CHAPTER 2

RESISTIVE CIRCUITS

Courtesy of Tesla Motors

THE LEARNING GOALS FOR THIS CHAPTER ARE:

- **Be able to use Ohm's law to solve electric circuits**
- **Be able to apply Kirchhoff's current law and Kirchhoff's voltage law to solve electric circuits**
- **Know how to analyze single-loop and single-node-pair circuits**
- **Know how to combine resistors in series and parallel**
- **Be able to use voltage and current division to solve simple electric circuits**
- **Understand when and how to apply wye-delta transformations in the analysis of electric circuits**
- **Know how to analyze electric circuits containing dependent sources**

Tesla Roadster Green technologies come in many colors. The 2010 Tesla Roadster, for example, comes in Fusion Red, Arctic White, Racing Green and Electric Blue, to name a few. An environmentally friendly sports car that seats two, this convertible has rocket acceleration and hugs the road like a dream; it's the world's first high-performance electric car. The Roadster contains over 6,800 safe, rechargeable lithium-ion batteries that weigh about 1,000 pounds in total. It is twice as efficient as hybrid cars that combine a gasoline engine and an electric motor to provide propulsion, but its fantastic performance comes at a cost of over $100,000.

Choosing between an all-electric vehicle and a hybrid requires trade-offs on a wide range of criteria: performance, cost, efficiency, effects on the environment, safety, and reliability. Handling qualities may be highly important to some, cost and efficiency to others.

As a student of circuit analysis, you will make trade-offs in choosing between methods of analysis for different circuit topologies. This chapter describes fundamental laws that apply to all circuits regardless of their complexity. Ohm's law governs the most common relationship between voltage and current for circuits that are linear. Circuits having a single power source with resistances having the same currents and others having the same voltage will be analyzed using the series-parallel method. You'll learn more techniques in the chapters that follow, as you begin to master the same principles used by the designers of the Tesla Roadster.

2.1 Ohm's Law

Ohm's law is named for the German physicist Georg Simon Ohm, who is credited with establishing the voltage–current relationship for resistance. As a result of his pioneering work, the unit of resistance bears his name.

Ohm's law states that the voltage across a resistance is directly proportional to the current flowing through it. The resistance, measured in ohms, is the constant of proportionality between the voltage and current.

[hint]

The passive sign convention will be employed in conjunction with Ohm's law.

A circuit element whose electrical characteristic is primarily resistive is called a resistor and is represented by the symbol shown in Fig. 2.1a. A resistor is a physical device that can be purchased in certain standard values in an electronic parts store. These resistors, which find use in a variety of electrical applications, are normally carbon composition or wire-wound. In addition, resistors can be fabricated using thick oxide or thin metal films for use in hybrid circuits, or they can be diffused in semiconductor integrated circuits. Some typical discrete resistors are shown in Fig. 2.1b.

The mathematical relationship of Ohm's law is illustrated by the equation

$$v(t) = R\,i(t), \text{ where } R \geqq 0 \qquad \textbf{2.1}$$

or equivalently, by the voltage–current characteristic shown in Fig. 2.2a. Note carefully the relationship between the polarity of the voltage and the direction of the current. In addition, note that we have tacitly assumed that the resistor has a constant value and therefore that the voltage–current characteristic is linear.

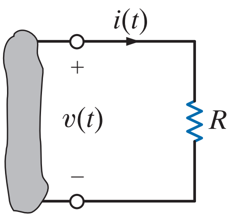
The symbol Ω is used to represent ohms, and therefore,

$$1\ \Omega = 1\ \text{V/A}$$

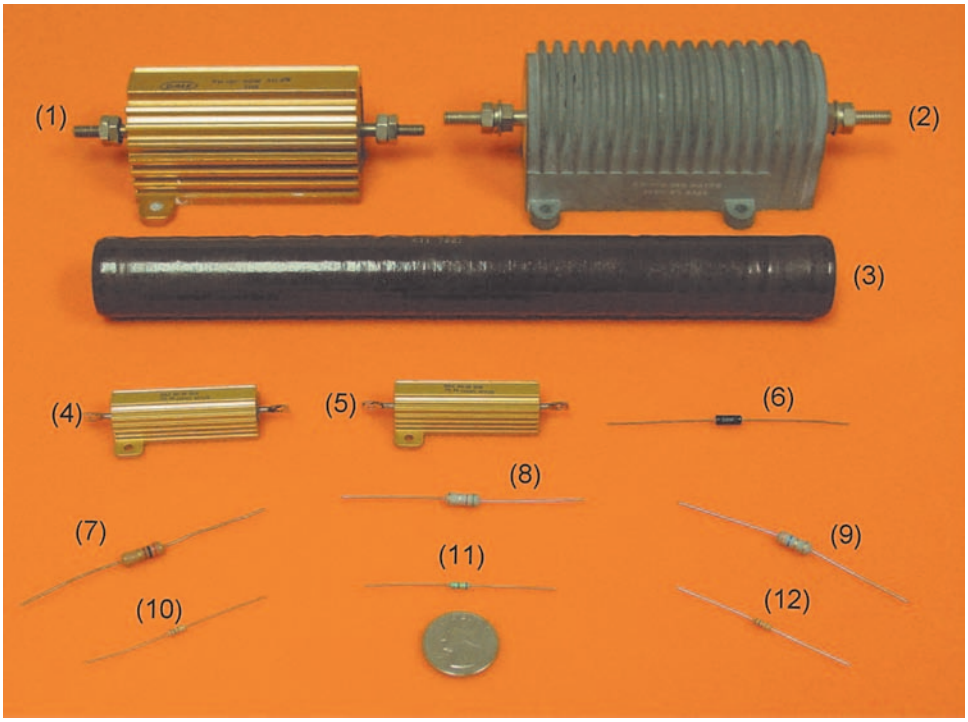
Although in our analysis we will always assume that the resistors are *linear* and are thus described by a straight-line characteristic that passes through the origin, it is important that readers realize that some very useful and practical elements do exist that exhibit a *nonlinear* resistance characteristic; that is, the voltage–current relationship is not a straight line.

Figure 2.1

(a) Symbol for a resistor; (b) some practical devices. (1), (2), and (3) are high-power resistors. (4) and (5) are high-wattage fixed resistors. (6) is a high-precision resistor. (7)–(12) are fixed resistors with different power ratings. (Photo courtesy of Mark Nelms and Jo Ann Loden)



(a)



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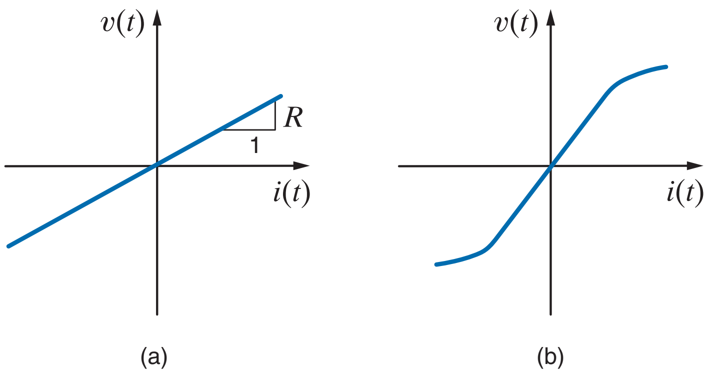


Figure 2.2
Graphical representation of the voltage–current relationship for (a) a linear resistor and (b) a light bulb.

The light bulb from the flashlight in Chapter 1 is an example of an element that exhibits a nonlinear characteristic. A typical characteristic for a light bulb is shown in Fig. 2.2b.

Since a resistor is a passive element, the proper current–voltage relationship is illustrated in Fig. 2.1a. The power supplied to the terminals is absorbed by the resistor. Note that the charge moves from the higher to the lower potential as it passes through the resistor and the energy absorbed is dissipated by the resistor in the form of heat. As indicated in Chapter 1, the rate of energy dissipation is the instantaneous power, and therefore

$$p(t) = v(t)i(t) \tag{2.2}$$

which, using Eq. (2.1), can be written as

$$p(t) = Ri^2(t) = \frac{v^2(t)}{R} \tag{2.3}$$

This equation illustrates that the power is a nonlinear function of either current or voltage and that it is always a positive quantity.

Conductance, represented by the symbol G, is another quantity with wide application in circuit analysis. By definition, conductance is the reciprocal of resistance; that is,

$$G = \frac{1}{R} \tag{2.4}$$

The unit of conductance is the siemens, and the relationship between units is

$$1\ \text{S} = 1\ \text{A/V}$$

Using Eq. (2.4), we can write two additional expressions,

$$i(t) = Gv(t) \tag{2.5}$$

and

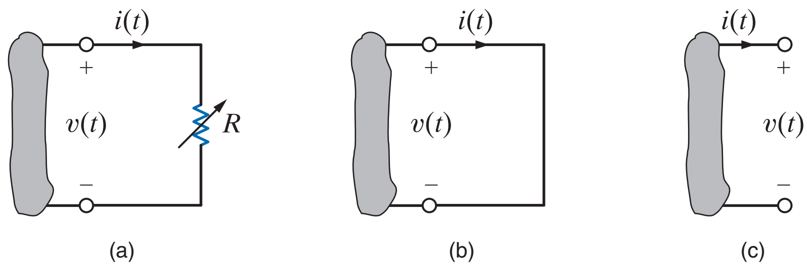
$$p(t) = \frac{i^2(t)}{G} = Gv^2(t) \tag{2.6}$$

Eq. (2.5) is another expression of Ohm's law.

Two specific values of resistance, and therefore conductance, are very important: $R = 0$ and $R = \infty$.

In examining the two cases, consider the network in Fig. 2.3a. The variable resistance symbol is used to describe a resistor such as the volume control on a radio or television set.

Figure 2.3
Short-circuit and open-circuit descriptions.



As the resistance is decreased and becomes smaller and smaller, we finally reach a point where the resistance is zero and the circuit is reduced to that shown in Fig. 2.3b; that is, the resistance can be replaced by a short circuit. On the other hand, if the resistance is increased and becomes larger and larger, we finally reach a point where it is essentially infinite and the resistance can be replaced by an open circuit, as shown in Fig. 2.3c. Note that in the case of a short circuit where $R = 0$,

$$\begin{aligned} v(t) &= Ri(t) \\ &= 0 \end{aligned}$$

Therefore, $v(t) = 0$, although the current could theoretically be any value. In the open-circuit case where $R = \infty$,

$$\begin{aligned} i(t) &= v(t)/R \\ &= 0 \end{aligned}$$

Therefore, the current is zero regardless of the value of the voltage across the open terminals.

EXAMPLE 2.1

In the circuit in Fig. 2.4a, determine the current and the power absorbed by the resistor.

SOLUTION

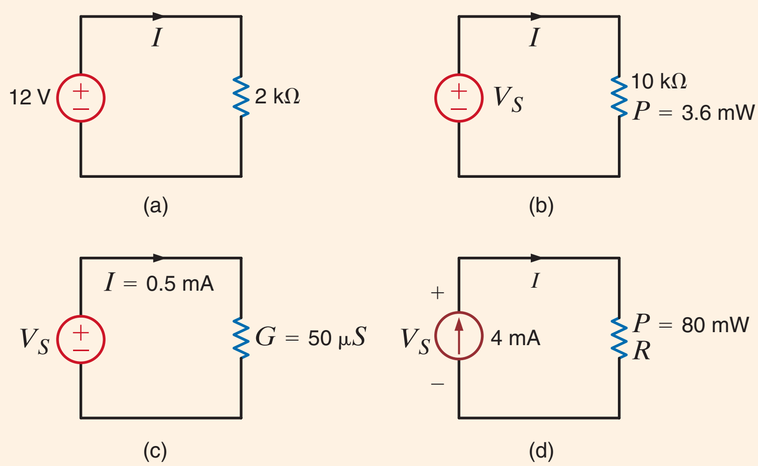
Using Eq. (2.1), we find the current to be

$$I = V/R = 12/2\text{k} = 6 \text{ mA}$$

Note that because many of the resistors employed in our analysis are in kΩ, we will use k in the equations in place of 1000. The power absorbed by the resistor is given by Eq. (2.2) or (2.3) as

$$\begin{aligned} P &= VI = (12)(6 \times 10^{-3}) = 0.072 \text{ W} \\ &= I^2R = (6 \times 10^{-3})^2(2\text{k}) = 0.072 \text{ W} \\ &= V^2/R = (12)^2/2\text{k} = 0.072 \text{ W} \end{aligned}$$

Figure 2.4
Circuits for Examples 2.1 to 2.4.



EXAMPLE 2.2

The power absorbed by the 10-kΩ resistor in Fig. 2.4b is 3.6 mW. Determine the voltage and the current in the circuit.

SOLUTION

Using the power relationship, we can determine either of the unknowns:

$$V_S^2/R = P$$
$$V_S^2 = (3.6 \times 10^{-3})(10\text{k})$$
$$V_S = 6 \text{ V}$$

and

$$I^2R = P$$
$$I^2 = (3.6 \times 10^{-3})/10\text{k}$$
$$I = 0.6 \text{ mA}$$

Furthermore, once V_S is determined, I could be obtained by Ohm's law, and likewise once I is known, then Ohm's law could be used to derive the value of V_S. Note carefully that the equations for power involve the terms I^2 and V_S^2. Therefore, $I = -0.6$ mA and $V_S = -6$ V also satisfy the mathematical equations and, in this case, the direction of *both* the voltage and current is reversed.

EXAMPLE 2.3

Given the circuit in Fig. 2.4c, we wish to find the value of the voltage source and the power absorbed by the resistance.

SOLUTION

The voltage is

$$V_S = I/G = (0.5 \times 10^{-3})/(50 \times 10^{-6}) = 10 \text{ V}$$

The power absorbed is then

$$P = I^2/G = (0.5 \times 10^{-3})^2/(50 \times 10^{-6}) = 5 \text{ mW}$$

Or we could simply note that

$$R = 1/G = 20 \text{ k}\Omega$$

and therefore

$$V_S = IR = (0.5 \times 10^{-3})(20\text{k}) = 10 \text{ V}$$

and the power could be determined using $P = I^2R = V_S^2/R = V_SI$.

EXAMPLE 2.4

Given the network in Fig. 2.4d, we wish to find R and V_S.

SOLUTION

Using the power relationship, we find that

$$R = P/I^2 = (80 \times 10^{-3})/(4 \times 10^{-3})^2 = 5 \text{ k}\Omega$$

The voltage can now be derived using Ohm's law as

$$V_S = IR = (4 \times 10^{-3})(5\text{k}) = 20 \text{ V}$$

The voltage could also be obtained from the remaining power relationships in Eqs. (2.2) and (2.3).

Before leaving this initial discussion of circuits containing sources and a single resistor, it is important to note a phenomenon that we will find to be true in circuits containing many sources and resistors. The presence of a voltage source between a pair of terminals tells us precisely what the voltage is between the two terminals regardless of what is happening in the balance of the network. What we do not know is the current in the voltage source. We must apply circuit analysis to the entire network to determine this current. Likewise, the presence of a current source connected between two terminals specifies the exact value of the current through the source between the terminals. What we do not know is the value of the voltage across the current source. This value must be calculated by applying circuit analysis to the entire network. Furthermore, it is worth emphasizing that when applying Ohm's law, the relationship $V = IR$ specifies a relationship between the voltage *directly across* a resistor R and the current that is *present* in this resistor. Ohm's law does not apply when the voltage is present in one part of the network and the current exists in another. This is a common mistake made by students who try to apply $V = IR$ to a resistor R in the middle of the network while using a V at some other location in the network.

Learning Assessments

E2.1 Given the circuits in Fig. E2.1, find (a) the current I and the power absorbed by the resistor in Fig. E2.1a, and (b) the voltage across the current source and the power supplied by the source in Fig. E2.1b.

ANSWER: (a) $I = 0.3$ mA, $P = 3.6$ mW; (b) $V_S = 3.6$ V, $P = 2.16$ mW.

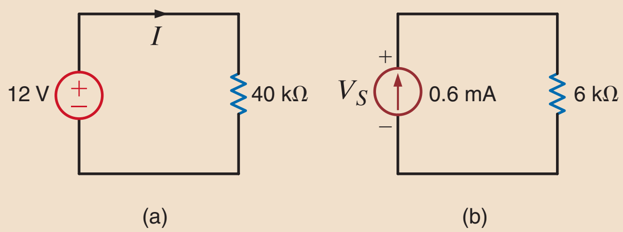


Figure E2.1

E2.2 Given the circuits in Fig. E2.2, find (a) R and V_S in the circuit in Fig. E2.2a, and (b) find I and R in the circuit in Fig. E2.2b.

ANSWER: (a) $R = 10\ \text{k}\Omega$, $V_S = 4$ V; (b) $I = 20.8$ mA, $R = 576\ \Omega$.

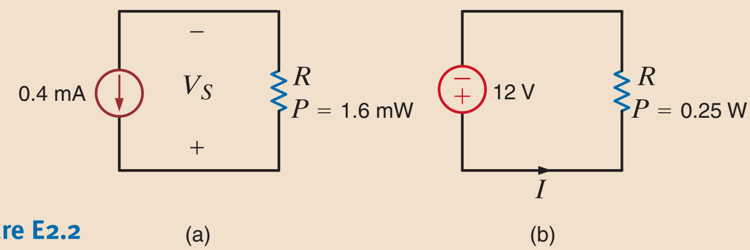


Figure E2.2

E2.3 The power absorbed by G_x in Fig. E2.3 is 50 mW. Find G_x.

ANSWER: $G_x = 500\ \mu\text{S}$.

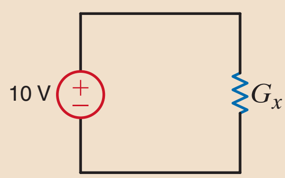


Figure E2.3

2.2 Kirchhoff's Laws

The circuits we have considered previously have all contained a single resistor, and we have analyzed them using Ohm's law. At this point we begin to expand our capabilities to handle more complicated networks that result from an interconnection of two or more of these simple elements. We will assume that the interconnection is performed by electrical conductors (wires) that have zero resistance—that is, perfect conductors. Because the wires have zero resistance, the energy in the circuit is in essence lumped in each element, and we employ the term *lumped-parameter circuit* to describe the network.

To aid us in our discussion, we will define a number of terms that will be employed throughout our analysis. As will be our approach throughout this text, we will use examples to illustrate the concepts and define the appropriate terms. For example, the circuit shown

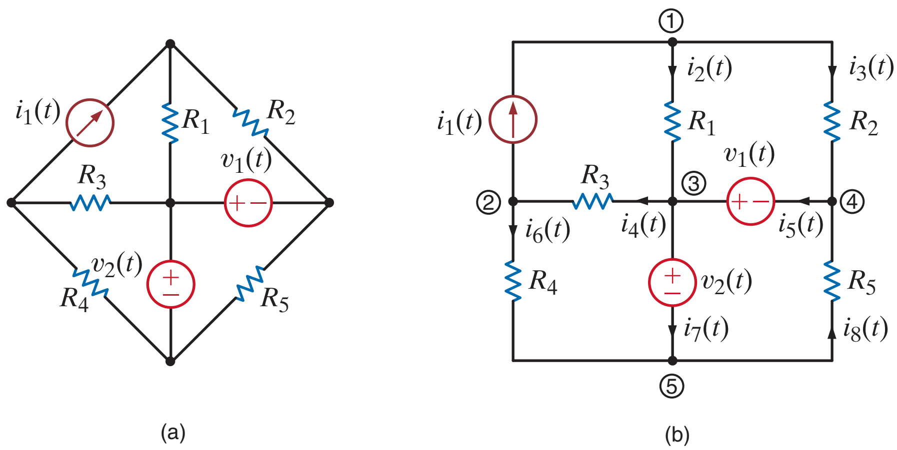


Figure 2.5

Circuit used to illustrate KCL.

in Fig. 2.5a will be used to describe the terms *node, loop*, and *branch*. A node is simply a point of connection of two or more circuit elements. The reader is cautioned to note that, although one node can be spread out with perfect conductors, it is still only one node. This is illustrated in Fig. 2.5b, where the circuit has been redrawn. Node 5 consists of the entire bottom connector of the circuit.

If we start at some point in the circuit and move along perfect conductors in any direction until we encounter a circuit element, the total path we cover represents a single node. Therefore, we can assume that a node is one end of a circuit element together with all the perfect conductors that are attached to it. Examining the circuit, we note that there are numerous paths through it. A *loop* is simply any *closed path* through the circuit in which no node is encountered more than once. For example, starting from node 1, one loop would contain the elements R_1, v_2, R_4, and i_1; another loop would contain R_2, v_1, v_2, R_4, and i_1; and so on. However, the path R_1, v_1, R_5, v_2, R_3, and i_1 is not a loop because we have encountered node 3 twice. Finally, a *branch* is a portion of a circuit containing only a single element and the nodes at each end of the element. The circuit in Fig. 2.5 contains eight branches.

Given the previous definitions, we are now in a position to consider Kirchhoff's laws, named after German scientist Gustav Robert Kirchhoff. These two laws are quite simple but extremely important. We will not attempt to prove them because the proofs are beyond our current level of understanding. However, we will demonstrate their usefulness and attempt to make the reader proficient in their use. The first law is *Kirchhoff's current law* (KCL), which states that *the algebraic sum of the currents entering any node is zero*. In mathematical form the law appears as

[hint]

KCL is an extremely important and useful law.

$$\sum_{j=1}^{N} i_j(t) = 0 \qquad \textbf{2.7}$$

where $i_j(t)$ is the jth current entering the node through branch j and N is the number of branches connected to the node. To understand the use of this law, consider node 3 shown in Fig. 2.5. Applying Kirchhoff's current law to this node yields

$$i_2(t) - i_4(t) + i_5(t) - i_7(t) = 0$$

We have assumed that the algebraic signs of the currents entering the node are positive and, therefore, that the signs of the currents leaving the node are negative.

If we multiply the foregoing equation by -1, we obtain the expression

$$-i_2(t) + i_4(t) - i_5(t) + i_7(t) = 0$$

which simply states that *the algebraic sum of the currents leaving a node is zero*. Alternatively, we can write the equation as

$$i_2(t) + i_5(t) = i_4(t) + i_7(t)$$

which states that *the sum of the currents entering a node is equal to the sum of the currents leaving the node*. Both of these italicized expressions are alternative forms of Kirchhoff's current law.

Once again it must be emphasized that the latter statement means that the sum of the *variables* that have been defined entering the node is equal to the sum of the *variables* that have been defined leaving the node, not the actual currents. For example, $i_j(t)$ may be defined entering the node, but if its actual value is negative, there will be positive charge leaving the node.

Note carefully that Kirchhoff's current law states that the *algebraic* sum of the currents either entering or leaving a node must be zero. We now begin to see why we stated in Chapter 1 that it is critically important to specify both the magnitude and the direction of a current. Recall that current is charge in motion. Based on our background in physics, charges cannot be stored at a node. In other words, if we have a number of charges entering a node, then an equal number must be leaving that same node. Kirchhoff's current law is based on this principle of conservation of charge.

Finally, it is possible to generalize Kirchhoff's current law to include a closed surface. By a closed surface we mean some set of elements completely contained within the surface that are interconnected. Since the current entering each element within the surface is equal to that leaving the element (i.e., the element stores no net charge), it follows that the current entering an interconnection of elements is equal to that leaving the interconnection. Therefore, Kirchhoff's current law can also be stated as follows: *The algebraic sum of the currents entering any closed surface is zero.*

EXAMPLE 2.5

Let us write KCL for every node in the network in Fig. 2.5, assuming that the currents leaving the node are positive.

SOLUTION

The KCL equations for nodes 1 through 5 are

$$\begin{aligned}
-i_1(t) + i_2(t) + i_3(t) &= 0\\
i_1(t) - i_4(t) + i_6(t) &= 0\\
-i_2(t) + i_4(t) - i_5(t) + i_7(t) &= 0\\
-i_3(t) + i_5(t) - i_8(t) &= 0\\
-i_6(t) - i_7(t) + i_8(t) &= 0
\end{aligned}$$

Note carefully that if we add the first four equations, we obtain the fifth equation. What does this tell us? Recall that this means that this set of equations is not linearly independent. We can show that the first four equations are, however, linearly independent. Store this idea in memory because it will become very important when we learn how to write the equations necessary to solve for all the currents and voltages in a network in the following chapter.

EXAMPLE 2.6

The network in Fig. 2.5 is represented by the topological diagram shown in Fig. 2.6. We wish to find the unknown currents in the network.

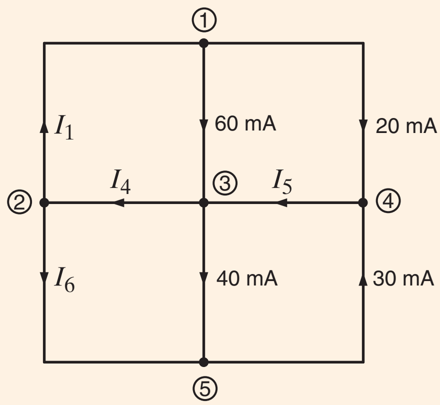


Figure 2.6 Topological diagram for the circuit in Fig. 2.5.

SOLUTION

Assuming the currents leaving the node are positive, the KCL equations for nodes 1 through 4 are

$$-I_1 + 0.06 + 0.02 = 0$$
$$I_1 - I_4 + I_6 = 0$$
$$-0.06 + I_4 - I_5 + 0.04 = 0$$
$$-0.02 + I_5 - 0.03 = 0$$

The first equation yields I_1 and the last equation yields I_5. Knowing I_5, we can immediately obtain I_4 from the third equation. Then the values of I_1 and I_4 yield the value of I_6 from the second equation. The results are $I_1 = 80$ mA, $I_4 = 70$ mA, $I_5 = 50$ mA, and $I_6 = -10$ mA.

As indicated earlier, dependent or controlled sources are very important because we encounter them when analyzing circuits containing active elements such as transistors. The following example presents a circuit containing a current-controlled current source.

EXAMPLE 2.7

Let us write the KCL equations for the circuit shown in Fig. 2.7.

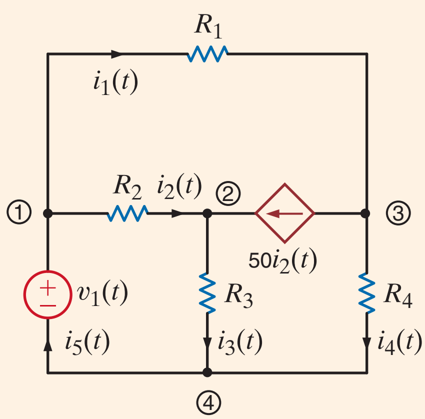


Figure 2.7 Circuit containing a dependent current source.

SOLUTION The KCL equations for nodes 1 through 4 follow:

$$i_1(t) + i_2(t) - i_5(t) = 0$$
$$-i_2(t) + i_3(t) - 50i_2(t) = 0$$
$$-i_1(t) + 50i_2(t) + i_4(t) = 0$$
$$i_5(t) - i_3(t) - i_4(t) = 0$$

If we added the first three equations, we would obtain the negative of the fourth. What does this tell us about the set of equations?

Kirchhoff's second law, called *Kirchhoff's voltage law* (KVL), states that the *algebraic sum of the voltages around any loop is zero*. As was the case with Kirchhoff's current law, we will defer the proof of this law and concentrate on understanding how to apply it. Once again the reader is cautioned to remember that we are dealing only with lumped-parameter circuits. These circuits are conservative, meaning that the work required to move a unit charge around any loop is zero.

In Chapter 1, we related voltage to the difference in energy levels within a circuit and talked about the energy conversion process in a flashlight. Because of this relationship between voltage and energy, Kirchhoff's voltage law is based on the conservation of energy.

Recall that in Kirchhoff's current law, the algebraic sign was required to keep track of whether the currents were entering or leaving a node. In Kirchhoff's voltage law, the algebraic sign is used to keep track of the voltage polarity. In other words, as we traverse the circuit, it is necessary to

EXAMPLE 2.8

Let us find I_4 and I_1 in the network represented by the topological diagram in Fig. 2.6.

SOLUTION This diagram is redrawn in Fig. 2.8; node 1 is enclosed in surface 1, and nodes 3 and 4 are enclosed in surface 2. A quick review of the previous example indicates that we derived a value for I_4 from the value of I_5. However, I_5 is now completely enclosed in surface 2. If we apply KCL to surface 2, assuming the currents out of the surface are positive, we obtain

$$I_4 - 0.06 - 0.02 - 0.03 + 0.04 = 0$$

or

$$I_4 = 70 \text{ mA}$$

which we obtained without any knowledge of I_5. Likewise for surface 1, what goes in must come out and, therefore, $I_1 = 80$ mA. The reader is encouraged to cut the network in Fig. 2.6 into two pieces in any fashion and show that KCL is always satisfied at the boundaries.

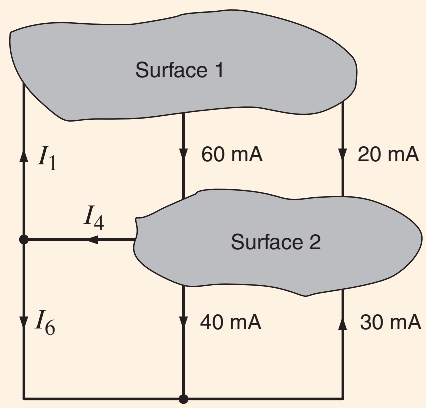


Figure 2.8 Diagram used to demonstrate KCL for a surface.

Learning Assessments

E2.4 Given the networks in Fig. E2.3, find (a) I_1 in Fig. E2.4a and (b) I_T in Fig. E2.4b.

ANSWER:
(a) $I_1 = -50$ mA;
(b) $I_T = 70$ mA.

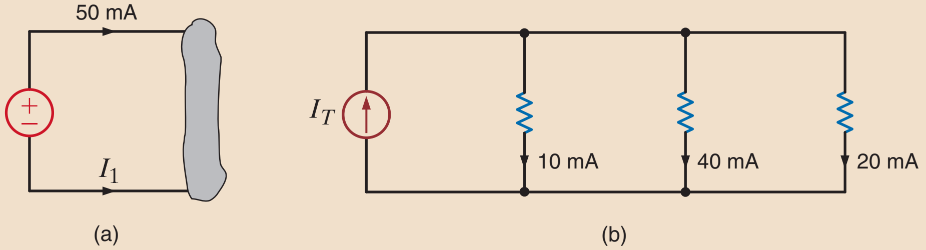


Figure E2.4

E2.5 Find (a) I_1 in the network in Fig. E2.5a and (b) I_1 and I_2 in the circuit in Fig. E2.5b.

ANSWER: (a) $I_1 = 6$ mA;
(b) $I_1 = 8$ mA and
$I_2 = 5$ mA.

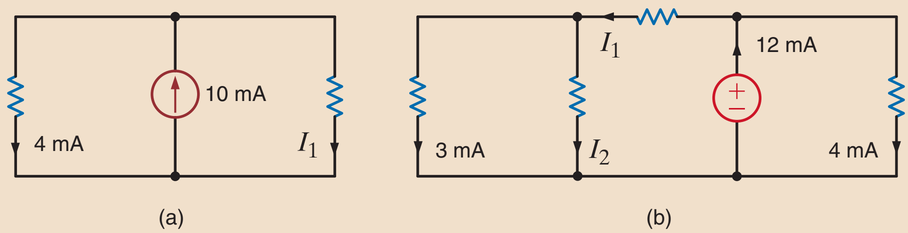


Figure E2.5

E2.6 Find the current i_x in the circuits in Fig. E2.6.

ANSWER: (a) $i_x = 4$ mA;
(b) $i_x = 12$ mA.

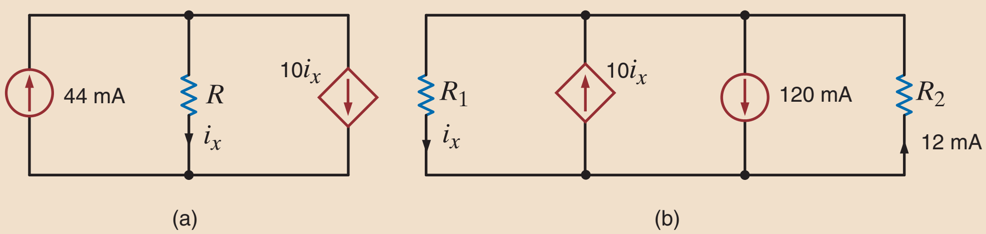


Figure E2.6

sum to zero the increases and decreases in energy level. Therefore, it is important we keep track of whether the energy level is increasing or decreasing as we go through each element.

In applying KVL, we must traverse any loop in the circuit and sum to zero the increases and decreases in energy level. At this point, we have a decision to make. Do we want to consider a decrease in energy level as positive or negative? We will adopt a policy of considering a decrease in energy level as positive and an increase in energy level as negative. As we move around a loop, we encounter the plus sign first for a decrease in energy level and a negative sign first for an increase in energy level.

Finally, we employ the convention V_{ab} to indicate the voltage of point a with respect to point b: that is, the variable for the voltage between point a and point b, with point a considered positive relative to point b. Since the potential is measured between two points, it is convenient to use an arrow between the two points, with the head of the arrow located at the positive node. Note that the double-subscript notation, the + and − notation, and the single-headed arrow notation are all the same if the head of the arrow is pointing toward the positive terminal and the first subscript in the double-subscript notation. All of these equivalent forms

EXAMPLE 2.9

Consider the circuit shown in Fig. 2.9. If V_{R_1} and V_{R_2} are known quantities, let us find V_{R_3}.

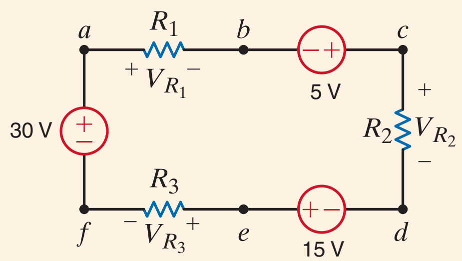


Figure 2.9 Circuit used to illustrate KVL.

SOLUTION Starting at point a in the network and traversing it in a clockwise direction, we obtain the equation

$$+V_{R_1} - 5 + V_{R_2} - 15 + V_{R_3} - 30 = 0$$

which can be written as

$$\begin{aligned} +V_{R_1} + V_{R_2} + V_{R_3} &= 5 + 15 + 30 \\ &= 50 \end{aligned}$$

Now suppose that V_{R_1} and V_{R_2} are known to be 18 V and 12 V, respectively. Then $V_{R_3} = 20$ V.

EXAMPLE 2.10

Consider the network in Fig. 2.10.

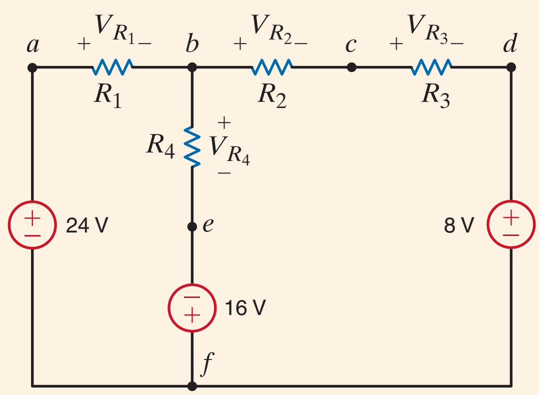


Figure 2.10 Circuit used to explain KVL.

Let us demonstrate that only two of the three possible loop equations are linearly independent.

SOLUTION Note that this network has three closed paths: the left loop, right loop, and outer loop. Applying our policy for writing KVL equations and traversing the left loop starting at point a, we obtain

$$V_{R_1} + V_{R_4} - 16 - 24 = 0$$

The corresponding equation for the right loop starting at point b is

$$V_{R_2} + V_{R_3} + 8 + 16 - V_{R_4} = 0$$

The equation for the outer loop starting at point a is

$$V_{R_1} + V_{R_2} + V_{R_3} + 8 - 24 = 0$$

Note that if we add the first two equations, we obtain the third equation. Therefore, as we indicated in Example 2.5, the three equations are not linearly independent. Once again, we will address this issue in the next chapter and demonstrate that we need only the first two equations to solve for the voltages in the circuit.

for labeling voltages are shown in Fig. 2.11. The usefulness of the arrow notation stems from the fact that we may want to label the voltage between two points that are far apart in a network. In this case, the other notations are often confusing.

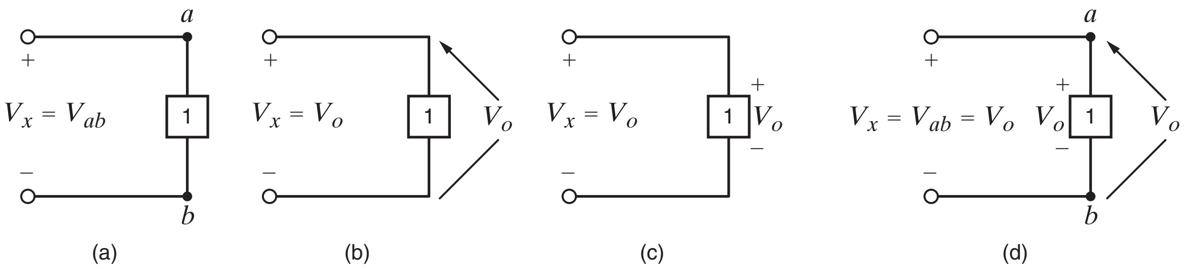


Figure 2.11 Equivalent forms for labeling voltage.

EXAMPLE 2.11

Consider the network in Fig. 2.12a. Let us apply KVL to determine the voltage between two points. Specifically, in terms of the double-subscript notation, let us find V_{ae} and V_{ec}.

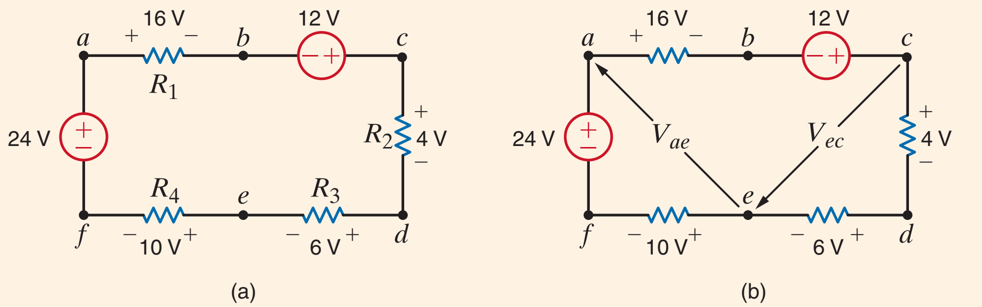


Figure 2.12 Network used in Example 2.11.

SOLUTION

The circuit is redrawn in Fig. 2.12b. Since points a and e as well as e and c are not physically close, the arrow notation is very useful. Our approach to determining the unknown voltage is to apply KVL with the unknown voltage in the closed path. Therefore, to determine V_{ae} we can use the path *aefa* or *abcdea*. The equations for the two paths in which V_{ae} is the only unknown are

$$V_{ae} + 10 - 24 = 0$$

and

$$16 - 12 + 4 + 6 - V_{ae} = 0$$

Note that both equations yield $V_{ae} = 14$ V. Even before calculating V_{ae}, we could calculate V_{ec} using the path *cdec* or *cefabc*. However, since V_{ae} is now known, we can also use the path *ceabc*. KVL for each of these paths is

$$4 + 6 + V_{ec} = 0$$
$$-V_{ec} + 10 - 24 + 16 - 12 = 0$$

and

$$-V_{ec} - V_{ae} + 16 - 12 = 0$$

Each of these equations yields $V_{ec} = -10$ V.

In general, the mathematical representation of Kirchhoff's voltage law is

$$\sum_{j=1}^{N} v_j(t) = 0 \qquad \textbf{2.8}$$

[hint] KVL is an extremely important and useful law.

where $v_j(t)$ is the voltage across the jth branch (with the proper reference direction) in a loop containing N voltages. This expression is analogous to Eq. (2.7) for Kirchhoff's current law.

EXAMPLE 2.12

Given the network in Fig. 2.13 containing a dependent source, let us write the KVL equations for the two closed paths *abda* and *bcdb*.

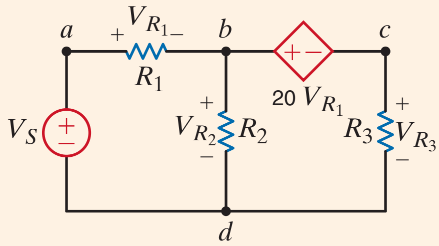


Figure 2.13 Network containing a dependent source.

SOLUTION The two KVL equations are

$$V_{R_1} + V_{R_2} - V_S = 0$$

$$20V_{R_1} + V_{R_3} - V_{R_2} = 0$$

Learning Assessments

E2.7 Find I_x and I_1 in Fig. E2.7.

ANSWER: $I_x = 2$ mA, $I_1 = 4$ mA.

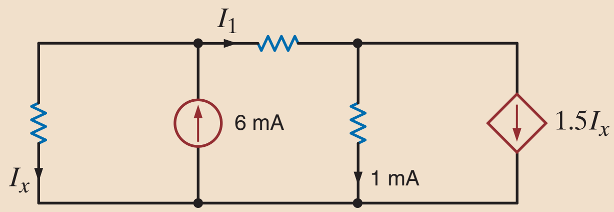


Figure E2.7

E2.8 Find V_{ad} and V_{eb} in the network in Fig. E2.8.

ANSWER: $V_{ad} = 26$ V, $V_{eb} = 10$ V.

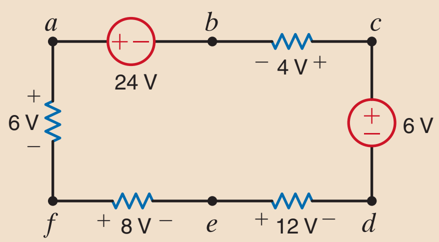


Figure E2.8

E2.9 Find V_{bd} in the circuit in Fig. E2.9.

ANSWER: $V_{bd} = 11$ V.

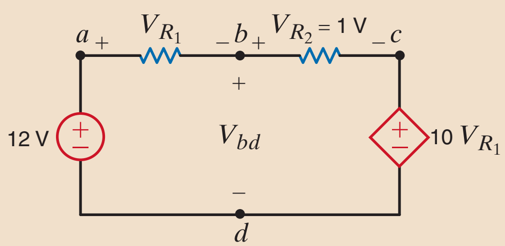


Figure E2.9

Before proceeding with the analysis of simple circuits, it is extremely important that we emphasize a subtle but very critical point. Ohm's law as defined by the equation $V = IR$ refers to the relationship between the voltage and current as defined in Fig. 2.14a. If the direction of either the current or the voltage, but not both, is reversed, the relationship between the current and the voltage would be $V = -IR$. In a similar manner, given the circuit in Fig. 2.14b, if the polarity of the voltage between the terminals A and B is specified as shown, then the direction of the current I is from point B through R to point A. Likewise, in Fig. 2.14c, if the direction of the current is specified as shown, then the polarity of the voltage must be such that point D is at a higher potential than point C and, therefore, the arrow representing the voltage V is from point C to point D.

[hint]

The subtleties associated with Ohm's law, as described here, are important and must be adhered to in order to ensure that the variables have the proper sign.

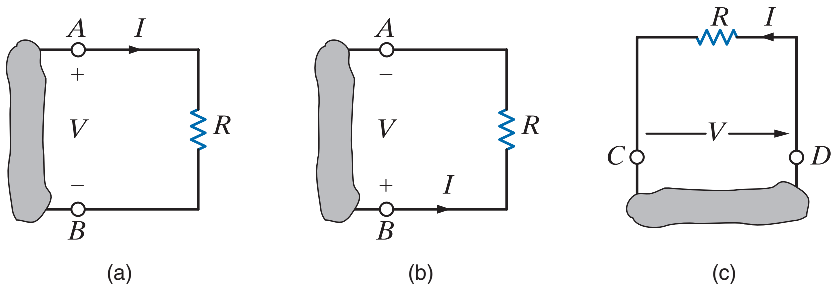


Figure 2.14 Circuits used to explain Ohm's law.

2.3 Single-Loop Circuits

VOLTAGE DIVISION At this point we can begin to apply the laws presented earlier to the analysis of simple circuits. To begin, we examine what is perhaps the simplest circuit—a single closed path, or loop, of elements.

Applying KCL to every node in a single-loop circuit reveals that the same current flows through all elements. We say that these elements are connected in series because they carry the same current. We will apply Kirchhoff's voltage law and Ohm's law to the circuit to determine various quantities in the circuit.

Our approach will be to begin with a simple circuit and then generalize the analysis to more complicated ones. The circuit shown in Fig. 2.15 will serve as a basis for discussion. This circuit consists of an independent voltage source that is in series with two resistors. We have assumed that the current flows in a clockwise direction. If this assumption is correct, the solution of the equations that yields the current will produce a positive value. If the current is actually flowing in the opposite direction, the value of the current variable will simply be negative, indicating that the current is flowing in a direction opposite to that assumed. We have also made voltage polarity assignments for v_{R_1} and v_{R_2}. These assignments have been made using the convention employed in our discussion of Ohm's law and our choice for the direction of $i(t)$—that is, the convention shown in Fig. 2.14a.

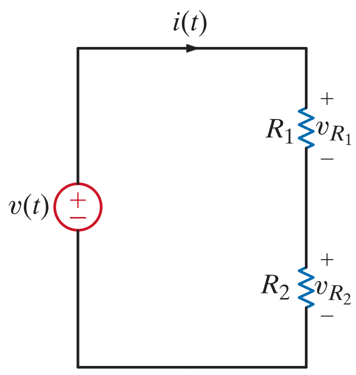


Figure 2.15 Single-loop circuit.

Applying Kirchhoff's voltage law to this circuit yields

$$-v(t) + v_{R_1} + v_{R_2} = 0$$

or

$$v(t) = v_{R_1} + v_{R_2}$$

However, from Ohm's law we know that

$$v_{R_1} = R_1 i(t)$$
$$v_{R_2} = R_2 i(t)$$

Therefore,

$$v(t) = R_1 i(t) + R_2 i(t)$$

Solving the equation for $i(t)$ yields

$$i(t) = \frac{v(t)}{R_1 + R_2} \quad \mathbf{2.9}$$

Knowing the current, we can now apply Ohm's law to determine the voltage across each resistor:

$$v_{R_1} = R_1 i(t) = R_1\left[\frac{v(t)}{R_1 + R_2}\right] = \frac{R_1}{R_1 + R_2} v(t) \tag{2.10}$$

[hint]

The manner in which voltage divides between two series resistors.

Similarly,

$$v_{R_2} = \frac{R_2}{R_1 + R_2} v(t) \tag{2.11}$$

Though simple, Eqs. (2.10) and (2.11) are very important because they describe the operation of what is called a *voltage divider*. In other words, the source voltage $v(t)$ *is divided between the resistors* R_1 *and* R_2 *in direct proportion to their resistances*.

In essence, if we are interested in the voltage across the resistor R_1, we bypass the calculation of the current $i(t)$ and simply multiply the input voltage $v(t)$ by the ratio

$$\frac{R_1}{R_1 + R_2}$$

As illustrated in Eq. (2.10), we are using the current in the calculation, but not explicitly.

Note that the equations satisfy Kirchhoff's voltage law, since

$$-v(t) + \frac{R_1}{R_1 + R_2} v(t) + \frac{R_2}{R_1 + R_2} v(t) = 0$$

EXAMPLE 2.13

Consider the circuit shown in Fig. 2.16. The circuit is identical to Fig. 2.15 except that R_1 is a variable resistor such as the volume control for a radio or television set. Suppose that $V_S = 9$ V, $R_1 = 90$ kΩ, and $R_2 = 30$ kΩ.

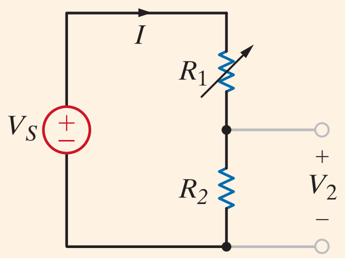


Figure 2.16 Voltage-divider circuit.

Let us examine the change in both the voltage across R_2 and the power absorbed in this resistor as R_1 is changed from 90 kΩ to 15 kΩ.

SOLUTION Since this is a voltage-divider circuit, the voltage V_2 can be obtained directly as

$$V_2 = \left[\frac{R_2}{R_1 + R_2}\right] V_S = \left[\frac{30\text{k}}{90\text{k} + 30\text{k}}\right](9) = 2.25 \text{ V}$$

Now suppose that the variable resistor is changed from 90 kΩ to 15 kΩ. Then

$$V_2 = \left[\frac{30\text{k}}{30\text{k} + 15\text{k}}\right]9$$
$$= 6 \text{ V}$$

The direct voltage-divider calculation is equivalent to determining the current I and then using Ohm's law to find V_2. Note that the larger voltage is across the larger resistance. This voltage-divider concept and the simple circuit we have employed to describe it are very useful because, as will be shown later, more complicated circuits can be reduced to this form.

Finally, let us determine the instantaneous power absorbed by the resistor R_2 under the two conditions $R_1 = 90\text{ k}\Omega$ and $R_1 = 15\text{ k}\Omega$. For the case $R_1 = 90\text{ k}\Omega$, the power absorbed by R_2 is

$$P_2 = I^2R_2 = \left(\frac{9}{120\text{k}}\right)^2(30\text{k})$$
$$= 0.169 \text{ mW}$$

In the second case

$$P_2 = \left(\frac{9}{45\text{k}}\right)^2(30\text{k})$$
$$= 1.2 \text{ mW}$$

The current in the first case is 75 μA, and in the second case it is 200 μA. Since the power absorbed is a function of the square of the current, the power absorbed in the two cases is quite different.

Let us now demonstrate the practical utility of this simple voltage-divider network.

EXAMPLE 2.14

Consider the circuit in Fig. 2.17a, which is an approximation of a high-voltage dc transmission facility. We have assumed that the bottom portion of the transmission line is a perfect conductor and will justify this assumption in the next chapter. The load can be represented by a resistor of value 183.5 Ω. Therefore, the equivalent circuit of this network is shown in Fig. 2.17b.

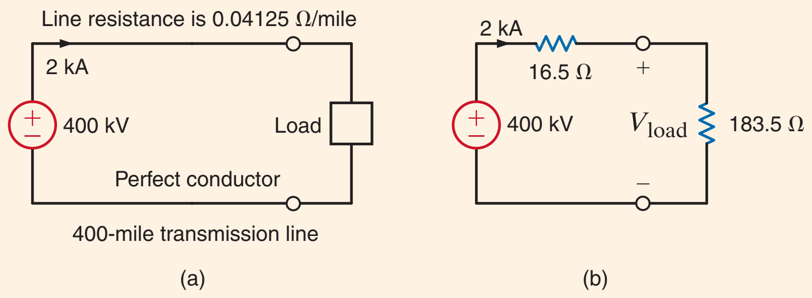


Figure 2.17 A high-voltage dc transmission facility.

Let us determine both the power delivered to the load and the power losses in the line.

SOLUTION

Using voltage division, the load voltage is

$$V_{\text{load}} = \left[\frac{183.5}{183.5 + 16.5}\right]400\text{k}$$
$$= 367 \text{ kV}$$

The input power is 800 MW and the power transmitted to the load is

$$P_{\text{load}} = I^2 R_{\text{load}} = 734 \text{ MW}$$

Therefore, the power loss in the transmission line is

$$P_{\text{line}} = P_{\text{in}} - P_{\text{load}} = I^2 R_{\text{line}} = 66 \text{ MW}$$

Since $P = VI$, suppose now that the utility company supplied power at 200 kV and 4 kA. What effect would this have on our transmission network? Without making a single calculation, we know that because power is proportional to the square of the current, there would be a large increase in the power loss in the line and, therefore, the efficiency of the facility would decrease substantially. That is why, in general, we transmit power at high voltage and low current.

MULTIPLE-SOURCE/RESISTOR NETWORKS At this point we wish to extend our analysis to include a multiplicity of voltage sources and resistors. For example, consider the circuit shown in Fig. 2.18a. Here we have assumed that the current flows in a clockwise direction, and we have defined the variable $i(t)$ accordingly. This may or may not be the case, depending on the value of the various voltage sources. Kirchhoff's voltage law for this circuit is

$$+v_{R_1} + v_2(t) - v_3(t) + v_{R_2} + v_4(t) + v_5(t) - v_1(t) = 0$$

or, using Ohm's law,

$$(R_1 + R_2)i(t) = v_1(t) - v_2(t) + v_3(t) - v_4(t) - v_5(t)$$

which can be written as

$$(R_1 + R_2)i(t) = v(t)$$

where

$$v(t) = v_1(t) + v_3(t) - [v_2(t) + v_4(t) + v_5(t)]$$

so that under the preceding definitions, Fig. 2.18a is equivalent to Fig. 2.18b. In other words, the sum of several voltage sources in series can be replaced by one source whose value is the algebraic sum of the individual sources. This analysis can, of course, be generalized to a circuit with N series sources.

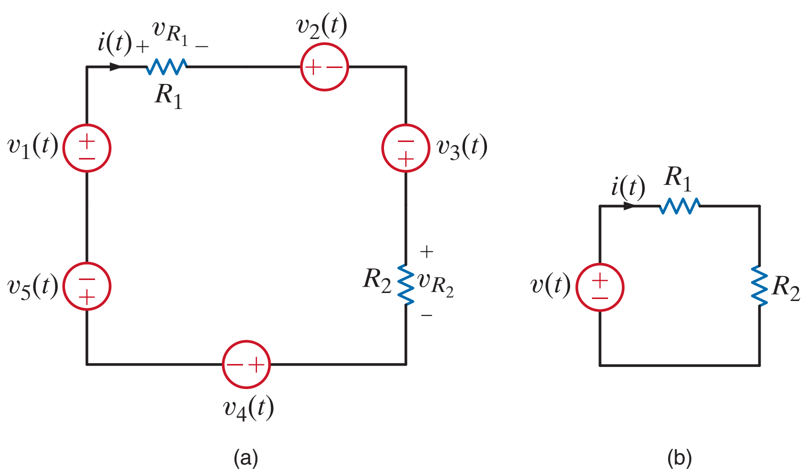


Figure 2.18 Equivalent circuits with multiple sources.

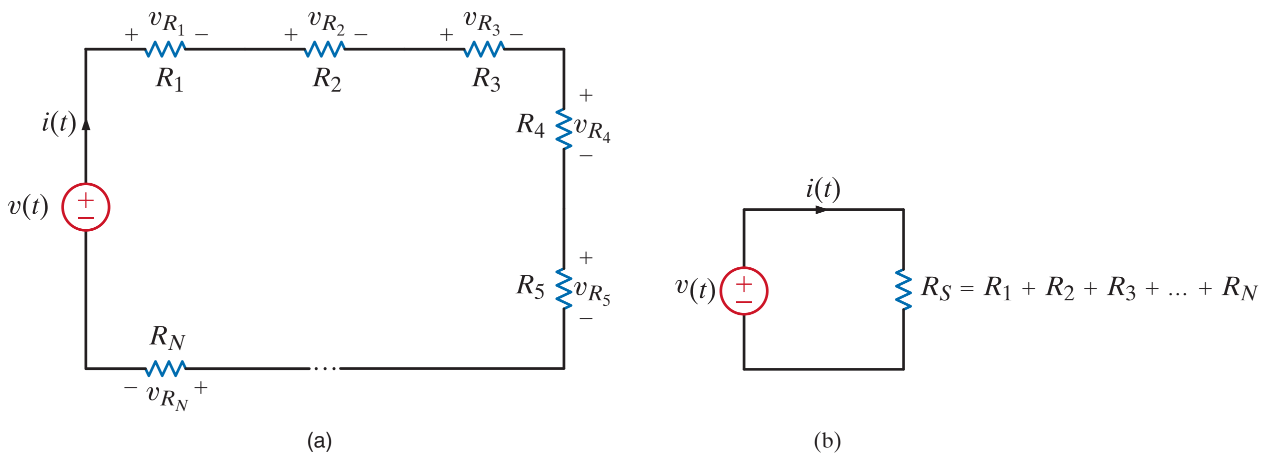


Figure 2.19

Equivalent circuits.

Now consider the circuit with N resistors in series, as shown in Fig. 2.19a. Applying Kirchhoff's voltage law to this circuit yields

$$\begin{aligned} v(t) &= v_{R_1} + v_{R_2} + \cdots + v_{R_N} \\ &= R_1 i(t) + R_2 i(t) + \cdots + R_N i(t) \end{aligned}$$

and therefore,

$$v(t) = R_S i(t) \tag{2.12}$$

where

$$R_S = R_1 + R_2 + \cdots + R_N \tag{2.13}$$

and hence,

$$i(t) = \frac{v(t)}{R_S} \tag{2.14}$$

Note also that for any resistor R_i in the circuit, the voltage across R_i is given by the expression

$$v_{R_i} = \frac{R_i}{R_S} v(t) \tag{2.15}$$

which is the voltage-division property for multiple resistors in series.

Equation (2.13) illustrates that *the equivalent resistance of N resistors in series is simply the sum of the individual resistances.* Thus, using Eq. (2.13), we can draw the circuit in Fig. 2.19b as an equivalent circuit for the one in Fig. 2.19a.

EXAMPLE 2.15

Given the circuit in Fig. 2.20a, let us find I, V_{bd}, and the power absorbed by the 30-kΩ resistor. Finally, let us use voltage division to find V_{bc}.

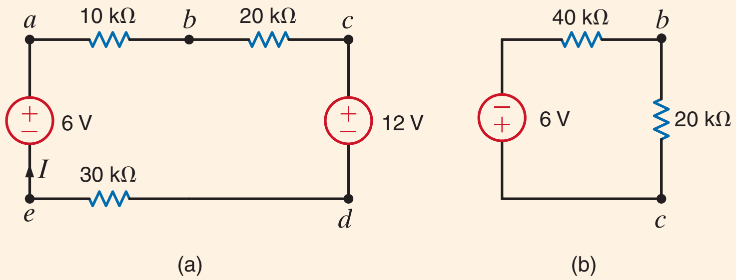


Figure 2.20

Circuit used in Example 2.15.

SOLUTION KVL for the network yields the equation

$$\begin{aligned} 10\text{k}I + 20\text{k}I + 12 + 30\text{k}I - 6 &= 0 \\ 60\text{k}I &= -6 \\ I &= -0.1 \text{ mA} \end{aligned}$$

Therefore, the magnitude of the current is 0.1 mA, but its direction is opposite to that assumed.

The voltage V_{bd} can be calculated using either of the closed paths *abdea* or *bcdb*. The equations for both cases are

$$10\text{k}I + V_{bd} + 30\text{k}I - 6 = 0$$

and

$$20\text{k}I + 12 - V_{bd} = 0$$

Using $I = -0.1$ mA in either equation yields $V_{bd} = 10$ V. Finally, the power absorbed by the 30-kΩ resistor is

$$P = I^2R = 0.3 \text{ mW}$$

Now from the standpoint of determining the voltage V_{bc}, we can simply add the sources since they are in series, add the remaining resistors since they are in series, and reduce the network to that shown in Fig. 2.20b. Then

$$\begin{aligned} V_{bc} &= \frac{20\text{k}}{20\text{k} + 40\text{k}}(-6) \\ &= -2 \text{ V} \end{aligned}$$

EXAMPLE 2.16

A dc transmission facility is modeled by the approximate circuit shown in Fig. 2.21. If the load voltage is known to be $V_{\text{load}} = 458.3$ kV, we wish to find the voltage at the sending end of the line and the power loss in the line.

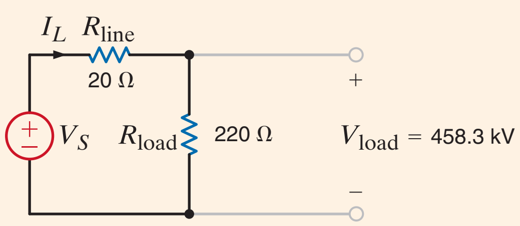


Figure 2.21 Circuit used in Example 2.16.

SOLUTION Knowing the load voltage and load resistance, we can obtain the line current using Ohm's law:

$$\begin{aligned} I_L &= 458.3\text{k}/220 \\ &= 2.083 \text{ kA} \end{aligned}$$

The voltage drop across the line is

$$\begin{aligned} V_{\text{line}} &= (I_L)(R_{\text{line}}) \\ &= 41.66 \text{ kV} \end{aligned}$$

Now, using KVL,

$$V_S = V_{\text{line}} + V_{\text{load}}$$
$$= 500 \text{ kV}$$

Note that since the network is simply a voltage-divider circuit, we could obtain V_S immediately from our knowledge of R_{line}, R_{load}, and V_{load}. That is,

$$V_{\text{load}} = \left[\frac{R_{\text{load}}}{R_{\text{load}} + R_{\text{line}}}\right] V_S$$

and V_S is the only unknown in this equation.

The power absorbed by the line is

$$P_{\text{line}} = I_L^2 R_{\text{line}}$$
$$= 86.79 \text{ MW}$$

Problem-Solving Strategy

Single-Loop Circuits

Step 1. Define a current $i(t)$. We know from KCL that there is only one current for a single-loop circuit. This current is assumed to be flowing either clockwise or counterclockwise around the loop.

Step 2. Using Ohm's law, define a voltage across each resistor in terms of the defined current.

Step 3. Apply KVL to the single-loop circuit.

Step 4. Solve the single KVL equation for the current $i(t)$. If $i(t)$ is positive, the current is flowing in the direction assumed; if not, then the current is actually flowing in the opposite direction.

Learning Assessments

E2.10 Find I and V_{bd} in the circuit in Fig. E2.10.

ANSWER: $I = -0.05$ mA and $V_{bd} = 10$ V.

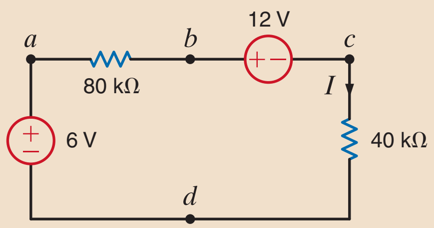


Figure E2.10

E2.11 In the network in Fig. E2.11, if V_{ad} is 3 V, find V_S.

ANSWER: $V_S = 9$ V.

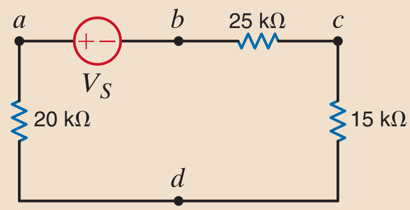


Figure E2.11

2.4 Single-Node-Pair Circuits

CURRENT DIVISION An important circuit is the single-node-pair circuit. If we apply KVL to every loop in a single-node-pair circuit, we discover that all of the elements have the same voltage across them and, therefore, are said to be connected in parallel. We will, however, apply Kirchhoff's current law and Ohm's law to determine various unknown quantities in the circuit.

Following our approach with the single-loop circuit, we will begin with the simplest case and then generalize our analysis. Consider the circuit shown in Fig. 2.22. Here we have an independent current source in parallel with two resistors.

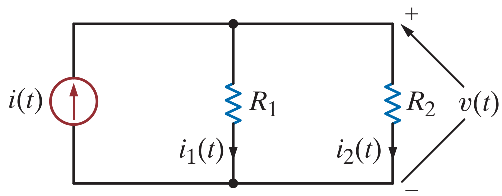


Figure 2.22
Simple parallel circuit.

Since all of the circuit elements are in parallel, the voltage $v(t)$ appears across each of them. Furthermore, an examination of the circuit indicates that the current $i(t)$ is into the upper node of the circuit and the currents $i_1(t)$ and $i_2(t)$ are out of the node. Since KCL essentially states that what goes in must come out, the question we must answer is how $i_1(t)$ and $i_2(t)$ divide the input current $i(t)$.

Applying Kirchhoff's current law to the upper node, we obtain

$$i(t) = i_1(t) + i_2(t)$$

and, employing Ohm's law, we have

$$\begin{aligned} i(t) &= \frac{v(t)}{R_1} + \frac{v(t)}{R_2} \\ &= \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v(t) \\ &= \frac{v(t)}{R_p} \end{aligned}$$

[hint]
The parallel resistance equation.

where

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} \tag{2.16}$$

$$R_p = \frac{R_1 R_2}{R_1 + R_2} \tag{2.17}$$

Therefore, the equivalent resistance of two resistors connected in parallel is equal to the product of their resistances divided by their sum. Note also that this equivalent resistance R_p is always less than either R_1 or R_2. Hence, by connecting resistors in parallel we reduce the overall resistance. In the special case when $R_1 = R_2$, the equivalent resistance is equal to half of the value of the individual resistors.

The manner in which the current $i(t)$ from the source divides between the two branches is called *current division* and can be found from the preceding expressions. For example,

$$\begin{aligned} v(t) &= R_p i(t) \\ &= \frac{R_1 R_2}{R_1 + R_2} i(t) \end{aligned} \tag{2.18}$$

and

$$i_1(t) = \frac{v(t)}{R_1}$$

$$i_1(t) = \frac{R_2}{R_1 + R_2} i(t) \quad \textbf{2.19}$$

and

$$i_2(t) = \frac{v(t)}{R_2}$$

$$= \frac{R_1}{R_1 + R_2} i(t) \quad \textbf{2.20}$$

Eqs. (2.19) and (2.20) are mathematical statements of the current-division rule.

[hint]

The manner in which current divides between two parallel resistors.

EXAMPLE 2.17

Given the network in Fig. 2.23a, let us find I_1, I_2, and V_o.

SOLUTION

First, it is important to recognize that the current source feeds two parallel paths. To emphasize this point, the circuit is redrawn as shown in Fig. 2.23b. Applying current division, we obtain

$$I_1 = \left[\frac{40\text{k} + 80\text{k}}{60\text{k} + (40\text{k} + 80\text{k})}\right](0.9 \times 10^{-3})$$

$$= 0.6 \text{ mA}$$

and

$$I_2 = \left[\frac{60\text{k}}{60\text{k} + (40\text{k} + 80\text{k})}\right](0.9 \times 10^{-3})$$

$$= 0.3 \text{ mA}$$

Note that the larger current flows through the smaller resistor, and vice versa. In addition, note that if the resistances of the two paths are equal, the current will divide equally between them. KCL is satisfied since $I_1 + I_2 = 0.9$ mA.

The voltage V_o can be derived using Ohm's law as

$$V_o = 80\text{k}I_2$$

$$= 24 \text{ V}$$

The problem can also be approached in the following manner. The total resistance seen by the current source is 40 kΩ, that is, 60 kΩ in parallel with the series combination of 40 kΩ and 80 kΩ, as shown in Fig. 2.23c. The voltage across the current source is then

$$V_1 = (0.9 \times 10^{-3})40\text{k}$$

$$= 36 \text{ V}$$

Now that V_1 is known, we can apply voltage division to find V_o:

$$V_o = \left(\frac{80\text{k}}{80\text{k} + 40\text{k}}\right)V_1$$

$$= \left(\frac{80\text{k}}{120\text{k}}\right)36$$

$$= 24 \text{ V}$$

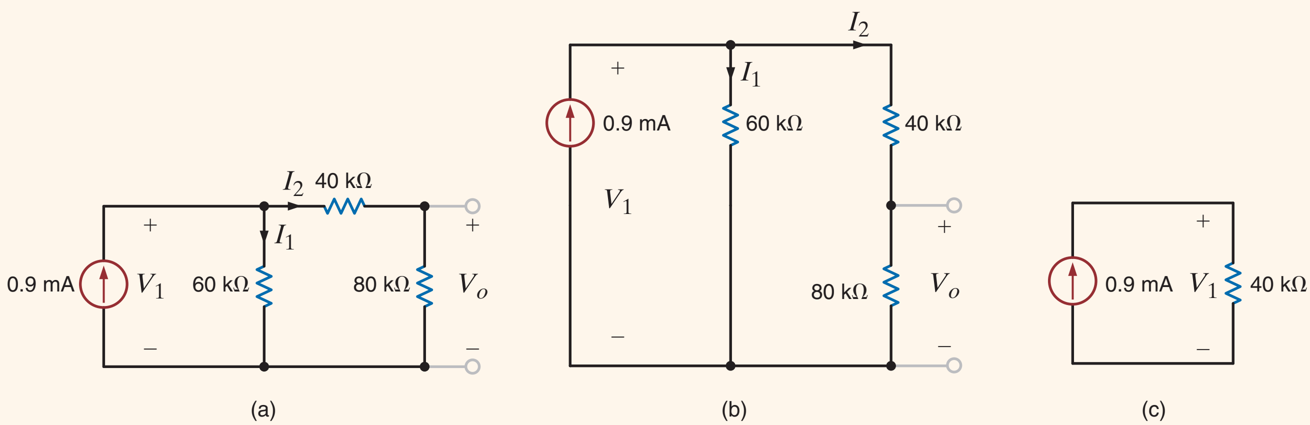


Figure 2.23

Circuits used in Example 2.17.

EXAMPLE 2.18

A typical car stereo consists of a 2-W audio amplifier and two speakers represented by the diagram shown in Fig. 2.24a. The output circuit of the audio amplifier is in essence a 430-mA current source, and each speaker has a resistance of 4 Ω. Let us determine the power absorbed by the speakers.

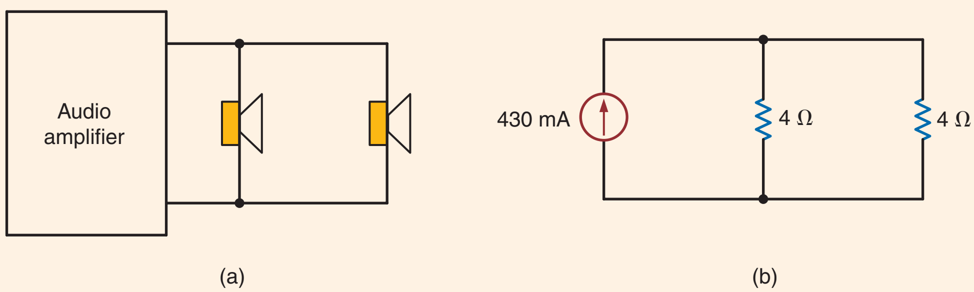
SOLUTION

The audio system can be modeled as shown in Fig. 2.24b. Since the speakers are both 4-Ω devices, the current will split evenly between them, and the power absorbed by each speaker is

$$
\begin{aligned}
P &= I^2 R \\
&= (215 \times 10^{-3})^2 (4) \\
&= 184.9 \text{ mW}
\end{aligned}
$$

Figure 2.24

Circuits used in Example 2.18.



Learning Assessment

E2.12 Find the currents I_1 and I_2 and the power absorbed by the 40-kΩ resistor in the network in Fig. E2.12.

ANSWER: $I_1 = 12$ mA, $I_2 = -4$ mA, and $P_{40\text{ k}\Omega} = 5.76$ W.

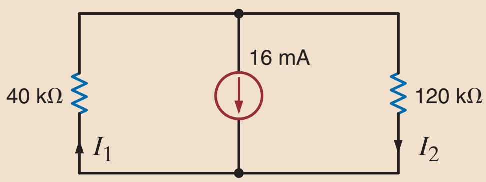


Figure E2.12

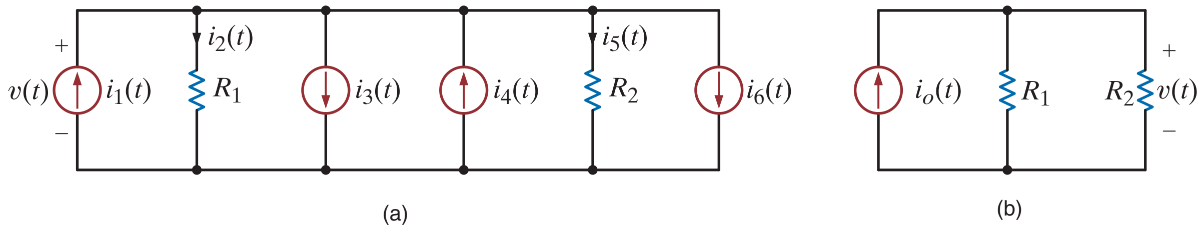


Figure 2.25
Equivalent circuits.

MULTIPLE-SOURCE/RESISTOR NETWORKS Let us now extend our analysis to include a multiplicity of current sources and resistors in parallel. For example, consider the circuit shown in Fig. 2.25a. We have assumed that the upper node is $v(t)$ volts positive with respect to the lower node. Applying Kirchhoff's current law to the upper node yields

$$i_1(t) - i_2(t) - i_3(t) + i_4(t) - i_5(t) - i_6(t) = 0$$

or

$$i_1(t) - i_3(t) + i_4(t) - i_6(t) = i_2(t) + i_5(t)$$

The terms on the left side of the equation all represent sources that can be combined algebraically into a single source; that is,

$$i_o(t) = i_1(t) - i_3(t) + i_4(t) - i_6(t)$$

which effectively reduces the circuit in Fig. 2.25a to that in Fig. 2.25b. We could, of course, generalize this analysis to a circuit with N current sources. Using Ohm's law, we can express the currents on the right side of the equation in terms of the voltage and individual resistances so that the KCL equation reduces to

$$i_o(t) = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)v(t)$$

Now consider the circuit with N resistors in parallel, as shown in Fig. 2.26a. Applying Kirchhoff's current law to the upper node yields

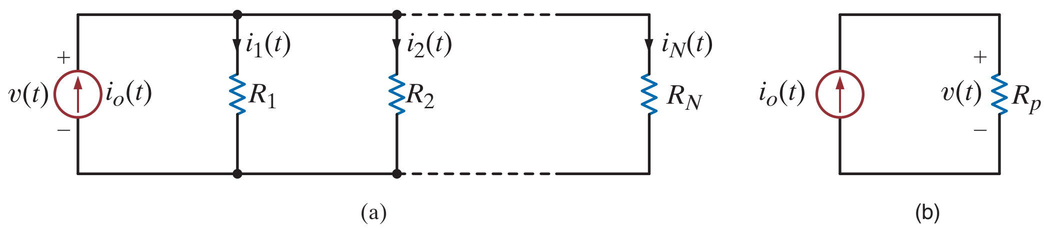


Figure 2.26
Equivalent circuits.

$$\begin{aligned} i_o(t) &= i_1(t) + i_2(t) + \cdots + i_N(t) \\ &= \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)v(t) \end{aligned} \qquad \textbf{2.21}$$

or

$$i_o(t) = \frac{v(t)}{R_p} \qquad \textbf{2.22}$$

where

$$\frac{1}{R_p} = \sum_{i=1}^{N} \frac{1}{R_i} \qquad \textbf{2.23}$$

so that as far as the source is concerned, Fig. 2.26a can be reduced to an equivalent circuit, as shown in Fig. 2.26b.

The current division for any branch can be calculated using Ohm's law and the preceding equations. For example, for the jth branch in the network of Fig. 2.26a,

$$i_j(t) = \frac{v(t)}{R_j}$$

Using Eq. (2.22), we obtain

$$i_j(t) = \frac{R_p}{R_j} i_o(t) \tag{2.24}$$

which defines the current-division rule for the general case.

EXAMPLE 2.19

Given the circuit in Fig. 2.27a, we wish to find the current in the 12-kΩ load resistor.

SOLUTION

To simplify the network in Fig. 2.27a, we add the current sources algebraically and combine the parallel resistors in the following manner:

$$\frac{1}{R_p} = \frac{1}{18\text{k}} + \frac{1}{9\text{k}} + \frac{1}{12\text{k}}$$

$$R_p = 4\ \text{k}\Omega$$

Using these values we can reduce the circuit in Fig. 2.27a to that in Fig. 2.27b. Now, applying current division, we obtain

$$I_L = -\left[\frac{4\text{k}}{4\text{k} + 12\text{k}}\right](1 \times 10^{-3})$$
$$= -0.25\ \text{mA}$$

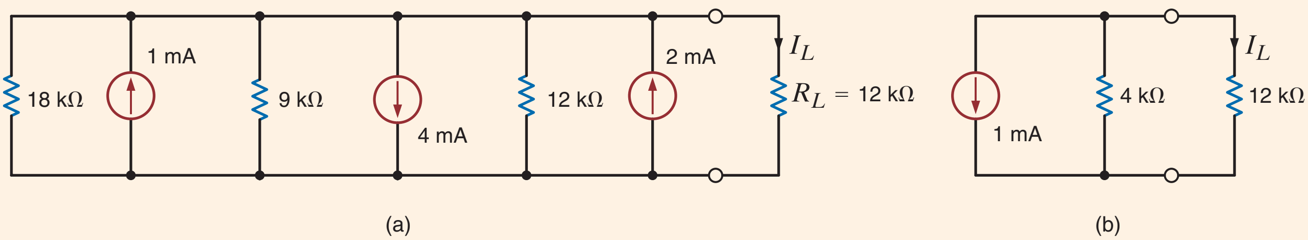


Figure 2.27 Circuits used in Example 2.19.

Problem-Solving Strategy

Single-Node-Pair Circuits

Step 1. Define a voltage $v(t)$ between the two nodes in this circuit. We know from KVL that there is only one voltage for a single-node-pair circuit. A polarity is assigned to the voltage such that one of the nodes is assumed to be at a higher potential than the other node, which we will call the reference node.

Step 2. Using Ohm's law, define a current flowing through each resistor in terms of the defined voltage.

Step 3. Apply KCL at one of the two nodes in the circuit.

Step 4. Solve the single KCL equation for $v(t)$. If $v(t)$ is positive, then the reference node is actually at a lower potential than the other node; if not, the reference node is actually at a higher potential than the other node

Learning Assessment

E2.13 Find the power absorbed by the 6-kΩ resistor in the network in Fig. E2.13.

ANSWER: $P = 2.67$ mW.

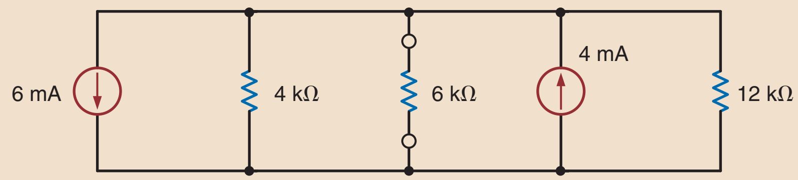


Figure E2.13

2.5 Series and Parallel Resistor Combinations

We have shown in our earlier developments that the equivalent resistance of N resistors in series is

$$R_S = R_1 + R_2 + \cdots + R_N \tag{2.25}$$

and the equivalent resistance of N resistors in parallel is found from

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} \tag{2.26}$$

Let us now examine some combinations of these two cases.

EXAMPLE 2.20

We wish to determine the resistance at terminals A-B in the network in Fig. 2.28a.

SOLUTION

Starting at the opposite end of the network from the terminals and combining resistors as shown in the sequence of circuits in Fig. 2.28, we find that the equivalent resistance at the terminals is 5 kΩ.

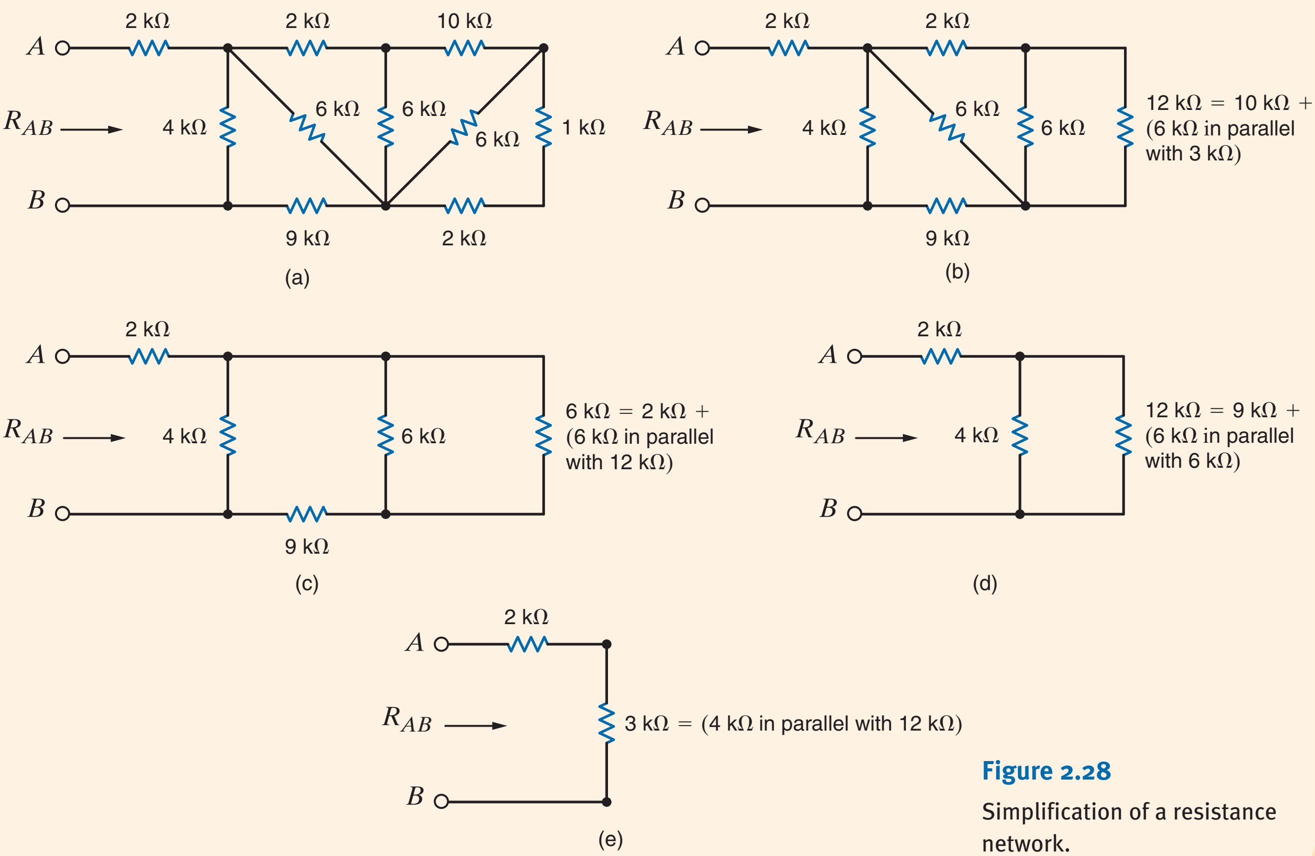


Figure 2.28 Simplification of a resistance network.

Learning Assessment

E2.14 Find the equivalent resistance at the terminals *A*-*B* in the network in Fig. E2.14.

ANSWER: $R_{AB} = 22\ \text{k}\Omega$.

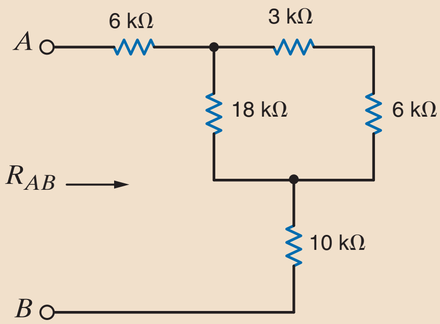


Figure E2.14

Problem-Solving Strategy

Simplifying Resistor Combinations

When trying to determine the equivalent resistance at a pair of terminals of a network composed of an interconnection of numerous resistors, it is recommended that the analysis begin at the end of the network opposite the terminals. Two or more resistors are combined to form a single resistor, thus simplifying the network by reducing the number of components as the analysis continues in a steady progression toward the terminals. The simplification involves the following:

Step 1. ***Resistors in series.*** Resistors R_1 and R_2 are in series if they are connected end to end with one common node and carry exactly the same current. They can then be combined into a single resistor R_S, where $R_S = R_1 + R_2$.

Step 2. ***Resistors in parallel.*** Resistors R_1 and R_2 are in parallel if they are connected to the same two nodes and have exactly the same voltage across their terminals. They can then be combined into a single resistor R_p, where $R_p = R_1R_2/(R_1 + R_2)$.

These two combinations are used repeatedly, as needed, to reduce the network to a single resistor at the pair of terminals.

Learning Assessment

E2.15 Find the equivalent resistance at the terminals *A*-*B* in the circuit in Fig. E2.15.

ANSWER: $R_{AB} = 3\ \text{k}\Omega$.

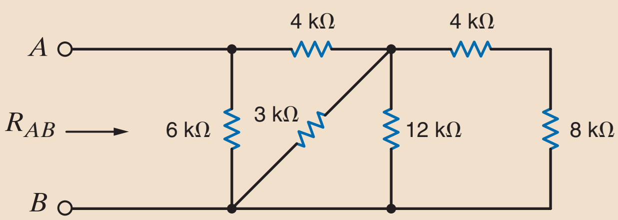


Figure E2.15

E2.16 Find R_{AB} in Fig. E2.16.

ANSWER: $R_{AB} = 12\ \text{k}\Omega$.

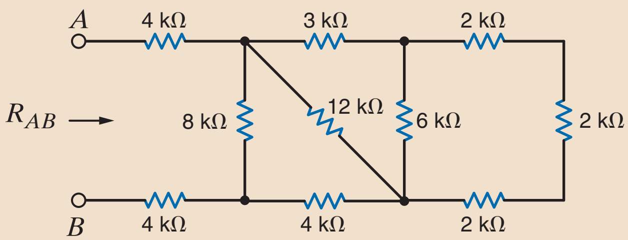


Figure E2.16

EXAMPLE 2.21

A standard dc current-limiting power supply shown in Fig. 2.29a provides 0–18 V at 3 A to a load. The voltage drop, V_R, across a resistor, R, is used as a current-sensing device, fed back to the power supply and used to limit the current I. That is, if the load is adjusted so that the current tries to exceed 3 A, the power supply will act to limit the current to that value. The feedback voltage, V_R, should typically not exceed 600 mV.

If we have a box of standard 0.1-Ω, 5-W resistors, let us determine the configuration of these resistors that will provide $V_R = 600$ mV when the current is 3 A.

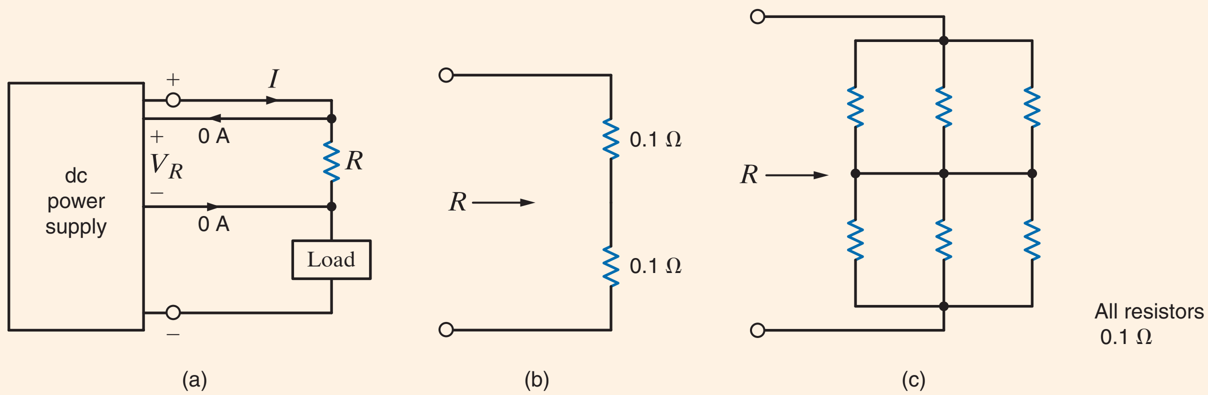


Figure 2.29 Circuits used in Example 2.21.

SOLUTION

Using Ohm's law, the value of R should be

$$\begin{aligned} R &= \frac{V_R}{I} \\ &= \frac{0.6}{3} \\ &= 0.2\ \Omega \end{aligned}$$

Therefore, two 0.1-Ω resistors connected in series, as shown in Fig. 2.29b, will provide the proper feedback voltage. Suppose, however, that the power supply current is to be limited to 9 A. The resistance required in this case to produce $V_R = 600$ mV is

$$\begin{aligned} R &= \frac{0.6}{9} \\ &= 0.0667\ \Omega \end{aligned}$$

We must now determine how to interconnect the 0.1-Ω resistor to obtain $R = 0.0667\ \Omega$. Since the desired resistance is less than the components available (i.e., 0.1-Ω), we must connect the resistors in some type of parallel configuration. Since all the resistors are of equal value, note that three of them connected in parallel would provide a resistance of one-third their value, or 0.0333 Ω. Then two such combinations connected in series, as shown in Fig. 2.29c, would produce the proper resistance.

Finally, we must check to ensure that the configurations in Figs. 2.29b and c have not exceeded the power rating of the resistors. In the first case, the current $I = 3$ A is present in each of the two series resistors. Therefore, the power absorbed in each resistor is

$$\begin{aligned} P &= I^2R \\ &= (3)^2(0.1) \\ &= 0.9 \text{ W} \end{aligned}$$

which is well within the 5-W rating of the resistors.

In the second case, the current $I = 9$ A. The resistor configuration for R in this case is a series combination of two sets of three parallel resistors of equal value. Using current division, we know that the current I will split equally among the three parallel paths and, hence, the current in each resistor will be 3 A. Therefore, once again, the power absorbed by each resistor is within its power rating.

RESISTOR SPECIFICATIONS Some important parameters that are used to specify resistors are the resistor's value, tolerance, and power rating. The tolerance specifications for resistors are typically 5% and 10%. A listing of standard resistor values with their specified tolerances is shown in Table 2.1.

The power rating for a resistor specifies the maximum power that can be dissipated by the resistor. Some typical power ratings for resistors are 1/4 W, 1/2 W, 1 W, 2 W, and so forth, up to very high values for high-power applications. Thus, in selecting a resistor for some particular application, one important selection criterion is the expected power dissipation.

TABLE 2.1 Standard resistor values for 5% and 10% tolerances (values available with a 10% tolerance shown in boldface)

1.0	**10**	**100**	**1.0k**	**10k**	**100k**	**1.0M**	**10M**
1.1	11	110	1.1k	11k	110k	**1.1M**	11M
1.2	**12**	**120**	**1.2k**	**12k**	**120k**	**1.2M**	**12M**
1.3	13	130	1.3k	13k	130k	1.3M	13M
1.5	**15**	**150**	**1.5k**	**15k**	**150k**	**1.5M**	**15M**
1.6	16	160	1.6k	16k	160k	1.6M	16M
1.8	**18**	**180**	**1.8k**	**18k**	**180k**	**1.8M**	**18M**
2.0	20	200	2.0k	20k	200k	2.0M	20M
2.2	**22**	**220**	**2.2k**	**22k**	**220k**	**2.2M**	**22M**
2.4	24	240	2.4k	24k	240k	2.4M	
2.7	**27**	**270**	**2.7k**	**27k**	**270k**	**2.7M**	
3.0	30	300	3.0k	30k	300k	3.0M	
3.3	**33**	**330**	**3.3k**	**33k**	**330k**	**3.3M**	
3.6	36	360	3.6k	36k	360k	3.6M	
3.9	**39**	**390**	**3.9k**	**39k**	**390k**	**3.9M**	
4.3	43	430	4.3k	43k	430k	4.3M	
4.7	**47**	**470**	**4.7k**	**47k**	**470k**	**4.7M**	
5.1	51	510	5.1k	51k	510k	5.1M	
5.6	**56**	**560**	**5.6k**	**56k**	**560k**	**5.6M**	
6.2	62	620	6.2k	62k	620k	6.2M	
6.8	**68**	**680**	**6.8k**	**68k**	**680k**	**6.8M**	
7.5	75	750	7.5k	75k	750k	7.5M	
8.2	**82**	**820**	**8.2k**	**82k**	**820k**	**8.2M**	
9.1	91	910	9.1k	91k	910k	9.1M	

EXAMPLE 2.22

Given the network in Fig. 2.30, we wish to find the range for both the current and power dissipation in the resistor if R is a 2.7-kΩ resistor with a tolerance of 10%.

SOLUTION

Using the equations $I = V/R = 10/R$ and $P = V^2/R = 100/R$, the minimum and maximum values for the resistor, current, and power are outlined next.

$$\text{Minimum resistor value} = R(1 - 0.1) = 0.9\,R = 2.43\text{ k}\Omega$$
$$\text{Maximum resistor value} = R(1 + 0.1) = 1.1\,R = 2.97\text{ k}\Omega$$
$$\text{Minimum current value} = 10/2970 = 3.37\text{ mA}$$
$$\text{Maximum current value} = 10/2430 = 4.12\text{ mA}$$
$$\text{Minimum power value} = 100/2970 = 33.7\text{ mW}$$
$$\text{Maximum power value} = 100/2430 = 41.2\text{ mW}$$

Thus, the ranges for the current and power are 3.37 mA to 4.12 mA and 33.7 mW to 41.2 mW, respectively.

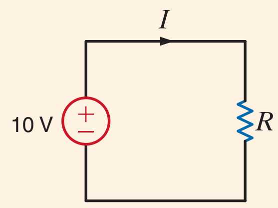


Figure 2.30
Circuit used in Example 2.22.

EXAMPLE 2.23

Given the network shown in Fig. 2.31: (a) find the required value for the resistor R; (b) use Table 2.1 to select a standard 10% tolerance resistor for R; (c) using the resistor selected in (b), determine the voltage across the 3.9-kΩ resistor; (d) calculate the percent error in the voltage V_1, if the standard resistor selected in (b) is used; and (e) determine the power rating for this standard component.

SOLUTION

a. Using KVL, the voltage across R is 19 V. Then using Ohm's law, the current in the loop is

$$\mathrm{I} = 5/3.9\text{k} = 1.282\text{ mA}$$

The required value of R is then

$$R = 19/0.001282 = 14.82\text{ k}\Omega$$

b. As shown in Table 2.1, the nearest standard 10% tolerance resistor is 15 kΩ.

c. Using the standard 15-kΩ resistor, the actual current in the circuit is

$$I = 24/18.9\text{k} = 1.2698\text{ mA}$$

and the voltage across the 3.9-kΩ resistor is

$$V = IR = (0.0012698)(3.9\text{k}) = 4.952\text{ V}$$

d. The percent error involved in using the standard resistor is

$$\%\text{ Error} = (4.952 - 5)/5 \times 100 = -0.96\%$$

e. The power absorbed by the resistor R is then

$$P = IR = (0.0012698)^2(15\text{k}) = 24.2\text{ mW}$$

Therefore, even a quarter-watt resistor is adequate in this application.

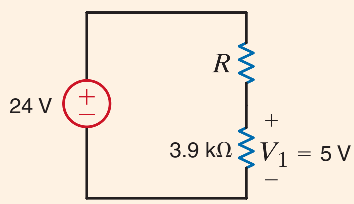


Figure 2.31
Circuit used in Example 2.23.

2.6 Circuits with Series-Parallel Combinations of Resistors

At this point we have learned many techniques that are fundamental to circuit analysis. Now we wish to apply them and show how they can be used in concert to analyze circuits. We will illustrate their application through a number of examples that will be treated in some detail.

EXAMPLE 2.24

We wish to find all the currents and voltages labeled in the ladder network shown in Fig. 2.32a.

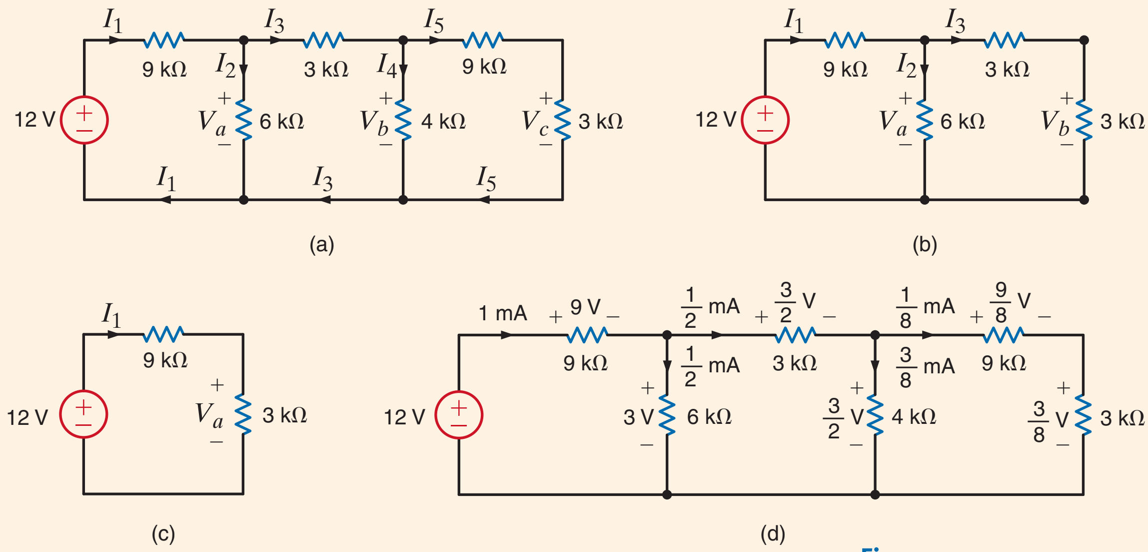


Figure 2.32

Analysis of a ladder network.

SOLUTION

To begin our analysis of the network, we start at the right end of the circuit and combine the resistors to determine the total resistance seen by the 12-V source. This will allow us to calculate the current I_1. Then employing KVL, KCL, Ohm's law, and/or voltage and current division, we will be able to calculate all currents and voltages in the network.

At the right end of the circuit, the 9-kΩ and 3-kΩ resistors are in series and, thus, can be combined into one equivalent 12-kΩ resistor. This resistor is in parallel with the 4-kΩ resistor, and their combination yields an equivalent 3-kΩ resistor, shown at the right edge of the circuit in Fig. 2.32b. In Fig. 2.32b the two 3-kΩ resistors are in series, and their combination is in parallel with the 6-kΩ resistor. Combining all three resistances yields the circuit shown in Fig. 2.32c.

Applying Kirchhoff's voltage law to the circuit in Fig. 2.32c yields

$$I_1(9\text{k} + 3\text{k}) = 12$$
$$I_1 = 1 \text{ mA}$$

V_a can be calculated from Ohm's law as

$$V_a = I_1(3\text{k})$$
$$= 3 \text{ V}$$

or, using Kirchhoff's voltage law,

$$V_a = 12 - 9\text{k}I_1$$
$$= 12 - 9$$
$$= 3 \text{ V}$$

Knowing I_1 and V_a, we can now determine all currents and voltages in Fig. 2.32b. Since $V_a = 3$ V, the current I_2 can be found using Ohm's law as

$$I_2 = \frac{3}{6\text{k}}$$
$$= \frac{1}{2} \text{ mA}$$

Then, using Kirchhoff's current law, we have

$$I_1 = I_2 + I_3$$
$$1 \times 10^{-3} = \frac{1}{2} \times 10^{-3} + I_3$$
$$I_3 = \frac{1}{2}\text{ mA}$$

Note that the I_3 could also be calculated using Ohm's law:

$$V_a = (3\text{k} + 3\text{k})I_3$$
$$I_3 = \frac{3}{6\text{k}}$$
$$= \frac{1}{2}\text{ mA}$$

Applying Kirchhoff's voltage law to the right-hand loop in Fig. 2.32b yields

$$V_a - V_b = 3\text{k}I_3$$
$$3 - V_b = \frac{3}{2}$$
$$V_b = \frac{3}{2}\text{ V}$$

or, since V_b is equal to the voltage drop across the 3-kΩ resistor, we could use Ohm's law as

$$V_b = 3\text{k}I_3$$
$$= \frac{3}{2}\text{ V}$$

We are now in a position to calculate the final unknown currents and voltages in Fig. 2.32a. Knowing V_b, we can calculate I_4 using Ohm's law as

$$V_b = 4\text{k}I_4$$
$$I_4 = \frac{\frac{3}{2}}{4\text{k}}$$
$$= \frac{3}{8}\text{ mA}$$

Then, from Kirchhoff's current law, we have

$$I_3 = I_4 + I_5$$
$$\frac{1}{2} \times 10^{-3} = \frac{3}{8} \times 10^{-3} + I_5$$
$$I_5 = \frac{1}{8}\text{ mA}$$

We could also have calculated I_5 using the current-division rule. For example,

$$I_5 = \frac{4\text{k}}{4\text{k} + (9\text{k} + 3\text{k})} I_3$$
$$= \frac{1}{8}\text{ mA}$$

Finally, V_c can be computed as

$$V_c = I_5(3\text{k}) \\ = \frac{3}{8}\text{ V}$$

V_c can also be found using voltage division (i.e., the voltage V_b will be divided between the 9-kΩ and 3-kΩ resistors). Therefore,

$$V_c = \left[\frac{3\text{k}}{3\text{k} + 9\text{k}}\right]V_b \\ = \frac{3}{8}\text{ V}$$

Note that Kirchhoff's current law is satisfied at every node and Kirchhoff's voltage law is satisfied around every loop, as shown in Fig. 2.32d.

The following example is, in essence, the reverse of the previous example in that we are given the current in some branch in the network and are asked to find the value of the input source.

EXAMPLE 2.25

Given the circuit in Fig. 2.33 and $I_4 = 1/2$ mA, let us find the source voltage V_o.

SOLUTION

If $I_4 = 1/2$ mA, then from Ohm's law, $V_b = 3$ V. V_b can now be used to calculate $I_3 = 1$ mA. Kirchhoff's current law applied at node y yields

$$I_2 = I_3 + I_4 \\ = 1.5\text{ mA}$$

Then, from Ohm's law, we have

$$V_a = (1.5 \times 10^{-3})(2\text{k}) \\ = 3\text{ V}$$

Since $V_a + V_b$ is now known, I_5 can be obtained:

$$I_5 = \frac{V_a + V_b}{3\text{k} + 1\text{k}} \\ = 1.5\text{ mA}$$

Applying Kirchhoff's current law at node x yields

$$I_1 = I_2 + I_5 \\ = 3\text{ mA}$$

Now KVL applied to any closed path containing V_o will yield the value of this input source. For example, if the path is the outer loop, KVL yields

$$-V_o + 6\text{k}I_1 + 3\text{k}I_5 + 1\text{k}I_5 + 4\text{k}I_1 = 0$$

Since $I_1 = 3$ mA and $I_5 = 1.5$ mA,

$$V_o = 36\text{ V}$$

If we had selected the path containing the source and the points x, y, and z, we would obtain

$$-V_o + 6kI_1 + V_a + V_b + 4kI_1 = 0$$

Once again, this equation yields

$$V_o = 36 \text{ V}$$

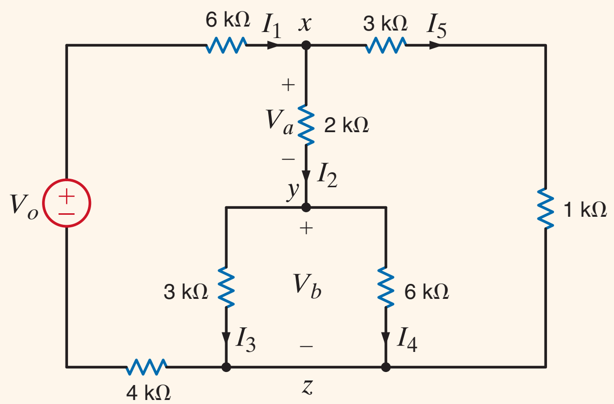


Figure 2.33 Example circuit for analysis.

Problem-Solving Strategy

Analyzing Circuits Containing a Single Source and a Series-Parallel Interconnection of Resistors

Step 1. Systematically reduce the resistive network so that the resistance seen by the source is represented by a single resistor.

Step 2. Determine the source current for a voltage source or the source voltage if a current source is present.

Step 3. Expand the network, retracing the simplification steps, and apply Ohm's law, KVL, KCL, voltage division, and current division to determine all currents and voltages in the network.

Learning Assessments

E2.17 Find V_o in the network in Fig. E2.17.

ANSWER: $V_o = 2$ V.

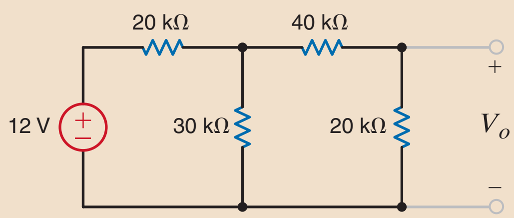


Figure E2.17

E2.18 Find V_S in the circuit in Fig. E2.18.

ANSWER: $V_S = 9$ V.

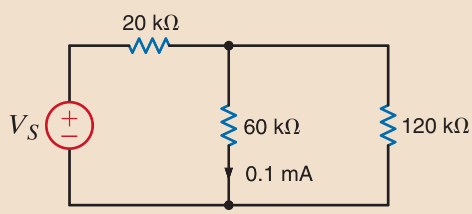


Figure E2.18

E2.19 Find I_S in the circuit in Fig. E2.19.

ANSWER: $I_S = 0.3$ mA.

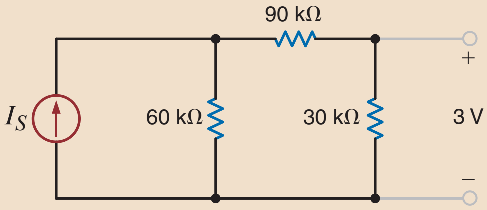


Figure E2.19

E2.20 Find V_1 in Fig. E2.20.

ANSWER: $V_1 = 12$ V.

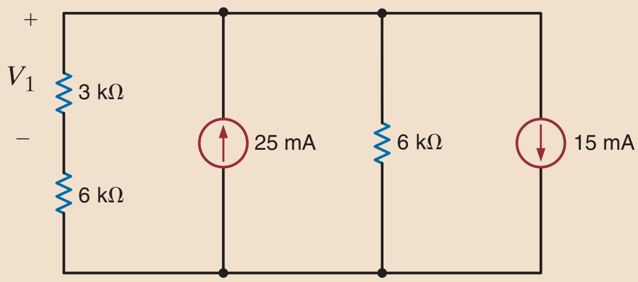


Figure E2.20

E2.21 Find I_0 in Fig. E2.21.

ANSWER: $I_0 = -4$ mA.

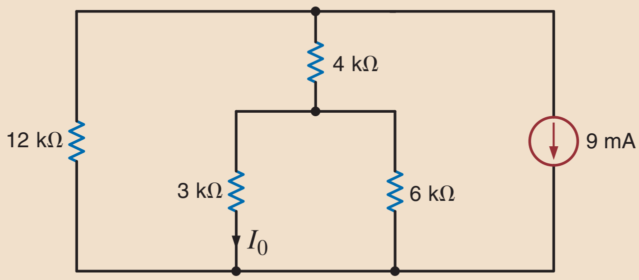


Figure E2.21

E2.22 Find V_o, V_1, and V_2 in Fig. E2.22.

ANSWER: $V_o = 3.33$ V, $V_1 = -4$ V, $V_2 = 4$ V.

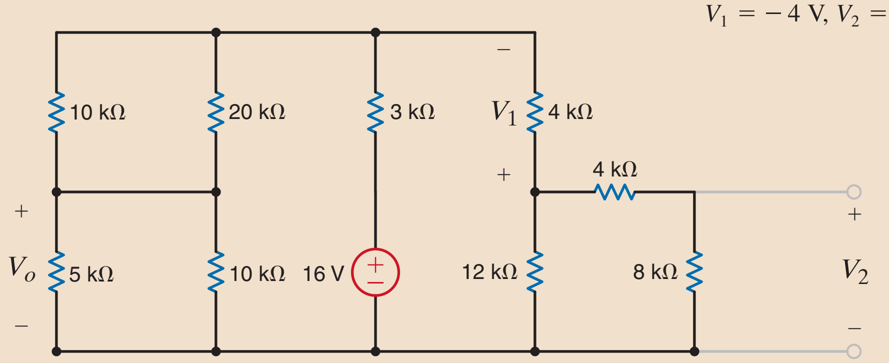


Figure E2.22

E2.23 Find V_0 and V_1 in Fig. E2.23.

ANSWER: $V_0 = -60$ V, $V_1 = 10$ V.

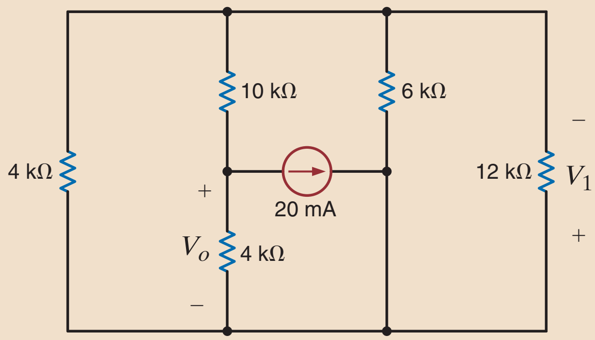


Figure E2.23

2.7 Wye ⇌ Delta Transformations

To provide motivation for this topic, consider the circuit in Fig. 2.34. Note that this network has essentially the same number of elements as contained in our recent examples. However, when we attempt to reduce the circuit to an equivalent network containing the source V_1 and an equivalent resistor R, we find that nowhere is a resistor in series or parallel with another. Therefore, we cannot attack the problem directly using the techniques that we have learned thus far. We can, however, replace one portion of the network with an equivalent circuit, and this conversion will permit us, with ease, to reduce the combination of resistors to a single equivalent resistance. This conversion is called the wye-to-delta or delta-to-wye transformation.

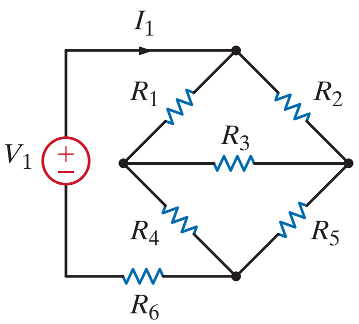


Figure 2.34

Network used to illustrate the need for the wye ⇌ delta transformation.

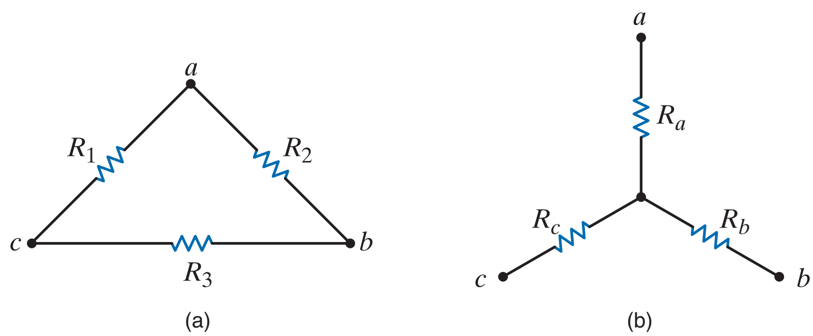


Figure 2.35

Delta and wye resistance networks.

Consider the networks shown in Fig. 2.35. Note that the resistors in Fig. 2.35a form a Δ (delta) and the resistors in Fig. 2.35b form a Y (wye). If both of these configurations are connected at only three terminals a, b, and c, it would be very advantageous if an equivalence could be established between them. It is, in fact, possible to relate the resistances of one network to those of the other such that their terminal characteristics are the same. This relationship between the two network configurations is called the Y-Δ transformation.

The transformation that relates the resistances R_1, R_2, and R_3 to the resistances R_a, R_b, and R_c is derived as follows. For the two networks to be equivalent at each corresponding pair of terminals, it is necessary that the resistance at the corresponding terminals be equal (e.g., the resistance at terminals a and b with c open-circuited must be the same for both networks).

Therefore, if we equate the resistances for each corresponding set of terminals, we obtain the following equations:

$$\begin{aligned} R_{ab} &= R_a + R_b = \frac{R_2(R_1 + R_3)}{R_2 + R_1 + R_3} \\ R_{bc} &= R_b + R_c = \frac{R_3(R_1 + R_2)}{R_3 + R_1 + R_2} \\ R_{ca} &= R_c + R_a = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} \end{aligned} \qquad \textbf{2.27}$$

Solving this set of equations for R_a, R_b, and R_c yields

$$\begin{aligned} R_a &= \frac{R_1 R_2}{R_1 + R_2 + R_3} \\ R_b &= \frac{R_2 R_3}{R_1 + R_2 + R_3} \\ R_c &= \frac{R_1 R_3}{R_1 + R_2 + R_3} \end{aligned} \qquad \textbf{2.28}$$

Similarly, if we solve Eq. (2.27) for R_1, R_2, and R_3, we obtain

$$\begin{aligned} R_1 &= \frac{R_a R_b + R_b R_c + R_a R_c}{R_b} \\ R_2 &= \frac{R_a R_b + R_b R_c + R_a R_c}{R_c} \\ R_3 &= \frac{R_a R_b + R_b R_c + R_a R_c}{R_a} \end{aligned} \qquad \textbf{2.29}$$

Equations (2.28) and (2.29) are general relationships and apply to any set of resistances connected in a Y or Δ. For the balanced case where $R_a = R_b = R_c$ and $R_1 = R_2 = R_3$, the equations above reduce to

$$R_Y = \frac{1}{3}\mathrm{R}_\Delta \qquad \textbf{2.30}$$

and

$$R_\Delta = 3R_Y \qquad \textbf{2.31}$$

It is important to note that it is not necessary to memorize the formulas in Eqs. (2.28) and (2.29). Close inspection of these equations and Fig. 2.35 illustrates a definite pattern to the relationships between the two configurations. For example, the resistance connected to point a in the wye (i.e., R_a) is equal to the product of the two resistors in the Δ that are connected to point a divided by the sum of all the resistances in the delta. R_b and R_c are determined in a similar manner. Similarly, there are geometrical patterns associated with the equations for calculating the resistors in the delta as a function of those in the wye.

Let us now examine the use of the delta $\rightleftharpoons$ wye transformation in the solution of a network problem.

EXAMPLE 2.26

Given the network in Fig. 2.36a, let us find the source current I_S.

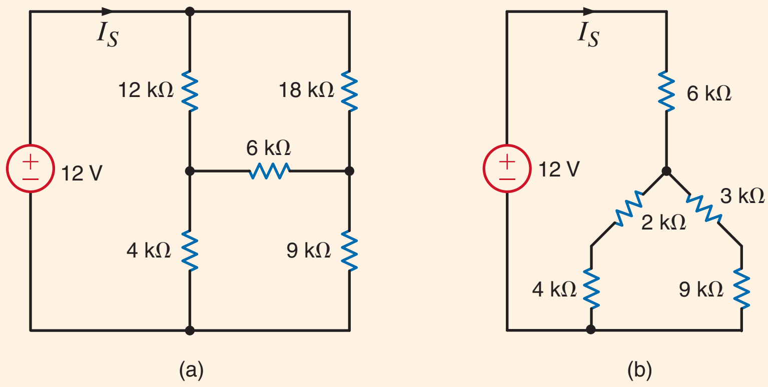


Figure 2.36 Circuits used in Example 2.26.

SOLUTION

Note that none of the resistors in the circuit are in series or parallel. However, careful examination of the network indicates that the 12k-, 6k-, and 18k-ohm resistors, as well as the 4k-, 6k-, and 9k-ohm resistors each form a delta that can be converted to a wye. Furthermore, the 12k-, 6k-, and 4k-ohm resistors, as well as the 18k-, 6k-, and 9k-ohm resistors, each form a wye that can be converted to a delta. Any one of these conversions will lead to a solution. We will perform a delta-to-wye transformation on the 12k-, 6k-, and 18k-ohm resistors, which leads to the circuit in Fig. 2.36b. The 2k- and 4k-ohm resistors, like the 3k- and 9k-ohm resistors, are in series and their parallel combination yields a 4k-ohm resistor. Thus, the source current is

$$\begin{aligned} I_S &= 12/(6\text{k} + 4\text{k}) \\ &= 1.2 \text{ mA} \end{aligned}$$

Learning Assessments

E2.24 Determine the total resistance R_T in the circuit in Fig. E2.24.

ANSWER: $R_T = 34\text{ k}\Omega$.

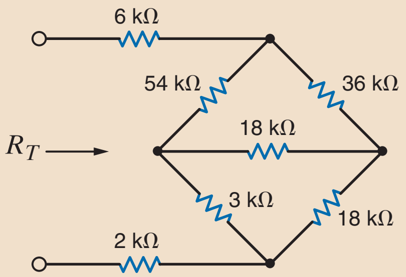


Figure E2.24

E2.25 Find V_o in the network in Fig. E2.25.

ANSWER: $V_o = 24$ V.

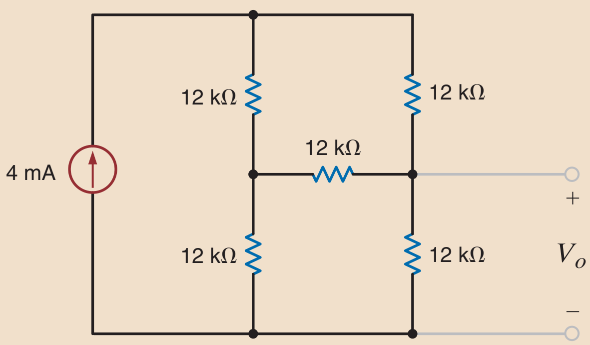


Figure E2.25

E2.26 Find I_1 in Fig. E2.26.

ANSWER: $I_1 = -1.2$ A.

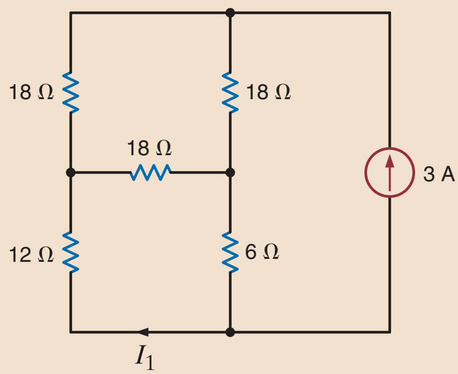


Figure E2.26

2.8 Circuits with Dependent Sources

In Chapter 1 we outlined the different kinds of dependent sources. These controlled sources are extremely important because they are used to model physical devices such as *npn* and *pnp* bipolar junction transistors (BJTs) and field-effect transistors (FETs) that are either metal-oxide-semiconductor field-effect transistors (MOSFETs) or insulated-gate field-effect transistors (IGFETs). These basic structures are, in turn, used to make analog and digital devices. A typical analog device is an operational amplifier (op-amp). This device is presented in Chapter 4. Typical digital devices are random access memories (RAMs), read-only memories (ROMs), and microprocessors. We will now show how to solve simple one-loop and one-node circuits that contain these dependent sources. Although the following examples are fairly simple, they will serve to illustrate the basic concepts.

Problem-Solving Strategy

Circuits with Dependent Sources

Step 1. When writing the KVL and/or KCL equations for the network, treat the dependent source as though it were an independent source.

Step 2. Write the equation that specifies the relationship of the dependent source to the controlling parameter.

Step 3. Solve the equations for the unknowns. Be sure that the number of linearly independent equations matches the number of unknowns.

The following four examples will each illustrate one of the four types of dependent sources: current-controlled voltage source, current-controlled current source, voltage-controlled voltage source, and voltage-controlled current source.

EXAMPLE 2.27

Let us determine the voltage V_o in the circuit in Fig. 2.37.

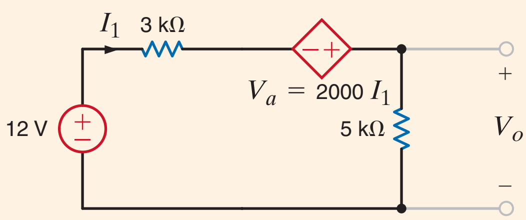


Figure 2.37 Circuit used in Example 2.27.

SOLUTION

Applying KVL, we obtain

$$-12 + 3kI_1 - V_A + 5kI_1 = 0$$

where

$$V_A = 2000I_1$$

and the units of the multiplier, 2000, are ohms. Solving these equations yields

$$I_1 = 2 \text{ mA}$$

Then

$$V_o = (5\text{ k})I_1$$
$$= 10 \text{ V}$$

EXAMPLE 2.28

Given the circuit in Fig. 2.38 containing a current-controlled current source, let us find the voltage V_o.

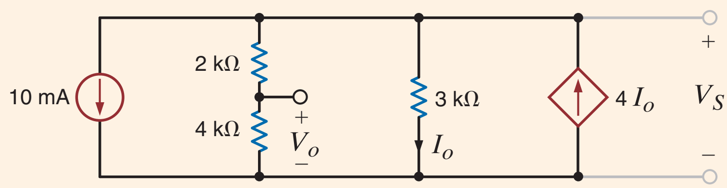


Figure 2.38 Circuit used in Example 2.28.

SOLUTION

Applying KCL at the top node, we obtain

$$10 \times 10^{-3} + \frac{V_S}{2k + 4k} + \frac{V_S}{3k} - 4I_o = 0$$

where

$$I_o = \frac{V_S}{3k}$$

Substituting this expression for the controlled source into the KCL equation yields

$$10^{-2} + \frac{V_S}{6k} + \frac{V_S}{3k} - \frac{4V_S}{3k} = 0$$

Solving this equation for V_S, we obtain

$$V_S = 12 \text{ V}$$

The voltage V_o can now be obtained using a simple voltage divider; that is,

$$V_o = \left[\frac{4k}{2k + 4k}\right]V_S$$
$$= 8 \text{ V}$$

EXAMPLE 2.29

The network in Fig. 2.39 contains a voltage-controlled voltage source. We wish to find V_o in this circuit.

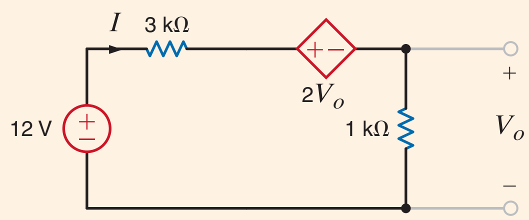


Figure 2.39 Circuit used in Example 2.29.

SOLUTION Applying KVL to this network yields

$$-12 + 3\text{k}I + 2V_o + 1\text{k}I = 0$$

where

$$V_o = 1\text{k}I$$

Hence, the KVL equation can be written as

$$-12 + 3\text{k}I + 2\text{k}I + 1\text{k}I = 0$$

or

$$I = 2 \text{ mA}$$

Therefore,

$$\begin{aligned} V_o &= 1\text{k}I \\ &= 2 \text{ V} \end{aligned}$$

EXAMPLE 2.30

An equivalent circuit for a FET common-source amplifier or BJT common-emitter amplifier can be modeled by the circuit shown in Fig. 2.40a. We wish to determine an expression for the gain of the amplifier, which is the ratio of the output voltage to the input voltage.

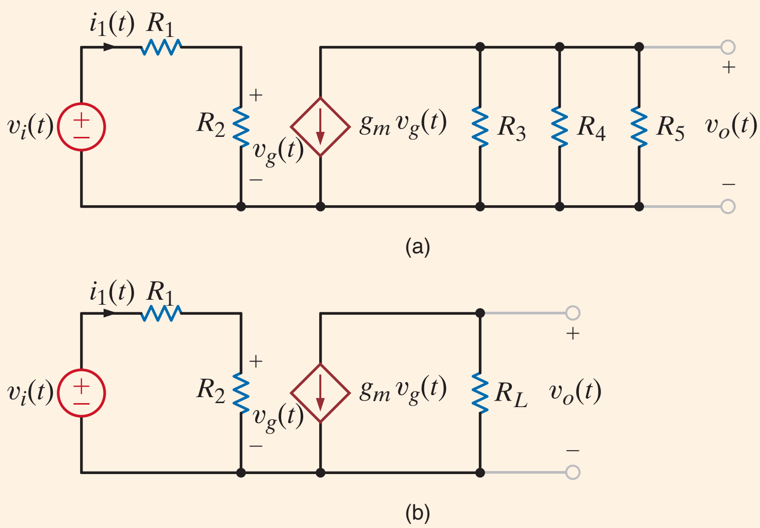


Figure 2.40 Example circuit containing a voltage-controlled current source.

SOLUTION Note that although this circuit, which contains a voltage-controlled current source, appears to be somewhat complicated, we are actually in a position now to solve it with techniques we have studied up to this point. The loop on the left, or input to the amplifier, is essentially detached from the output portion of the amplifier on the right. The voltage across R_2 is $v_g(t)$, which controls the dependent current source.

To simplify the analysis, let us replace the resistors R_3, R_4, and R_5 with R_L such that

$$\frac{1}{R_L} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}$$

Then the circuit reduces to that shown in Fig. 2.40b. Applying Kirchhoff's voltage law to the input portion of the amplifier yields

$$v_i(t) = i_1(t)(R_1 + R_2)$$

and

$$v_g(t) = i_1(t)R_2$$

Solving these equations for $v_g(t)$ yields

$$v_g(t) = \frac{R_2}{R_1 + R_2} v_i(t)$$

From the output circuit, note that the voltage $v_o(t)$ is given by the expression

$$v_o(t) = -g_m v_g(t) R_L$$

Combining this equation with the preceding one yields

$$v_o(t) = \frac{-g_m R_L R_2}{R_1 + R_2} v_i(t)$$

Therefore, the amplifier gain, which is the ratio of the output voltage to the input voltage, is given by

$$\frac{v_o(t)}{v_i(t)} = -\frac{g_m R_L R_2}{R_1 + R_2}$$

Reasonable values for the circuit parameters in Fig. 2.40a are $R_1 = 100\ \Omega$, $R_2 = 1\ \text{k}\Omega$, $g_m = 0.04$ S, $R_3 = 50\ \text{k}\Omega$, and $R_4 = R_5 = 10\ \text{k}\Omega$. Hence, the gain of the amplifier under these conditions is

$$\begin{aligned}\frac{v_o(t)}{v_i(t)} &= \frac{-(0.04)(4.545)(10^3)(1)(10^3)}{(1.1)(10^3)} \\ &= -165.29\end{aligned}$$

Thus, the magnitude of the gain is 165.29.

At this point it is perhaps helpful to point out again that when analyzing circuits with dependent sources, we first treat the dependent source as though it were an independent source when we write a Kirchhoff's current or voltage law equation. Once the equation is written, we then write the controlling equation that specifies the relationship of the dependent source to the unknown variable. For instance, the first equation in Example 2.28 treats the dependent source like an independent source. The second equation in the example specifies the relationship of the dependent source to the voltage, which is the unknown in the first equation.

Learning Assessments

E2.27 Find V_o in the circuit in Fig. E2.27.

ANSWER: $V_o = 12$ V.

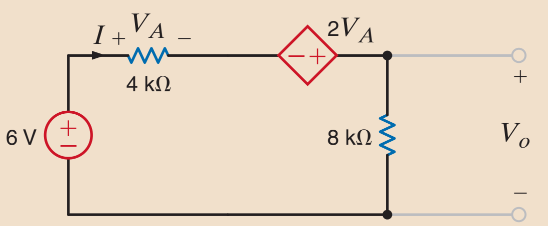


Figure E2.27

E2.28 Find V_o in the network in Fig. E2.28.

ANSWER: $V_o = 8$ V.

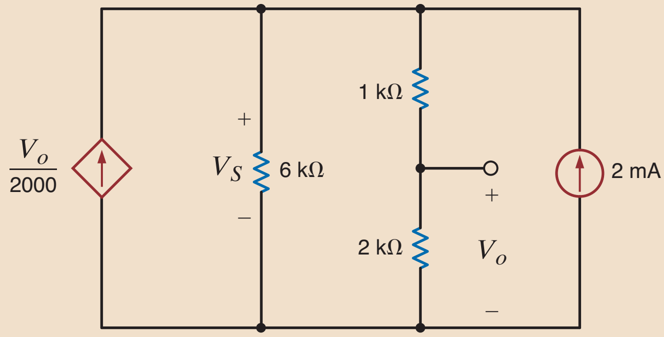


Figure E2.28

E2.29 Find V_A in Fig. E2.29.

ANSWER: $V_A = -12$ V.

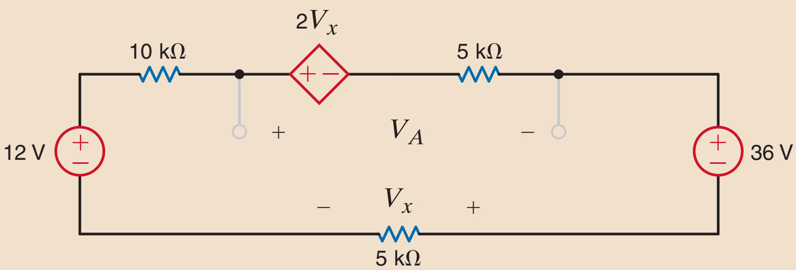


Figure E2.29

E2.30 Find V_1 in Fig. E2.30.

ANSWER: $V_1 = -32/3$ V.

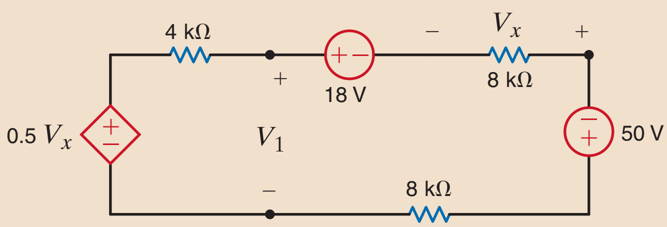


Figure E2.30

E2.31 Find I_x in Fig. E2.31.

ANSWER: $I_x = -1.5$ mA.

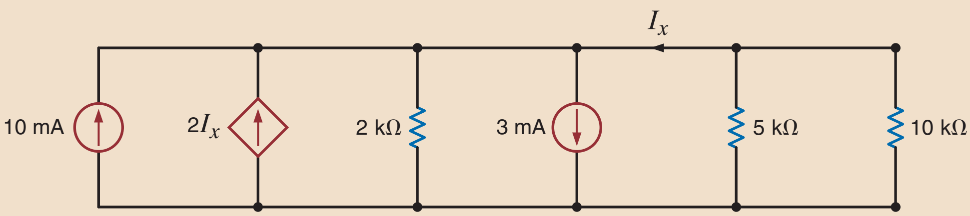


Figure E2.31

E2.32 Find V_o in Fig. E2.32.

ANSWER: $V_o = 16$ V.

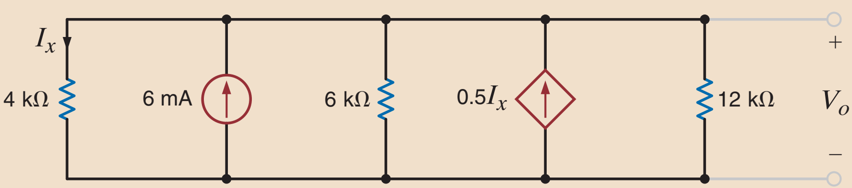


Figure E2.32

E2.33 If the power supplied by the 3-A current source in Fig. E2.33 is 12 W, find V_S and the power supplied by the 10-V source.

ANSWER: $V_S = 42$ V, −30 W.

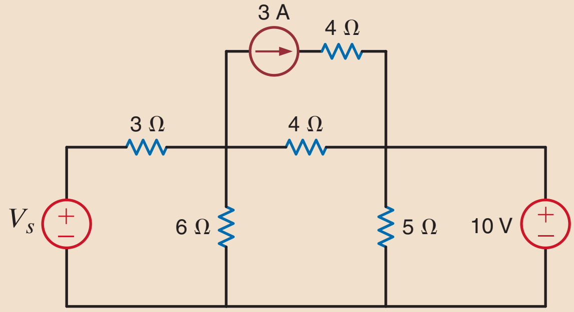


Figure E2.33

2.9 Resistor Technologies for Electronic Manufacturing

In addition to the resistors shown in Fig 2.1, three types are employed in the modern electronics industry: thick-film, thin-film, and silicon-diffused resistors.

THICK-FILM RESISTORS Thick-film resistor components are found on all modern surface mount technology (SMT) printed circuit boards. They come in a variety of shapes, sizes, and values. A table of standard sizes for thick-film chip resistors is shown in Table 2.2, and some examples of surface mount thick-film ceramic resistors can be seen in Fig. 2.41.

Thick-film resistors are considered "low-tech," when compared with thin-film and silicon-diffused components, because they are manufactured using a screen printing process similar to that used with T-shirts. The screens utilized in thick-film manufacturing use a much finer mesh and are typically made of stainless steel for a longer lifetime. The paste used in screen printing resistors consists of a mixture of ruthenium oxides (RuO_2) and glass.

TABLE 2.2 Thick-film chip resistor standard sizes

SIZE CODE	SIZE (MILS)	POWER RATING (WATTS)
0201	20 × 10	1/20
0402	40 × 20	1/16
0603	60 × 30	1/10
0805	80 × 50	1/8
1206	120 × 60	1/4
2010	200 × 100	1/2
2512	250 × 120	1

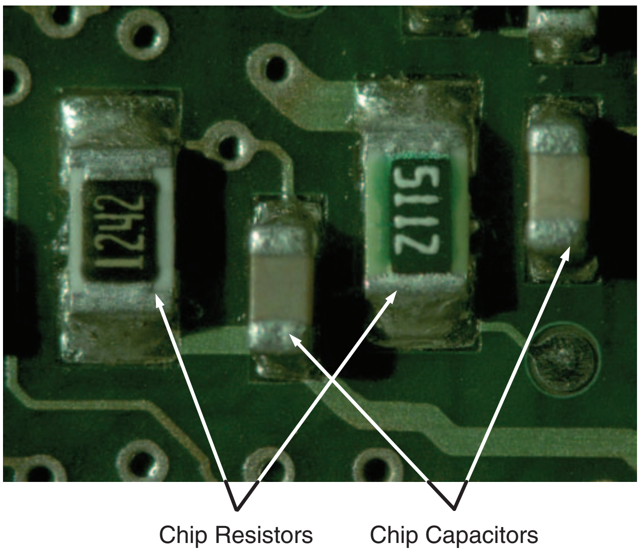


Figure 2.41

A printed circuit board showing surface mount thick-film ceramic resistors. (Courtesy of Mike Palmer)

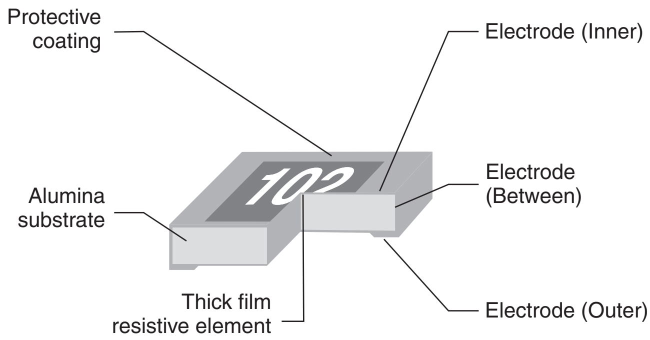


Figure 2.42

Thick-film chip resistor cross-section.

Once the paste is screen printed, it is fired at temperatures around 850°C, causing the organic binders to vaporize and to allow the glass to melt and bind the metal and glass filler to the substrate. The substrates are typically 95% alumina ceramic. After firing, conductors are screen printed and fired to form the contacts used to solder the resistors. A second layer of glass is screen printed and fired to seal and protect the resistor. A cross-section of a typical thick-film resistor is shown in Fig. 2.42. Notice that the conductors are "wrapped" around the substrate to allow them to be soldered from the bottom or top and allow the solder to "wic" up the side to form a more reliable mechanical and electrical contact.

Thick-film resistors have typical "as fired" tolerances of +/− 10% to +/− 20%. These wide tolerances are due to the fact that the screen printing process does not afford good geometry transfer or consistent thickness. To obtain a better tolerance (i.e., +/− 0.5% to +/− 1.0%), the resistors can be trimmed with a YAG laser to remove a portion of the resistor and change its value. The resistor is constantly measured during the cutting process to make sure the resistance is within the specified tolerance.

THIN-FILM RESISTORS Thin-film resistors are fabricated by depositing a thin layer (hundreds of angstroms, where one angstrom is one ten-billionth of a meter) of Tantalum Nitride (TaN) or Nichrome (NiCr) onto a silicon or highly polished alumina ceramic substrate. Using a photolithography process, the metal film is patterned and etched to form the resistor structure. Thin-film metals have a limited resistivity (the reciprocal of conductivity—a measure of a material's ability to carry an electric current). This low resistivity limits the practical range of thin-film resistors due to the large areas required. Both TaN and NiCr have similar characteristics, but TaN is more chemically and thermally resistant and will hold up better to harsh environments. Sputtered metal thin films are continuous and virtually defect free, which makes them very stable, low-noise components that have negligible nonlinearity when compared with the more porous thick-film materials.

Thin-film resistors are available in standard SMT packages, but are also available as wire-bondable chips that can be directly patterned onto integrated circuits. A cross-sectional drawing of the thin-film chip on ceramic or silicon is shown in Fig 2.43. Because of the additional sophistication involved in fabrication, thin-film resistors are more expensive than thick-film resistors. However, they have a number of important characteristics that make them the preferred devices for a number of microwave applications. Like thick-film resistors, these components can also be laser-trimmed to obtain a desired value within a specified tolerance. Since the sputtered metal film is extremely thin, the power requirement for the laser is very low, which in turn ensures that there will be minimal micro-cracking and therefore an increased level of stability.

SILICON-DIFFUSED RESISTORS Silicon-diffused resistors are part of virtually all integrated circuits (ICs). They are passive devices that are implemented to support or enhance the capabilities of active devices, such as transistors and diodes. Both passive and active devices are manufactured at the same time using the same technology (e.g., CMOS—complementary metal-oxide semiconductor). The resistors are made by diffusing a dopant, such as boron or phosphorus, into a silicon substrate at high temperature. This process is very

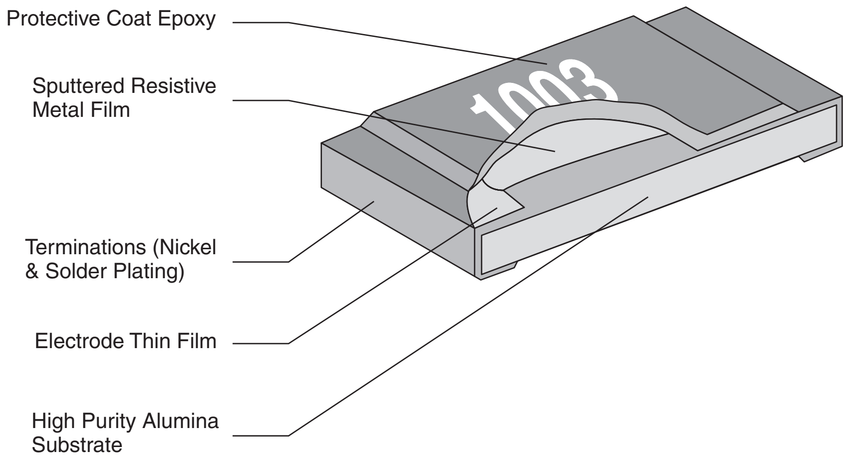


Figure 2.43
Thin-film chip resistor cross-section.

expensive and is the reason silicon-diffused resistors cost more than thin- or thick-film resistors. A photo of an integrated silicon resistor is shown in Fig. 2.44. Notice that the resistor is completely integrated within a larger circuit, because it is not economically feasible to make discrete silicon-diffused resistors. Table 2.3 compares some of the characteristics of thick-film, thin-film, and silicon-diffused resistors.

Silicon resistors have a resistance range on the order of 5–6k ohms/sq. The term "ohms per square" means a dimensionless square area of resistive material, having an ohmic value equal

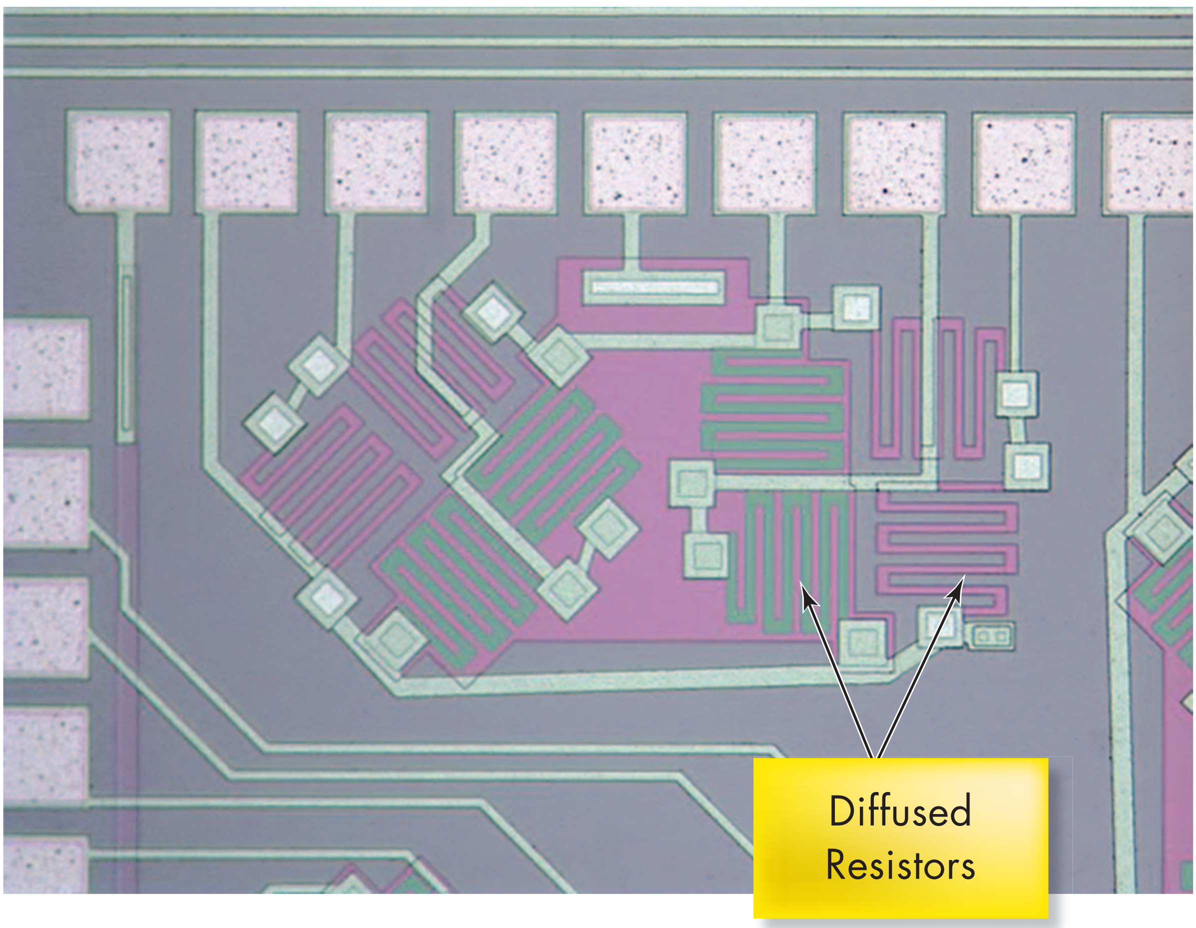


Figure 2.44
Silicon-diffused resistors.

to the sheet resistivity of the material. For example, a 10-ohm sheet resistivity material would constitute a 10-ohm resistor whether the material was 1 mil by 1 mil or 1 inch by 1 inch. Dividing the length of the resistor by its width yields the number of squares, and multiplying the number of squares by the sheet resistance yields the resistance value. The total resistance values are limited because of the high cost of silicon area, and there are other circuit design techniques for implementing high-valued resistors through the judicious use of transistors. These devices suffer from large changes in value over temperature and some resistance change with applied voltage. As a result of these poor characteristics, thin-film resistors mounted on the surface of the silicon are used in place of diffused resistors in critical applications.

TABLE 2.3 Characteristics of resistor types

CHARACTERISTIC	THICK-FILM	THIN-FILM	SILICON-DIFFUSED
Sheet resistance	$5-500$k ohms/sq	$25-300$ ohms/sq	$5-6$k ohms/sq
Sheet tolerance (as fired)	$+/-20\%$	$+/-10\%$	$+/-2\%$
Sheet tolerance (final)	$+/-1\%$	$+/-1\%$	N/A
Relative cost	Low	High	Higher

2.10 Application Examples

Throughout this book we endeavor to present a wide variety of examples that demonstrate the usefulness of the material under discussion in a practical environment. To enhance our presentation of the practical aspects of circuit analysis and design, we have dedicated sections, such as this one, in most chapters for the specific purpose of presenting additional application-oriented examples.

APPLICATION EXAMPLE 2.31

The eyes (heating elements) of an electric range are frequently made of resistive nichrome strips. Operation of the eye is quite simple. A current is passed through the heating element causing it to dissipate power in the form of heat. Also, a four-position selector switch, shown in Fig. 2.45, controls the power (heat) output. In this case the eye consists of two nichrome strips modeled by the resistors R_1 and R_2, where $R_1 < R_2$.

1. How should positions A, B, C, and D be labeled with regard to high, medium, low, and off settings?
2. If we desire that high and medium correspond to 2000 W and 1200 W power dissipation, respectively, what are the values of R_1 and R_2?
3. What is the power dissipation at the low setting?

SOLUTION Position A is the off setting since no current flows to the heater elements. In position B, current flows through R_2 only, while in position C current flows through R_1 only. Since $R_1 < R_2$, more power will be dissipated when the switch is at position C. Thus, position C is the medium setting, B is the low setting, and, by elimination, position D is the high setting.

When the switch is at the medium setting, only R_1 dissipates power, and we can write R_1 as

$$R_1 = \frac{V_S^2}{P_1} = \frac{230^2}{1200}$$

or

$$R_1 = 44.08\ \Omega$$

On the high setting, 2000 W of total power is delivered to R_1 and R_2. Since R_1 dissipates 1200 W, R_2 must dissipate the remaining 800 W. Therefore, R_2 is

$$R_2 = \frac{V_S^2}{P_2} = \frac{230^2}{800}$$

or

$$R_2 = 66.13\ \Omega$$

Finally, at the low setting, only R_2 is connected to the voltage source; thus, the power dissipation at this setting is 800 W.

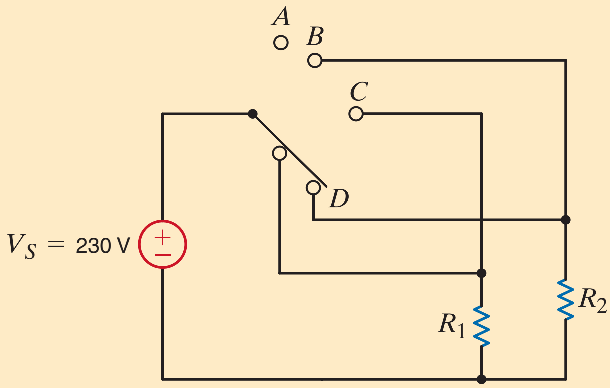


Figure 2.45

Simple resistive heater selector circuit.

APPLICATION EXAMPLE 2.32

Have you ever cranked your car with the headlights on? While the starter kicked the engine, you probably saw the headlights dim then return to normal brightness once the engine was running on its own. Can we create a model to predict this phenomenon?

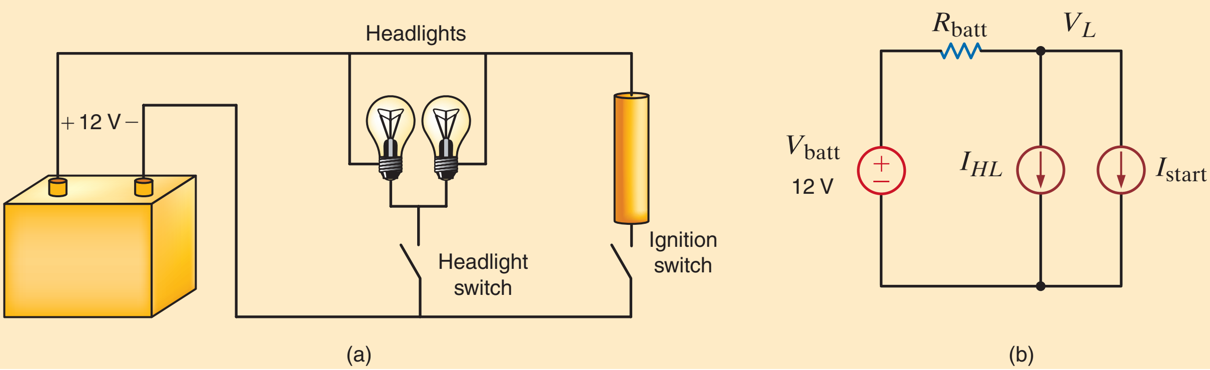
SOLUTION

Yes, we can. Consider the conceptual circuit in Fig. 2.46a and the model circuit in Fig. 2.46b, which isolates just the battery, headlights, and starter. Note the resistor R_{batt}. It is included to model several power loss mechanisms that can occur between the battery and the loads, that is, the headlights and starter. First, there are the chemical processes within the battery itself which are not 100% efficient. Second, there are the electrical connections at both the battery posts and the loads. Third, the wiring itself has some resistance, although this is usually so small that it is negligible. The sum of these losses is modeled by R_{batt}, and we expect the value of R_{batt} to be small. A reasonable value is 25 mΩ.

Next we address the starter. When energized, a typical automobile starter will draw between 90 and 120 A. We will use 100 A as a typical number. Finally, the headlights will draw much less current—perhaps only 1 A. Now we have values to use in our model circuit.

Figure 2.46

A conceptual (a) model and (b) circuit for examining the effect of starter current on headlight intensity.



Assume first that the starter is off. By applying KCL at the node labeled V_L, we find that the voltage applied to the headlights can be written as

$$V_L = V_{\text{batt}} - I_{HL}R_{\text{batt}}$$

Substituting our model values into this equation yields $V_L = 11.75$ V—very close to 12 V. Now we energize the starter and apply KCL again:

$$V_L = V_{\text{batt}} - (I_{HL} + I_{\text{start}})R_{\text{batt}}$$

Now the voltage across the headlights is only 9.25 V. No wonder the headlights dim! How would corrosion or loose connections on the battery posts change the situation? In this case, we would expect the quality of the connection from battery to load to deteriorate, increasing R_{batt} and compounding the headlight dimming issue.

APPLICATION EXAMPLE 2.33

A Wheatstone bridge circuit is an accurate device for measuring resistance. This circuit, shown in Fig. 2.47, is used to measure the unknown resistor R_x. The center leg of the circuit contains a galvanometer, which is a very sensitive device that can be used to measure current in the microamp range. When the unknown resistor is connected to the bridge, R_3 is adjusted until the current in the galvanometer is zero, at which point the bridge is balanced. In this balanced condition

$$\frac{R_1}{R_3} = \frac{R_2}{R_x}$$

so that

$$R_x = \left(\frac{R_2}{R_1}\right)R_3$$

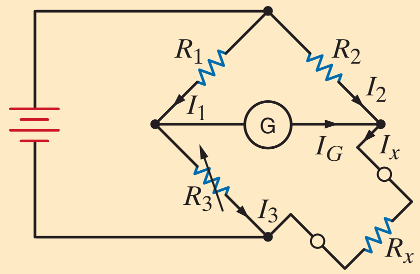


Figure 2.47 The Wheatstone bridge circuit.

Engineers also use this bridge circuit to measure strain in solid material. For example, a system used to determine the weight of a truck is shown in Fig. 2.48a. The platform is supported by cylinders on which strain gauges are mounted. The strain gauges, which measure strain when the cylinder deflects under load, are connected to a Wheatstone bridge as shown in Fig. 2.48b. The strain gauge has a resistance of 120 Ω under no-load conditions and changes value under load. The variable resistor in the bridge is a calibrated precision device.

Weight is determined in the following manner. The ΔR_3 required to balance the bridge represents the Δ strain, which when multiplied by the modulus of elasticity yields the Δ stress. The Δ stress multiplied by the cross-sectional area of the cylinder produces the Δ load, which is used to determine weight.

Let us determine the value of R_3 under no load when the bridge is balanced and its value when the resistance of the strain gauge changes to 120.24 Ω under load.

SOLUTION Using the balance equation for the bridge, the value of R_3 at no load is

$$R_3 = \left(\frac{R_1}{R_2}\right)R_x$$

$$= \left(\frac{100}{110}\right)(120)$$

$$= 109.0909\ \Omega$$

Under load, the value of R_3 is

$$R_3 = \left(\frac{100}{110}\right)(120.24)$$

$$= 109.3091\ \Omega$$

Therefore, the ΔR_3 is

$$\Delta R_3 = 109.3091 - 109.0909$$

$$= 0.2182\ \Omega$$

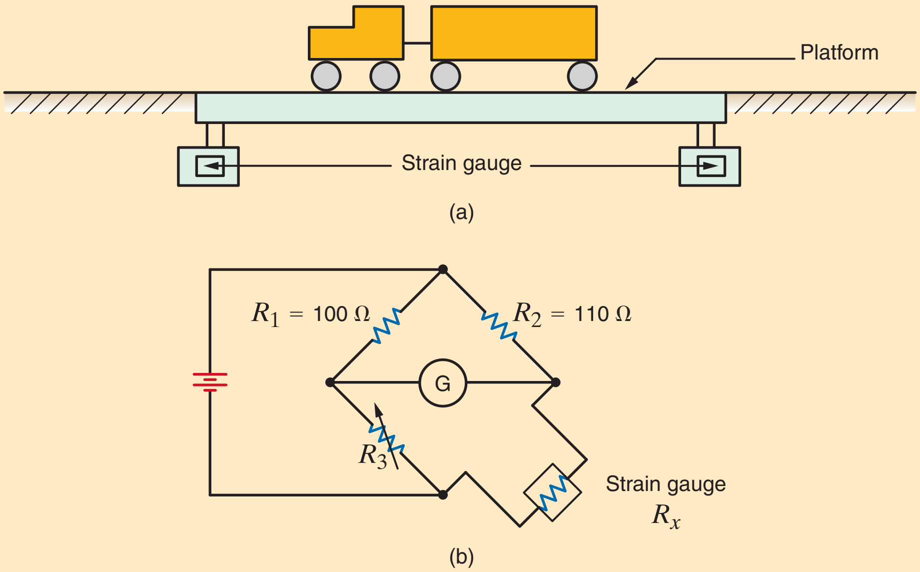


Figure 2.48
Diagrams used in Example 2.33.

2.11 Design Examples

Most of this text is concerned with circuit analysis; that is, given a circuit in which all the components are specified, analysis involves finding such things as the voltage across some element or the current through another. Furthermore, the solution of an analysis problem is generally unique. In contrast, design involves determining the circuit configuration that will meet certain specifications. In addition, the solution is generally not unique in that there may be many ways to satisfy the circuit/performance specifications. It is also possible that there is no solution that will meet the design criteria.

In addition to meeting certain technical specifications, designs normally must also meet other criteria, such as economic, environmental, and safety constraints. For example, if a circuit design that meets the technical specifications is either too expensive or unsafe, it is not viable regardless of its technical merit.

At this point, the number of elements that we can employ in circuit design is limited primarily to the linear resistor and the active elements we have presented. However, as we progress through the text we will introduce a number of other elements (for example, the op-amp, capacitor, and inductor), which will significantly enhance our design capability.

We begin our discussion of circuit design by considering a couple of simple examples that demonstrate the selection of specific components to meet certain circuit specifications.

DESIGN EXAMPLE 2.34

An electronics hobbyist who has built his own stereo amplifier wants to add a back-lit display panel to his creation for that professional look. His panel design requires seven light bulbs—two operate at 12 V/15 mA and five at 9 V/5 mA. Luckily, his stereo design already has a quality 12-V dc supply; however, there is no 9-V supply. Rather than building a new dc power supply, let us use the inexpensive circuit shown in Fig. 2.49a to design a 12-V to 9-V converter with the restriction that the variation in V_2 be no more than ±5%. In particular, we must determine the necessary values of R_1 and R_2.

SOLUTION

First, lamps L_1 and L_2 have no effect on V_2. Second, when lamps L_3–L_7 are on, they each have an equivalent resistance of

$$R_{eq} = \frac{V_2}{I} = \frac{9}{0.005} = 1.8\text{ k}\Omega$$

As long as V_2 remains fairly constant, the lamp resistance will also be fairly constant. Thus, the requisite model circuit for our design is shown in Fig. 2.49b. The voltage V_2 will be at its maximum value of 9 + 5% = 9.45 V when L_3–L_7 are all off. In this case R_1 and R_2 are in series, and V_2 can be expressed by simple voltage division as

$$V_2 = 9.45 = 12\left[\frac{R_2}{R_1 + R_2}\right]$$

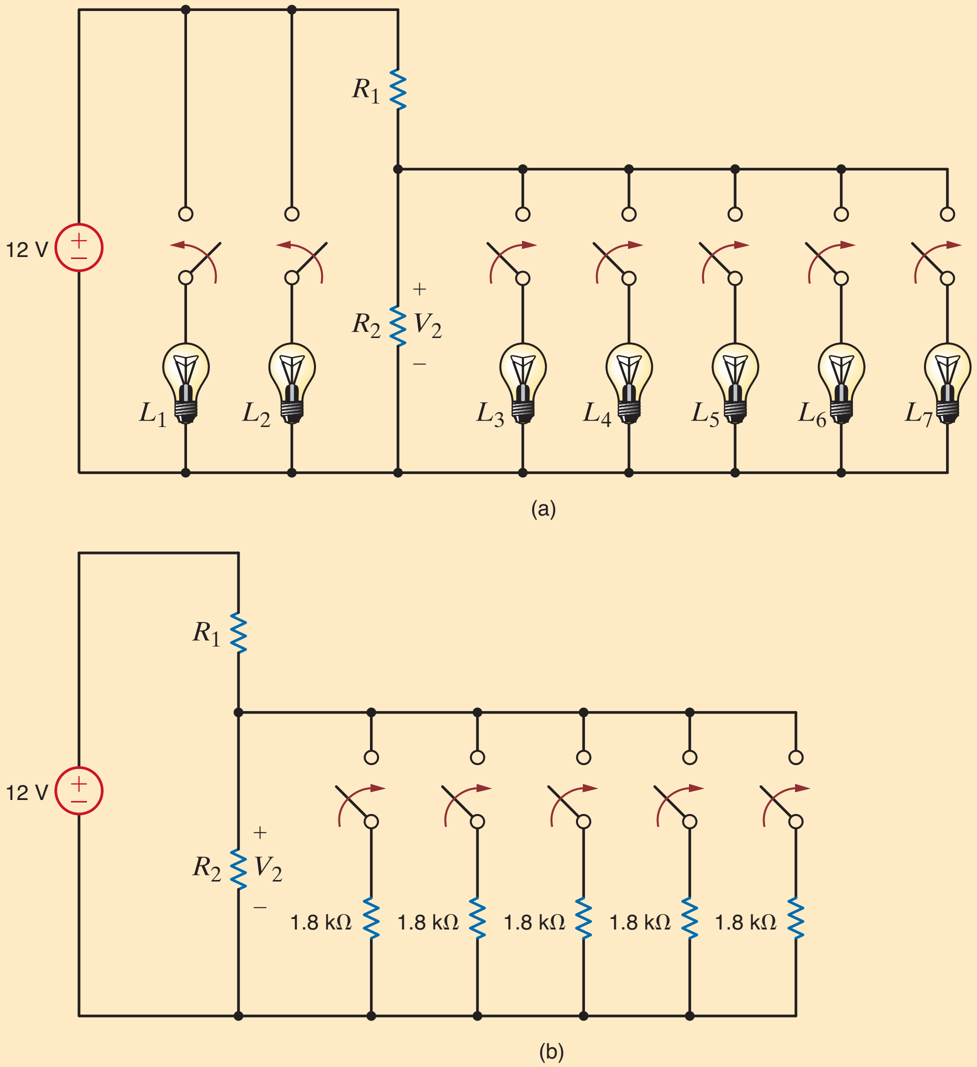


Figure 2.49
12-V to 9-V converter circuit for powering panel lighting.

Rearranging the equation yields

$$\frac{R_1}{R_2} = 0.27$$

A second expression involving R_1 and R_2 can be developed by considering the case when L_3–L_7 are all on, which causes V_2 to reach its minimum value of 9–5%, or 8.55 V. Now, the effective resistance of the lamps is five 1.8-kΩ resistors in parallel, or 360 Ω. The corresponding expression for V_2 is

$$V_2 = 8.55 = 12\left[\frac{R_2//360}{R_1 + (R_2//360)}\right]$$

which can be rewritten in the form

$$\frac{\dfrac{360R_1}{R_2} + 360 + R_1}{360} = \frac{12}{8.55} = 1.4$$

Substituting the value determined for R_1/R_2 into the preceding equation yields

$$R_1 = 360[1.4 - 1 - 0.27]$$

or

$$R_1 = 48.1\ \Omega$$

and so for R_2

$$R_2 = 178.3\ \Omega$$

DESIGN EXAMPLE 2.35

Let's design a circuit that produces a 5-V output from a 12-V input. We will arbitrarily fix the power consumed by the circuit at 240 mW. Finally, we will choose the best possible standard resistor values from Table 2.1 and calculate the percent error in the output voltage that results from that choice.

SOLUTION

The simple voltage divider, shown in Fig. 2.50, is ideally suited for this application. We know that V_o is given by

$$V_o = V_{\text{in}}\left[\frac{R_2}{R_1 + R_2}\right]$$

which can be written as

$$R_1 = R_2\left[\frac{V_{\text{in}}}{V_o} - 1\right]$$

Since all of the circuit's power is supplied by the 12-V source, the total power is given by

$$P = \frac{V_{\text{in}}^2}{R_1 + R_2} \leq 0.24$$

Using the second equation to eliminate R_1, we find that R_2 has a lower limit of

$$R_2 \geq \frac{V_o V_{\text{in}}}{P} = \frac{(5)(12)}{0.24} = 250\ \Omega$$

Substituting these results into the second equation yields the lower limit of R_1, that is

$$R_1 = R_2\left[\frac{V_{\text{in}}}{V_o} - 1\right] \geq 350\ \Omega$$

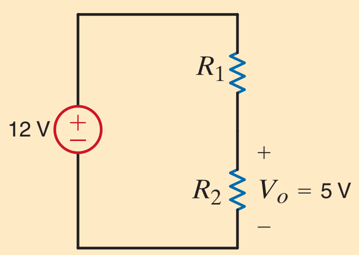


Figure 2.50

A simple voltage divider

Thus, we find that a significant portion of Table 2.1 is not applicable to this design. However, determining the best pair of resistor values is primarily a trial-and-error operation that can be enhanced by using an Excel spreadsheet as shown in Table 2.4. Standard resistor values from Table 2.1 were entered into Column A of the spreadsheet for R_2. Using the equation above, theoretical values for R_1 were calculated using $R_1 = 1.4 \cdot R_2$. A standard resistor value was selected from Table 2.1 for R_1 based on the theoretical calculation in Column B. V_0 was calculated using the simple voltage-divider equation, and the power absorbed by R_1 and R_2 was calculated in Column E.

Note that a number of combinations of R_1 and R_2 satisfy the power constraint for this circuit. The power absorbed decreases as R_1 and R_2 increase. Let's select $R_1 = 1800\ \Omega$ and $R_2 = 1300\ \Omega$, because this combination yields an output voltage of 5.032 V that is closest to the desired value of 5 V. The resulting error in the output voltage can be determined from the expression

$$\text{Percent error} = \left[\frac{5.032 - 5}{5}\right]100\% = 0.64\%$$

It should be noted, however, that these resistor values are nominal, that is, typical values. To find the worst-case error, we must consider that each resistor as purchased may be as much as ±5% off the nominal value. In this application, since V_0 is already greater than the target of 5 V, the worst-case scenario occurs when V_0 increases even further, that is, R_1 is 5% too low $(1710\ \Omega)$ and R_2 is 5% too high $(1365\ \Omega)$. The resulting output voltage is 5.32 V, which yields a percent error of 6.4%. Of course, most resistor values are closer to the nominal value than to the guaranteed maximum/minimum values. However, if we intend to build this circuit with a guaranteed tight output error such as 5% we should use resistors with lower tolerances.

How much lower should the tolerances be? Our first equation can be altered to yield the worst-case output voltage by adding a tolerance, Δ, to R_2 and subtracting the tolerance from R_1. Let's choose a worst-case output voltage of $V_{0max} = 5.25$ V, that is, a 5% error:

TABLE 2.4 Spreadsheet calculations for simple voltage divider

	A	B	C	D	E
1	R2	R1 theor	R1	Vo	Pabs
2	300	420	430	4.932	0.197
3	330	462	470	4.950	0.180
4	360	504	510	4.966	0.166
5	390	546	560	4.926	0.152
6	430	602	620	4.914	0.137
7	470	658	680	4.904	0.125
8	510	714	750	4.857	0.114
9	560	784	750	5.130	0.110
10	620	868	910	4.863	0.094
11	680	952	910	5.132	0.091
12	750	1050	1000	5.143	0.082
13	820	1148	1100	5.125	0.075
14	910	1274	1300	4.941	0.065
15	1000	1400	1300	5.217	0.063
16	1100	1540	1500	5.077	0.055
17	1200	1680	1600	5.143	0.051
18	1300	1820	1800	5.032	0.046
19	1500	2100	2000	5.143	0.041
20	1600	2240	2200	5.053	0.038
21	1800	2520	2400	5.143	0.034
22	2000	2800	2700	5.106	0.031
23	2200	3080	3000	5.077	0.028
24	2400	3360	3300	5.053	0.025

$$V_{0\max} = 5.25 = V_{in}\left[\frac{R_2(1+\Delta)}{R_1(1-\Delta)+R_2(1+\Delta)}\right] = 12\left[\frac{1300(1+\Delta)}{1800(1-\Delta)+1300(1+\Delta)}\right]$$

The resulting value of Δ is 0.037, or 3.7%. Standard resistors are available in tolerances of 10, 5, 2, and 1%. Tighter tolerances are available but very expensive. Thus, based on nominal values of 1300 Ω and 1800 Ω, we should utilize 2% resistors to ensure an output voltage error less than 5%.

DESIGN EXAMPLE 2.36

In factory instrumentation, process parameters such as pressure and flowrate are measured, converted to electrical signals, and sent some distance to an electronic controller. The controller then decides what actions should be taken. One of the main concerns in these systems is the physical distance between the sensor and the controller. An industry standard format for encoding the measurement value is called the 4–20 mA standard, where the parameter range is linearly distributed from 4 to 20 mA. For example, a 100 psi pressure sensor would output 4 mA if the pressure were 0 psi, 20 mA at 100 psi, and 12 mA at 50 psi. But most instrumentation is based on voltages between 0 and 5 V, not on currents.

Therefore, let us design a current-to-voltage converter that will output 5 V when the current signal is 20 mA.

SOLUTION

The circuit in Fig. 2.51a is a very accurate model of our situation. The wiring from the sensor unit to the controller has some resistance, R_{wire}. If the sensor output were a voltage proportional to pressure, the voltage drop in the line would cause measurement error even if the sensor output were an ideal source of voltage. But, since the data are contained in the current value, R_{wire} does not affect the accuracy at the controller as long as the sensor acts as an ideal current source.

As for the current-to-voltage converter, it is extremely simple—a resistor. For 5 V at 20 mA, we employ Ohm's law to find

$$R = \frac{5}{0.02} = 250\ \Omega$$

The resulting converter is added to the system in Fig. 2.51b, where we tacitly assume that the controller does not load the remaining portion of the circuit.

Note that the model indicates that the distance between the sensor and controller could be infinite. Intuitively, this situation would appear to be unreasonable, and it is. Losses that would take place over distance can be accounted for by using a more accurate model of the sensor, as shown in Fig. 2.52. The effect of this new sensor model can be seen from the equations that describe this new network. The model equations are

$$I_S = \frac{V_S}{R_S} + \frac{V_S}{R_{\text{wire}} + 250}$$

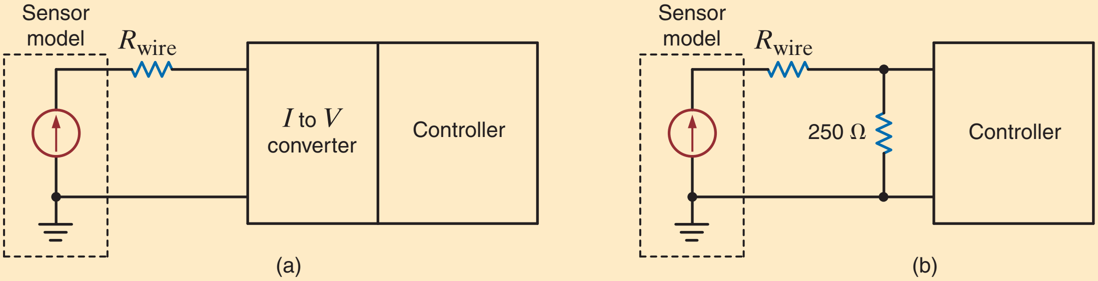


Figure 2.51 The 4- to 20-mA control loop (a) block diagram, (b) with the current-to-voltage converter.

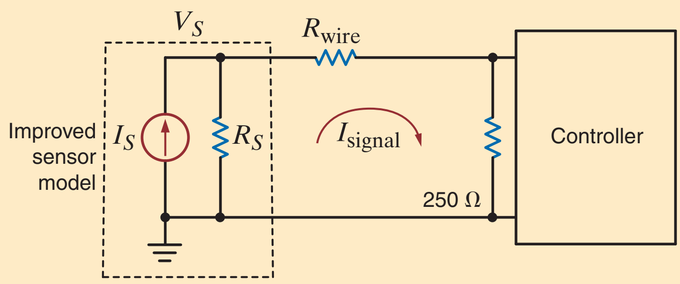


Figure 2.52
A more accurate model for the 4- to 20-mA control loop.

and

$$I_{signal} = \frac{V_S}{R_{wire} + 250}$$

Combining these equations yields

$$\frac{I_{signal}}{I_S} = \frac{1}{1 + \dfrac{R_{wire} + 250}{R_S}}$$

Thus, we see that it is the size of R_S relative to $(R_{wire} + 250\ \Omega)$ that determines the accuracy of the signal at the controller. Therefore, we want R_S as large as possible. Both the maximum sensor output voltage and output resistance, R_S, are specified by the sensor manufacturer.

We will revisit this current-to-voltage converter in Chapter 4.

DESIGN EXAMPLE 2.37

The network in Fig. 2.53 is an equivalent circuit for a transistor amplifier used in a stereo preamplifier. The input circuitry, consisting of a 2-mV source in series with a 500-Ω resistor, models the output of a compact disk player. The dependent source, R_{in}, and R_o model the transistor, which amplifies the signal and then sends it to the power amplifier. The 10-kΩ load resistor models the input to the power amplifier that actually drives the speakers. We must design a transistor amplifier as shown in Fig. 2.53 that will provide an overall gain of −200. In practice we do not actually vary the device parameters to achieve the desired gain; rather, we select a transistor from the manufacturer's data books that will satisfy the required specification. The model parameters for three different transistors are listed as follows:

Manufacturer's transistor parameter values

Part Number	R_{in} (kΩ)	R_o (kΩ)	g_m (mA/V)
1	1.0	50	50
2	2.0	75	30
3	8.0	80	20

Design the amplifier by choosing the transistor that produces the most accurate gain. What is the percent error of your choice?

SOLUTION The output voltage can be written

$$V_o = -g_m V(R_o // R_L)$$

Using voltage division at the input to find V,

$$V = V_S\left(\frac{R_{\text{in}}}{R_{\text{in}} + R_S}\right)$$

Combining these two expressions, we can solve for the gain:

$$A_V = \frac{V_o}{V_S} = -g_m\left(\frac{R_{\text{in}}}{R_{\text{in}} + R_S}\right)(R_o//R_L)$$

Using the parameter values for the three transistors, we find that the best alternative is transistor number 2, which has a gain error of

$$\text{Percent error} = \left(\frac{211.8 - 200}{200}\right) \times 100\% = 5.9\%$$

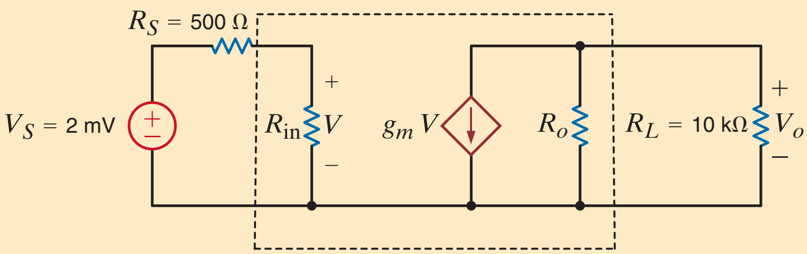


Figure 2.53
Transistor amplifier circuit model.

SUMMARY

- ***Ohm's law*** $V = IR$
- ***The passive sign convention with Ohm's law*** The current enters the resistor terminal with the positive voltage reference.
- ***Kirchhoff's current law (KCL)*** The algebraic sum of the currents leaving (entering) a node is zero.
- ***Kirchhoff's voltage law (KVL)*** The algebraic sum of the voltages around any closed path is zero.
- ***Solving a single-loop circuit*** Determine the loop current by applying KVL and Ohm's law.
- ***Solving a single-node-pair circuit*** Determine the voltage between the pair of nodes by applying KCL and Ohm's law.
- ***The voltage-division rule*** The voltage is divided between two series resistors in direct proportion to their resistance.
- ***The current-division rule*** The current is divided between two parallel resistors in reverse proportion to their resistance.
- ***The equivalent resistance of a network of resistors*** Combine resistors in series by adding their resistances. Combine resistors in parallel by adding their conductances. The wye-to-delta and delta-to-wye transformations are also an aid in reducing the complexity of a network.
- ***Short circuit*** Zero resistance, zero voltage; the current in the short is determined by the rest of the circuit.
- ***Open circuit*** Zero conductance, zero current; the voltage across the open terminals is determined by the rest of the circuit.

PROBLEMS

2.1 Determine the current and power dissipated in the resistor in Fig. P2.1.

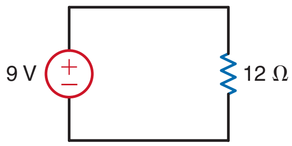


Figure P2.1

2.2 Determine the current and power dissipated in the resistors in Fig. P2.2.

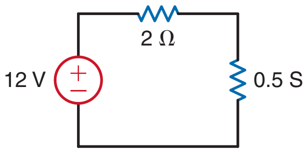


Figure P2.2

2.3 Determine the voltage across the resistor in Fig. P2.3 and the power dissipated.

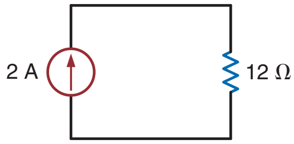


Figure P2.3

2.4 Given the circuit in Fig. P2.4, find the voltage across each resistor and the power dissipated in each.

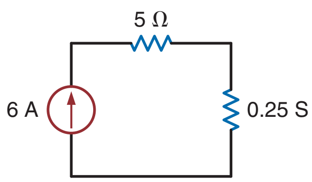


Figure P2.4

2.5 In the network in Fig. P2.5, the power absorbed by R_x is 20 mW. Find R_x.

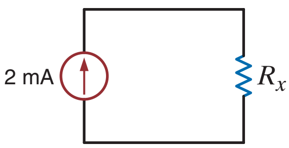


Figure P2.5

2.6 In the network in Fig. P2.6, the power absorbed by G_x is 20 mW. Find G_x.

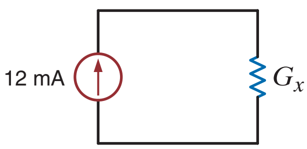


Figure P2.6

2.7 A model for a standard two D-cell flashlight is shown in Fig. P2.7. Find the power dissipated in the lamp.

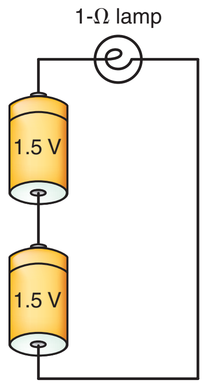


Figure P2.7

2.8 An automobile uses two halogen headlights connected as shown in Fig. P2.8. Determine the power supplied by the battery if each headlight draws 3 A of current.

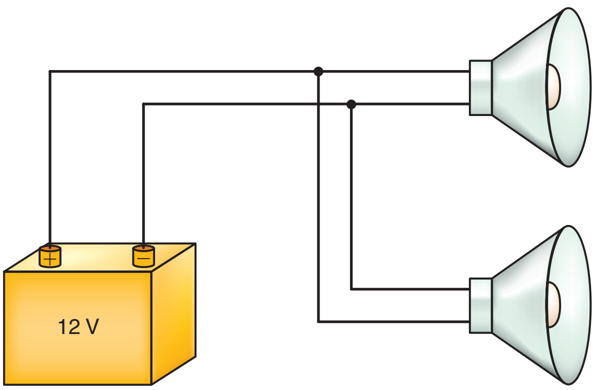


Figure P2.8

2.9 Many years ago a string of Christmas tree lights was manufactured in the form shown in Fig. P2.9a. Today the lights are manufactured as shown in Fig. P2.9b. Is there a good reason for this change?

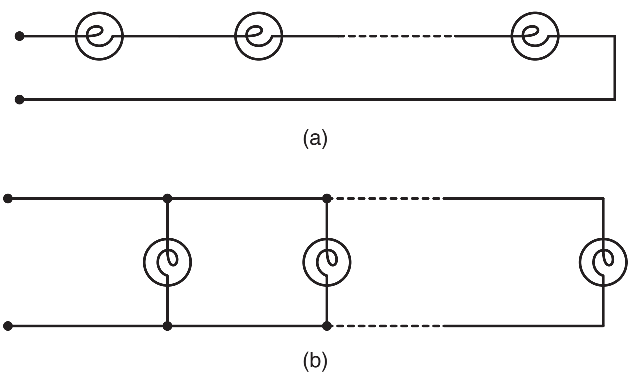


Figure P2.9

2.10 Find I_1 in the network in Fig. P2.10.

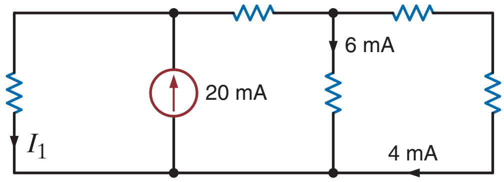


Figure P2.10

2.11 Find I_1 in the network in Fig. P2.11.

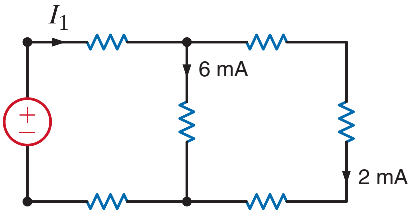


Figure P2.11

2.12 Find I_1 and I_2 in the network in Fig. P2.12.

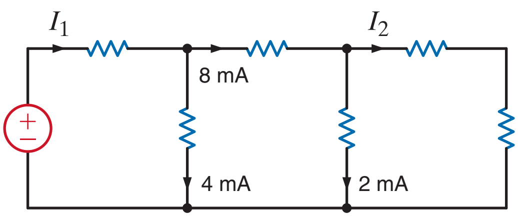


Figure P2.12

2.13 Find I_1 in the circuit in Fig. P2.13.

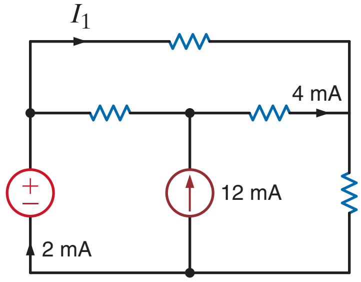


Figure P2.13

2.14 Find I_x in the network in Fig. P2.14.

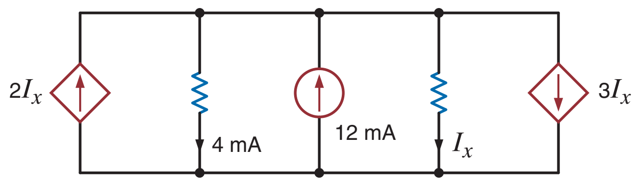


Figure P2.14

2.15 Determine I_L in the circuit in Fig. P2.15.

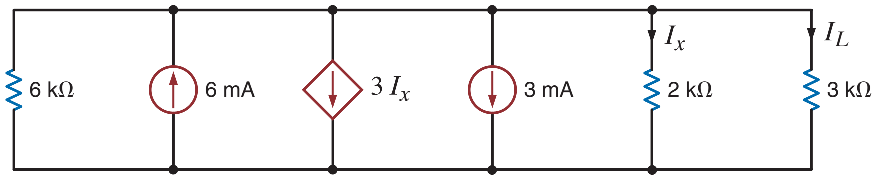


Figure P2.15

2.16 Find I_o and I_1 in the circuit in Fig. P2.16.

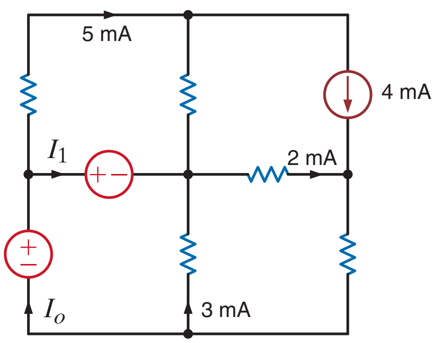


Figure P2.16

2.17 Find I_1 in the network in Fig. P2.17.

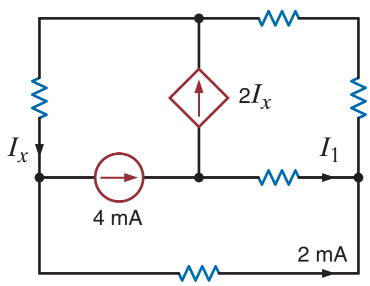


Figure P2.17

2.18 Find I_x, I_y, and I_z in the network in Fig. P2.18.

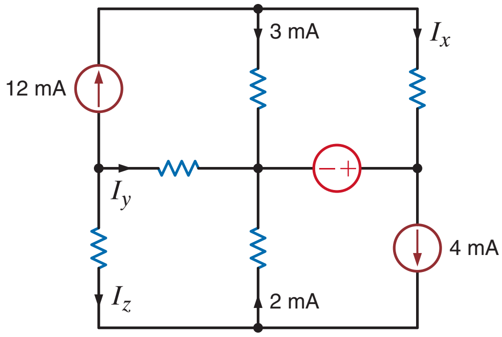


Figure P2.18

2.19 Find I_1 in the circuit in Fig. P2.19.

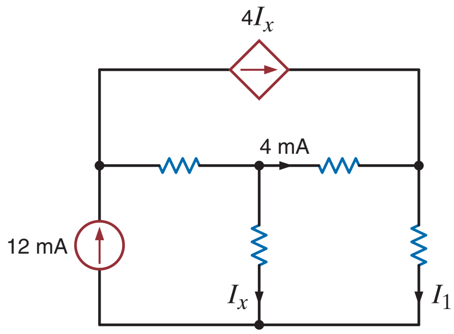


Figure P2.19

2.20 Find I_1 in the network in Fig. P2.20.

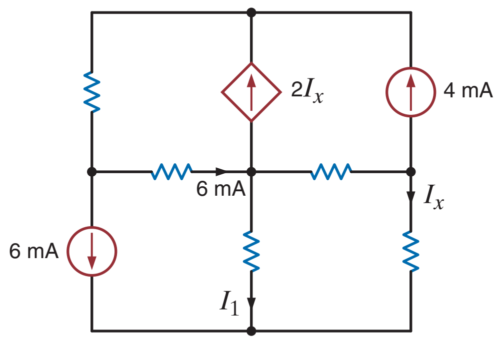


Figure P2.20

2.21 Find I_1, I_2, and I_3 in the network in Fig. P2.21.

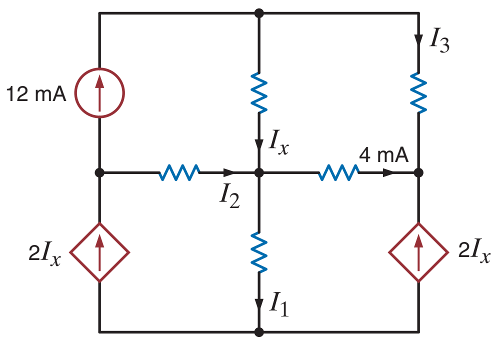


Figure P2.21

2.22 In the network in Fig. P2.22, Find I_1, I_2 and I_3 and show that KCL is satisfied at the boundary.

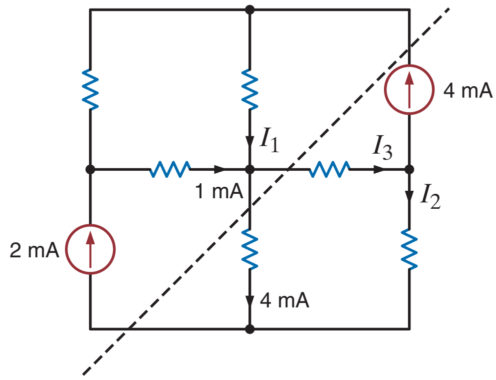


Figure P2.22

2.23 Find V_{bd} in the circuit in Fig. P2.23.

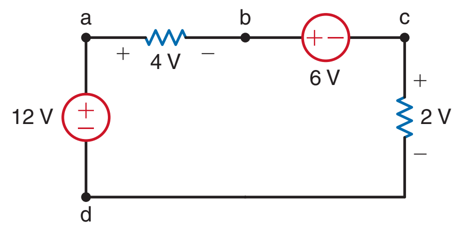


Figure P2.23

2.24 Find V_{ad} in the network in Fig. P2.24.

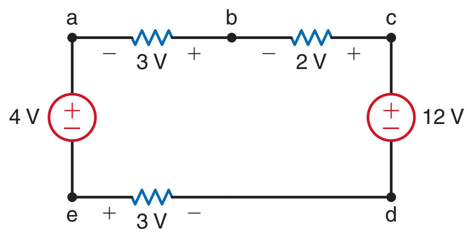


Figure P2.24

2.25 Find V_{fb} and V_{ec} in the circuit in Fig. P2.25.

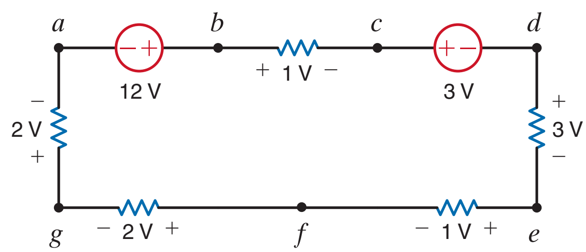


Figure P2.25

2.26 Find V_{ae} and V_{cf} in the circuit in Fig. P2.26.

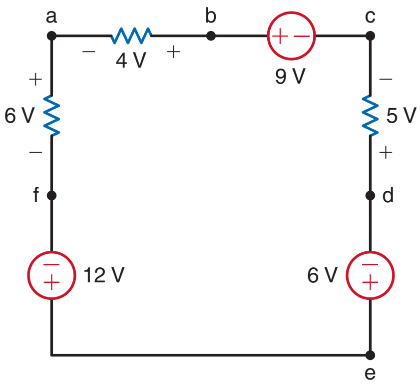


Figure P2.26

2.27 Given the circuit diagram in Fig. P2.27, find the following voltages: V_{da}, V_{bh}, V_{gc}, V_{di}, V_{fa}, V_{ac}, V_{ai}, V_{hf}, V_{fb}, and V_{dc}.

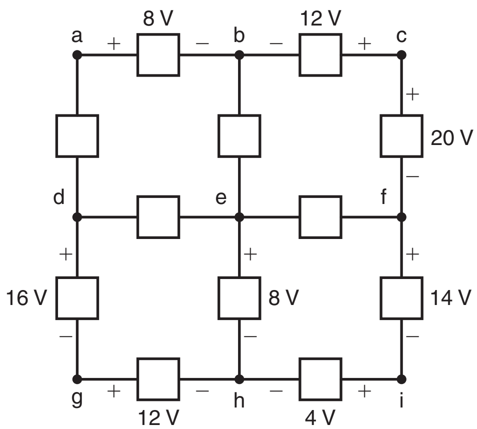


Figure P2.27

2.28 Find V_x and V_y in the circuit in Fig. P2.28.

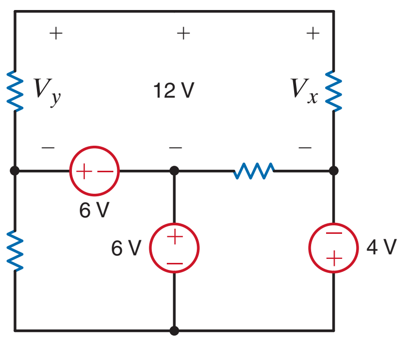


Figure P2.28

2.29 Find V_x and V_y in the circuit in Fig. P2.29.

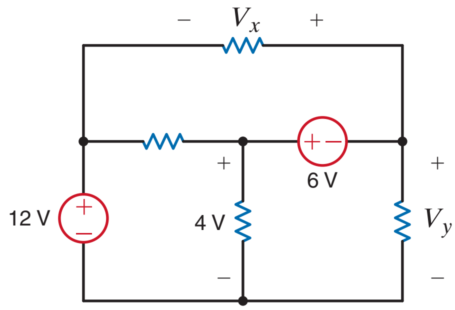


Figure P2.29

2.30 Find V_1, V_2 and V_3 in the network in Fig. P2.30.

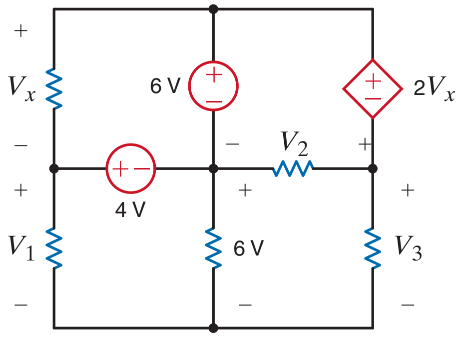


Figure P2.30

2.31 Find V_o in the network in Fig. P2.31.

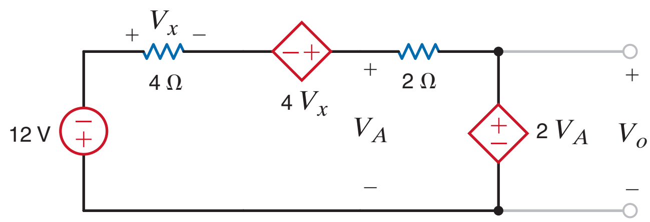


Figure P2.31

2.32 Find V_o in the circuit in Fig. P2.32.

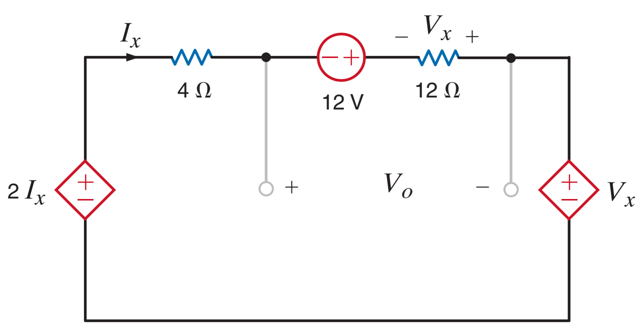


Figure P2.32

2.33 The 10-V source absorbs 2.5 mW of power. Calculate V_{ba} and the power absorbed by the dependent voltage source in Fig. P2.33.

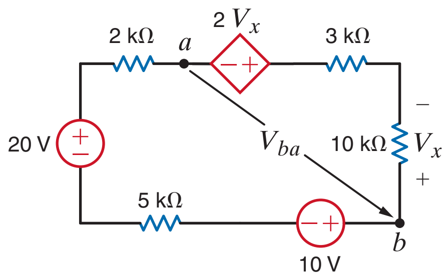


Figure P2.33

2.34 Find V_1, V_2, and V_3 in the network in Fig. P2.34.

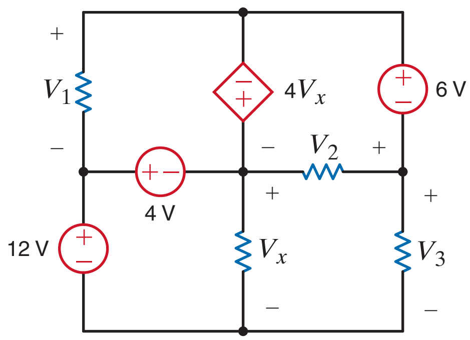


Figure P2.34

2.35 The 10-V source in Fig. P.2.35 is supplying 50 W. Determine R_1.

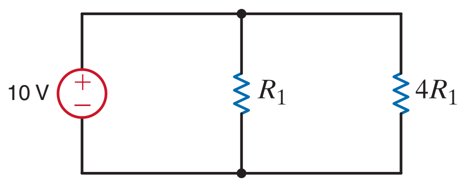


Figure P2.35

2.36 Find V_1 and V_2 in Fig. P2.36.

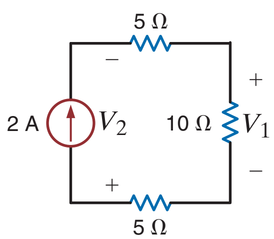


Figure P2.36

2.37 Find V_{bd} in the network in Fig. P2.37.

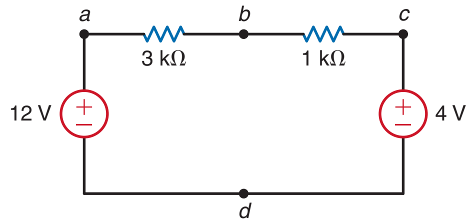


Figure P2.37

2.38 Find V_x in the circuit in Fig. P2.38.

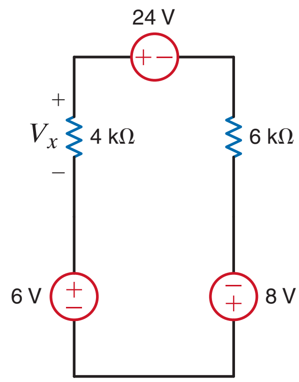


Figure P2.38

2.39 Find V_{ab} in the network in Fig. P2.39.

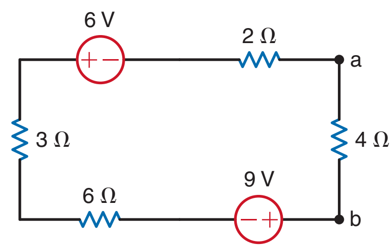


Figure P2.39

2.40 Find V_x and the power supplied by the 15-V source in the circuit in Fig. P2.40.

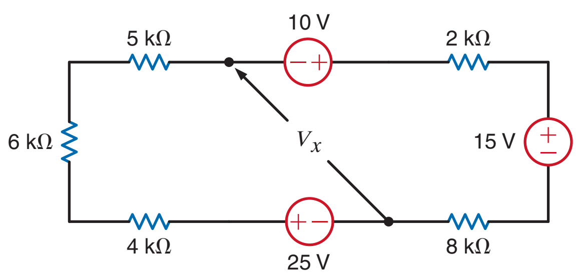


Figure P2.40

2.41 Find V_1 in the network in Fig. P2.41.

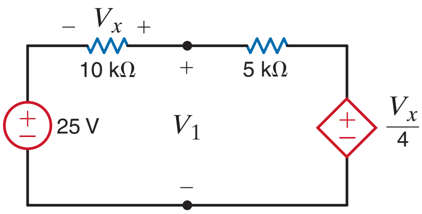


Figure P2.41

2.42 Find the power supplied by each source, including the dependent source, in Fig. P2.42.

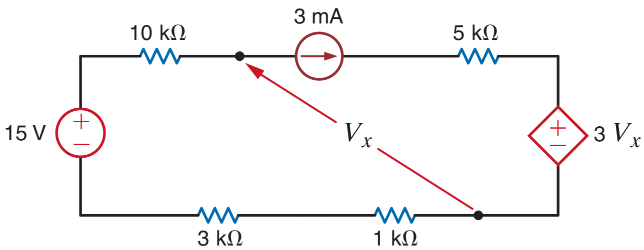


Figure P2.42

2.43 Find the power absorbed by the dependent voltage source in the circuit in Fig. P2.43.

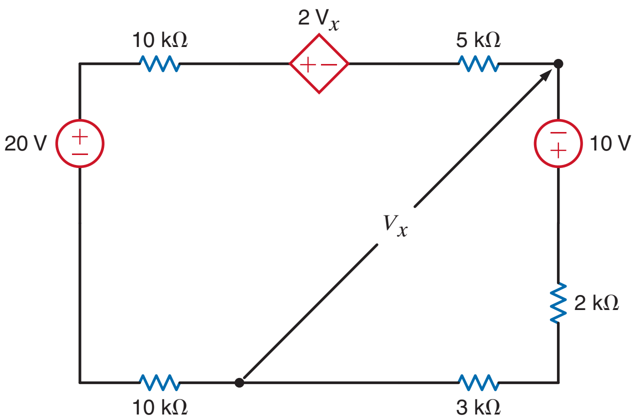


Figure P2.43

2.44 Find the power absorbed by the dependent source in the circuit in Fig. P2.44.

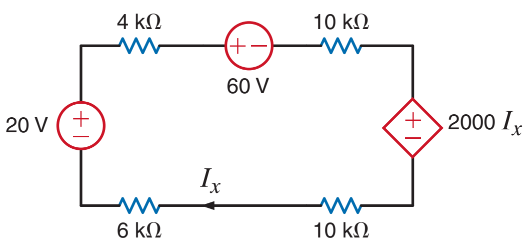


Figure P2.44

2.45 The 100-V source in the circuit in Fig. P2.45 is supplying 200 W. Solve for V_2.

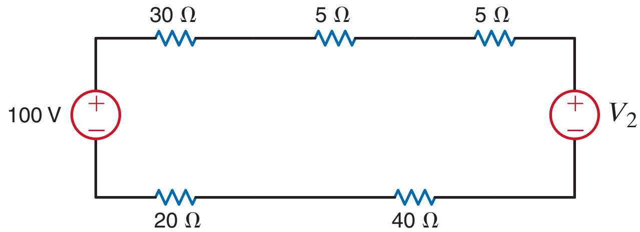


Figure P2.45

2.46 Find the value of V_2 in Fig. P2.46 such that $V_1 = 0$.

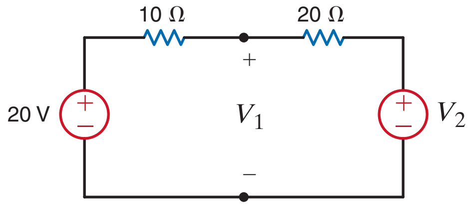


Figure P2.46

2.47 Find I_o in the network in Fig. P2.47.

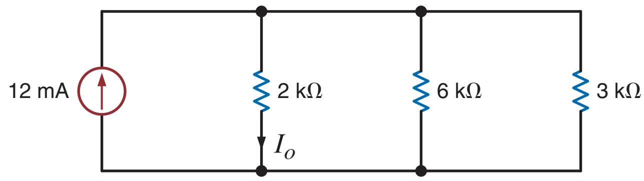


Figure P2.47

2.48 Find I_o in the network in Fig. P2.48.

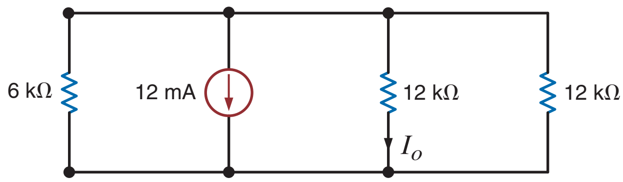


Figure P2.48

2.49 Find the power supplied by each source in the circuit in Fig. P2.49.

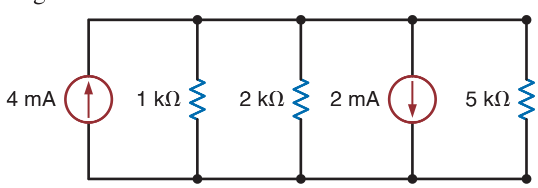


Figure P2.49

2.50 Find the current I_A in the circuit in Fig. P2.50.

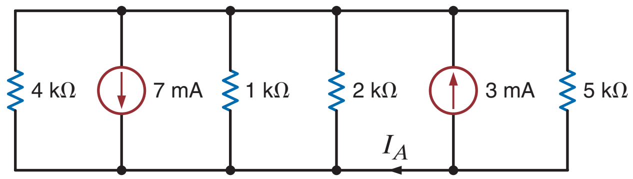


Figure P2.50

2.51 Find I_o in the network in Fig. P2.51.

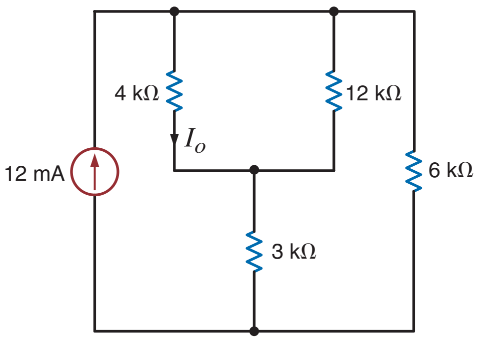


Figure P2.51

2.52 Find I_o in the network in Fig. P2.52.

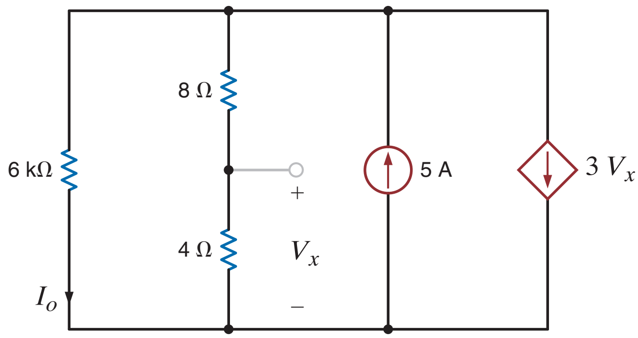


Figure P2.52

2.53 Determine I_L in the circuit in Fig. P2.53.

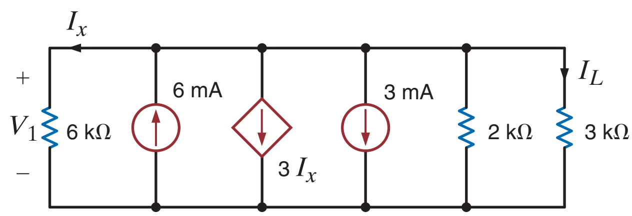


Figure P2.53

2.54 Find the power absorbed by the dependent source in the network in Fig. P2.54.

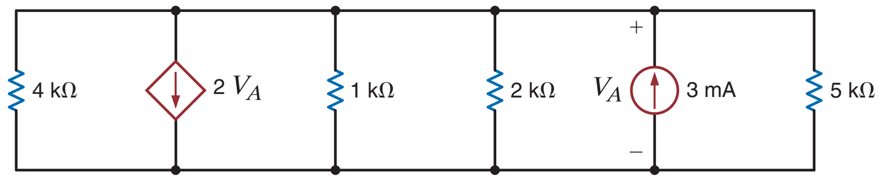


Figure P2.54

2.55 Find R_{AB} in the circuit in Fig. P2.55.

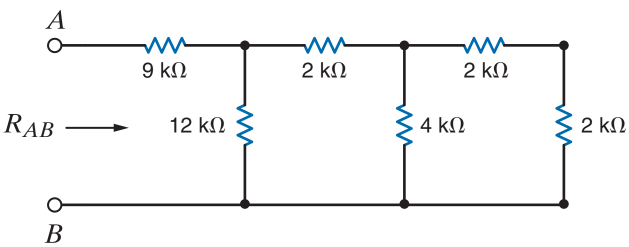


Figure P2.55

2.56 Find R_{AB} in the network in Fig. P2.56.

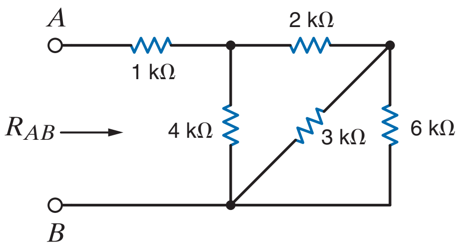


Figure P2.56

2.57 Find R_{AB} in the circuit in Fig. P2.57.

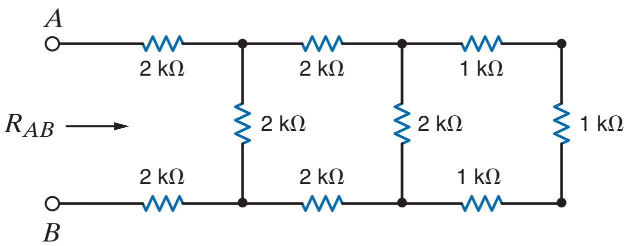


Figure P2.57

2.58 Find R_{AB} in the network in Fig. P2.58.

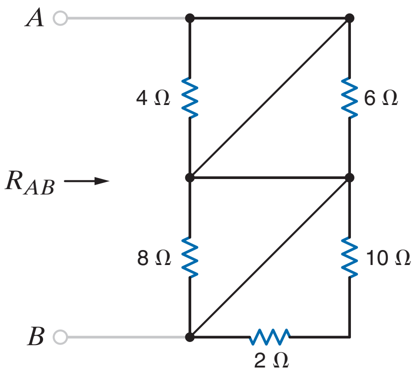


Figure P2.58

2.59 Find R_{AB} in the circuit in Fig. P2.59.

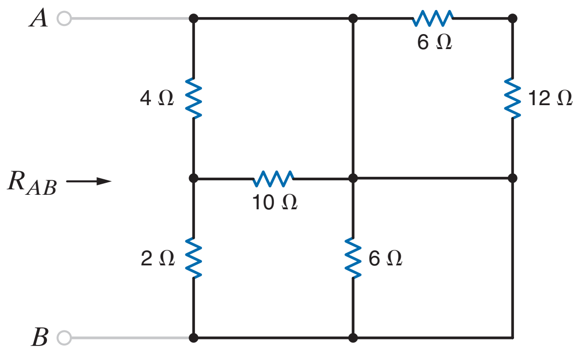


Figure P2.59

2.60 Find R_{AB} in the network in Fig. P2.60.

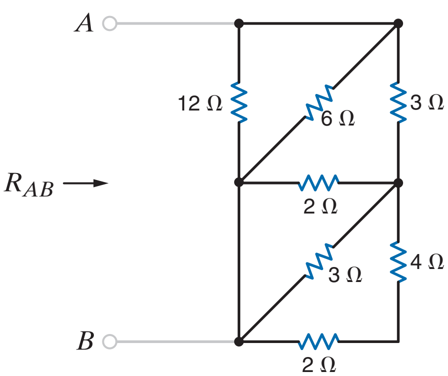


Figure P2.60

2.61 Find R_{AB} in the circuit in Fig. P2.61.

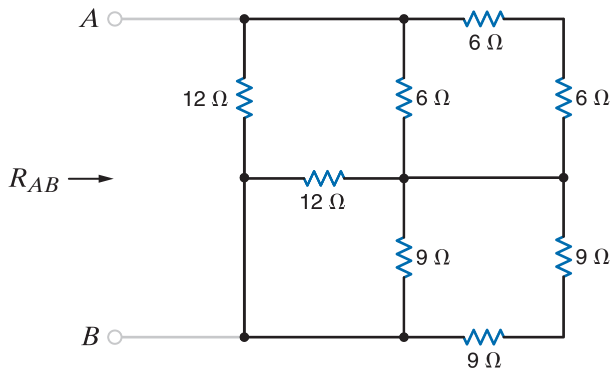


Figure P2.61

2.62 Find R_{AB} in the network in Fig. P2.62.

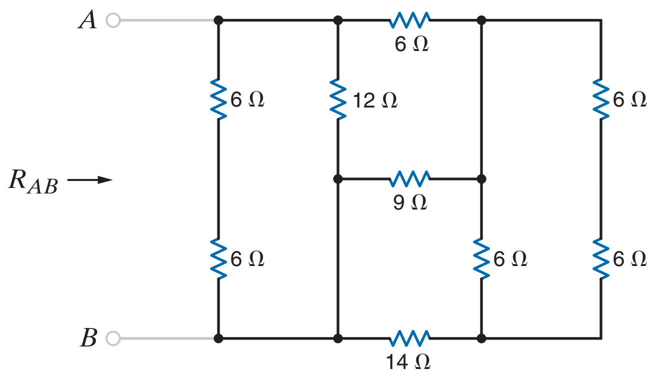


Figure P2.62

2.63 Find the equivalent resistance R_{eq} in the network in Fig. P2.63.

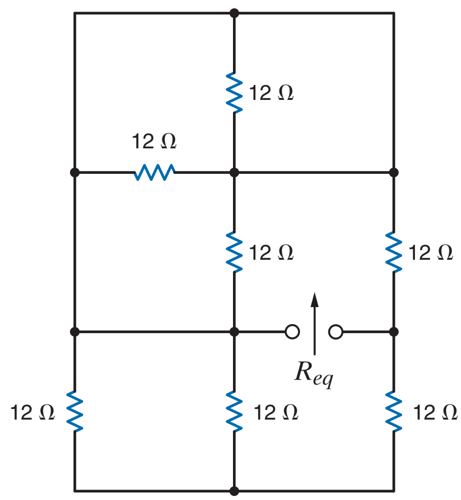


Figure P2.63

2.64 Find the equivalent resistance looking in at terminals a-b in the circuit in Fig. P2.64.

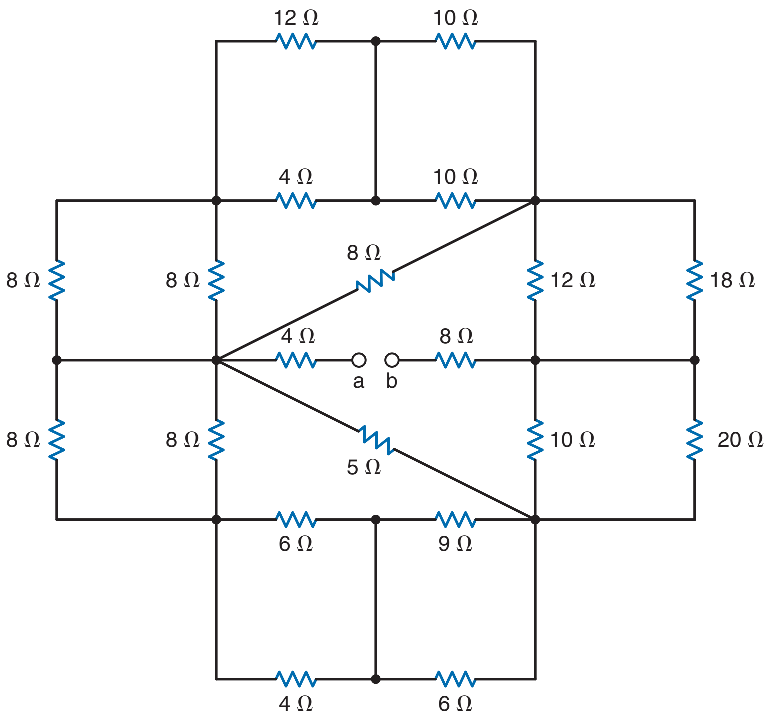


Figure P2.64

2.65 Given the resistor configuration shown in Fig. P2.65, find the equivalent resistance between the following sets of terminals: (1) a and b, (2) b and c, (3) a and c, (4) d and e, (5) a and e, (6) c and d, (7) a and d, (8) c and e, (9) b and d, and (10) b and e.

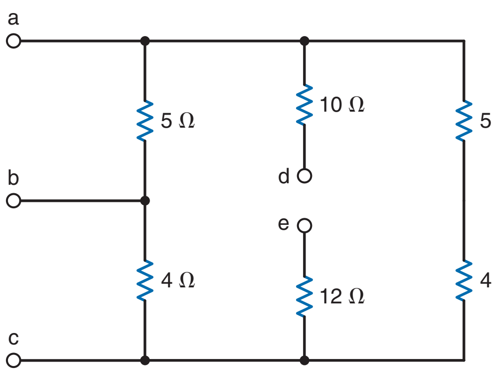


Figure P2.65

2.66 Seventeen possible equivalent resistance values may be obtained using three resistors. Determine the seventeen different values if you are given resistors with standard values: 47 Ω, 33 Ω, and 15 Ω.

2.67 Find I_1 and V_0 in the circuit in Fig. P2.67.

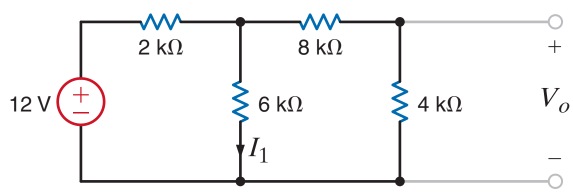


Figure P2.67

2.68 Find I_1 and V_0 in the circuit in Fig. P2.68.

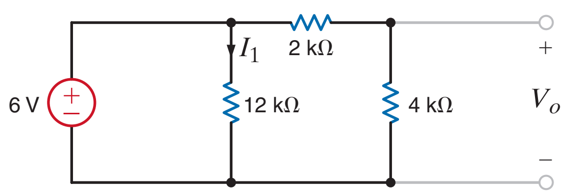


Figure P2.68

2.69 Find V_{ab} and V_{dc} in the circuit in Fig. P2.69.

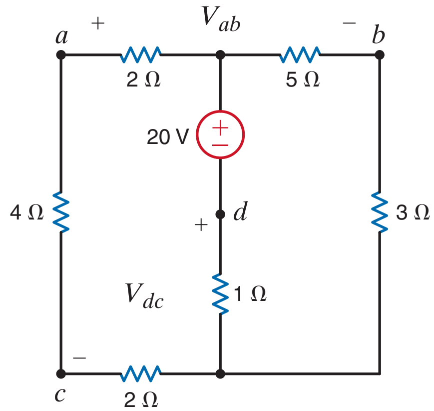


Figure P2.69

2.70 Find V_1 and I_A in the network in Fig. P2.70.

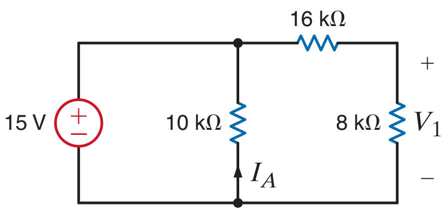


Figure P2.70

2.71 Find I_o in the network in Fig. P2.71.

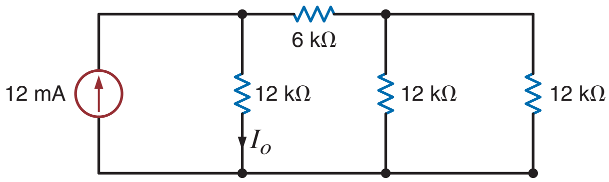


Figure P2.71

2.72 Determine I_o in the circuit in Fig. P2.72.

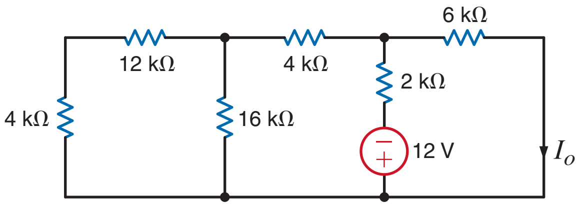


Figure P2.72

2.73 Determine V_o in the network in Fig. P2.73.

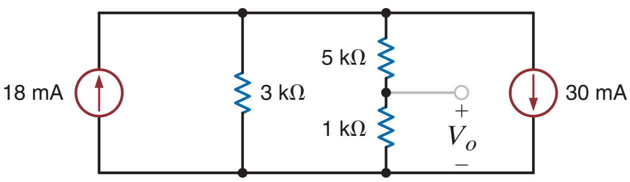


Figure P2.73

2.74 Calculate V_{ab} in Fig. P2.74.

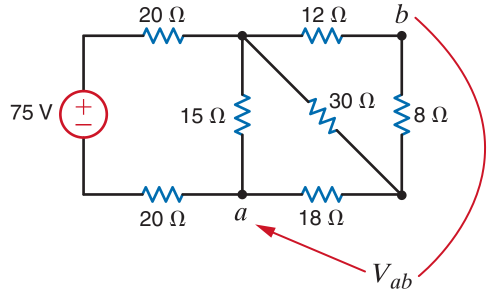


Figure P2.74

2.75 Calculate V_{AB} in Fig. P2.75.

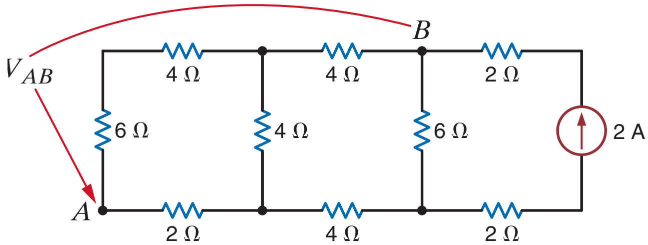


Figure P2.75

2.76 Calculate V_{ab} and V_1 in Fig. P2.76.

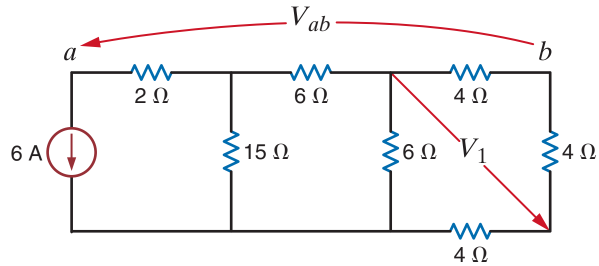


Figure P2.76

2.77 Calculate V_{AB} in Fig. P2.77.

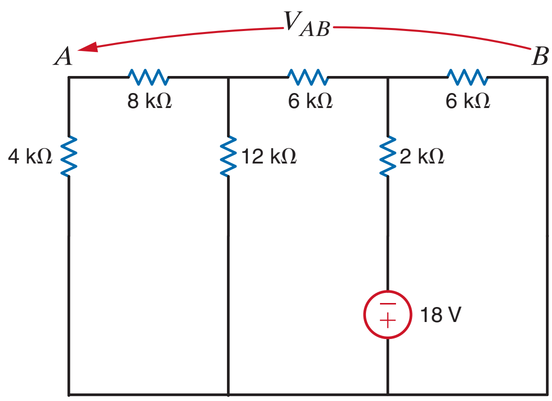


Figure P2.77

2.78 Calculate V_{AB} and I_1 in Fig. P2.78.

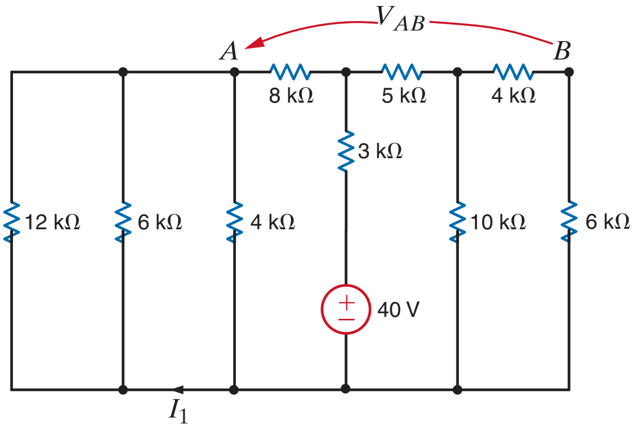


Figure P2.78

2.79 Calculate V_{AB} and I_1 in Fig. P2.79.

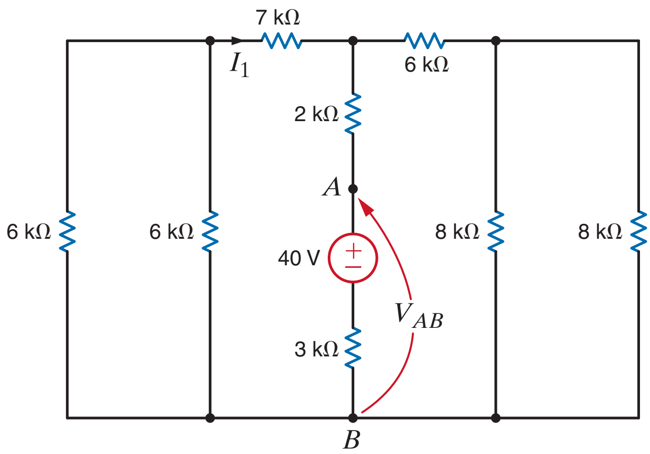


Figure P2.79

2.80 Find V_{ab} in Fig. P2.80.

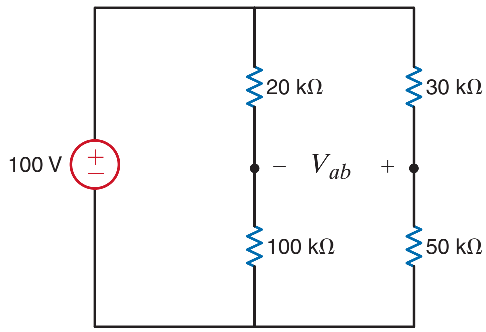


Figure P2.80

2.81 If $V_o = 4$ V in the network in Fig. P2.81, find V_S.

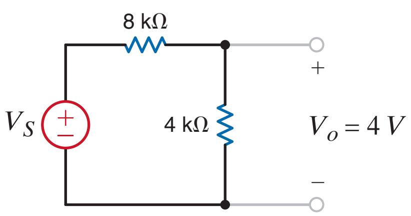


Figure P2.81

2.82 If $I_o = 5$ mA in the circuit in Fig. P2.82, find I_S.

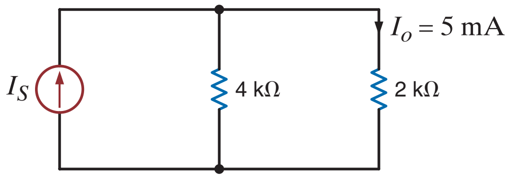


Figure P2.82

2.83 If $I_o = 2$ mA in the circuit in Fig. P2.83, find V_S.

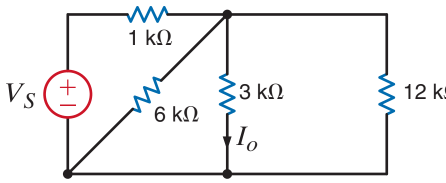


Figure P2.83

2.84 Find the value of V_S in the network in Fig. P2.84 such that the power supplied by the current source is 0.

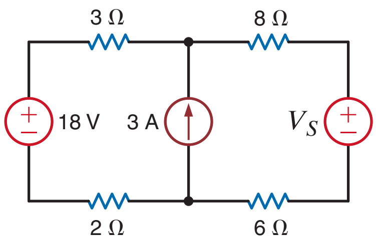


Figure P2.84

2.85 In the network in Fig. P2.85, $V_o = 6$ V. Find I_S.

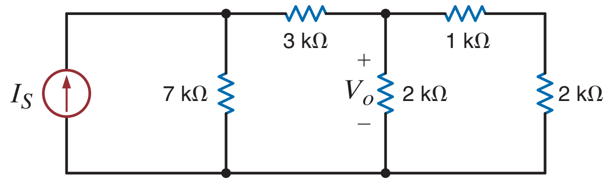


Figure P2.85

2.86 Find the value of V_1 in the network in Fig. P2.86 such that $V_a = 0$.

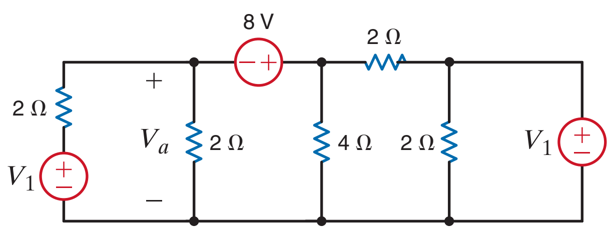


Figure P2.86

2.87 If $V_1 = 5$ V in the circuit in Fig. P2.87, find I_S.

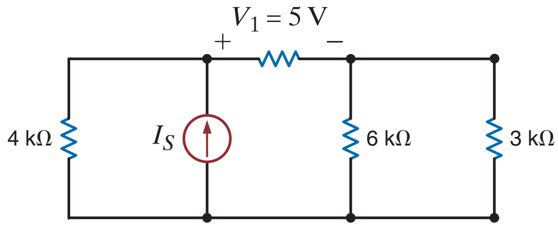


Figure P2.87

2.88 In the network in Fig. P2.88, $V_1 = 12$ V. Find V_S.

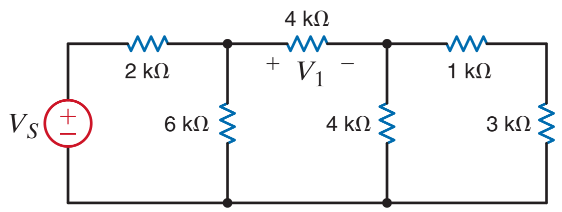


Figure P2.88

2.89 Given that $V_o = 4$ V in the network in Fig. P2.89, find V_S.

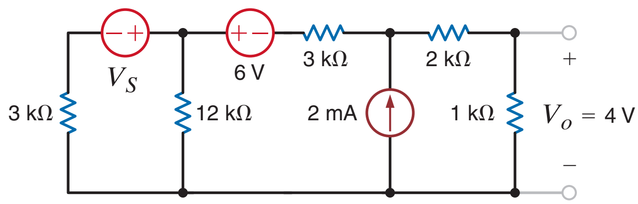


Figure P2.89

2.90 If $V_R = 15$ V, find V_x in Fig. P2.90.

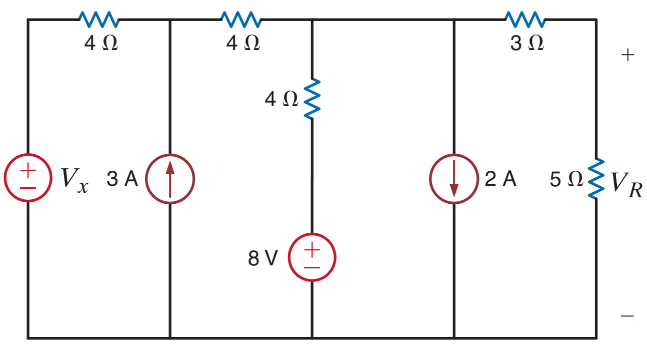


Figure P2.90

2.91 If $V_2 = 4$ V in Fig. P2.91, calculate V_x.

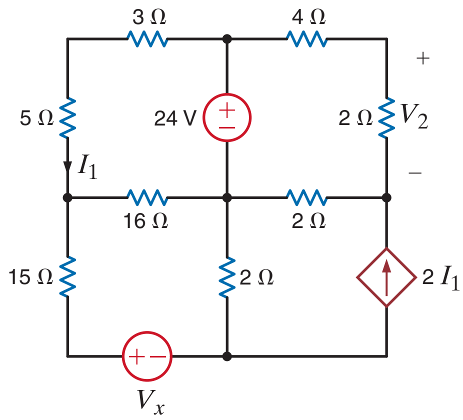


Figure P2.91

2.92 Find the value of I_A in the network in Fig. P2.92.

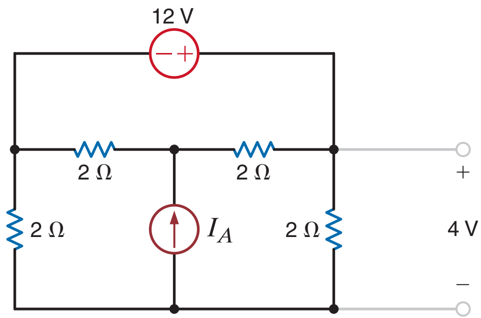


Figure P2.92

2.93 Find the value of I_A in the circuit in Fig. P2.93.

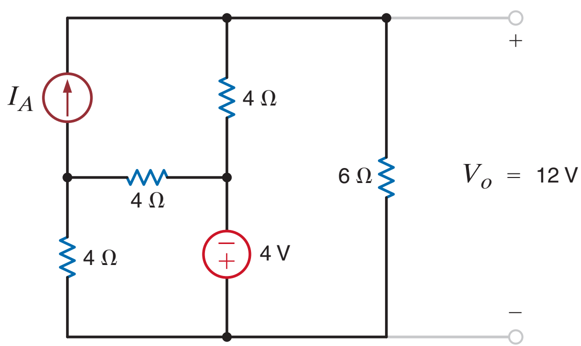


Figure P2.93

2.94 Find in value of the current source I_A in the network in Fig. P2.94.

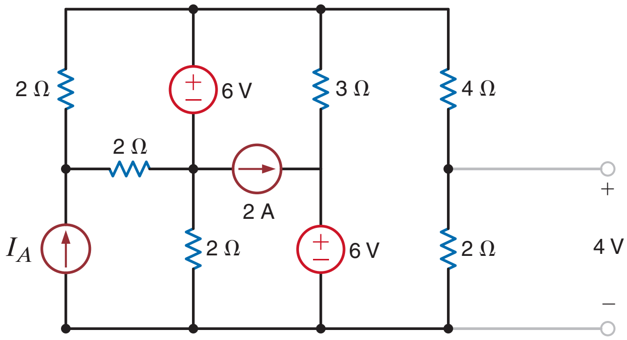


Figure P2.94

2.95 Given $V_o = 12$ V, find the value of I_A in the circuit in Fig. P2.95.

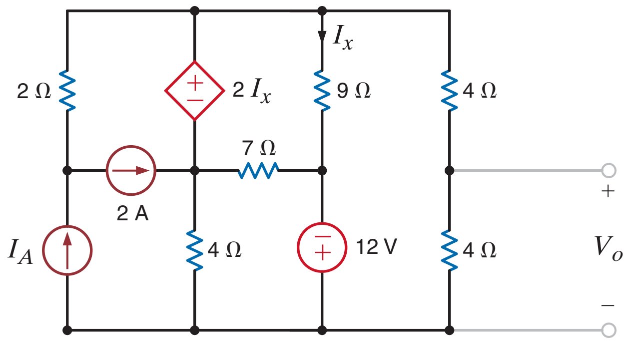


Figure P2.95

2.96 Find the value of V_x in the network in Fig. P2.96, such that the 5-A current source supplies 50 W.

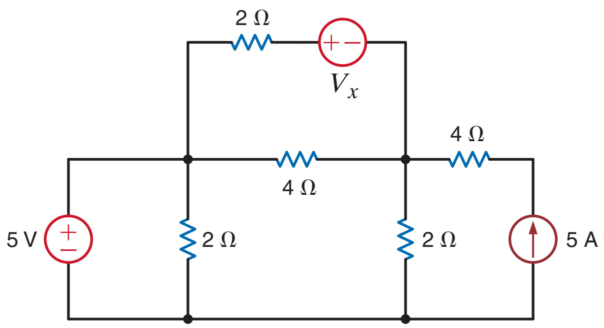


Figure P2.96

2.97 The 5-A current source in Fig. P2.97 supplies 150 W. Calculate V_A.

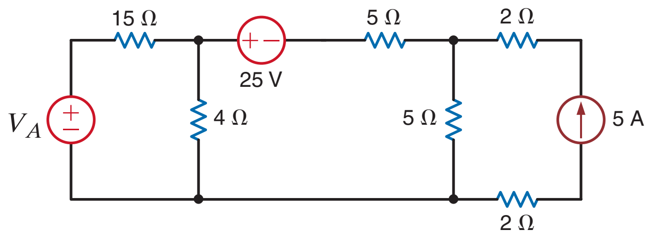


Figure P2.97

2.98 Given $I_o = 2$ mA in the circuit in Fig. P2.98, find I_A.

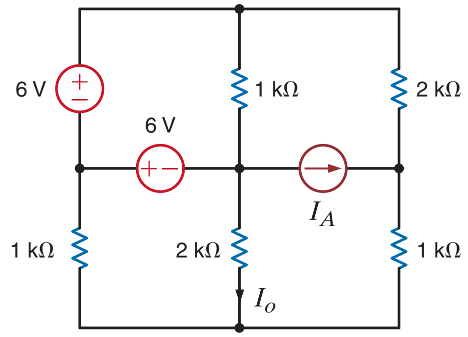


Figure P2.98

2.99 Given $I_o = 2$ mA in the network in Fig. P2.99, find V_A.

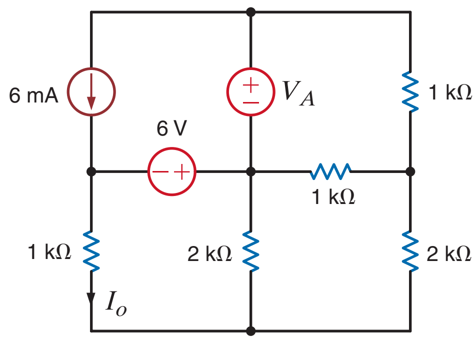


Figure P2.99

2.100 Given V_o in the network in Fig. P2.100, find I_A.

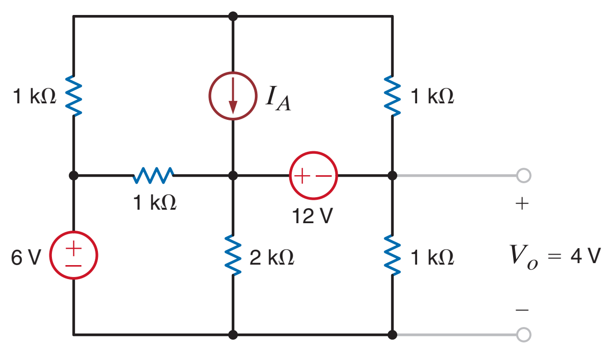


Figure P2.100

2.101 Find the value of V_x in the circuit in Fig. P2.101 such that the power supplied by the 5-A source is 60 W.

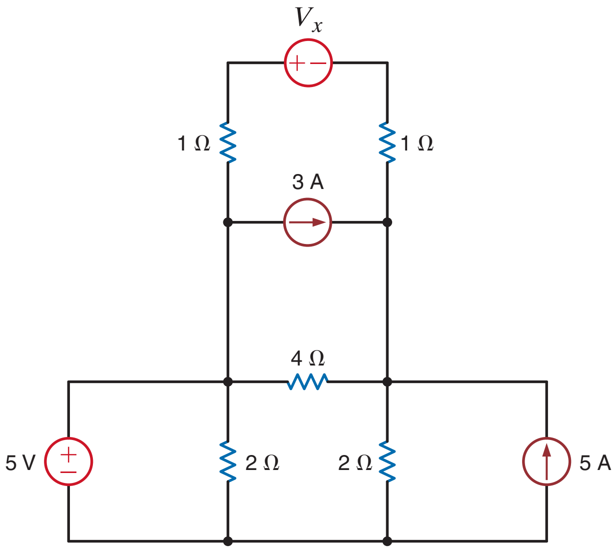


Figure P2.101

2.102 The 3-A current source in Fig. P2.102 is absorbing 12 W. Determine R.

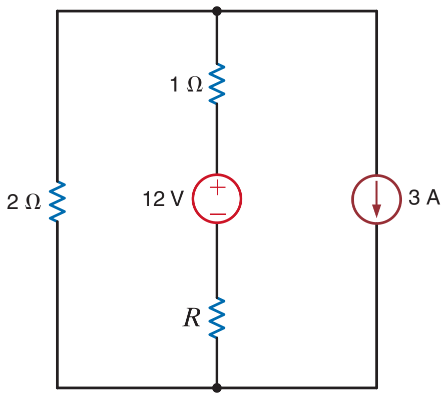


Figure P2.102

2.103 If the power supplied by the 50-V source in Fig. P2.103 is 100 W, find R.

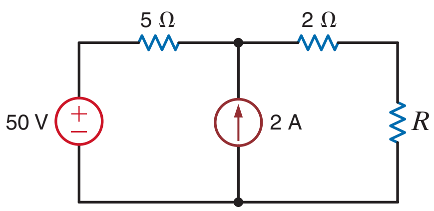


Figure P2.103

2.104 Given that $V_1 = 4$ V, find V_A and R_B in the circuit in Fig. P2.104.

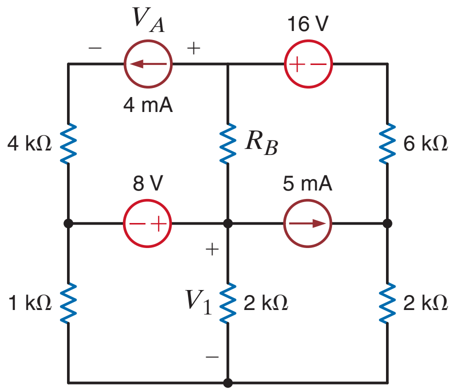


Figure P2.104

2.105 Find the power absorbed by the network in Fig. P2.105.

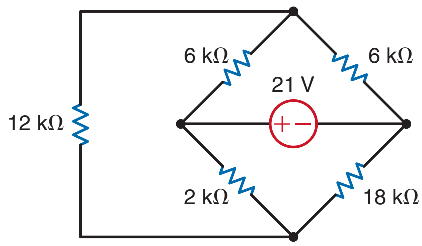


Figure P2.105

2.106 Find the value of g in the network in Fig. P2.106 such that the power supplied by the 3-A source is 20 W.

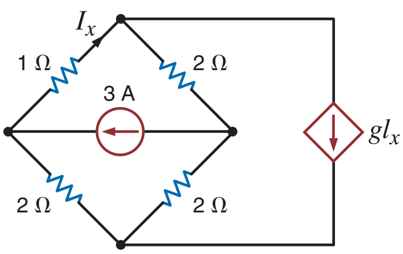


Figure P2.106

2.107 Find the power supplied by the 24-V source in the circuit in Fig. P2.107.

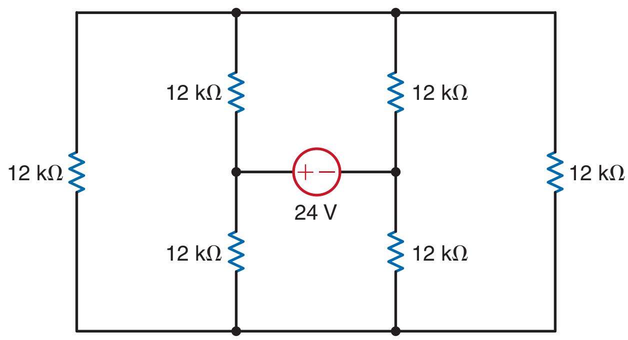


Figure P2.107

2.108 Find I_o in the circuit in Fig. P2.108.

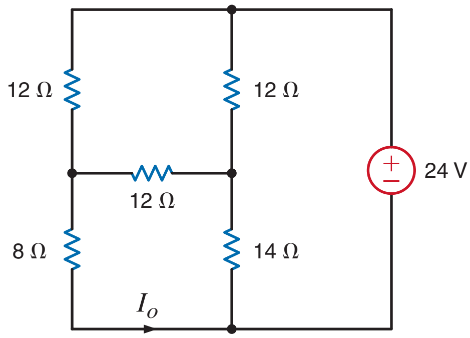


Figure P2.108

2.109 Find I_o in the circuit in Fig. P2.109.

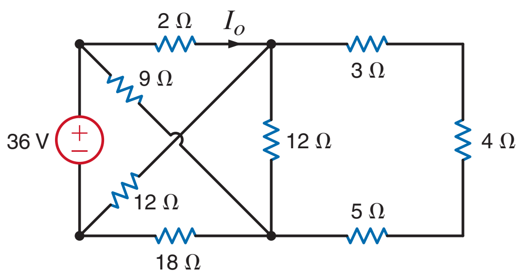


Figure P2.109

2.110 Determine the value of V_o in the network in Fig. P2.110.

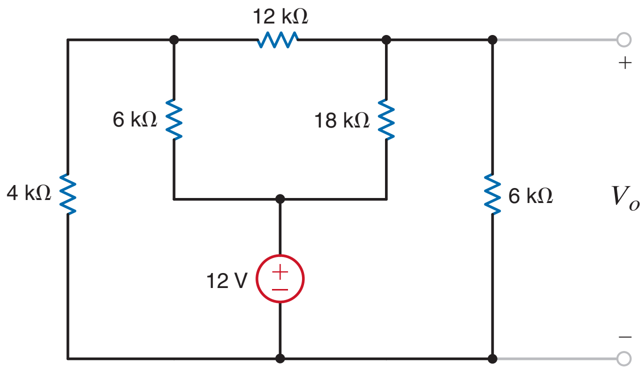


Figure P2.110

2.111 Find V_o in the circuit in Fig. P2.111.

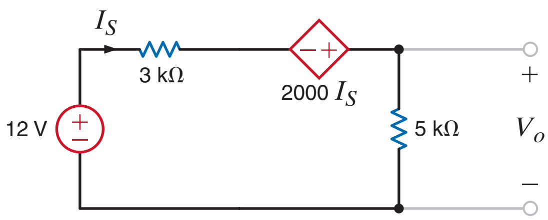


Figure P2.111

2.112 Find V_o in the network in Fig. P2.112.

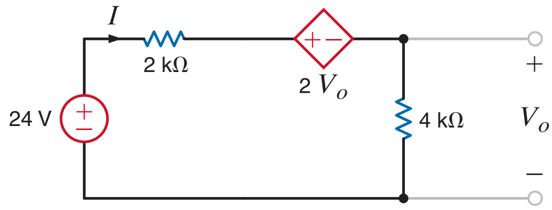


Figure P2.112

2.113 Find I_o in the circuit in Fig. P2.113.

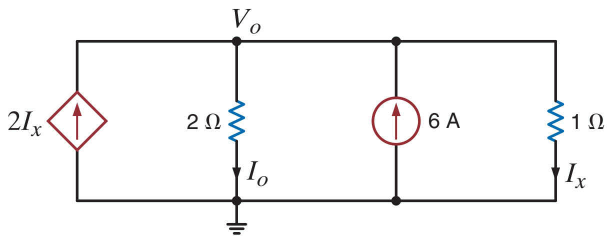


Figure P2.113

2.114 Find I_o in the circuit in Fig. P2.114.

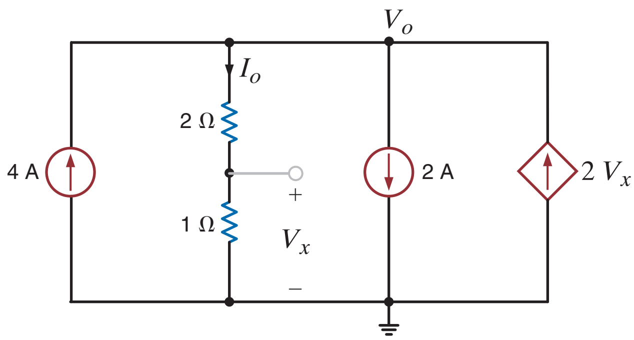


Figure P2.114

2.115 Find V_o in the circuit in Fig. P2.115.

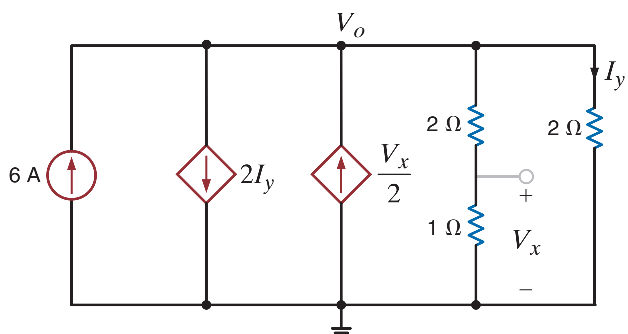


Figure P2.115

2.116 Find V_x in the network in Fig. P2.116.

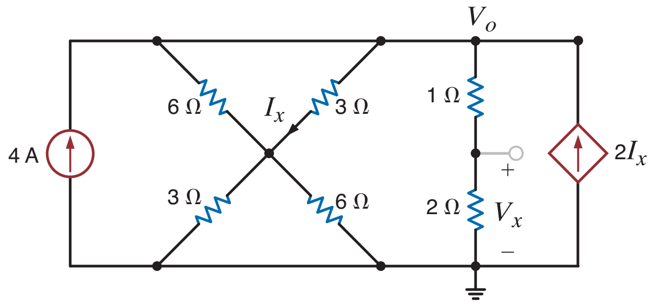


Figure P2.116

2.117 Find V_o in the network in Fig. P2.117.

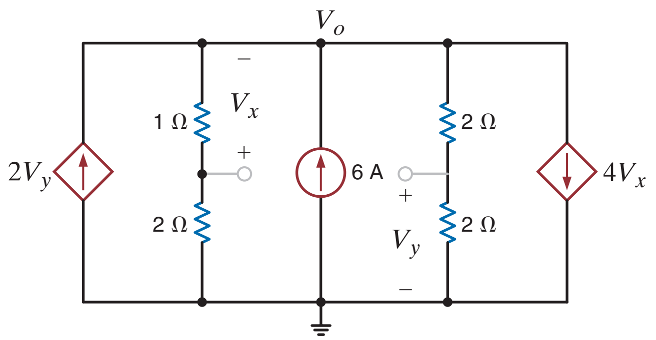


Figure P2.117

2.118 Find I_1, I_2, and I_3 in the circuit in Fig. P2.118.

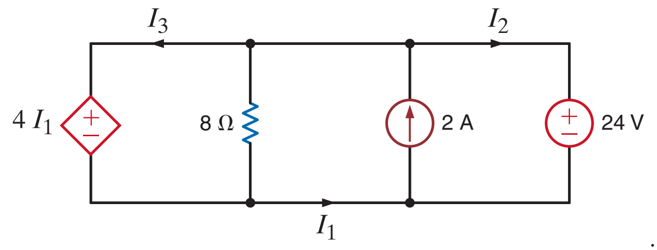


Figure P2.118

2.119 Find I_o in the network in Fig. P2.119.

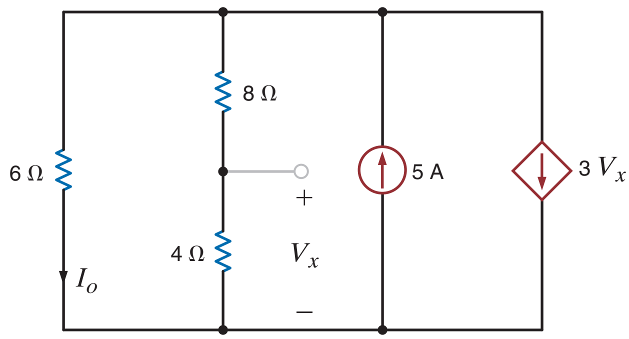


Figure P2.119

2.120 A typical transistor amplifier is shown in Fig. P2.120. Find the amplifier gain G (i.e., the ratio of the output voltage to the input voltage).

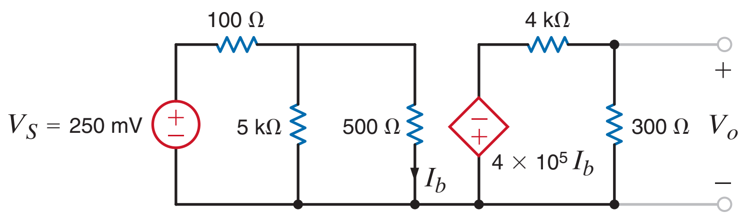


Figure P2.120

2.121 Find the value of k in the network in Fig. P2.121, such that the power supplied by the 6-A source is 108 W.

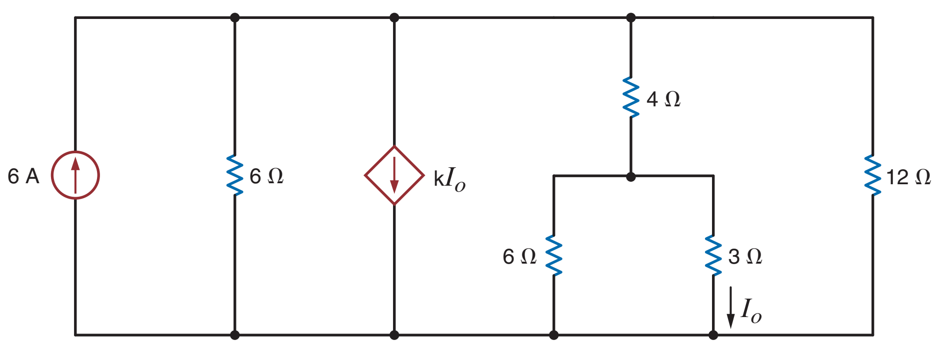


Figure P2.121

2.122 Find the power supplied by the dependent current source in Fig. P2.122.

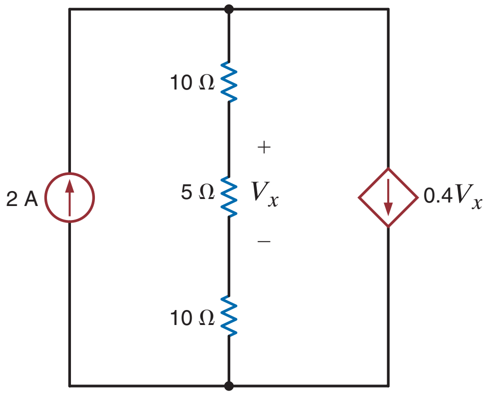


Figure P2.122

2.123 If the power absorbed by the 10-V source in Fig. P2.123 is 40 W, calculate I_S.

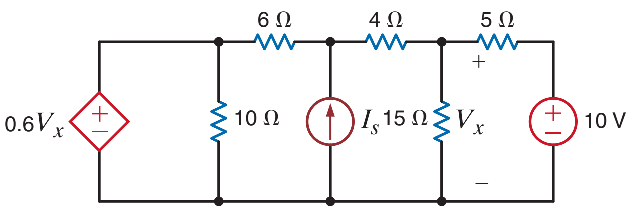


Figure P2.123

2.124 The power supplied by the 2-A current source in Fig. P2.124 is 50 W, calculate k.

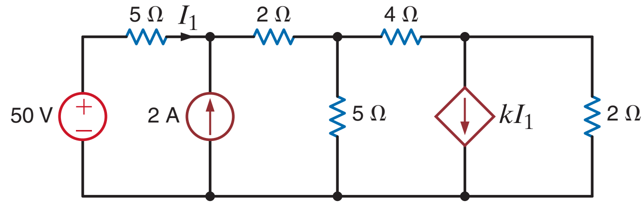


Figure P2.124

2.125 Given the circuit in Fig. P2.125, solve for the value of k.

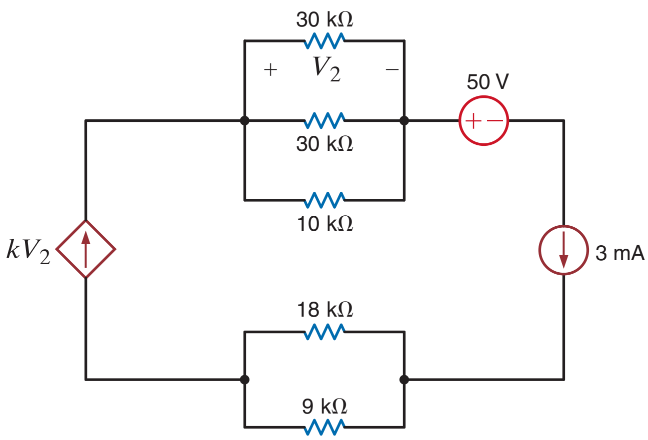


Figure P2.125

TYPICAL PROBLEMS FOUND ON THE FE EXAM

2FE-1 What is the power generated by the source in the network in Fig. 2PFE-1?

a. 2.8 W

b. 1.2 W

c. 3.6 W

d. 2.4 W

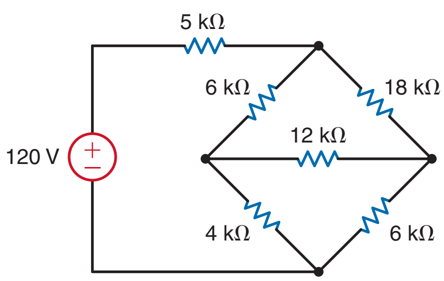


Figure 2PFE-1

2FE-2 Find V_{ab} in the circuit in Fig. 2PFE-2.

a. −5 V

b. 10 V

c. 15 V

d. −10 V

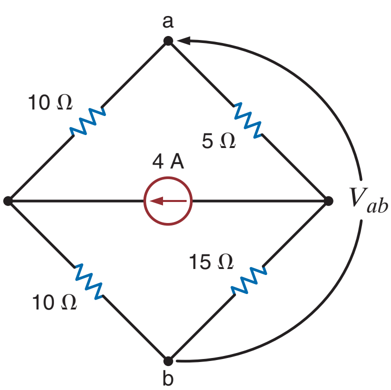


Figure 2PFE-2

2FE-3 If $R_{eq} = 10.8\ \Omega$ in the circuit in Fig. 2PFE-3, what is R_2 ?

a. 12 Ω
b. 20 Ω
c. 8 Ω
d. 18 Ω

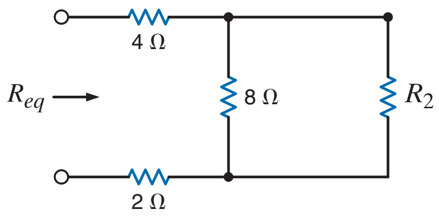


Figure 2PFE-3

2FE-4 Find the equivalent resistance of the circuit in Fig. 2PFE-4 at the terminals *A*-*B*.

a. 4 kΩ
b. 12 kΩ
c. 8 kΩ
d. 20 kΩ

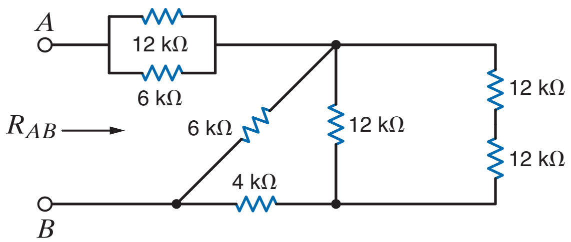


Figure 2PFE-4

2FE-5 The 100 V source is absorbing 50 W of power in the network in Fig. 2PFE-5. What is *R*?

a. 17.27 Ω
b. 9.42 Ω
c. 19.25 Ω
d. 15.12 Ω

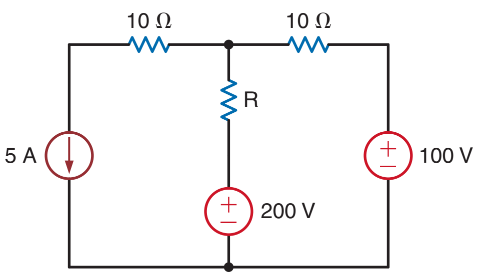


Figure 2PFE-5

2FE-6 Find the power supplied by the 40 V source in the circuit in Fig. 2PFE-6.

a. 120 W
b. 232 W
c. 212 W
d. 184 W

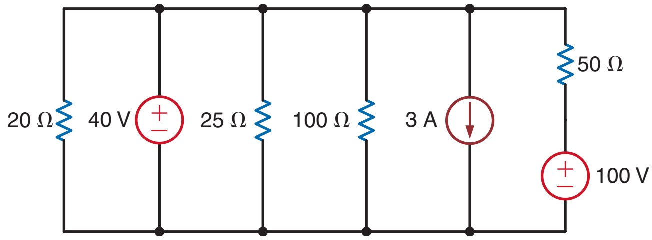


Figure 2PFE-6

2FE-7 What is the current I_o in the circuit in Fig. 2PFE-7?

a. 0.84 mA
b. −1.25 mA
c. 2.75 mA
d. −0.22 mA

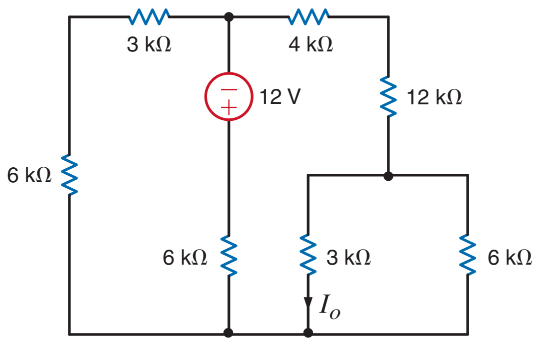


Figure 2PFE-7

2FE-8 Find the voltage V_o in the network in Fig. 2PFE-8.

a. 24 V
b. 10 V
c. 36 V
d. 12 V

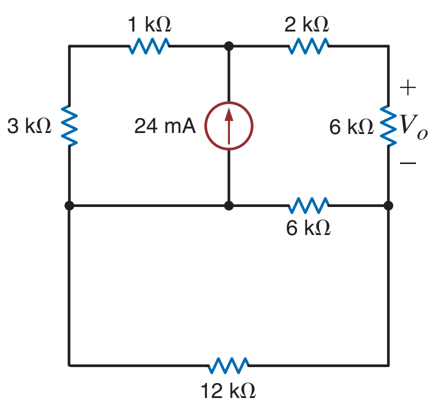


Figure 2PFE-8

2FE-9 What is the voltage V_o in the circuit in Fig. 2PFE-9?

a. 2 V
b. 8 V
c. 5 V
d. 12 V

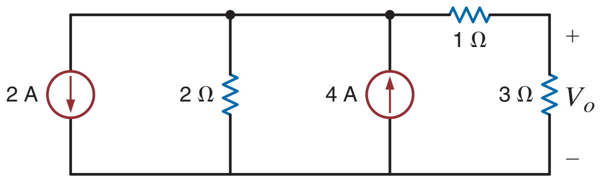


Figure 2PFE-9

2FE-10 Find the current I_x in Fig. 2PFE-10.

a. 1/2 A
b. 5/3 A
c. 3/2 A
d. 8/3 A

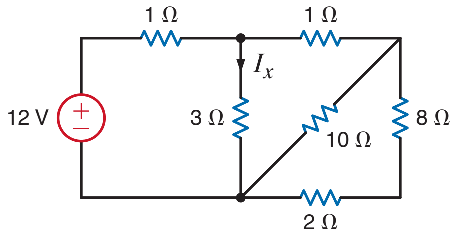


Figure 2PFE-10