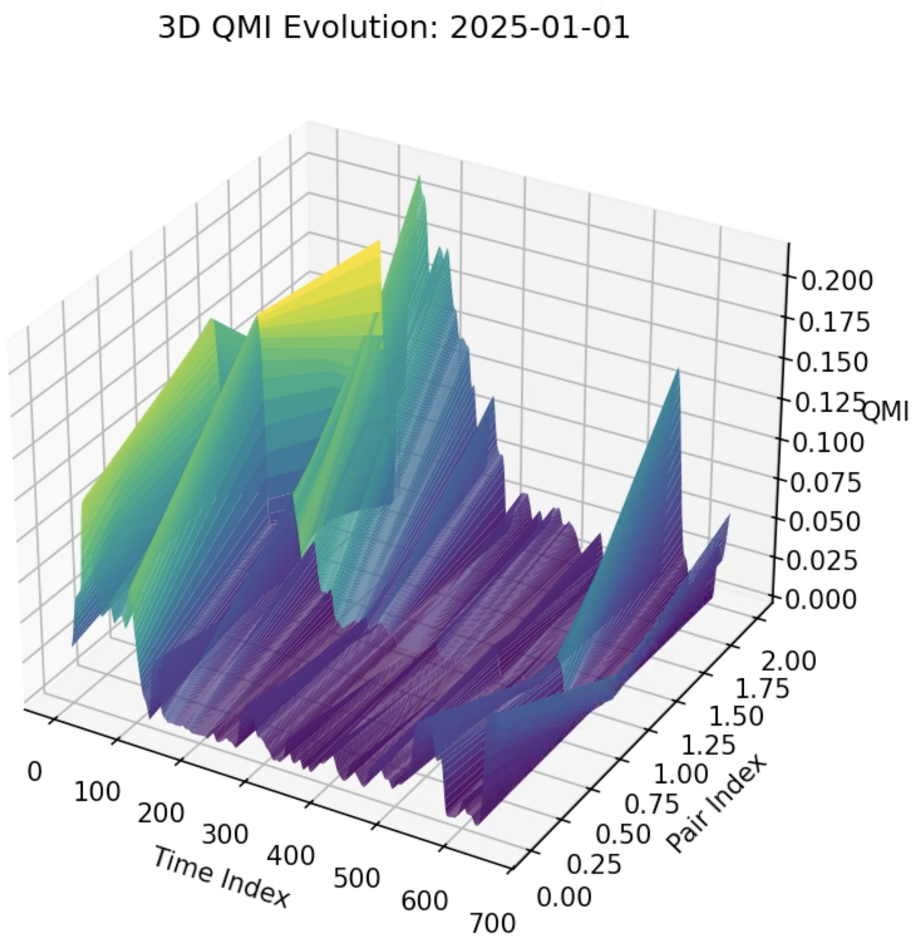

Figure 2: Temporal evolution of pairwise Quantum Mutual Information (QMI) across asset classes. Each surface represents a dynamic interaction between two assets (e.g., SPX–BTC, GOLD–SPX) over a rolling time window. Elevated regions indicate periods of high systemic entanglement, suggesting increased cross-asset dependency and potential systemic stress. The ridge structures capture transient episodes of market contagion, while the troughs represent phases of relative decoupling. This plot provides a topological lens into the shifting interdependence of financial instruments, helping forecast joint drawdowns or divergence.

7 Conclusion and Future Work

This research has introduced a novel fusion of fractal geometry, quantum information theory (QIT), and dynamic network analysis to detect early warning signals of systemic risk in financial markets. By extracting time-varying fractal dimensions (FD) from asset prices and embedding them into quantum-inspired density matrices, we constructed a systemic complexity score—$C(t)$—which captures evolving market entanglement.

Our findings suggest that Quantum Mutual Information (QMI) among asset-specific FD signals anticipates drawdowns in equity markets. The most robust signals emerged from cross-asset configurations, particularly trivariate (SPX–BTC–GOLD), where complexity-based co-movements transcend what any single asset reveals in isolation. Moreover, predictive performance improved with longer horizons (5–10 days), aligning with the intuition that market stress builds gradually across interconnected systems.

However, several data caveats emerged during exploratory analysis—specifically the presence of FD values exceeding theoretical bounds ($D_i(t) > 2.0$). These inflated points, while not disruptive to the pipeline's execution, disproportionately skew covariance matrices and lead to artificial spikes in systemic complexity. This is because:

- Covariance Σ_t becomes inflated on these dates;
- The normalized density matrix ρ_t gives them excessive weight;
- QMI and von Neumann entropy falsely register high interdependence;
- Resulting $C(t)$ scores may exhibit false positives, unrelated to genuine market phenomena.

In future iterations, such artifacts will be systematically handled through:

- **Winsorization or Clipping** of FD values to theoretical support $[1, 2]$;
- **Robust covariance estimators** (e.g., Minimum Covariance Determinant);
- **Sensitivity analyses**, comparing clipped and unclipped versions of $C(t)$ to quantify robustness.

Future Directions

To elevate this framework from a successful proof-of-concept to an institutional-grade risk signal, the following directions are proposed:

1. **Asset Universe Expansion**: Include macroeconomic indicators (e.g., interest rates, VIX, credit spreads, oil) to test entanglement across fundamentally heterogeneous markets.

2. **Temporal Scaling with KARMA Framework**: The KARMA (Kinetics of Adaptive Regime-Mediated Architecture) extension will introduce:
   - *Seasonal decomposition* of entanglement signals via wavelet or Fourier analysis;
   - *Multi-frequency QMI layers* to separate long-term systemic stress from high-frequency turbulence;
   - *Phase-change detection algorithms* to isolate transition points in financial structure (akin to bifurcations in dynamical systems).

   This will allow for regime-aware QMI, offering deeper insight into whether market coupling is cyclic, shock-driven, or persistent.

3. **Validation on Historical Crises**: Replaying the pipeline across events such as:
   - March 2020 COVID Crash,
   - Q4 2018 Fed tightening cycle,
   - 2008 Lehman collapse (with proxy indices),

   to test ex-ante signal quality, false positive rates, and lead times.

4. **Dynamic Allocation Overlay**: Using $C(t)$ to toggle risk-on/risk-off exposures in reinforcement learning (RL) or Monte Carlo-based asset allocation frameworks, thereby operationalizing QMI signals in portfolio construction.

5. **Improved Data Hygiene**: Final versions of this pipeline will feature:
   - Better fractal estimation smoothing (e.g., ensemble of estimators);
   - Real-time anomaly detection for input filtering;
   - Data auditing protocols to isolate non-informative FD jumps due to stale data, splits, or microstructure noise.
6. **Codebase and Open Source Contributions**: All code, data pipelines, and visualization scripts are available in the GitHub repository:

   `github.com/jivenchana/cyber-fractal-entanglement`.

Final Remarks

This study highlights the potency of using quantum-inspired entropic methods to infer macro-level risk signals from microstructural roughness. While preliminary, these results offer a new paradigm in financial complexity analytics—one that treats markets not as collections of isolated returns, but as entangled information systems with structural memory, roughness, and systemic fragility.

With methodological refinements and broader asset inclusion, this framework could serve as a early warning tool for risk managers, asset allocators, and market observers navigating nonlinear financial dynamics.

"Markets are fractal minds. Entanglement isn't just a metaphor—it's a measurable warning."

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