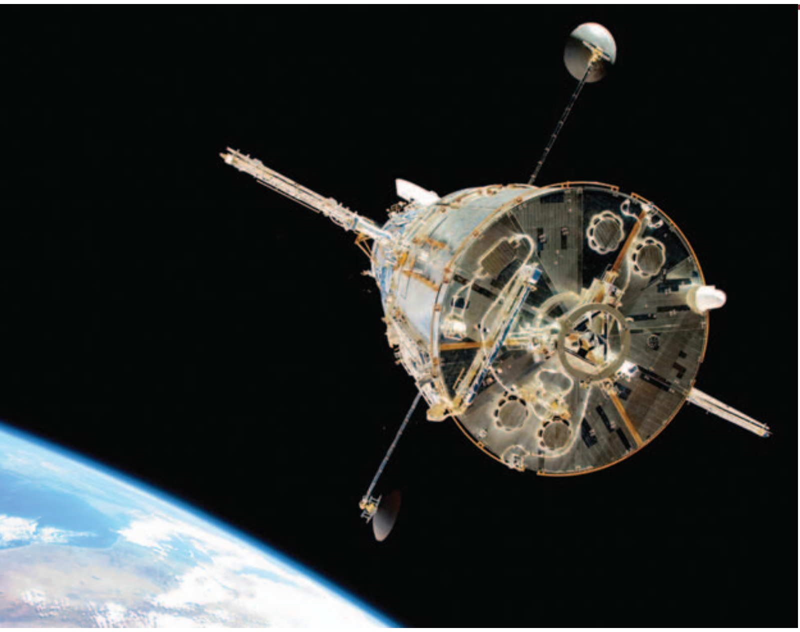
CHAPTER 1

BASIC CONCEPTS



Courtesy NASA, 2009

THE LEARNING GOALS FOR THIS CHAPTER ARE:

- **Review the SI system of units and standard prefixes**
- **Know the definitions of basic electrical quantities: voltage, current, and power**
- **Know the symbols for and definitions of independent and dependent sources**
- **Be able to calculate the power absorbed by a circuit element using the passive sign convention**

Hubble Space Telescope If you were asked to identify the top engineering achievements that depend on currents, voltages, and power in electrical systems, would NASA's Hubble Space Telescope make your list? It should. Launched over 20 years ago into an orbit 375 miles above the Earth's surface, the Hubble Telescope avoids distorting effects of the atmosphere and gives significant new data about the universe. It features multiple channels having many intricate electrical systems that detect different wavelengths of light and enables us to examine our solar system as well as remote galaxies. The success of the Hubble Space Telescope program has led to other NASA plans. In February 2010, the Solar Dynamics Observatory was launched to aid in studying our sun's dynamic processes including high resolution measurements of solar flares; it is the first mission of NASA's Living with a Star program.

Sophisticated as it is, the power of the Hubble Space Telescope is rooted in the fundamental concepts you will begin to study in this chapter—charge, current, voltage, power, and batteries. These core principles are the fundamental building blocks of your understanding of electrical engineering and your ability to analyze and design more complicated electrical systems. Just as the Hubble has led to even greater innovations, we cannot imagine today what else may lie ahead for you.

1.1 System of Units

The system of units we employ is the international system of units, the Système International des Unités, which is normally referred to as the SI standard system. This system, which is composed of the basic units meter (m), kilogram (kg), second (s), ampere (A), kelvin (K), and candela (cd), is defined in all modern physics texts and therefore will not be defined here. However, we will discuss the units in some detail as we encounter them in our subsequent analyses.

The standard prefixes that are employed in SI are shown in Fig. 1.1. Note the decimal relationship between these prefixes. These standard prefixes are employed throughout our study of electric circuits.

Circuit technology has changed drastically over the years. For example, in the early 1960s the space on a circuit board occupied by the base of a single vacuum tube was about the size of a quarter (25-cent coin). Today that same space could be occupied by an Intel Pentium integrated circuit chip containing 50 million transistors. These chips are the engine for a host of electronic equipment.

Figure 1.1 Standard SI prefixes.

1.2 Basic Quantities

Before we begin our analysis of electric circuits, we must define terms that we will employ. However, in this chapter and throughout the book our definitions and explanations will be as simple as possible to foster an understanding of the use of the material. No attempt will be made to give complete definitions of many of the quantities because such definitions are not only unnecessary at this level but are often confusing. Although most of us have an intuitive concept of what is meant by a circuit, we will simply refer to an *electric circuit* as an interconnection of electrical components, each of which we will describe with a mathematical model.

The most elementary quantity in an analysis of electric circuits is the electric *charge*. Our interest in electric charge is centered around its motion, since charge in motion results in an energy transfer. Of particular interest to us are those situations in which the motion is confined to a definite closed path.

An electric circuit is essentially a pipeline that facilitates the transfer of charge from one point to another. The time rate of change of charge constitutes an electric *current*. Mathematically, the relationship is expressed as

$$i(t) = \frac{dq(t)}{dt} \quad \text{or} \quad q(t) = \int_{-\infty}^{t} i(x)\,dx \qquad \textbf{1.1}$$

where i and q represent current and charge, respectively (lowercase letters represent time dependency, and capital letters are reserved for constant quantities). The basic unit of current is the ampere (A), and 1 ampere is 1 coulomb per second.

Although we know that current flow in metallic conductors results from electron motion, the conventional current flow, which is universally adopted, represents the movement of positive charges. It is important that the reader think of current flow as the movement of positive charge regardless of the physical phenomena that take place. The symbolism that will be used to represent current flow is shown in Fig. 1.2. $I_1 = 2$ A in Fig. 1.2a indicates that at any point in the wire shown, 2 C of charge pass from left to right each second. $I_2 = -3$ A in Fig. 1.2b indicates that at any point in the wire shown, 3 C of charge pass from right to left each second. Therefore, it is important to specify not only the magnitude of the variable representing the current but also its direction.

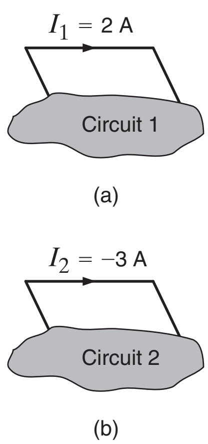


Figure 1.2 Conventional current flow: (a) positive current flow; (b) negative current flow.

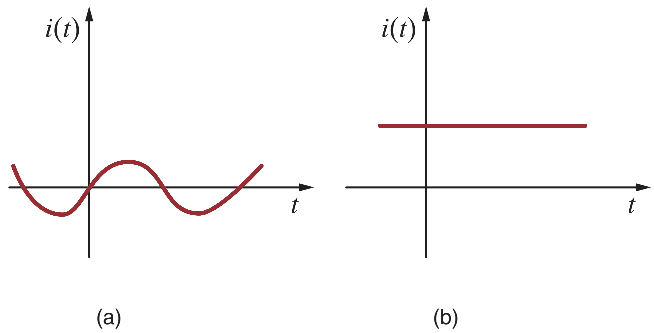


Figure 1.3
Two common types of current: (a) alternating current (ac); (b) direct current (dc).

The two types of current that we encounter often in our daily lives, alternating current (ac) and direct current (dc), are shown as a function of time in Fig. 1.3. *Alternating current* is the common current found in every household and is used to run the refrigerator, stove, washing machine, and so on. Batteries, which are used in automobiles and flashlights, are one source of *direct current*. In addition to these two types of currents, which have a wide variety of uses, we can generate many other types of currents. We will examine some of these other types later in the book. In the meantime, it is interesting to note that the magnitude of currents in elements familiar to us ranges from soup to nuts, as shown in Fig. 1.4.

We have indicated that charges in motion yield an energy transfer. Now we define the *voltage* (also called the *electromotive force*, or *potential*) between two points in a circuit as the difference in energy level of a unit charge located at each of the two points. Voltage is very similar to a gravitational force. Think about a bowling ball being dropped from a ladder into a tank of water. As soon as the ball is released, the force of gravity pulls it toward the bottom of the tank. The potential energy of the bowling ball decreases as it approaches the bottom. The gravitational force is pushing the bowling ball through the water. Think of the bowling ball as a charge and the voltage as the force pushing the charge through a circuit. Charges in motion represent a current, so the motion of the bowling ball could be thought of as a current. The water in the tank will resist the motion of the bowling ball. The motion of charges in an electric circuit will be impeded or resisted as well. We will introduce the concept of resistance in Chapter 2 to describe this effect.

Work or energy, $w(t)$ or W, is measured in joules (J); 1 joule is 1 newton meter ($N \cdot m$). Hence, voltage [$v(t)$ or V] is measured in volts (V) and 1 volt is 1 joule per coulomb; that is, 1 volt = 1 joule per coulomb = 1 newton meter per coulomb. If a unit positive charge is moved between two points, the energy required to move it is the difference in energy level between the two points and is the defined voltage. It is extremely important that the variables used to represent voltage between two points be defined in such a way that the solution will let us interpret which point is at the higher potential with respect to the other.

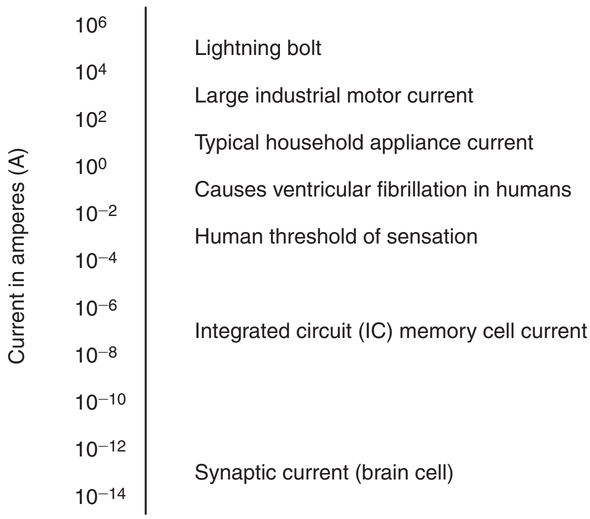
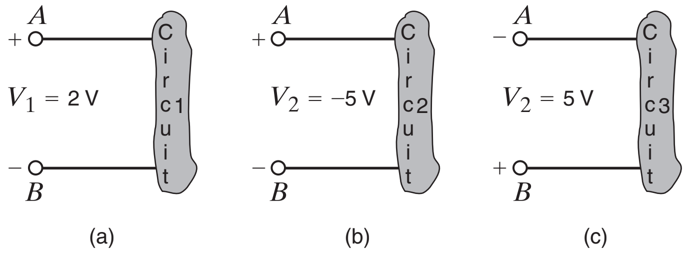


Figure 1.4
Typical current magnitudes.

Figure 1.5
Voltage representations.



In Fig. 1.5a the variable that represents the voltage between points A and B has been defined as V_1, and it is assumed that point A is at a higher potential than point B, as indicated by the + and − signs associated with the variable and defined in the figure. The + and − signs define a reference direction for V_1. If $V_1 = 2$ V, then the difference in potential of points A and B is 2 V and point A is at the higher potential. If a unit positive charge is moved from point A through the circuit to point B, it will give up energy to the circuit and have 2 J less energy when it reaches point B. If a unit positive charge is moved from point B to point A, extra energy must be added to the charge by the circuit, and hence the charge will end up with 2 J more energy at point A than it started with at point B.

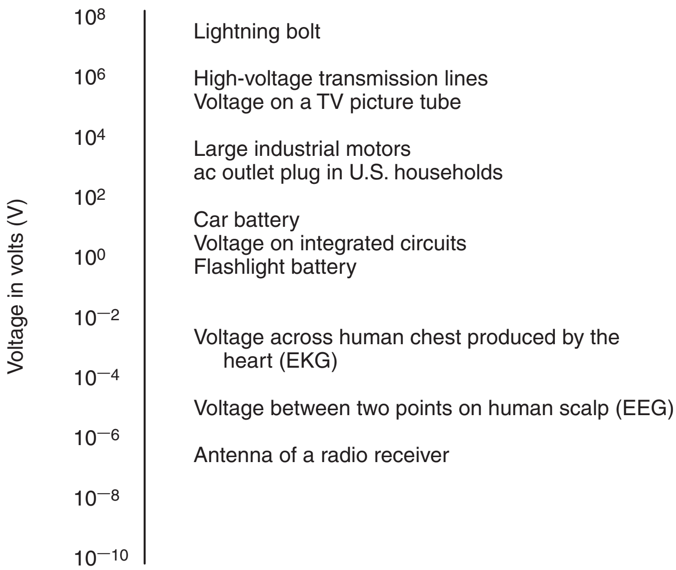
For the circuit in Fig. 1.5b, $V_2 = -5$ V means that the potential between points A and B is 5 V and point B is at the higher potential. The voltage in Fig. 1.5b can be expressed as shown in Fig. 1.5c. In this equivalent case, the difference in potential between points A and B is $V_2 = 5$ V, and point B is at the higher potential.

Note that it is important to define a variable with a reference direction so that the answer can be interpreted to give the physical condition in the circuit. We will find that it is not possible in many cases to define the variable so that the answer is positive, and we will also find that it is not necessary to do so.

As demonstrated in Figs. 1.5b and c, a negative number for a given variable, for example, V_2 in Fig. 1.5b, gives exactly the same information as a positive number, that is, V_2 in Fig. 1.5c, except that it has an opposite reference direction. Hence, when we define either current or voltage, it is absolutely necessary that we specify both magnitude and direction. Therefore, it is incomplete to say that the voltage between two points is 10 V or the current in a line is 2 A, since only the magnitude and not the direction for the variables has been defined.

The range of magnitudes for voltage, equivalent to that for currents in Fig. 1.4, is shown in Fig. 1.6. Once again, note that this range spans many orders of magnitude.

Figure 1.6
Typical voltage magnitudes.



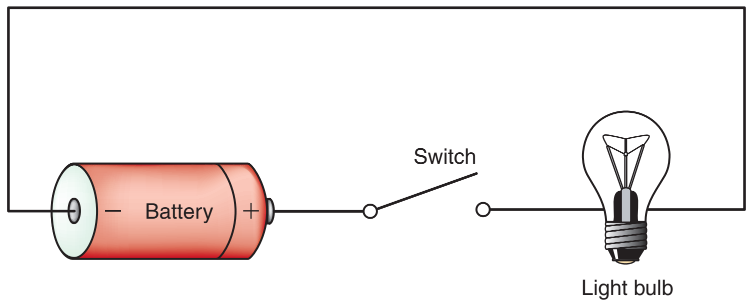


Figure 1.7

Flashlight circuit.

At this point we have presented the conventions that we employ in our discussions of current and voltage. *Energy* is yet another important term of basic significance. Let's investigate the voltage–current relationships for energy transfer using the flashlight shown in Fig. 1.7. The basic elements of a flashlight are a battery, a switch, a light bulb, and connecting wires. Assuming a good battery, we all know that the light bulb will glow when the switch is closed. A current now flows in this closed circuit as charges flow out of the positive terminal of the battery through the switch and light bulb and back into the negative terminal of the battery. The current heats up the filament in the bulb, causing it to glow and emit light. The light bulb converts electrical energy to thermal energy; as a result, charges passing through the bulb lose energy. These charges acquire energy as they pass through the battery as chemical energy is converted to electrical energy. An energy conversion process is occurring in the flashlight as the chemical energy in the battery is converted to electrical energy, which is then converted to thermal energy in the light bulb.

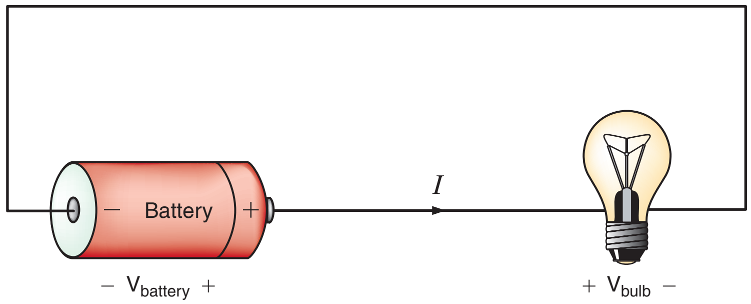


Figure 1.8

Flashlight circuit with voltages and current.

Let's redraw the flashlight as shown in Fig. 1.8. There is a current I flowing in this diagram. Since we know that the light bulb uses energy, the charges coming out of the bulb have less energy than those entering the light bulb. In other words, the charges expend energy as they move through the bulb. This is indicated by the voltage shown across the bulb. The charges gain energy as they pass through the battery, which is indicated by the voltage across the battery. Note the voltage–current relationships for the battery and bulb. We know that the bulb is absorbing energy; the current is entering the positive terminal of the voltage. For the battery, the current is leaving the positive terminal, which indicates that energy is being supplied.

This is further illustrated in Fig. 1.9, where a circuit element has been extracted from a larger circuit for examination. In Fig. 1.9a, energy is being supplied *to* the element by whatever is attached to the terminals. Note that 2 A, that is, 2 C of charge are moving from point A to point B through the element each second. Each coulomb loses 3 J of energy as it passes through the element from point A to point B. Therefore, the element is absorbing 6 J of energy per second. Note that when the element is *absorbing* energy, a positive current enters the positive terminal. In Fig. 1.9b energy is being supplied *by* the element to whatever is connected to terminals A-B. In this case, note that when the element is *supplying* energy, a positive current enters the negative terminal and leaves via the positive terminal. In this convention, a negative current in one direction is equivalent to a positive current in the opposite direction, and vice versa. Similarly, a negative voltage in one direction is equivalent to a positive voltage in the opposite direction.

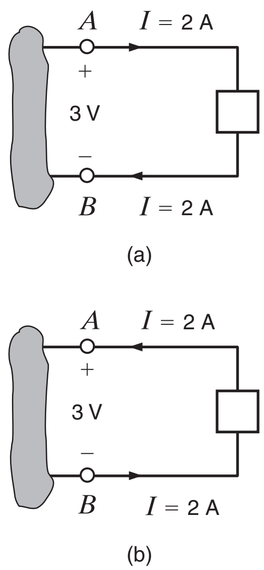


Figure 1.9

Voltage–current relationships for (a) energy absorbed and (b) energy supplied.

EXAMPLE 1.1

Suppose that your car will not start. To determine whether the battery is faulty, you turn on the light switch and find that the lights are very dim, indicating a weak battery. You borrow a friend's car and a set of jumper cables. However, how do you connect his car's battery to yours? What do you want his battery to do?

SOLUTION

Essentially, his car's battery must supply energy to yours, and therefore it should be connected in the manner shown in Fig. 1.10. Note that the positive current leaves the positive terminal of the good battery (supplying energy) and enters the positive terminal of the weak battery (absorbing energy). Note that the same connections are used when charging a battery.

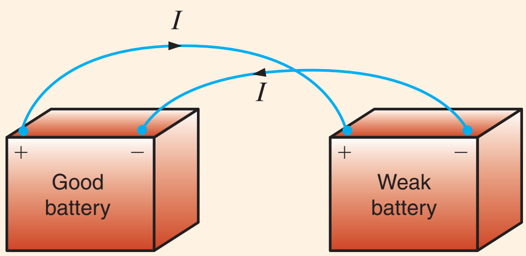


Figure 1.10

Diagram for Example 1.1.

In practical applications there are often considerations other than simply the electrical relations (e.g., safety). Such is the case with jump-starting an automobile. Automobile batteries produce explosive gases that can be ignited accidentally, causing severe physical injury. Be safe—follow the procedure described in your auto owner's manual.

We have defined voltage in joules per coulomb as the energy required to move a positive charge of 1 C through an element. If we assume that we are dealing with a differential amount of charge and energy, then

$$v = \frac{dw}{dq} \qquad \textbf{1.2}$$

Multiplying this quantity by the current in the element yields

$$vi = \frac{dw}{dq}\left(\frac{dq}{dt}\right) = \frac{dw}{dt} = p \qquad \textbf{1.3}$$

which is the time rate of change of energy or power measured in joules per second, or watts (W). Since, in general, both v and i are functions of time, p is also a time-varying quantity. Therefore, the change in energy from time t_1 to time t_2 can be found by integrating Eq. (1.3); that is,

$$\Delta w = \int_{t_1}^{t_2} p\,dt = \int_{t_1}^{t_2} vi\,dt \qquad \textbf{1.4}$$

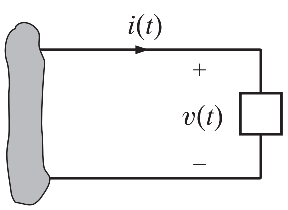


Figure 1.11

Sign convention for power.

[hint]

The passive sign convention is used to determine whether power is being absorbed or supplied.

At this point, let us summarize our sign convention for power. To determine the sign of any of the quantities involved, the variables for the current and voltage should be arranged as shown in Fig. 1.11. The variable for the voltage $v(t)$ is defined as the voltage across the element with the positive reference at the same terminal that the current variable $i(t)$ is entering. This convention is called the *passive sign convention* and will be so noted in the remainder of this book. The product of v and i, with their attendant signs, will determine the magnitude and sign of the power. If the sign of the power is positive, power is being absorbed by the element; if the sign is negative, power is being supplied by the element.

EXAMPLE 1.2

Given the two diagrams shown in Fig. 1.12, determine whether the element is absorbing or supplying power and how much.

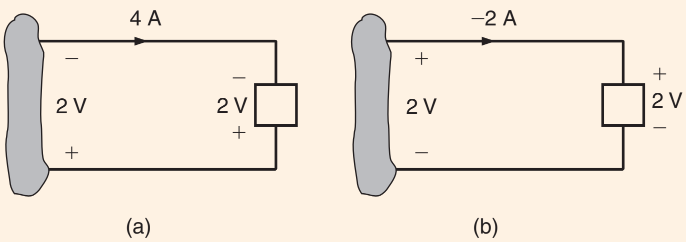


Figure 1.12
Elements for Example 1.2.

SOLUTION

In Fig. 1.12a the power is $P = (2\text{ V})(-4\text{ A}) = -8$ W. Therefore, the element is supplying power. In Fig. 1.12b, the power is $P = (2\text{ V})(-2\text{ A}) = -4$ W. Therefore, the element is supplying power.

Learning Assessment

E1.1 Determine the amount of power absorbed or supplied by the elements in Fig. E1.1.

ANSWER:
(a) $P = -48$ W;
(b) $P = 8$ W.

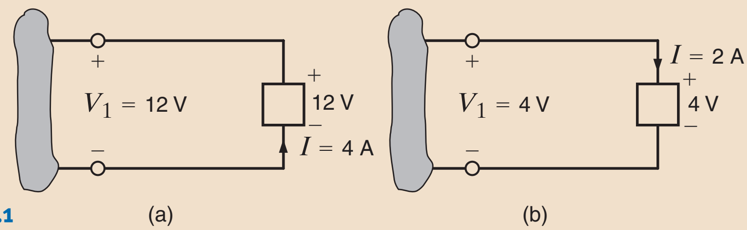


Figure E1.1

EXAMPLE 1.3

We wish to determine the unknown voltage or current in Fig. 1.13.

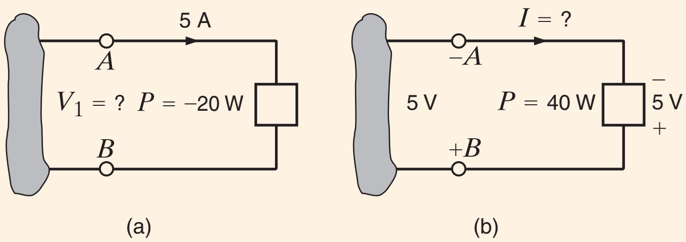


Figure 1.13
Elements for Example 1.3.

SOLUTION

In Fig. 1.13a, a power of −20 W indicates that the element is delivering power. Therefore, the current enters the negative terminal (terminal A), and from Eq. (1.3) the voltage is 4 V. Thus, B is the positive terminal, A is the negative terminal, and the voltage between them is 4 V.

In Fig 1.13b, a power of +40 W indicates that the element is absorbing power and, therefore, the current should enter the positive terminal B. The current thus has a value of −8 A, as shown in the figure.

Learning Assessment

E1.2 Determine the unknown variables in Fig. E1.2.

ANSWER:
(a) $V_1 = -20$ V;
(b) $I = -5$ A.

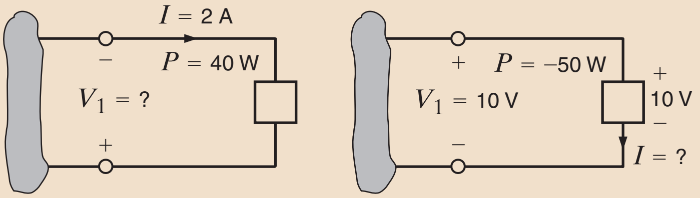


Figure E1.2 (a) (b)

Finally, it is important to note that our electrical networks satisfy the principle of conservation of energy. Because of the relationship between energy and power, it can be implied that power is also conserved in an electrical network. This result was formally stated in 1952 by B. D. H. Tellegen and is known as Tellegen's theorem—the sum of the powers absorbed by all elements in an electrical network is zero. Another statement of this theorem is that the power supplied in a network is exactly equal to the power absorbed. Checking to verify that Tellegen's theorem is satisfied for a particular network is one way to check our calculations when analyzing electrical networks.

1.3 Circuit Elements

Thus far we have defined voltage, current, and power. In the remainder of this chapter we will define both independent and dependent current and voltage sources. Although we will assume ideal elements, we will try to indicate the shortcomings of these assumptions as we proceed with the discussion.

In general, the elements we will define are terminal devices that are completely characterized by the current through the element and/or the voltage across it. These elements, which we will employ in constructing electric circuits, will be broadly classified as being either active or passive. The distinction between these two classifications depends essentially on one thing—whether they supply or absorb energy. As the words themselves imply, an *active* element is capable of generating energy and a *passive* element cannot generate energy.

However, later we will show that some passive elements are capable of storing energy. Typical active elements are batteries and generators. The three common passive elements are resistors, capacitors, and inductors.

In Chapter 2 we will launch an examination of passive elements by discussing the resistor in detail. Before proceeding with that element, we first present some very important active elements.

1. Independent voltage source
2. Independent current source
3. Two dependent voltage sources
4. Two dependent current sources

INDEPENDENT SOURCES An *independent voltage source* is a two-terminal element that maintains a specified voltage between its terminals *regardless of the current through it* as shown by the v-i plot in Fig. 1.14a. The general symbol for an independent source, a circle, is also shown in Fig. 1.14a. As the figure indicates, terminal A is $v(t)$ volts positive with respect to terminal B.

In contrast to the independent voltage source, the *independent current source* is a two-terminal element that maintains a specified current *regardless of the voltage across its terminals*, as illustrated by the v-i plot in Fig. 1.14b. The general symbol for an independent current source is also shown in Fig. 1.14b, where $i(t)$ is the specified current and the arrow indicates the positive direction of current flow.

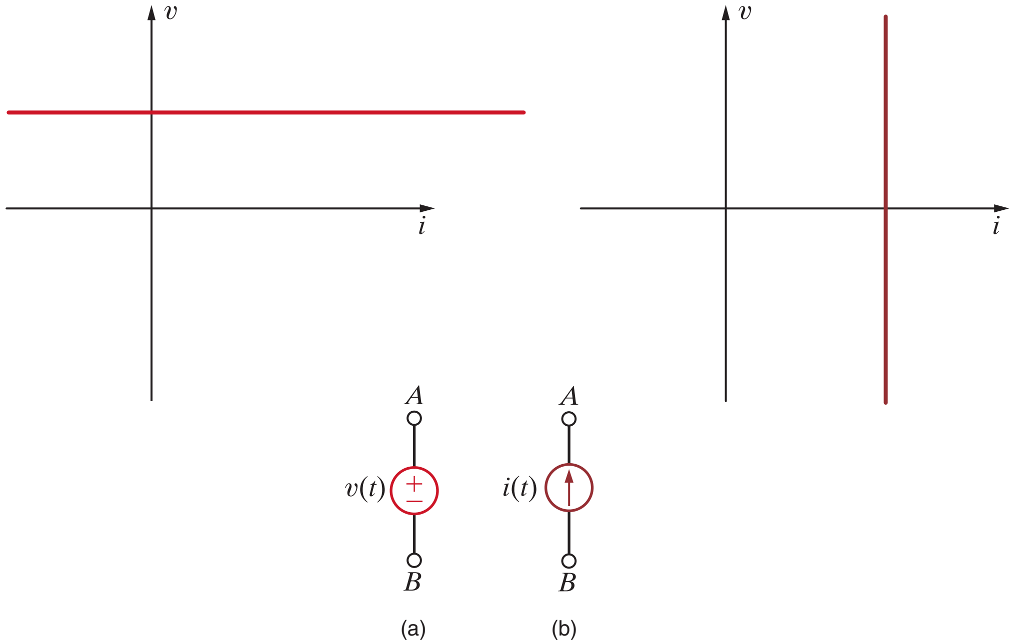


Figure 1.14
Symbols for (a) independent voltage source, (b) independent current source.

In their normal mode of operation, independent sources supply power to the remainder of the circuit. However, they may also be connected into a circuit in such a way that they absorb power. A simple example of this latter case is a battery-charging circuit such as that shown in Example 1.1.

It is important that we pause here to interject a comment concerning a shortcoming of the models. In general, mathematical models approximate actual physical systems only under a certain range of conditions. Rarely does a model accurately represent a physical system under every set of conditions. To illustrate this point, consider the model for the voltage source in Fig. 1.14a. We assume that the voltage source delivers v volts regardless of what is connected to its terminals. Theoretically, we could adjust the external circuit so that an infinite amount of current would flow, and therefore the voltage source would deliver an infinite amount of power. This is, of course, physically impossible. A similar argument could be made for the independent current source. Hence, the reader is cautioned to keep in mind that models have limitations and thus are valid representations of physical systems only under certain conditions.

For example, can the independent voltage source be utilized to model the battery in an automobile under all operating conditions? With the headlights on, turn on the radio. Do the headlights dim with the radio on? They probably won't if the sound system in your automobile was installed at the factory. If you try to crank your car with the headlights on, you will notice that the lights dim. The starter in your car draws considerable current, thus causing the voltage at the battery terminals to drop and dimming the headlights. The independent voltage source is a good model for the battery with the radio turned on; however, an improved model is needed for your battery to predict its performance under cranking conditions.

EXAMPLE 1.4

Determine the power absorbed or supplied by the elements in the network in Fig. 1.15.

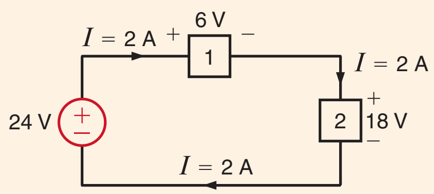


Figure 1.15
Network for Example 1.4.

SOLUTION

[hint]

Elements that are connected in series have the same current.

The current flow is out of the positive terminal of the 24-V source, and therefore this element is supplying (2)(24) = 48 W of power. The current is into the positive terminals of elements 1 and 2, and therefore elements 1 and 2 are absorbing (2)(6) = 12 W and (2)(18) = 36 W, respectively. Note that the power supplied is equal to the power absorbed.

Learning Assessment

E1.3 Find the power that is absorbed or supplied by the elements in Fig. E1.3.

ANSWER: Current source supplies 36 W, element 1 absorbs 54 W, and element 2 supplies 18 W.

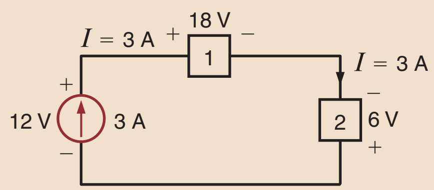


Figure E1.3

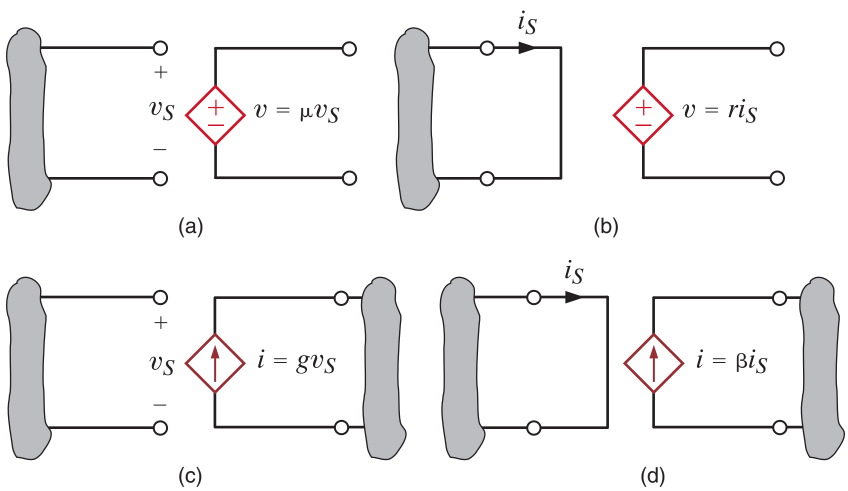
DEPENDENT SOURCES In contrast to the independent sources, which produce a particular voltage or current completely unaffected by what is happening in the remainder of the circuit, dependent sources generate a voltage or current that is determined by a voltage or current at a specified location in the circuit. These sources are very important because they are an integral part of the mathematical models used to describe the behavior of many electronic circuit elements.

For example, metal-oxide-semiconductor field-effect transistors (MOSFETs) and bipolar transistors, both of which are commonly found in a host of electronic equipment, are modeled with dependent sources, and therefore the analysis of electronic circuits involves the use of these controlled elements.

In contrast to the circle used to represent independent sources, a diamond is used to represent a dependent or controlled source. Fig. 1.16 illustrates the four types of dependent sources. The input terminals on the left represent the voltage or current that controls the dependent source, and the output terminals on the right represent the output current or voltage of the controlled source. Note that in Figs. 1.16a and d, the quantities μ and β are dimensionless constants because we are transforming voltage to voltage and current to current. This is not the case in Figs. 1.16b and c; hence, when we employ these elements a short time later, we must describe the units of the factors r and g.

Figure 1.16

Four different types of dependent sources.



EXAMPLE 1.5

Given the two networks shown in Fig. 1.17, we wish to determine the outputs.

SOLUTION

In Fig. 1.17a the output voltage is $V_o = \mu V_S$ or $V_o = 20\,V_S = (20)(2\text{ V}) = 40$ V. Note that the output voltage has been amplified from 2 V at the input terminals to 40 V at the output terminals; that is, the circuit is a voltage amplifier with an amplification factor of 20.

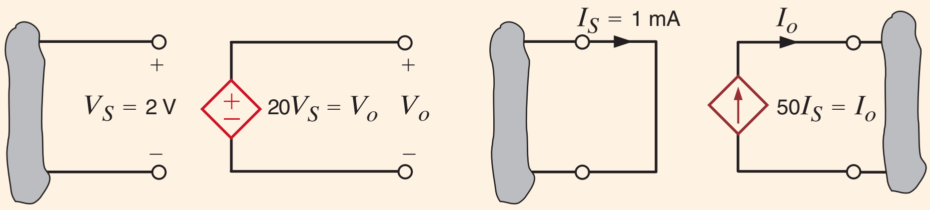


Figure 1.17
Circuits for Example 1.5.

In Fig. 1.17b, the output current is $I_o = \beta I_S = (50)(1\text{ mA}) = 50$ mA; that is, the circuit has a current gain of 50, meaning that the output current is 50 times greater than the input current.

Learning Assessment

E1.4 Determine the power supplied by the dependent sources in Fig. E1.4.

ANSWER:
(a) Power supplied = 80 W;
(b) power supplied = 160 W.

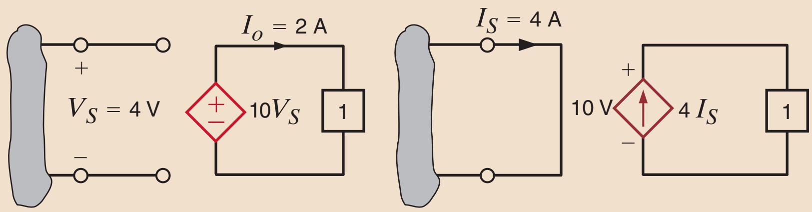


Figure E1.4

EXAMPLE 1.6

Calculate the power absorbed by each element in the network of Fig. 1.18. Also verify that Tellegen's theorem is satisfied by this network.

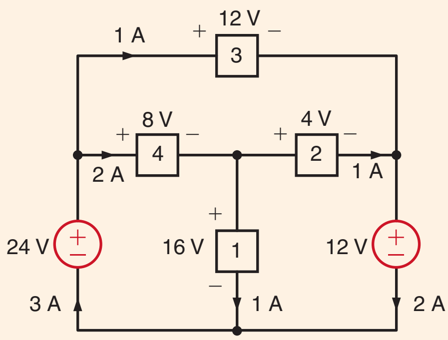


Figure 1.18
Circuit used in Example 1.6.

SOLUTION

Let's calculate the power absorbed by each element using the sign convention for power.

$$P_1 = (16)(1) = 16\text{ W}$$
$$P_2 = (4)(1) = 4\text{ W}$$
$$P_3 = (12)(1) = 12\text{ W}$$

$$P_4 = (8)(2) = 16 \text{ W}$$
$$P_{12V} = (12)(2) = 24 \text{ W}$$
$$P_{24V} = (24)(-3) = -72 \text{ W}$$

Note that to calculate the power absorbed by the 24-V source, the current of 3 A flowing up through the source was changed to a current −3 A flowing down through the 24-V source.

Let's sum up the power absorbed by all elements: 16 + 4 + 12 + 16 + 24 − 72 = 0

This sum is zero, which verifies that Tellegen's theorem is satisfied.

EXAMPLE 1.7

Use Tellegen's theorem to find the current I_o in the network in Fig. 1.19.

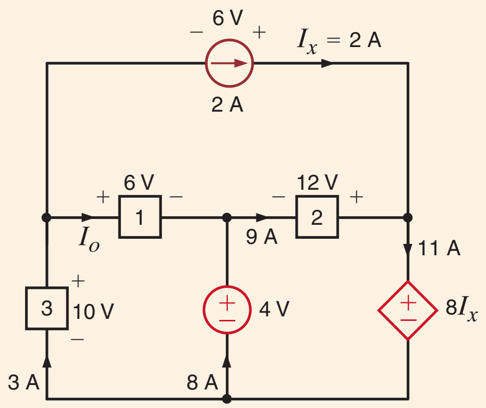


Figure 1.19
Circuit used in Example 1.7.

SOLUTION

First, we must determine the power absorbed by each element in the network. Using the sign convention for power, we find

$$P_{2\text{ A}} = (6)(-2) = -12 \text{ W}$$
$$P_1 = (6)(I_o) = 6I_o \text{ W}$$
$$P_2 = (12)(-9) = -108 \text{ W}$$
$$P_3 = (10)(-3) = -30 \text{ W}$$
$$P_{4\text{ V}} = (4)(-8) = -32 \text{ W}$$
$$P_{DS} = (8I_x)(11) = (16)(11) = 176 \text{ W}$$

Applying Tellegen's theorem yields

$$-12 + 6I_o - 108 - 30 - 32 + 176 = 0$$

or

$$6I_o + 176 = 12 + 108 + 30 + 32$$

Hence,

$$I_o = 1\text{A}$$

Learning Assessment

E1.5 Find the power that is absorbed or supplied by the circuit elements in the network in Fig. E1.5.

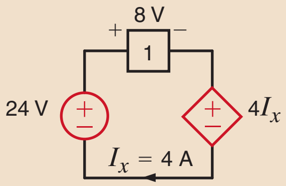


Figure E1.5

ANSWER:
$P_{24\text{ V}}$ = 96 W supplied;
P_1 = 32 W absorbed;
P_{4I_x} = 64 W absorbed.

E1.6 Find the power that is absorbed or supplied by the network elements in Fig. E1.6.

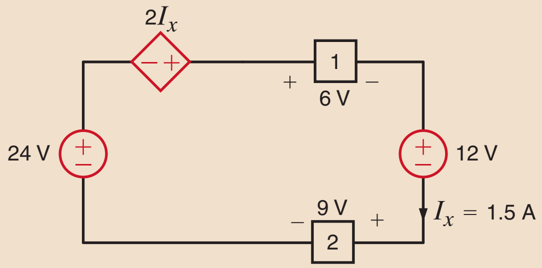


Figure E1.6

ANSWER:
$P_{24V} = 36$ W supplied,
$P_{12V} = 18$ W absorbed,
$P_{2I_x} = 4.5$ W supplied,
$P_1 = 9$ W absorbed,
$P_2 = 13.5$ W absorbed.

E1.7 Find I_x in Fig. E1.7 using Tellegen's theorem.

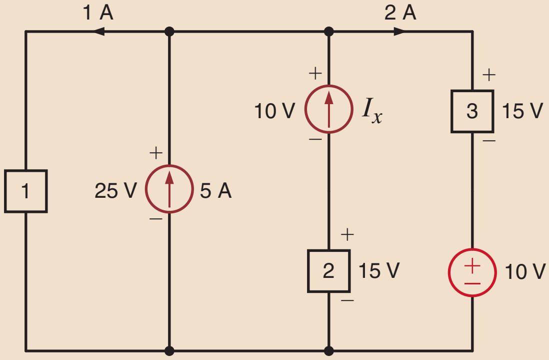


Figure E1.7

ANSWER:
$I_x = -2$ A.

EXAMPLE 1.8

The charge that enters the BOX is shown in Fig. 1.20. Calculate and sketch the current flowing into and the power absorbed by the BOX between 0 and 10 milliseconds.

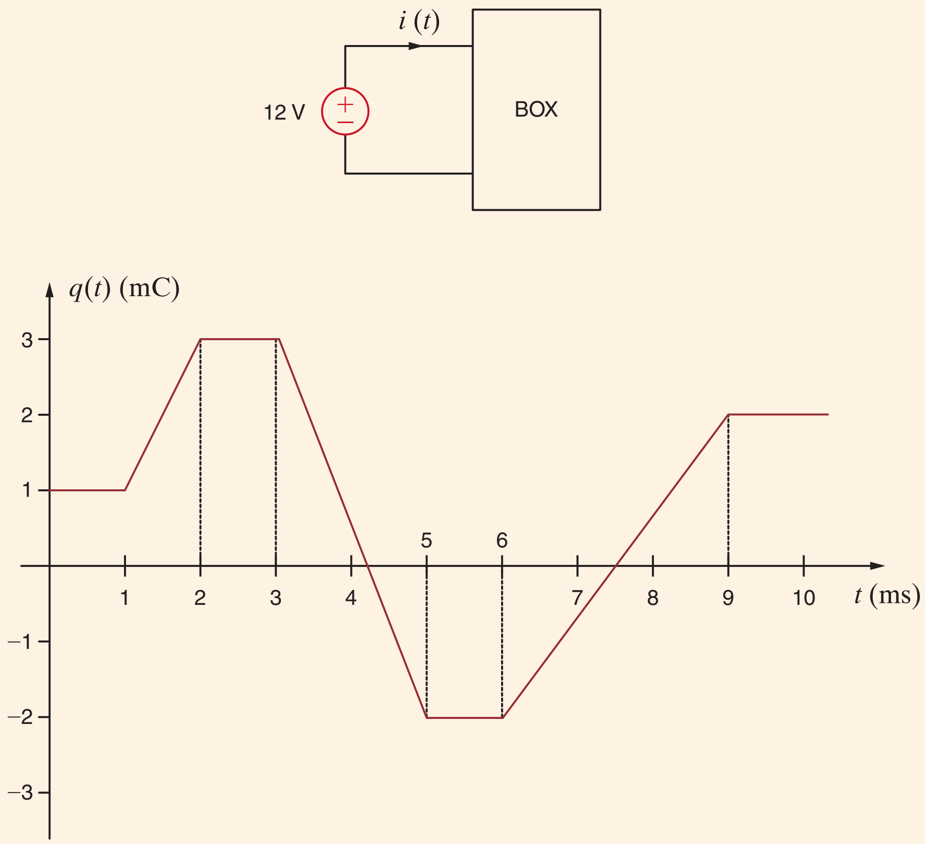


Figure 1.20

Diagrams for Example 1.8.

SOLUTION Recall that current is related to charge by $i(t) = \frac{dq(t)}{dt}$. The current is equal to the slope of the charge waveform.

$$i(t) = 0 \qquad 0 \leq t \leq 1 \text{ ms}$$

$$i(t) = \frac{3 \times 10^{-3} - 1 \times 10^{-3}}{2 \times 10^{-3} - 1 \times 10^{-3}} = 2\text{A} \qquad 1 \leq t \leq 2 \text{ ms}$$

$$i(t) = 0 \qquad 2 \leq t \leq 3 \text{ ms}$$

$$i(t) = \frac{-2 \times 10^{-3} - 3 \times 10^{-3}}{5 \times 10^{-3} - 3 \times 10^{-3}} = -2.5 \text{ A} \qquad 3 \leq t \leq 5 \text{ ms}$$

$$i(t) = 0 \qquad 5 \leq t \leq 6 \text{ ms}$$

$$i(t) = \frac{2 \times 10^{-3} - (-2 \times 10^{-3})}{9 \times 10^{-3} - 6 \times 10^{-3}} = 1.33 \text{ A} \qquad 6 \leq t \leq 9 \text{ ms}$$

$$i(t) = 0 \qquad t \geq 9 \text{ ms}$$

The current is plotted with the charge waveform in Fig. 1.21. Note that the current is zero during times when the charge is a constant value. When the charge is increasing, the current is positive, and when the charge is decreasing, the current is negative.

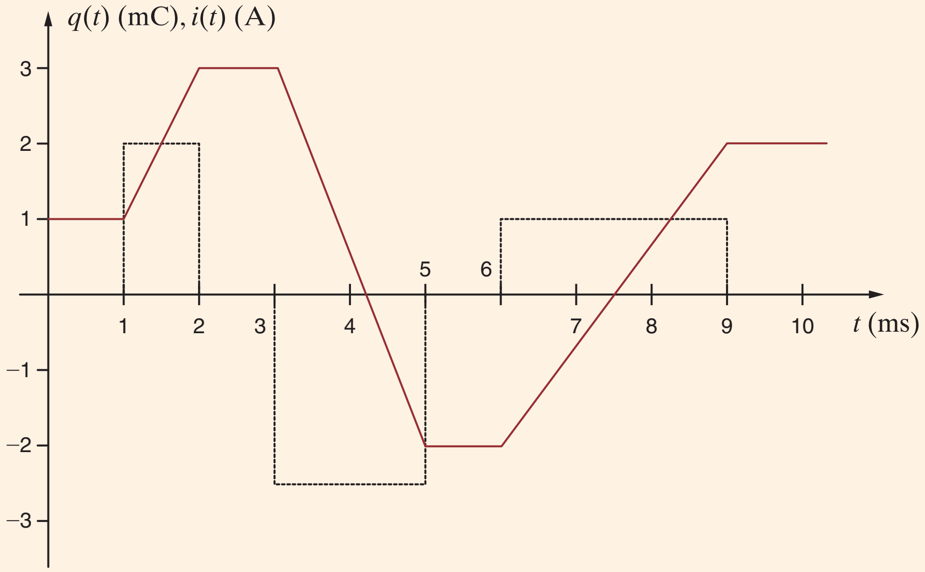


Figure 1.21
Charge and current waveforms for Example 1.8.

The power absorbed by the BOX is $12 \cdot i(t)$.

$$p(t) = 12*0 = 0 \qquad 0 \leq t \leq 1 \text{ ms}$$
$$p(t) = 12*2 = 24 \text{ W} \qquad 1 \leq t \leq 2 \text{ ms}$$
$$p(t) = 12*0 = 0 \qquad 2 \leq t \leq 3 \text{ ms}$$
$$p(t) = 12*(-2.5) = -30 \text{ W} \qquad 3 \leq t \leq 5 \text{ ms}$$
$$p(t) = 12*0 = 0 \qquad 5 \leq t \leq 6 \text{ ms}$$
$$p(t) = 12*1.33 = 16 \text{ W} \qquad 6 \leq t \leq 9 \text{ ms}$$
$$p(t) = 12*0 = 0 \qquad t \geq 9 \text{ ms}$$

The power absorbed by the BOX is plotted in Fig. 1.22. For the time intervals, $1 \leq t \leq 2$ ms and $6 \leq t \leq 9$ ms, the BOX is absorbing power. During the time interval $3 \leq t \leq 5$ ms, the power absorbed by the BOX is negative, which indicates that the BOX is supplying power to the 12-V source.

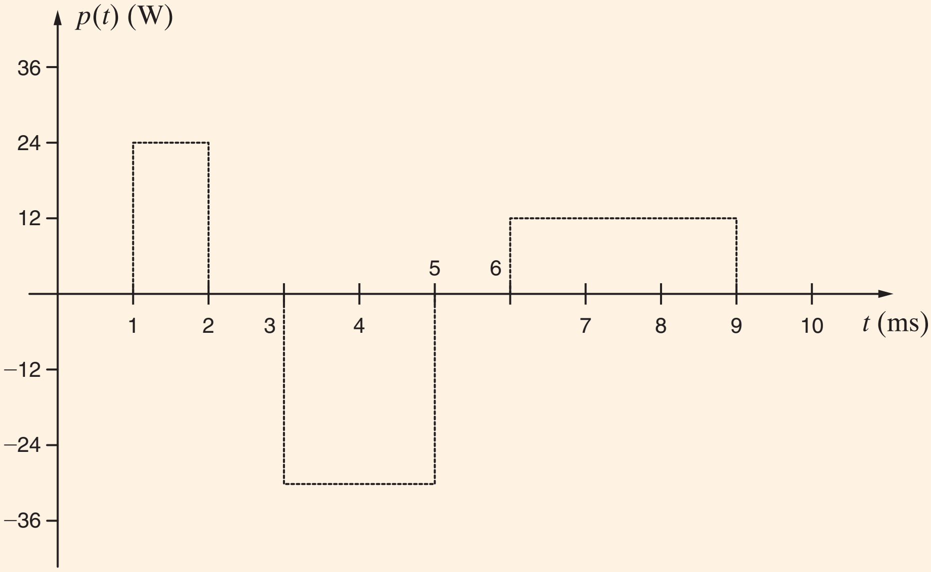


Figure 1.22
Power waveform for Example 1.8.

Learning Assessment

E1.8 The power absorbed by the BOX in Fig. El.8 is $p(t) = 2.5e^{-4t}$ W. Compute the energy and charge delivered to the BOX in the time interval $0 < t < 250$ ms.

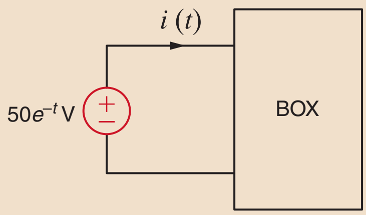


Figure E1.8

ANSWER: 395.1 mJ, 8.8 mC.

E1.9 The energy absorbed by the BOX in Fig. El.9 is given below. Calculate and sketch the current flowing into the BOX. Also calculate the charge that enters the BOX between 0 and 12 seconds.

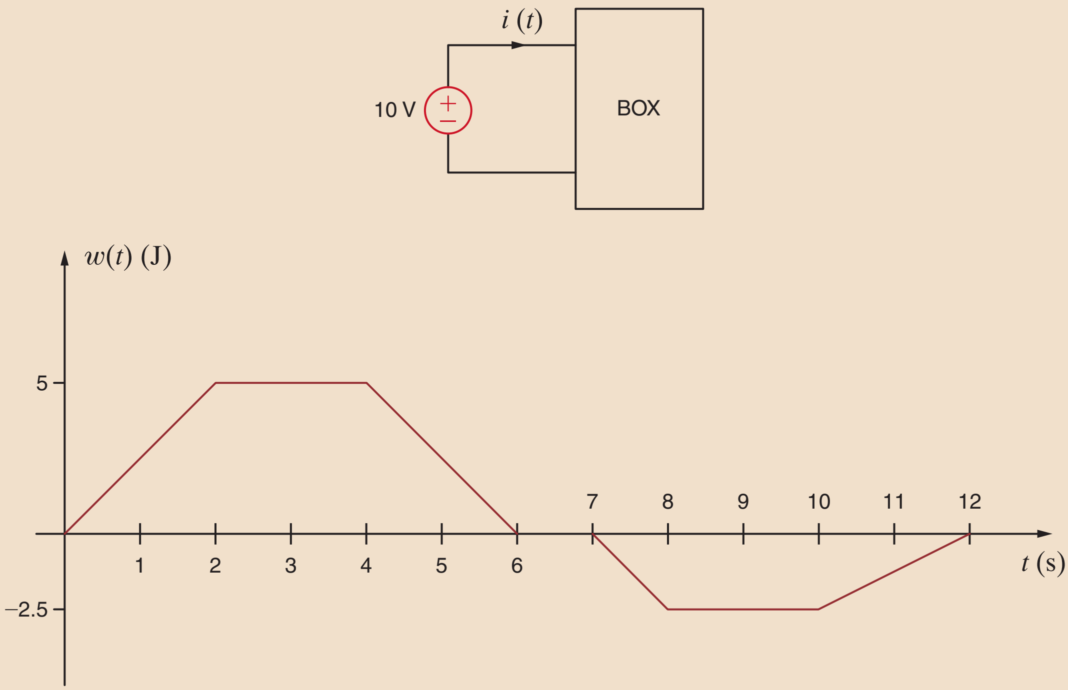
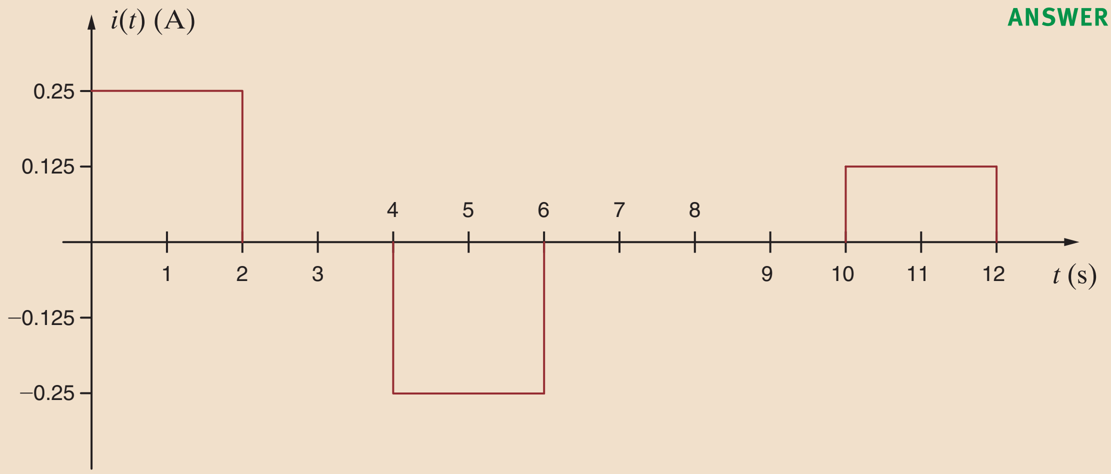


Figure E1.9



ANSWER: $Q = 0$.

EXAMPLE 1.9

A Universal Serial Bus (USB) port is a common feature on both desktop and notebook computers as well as many handheld devices such as MP3 players, digital cameras, and cell phones. The USB 2.0 specification (www.usb.org) permits data transfer between a computer and a peripheral device at rates up to 480 megabits per second. One important feature of USB is the ability to swap peripherals without having to power down a computer. USB ports are also capable of supplying power to external peripherals. Fig. 1.23 shows a Motorola RAZR® and an Apple iPod® being charged from the USB ports on a notebook computer. A USB cable is a four-conductor cable with two signal conductors and two conductors for providing power. The amount of current that can be provided over a USB port is defined in the USB specification in terms of unit loads, where one unit load is specified to be 100 mA. All USB ports default to low-power ports at one unit load, but can be changed under software control to high-power ports capable of supplying up to five unit loads or 500 mA.

Figure 1.23
Charging a Motorola RAZR® and Apple iPod® from USB ports. (Courtesy of Mark Nelms and Jo Ann Loden)

1. A 680 mAh lithium-ion battery is standard in a Motorola RAZR®. If this battery is completely discharged (i.e., 0 mAh), how long will it take to recharge the battery to its full capacity of 680 mAh from a low-power USB port? How much charge is stored in the battery at the end of the charging process?

2. A third-generation iPod® with a 630 mAh lithium-ion battery is to be recharged from a high-power USB port supplying 150 mA of current. At the beginning of the recharge, 7.8 C of charge are stored in the battery. The recharging process halts when the stored charge reaches 35.9 C. How long does it take to recharge the battery?

SOLUTION

1. A low-power USB port operates at 100 mA. Assuming that the charging current from the USB port remains at 100 mA throughout the charging process, the time required to recharge the battery is $680 \text{ mAh}/100 \text{ mA} = 6.8 \text{ h}$. The charge stored in the battery when fully charged is $680\text{mAh} \cdot 60 \text{ s/h} = 40{,}800 \text{ mAs} = 40.8 \text{ As} = 40.8 \text{ C}$.

2. The charge supplied to the battery during the recharging process is $35.9 - 7.8 = 28.1 \text{ C}$. This corresponds to $28.1 \text{ As} = 28{,}100 \text{ mAs} \cdot 1\text{h}/60\text{s} = 468.3 \text{ mAh}$. Assuming a constant charging current of 150 mA from the high-power USB port, the time required to recharge the battery is $468.3 \text{ mAh}/150 \text{ mA} = 3.12 \text{ h}$.

SUMMARY

- ***The standard prefixes employed***

$$p = 10^{-12} \qquad k = 10^{3}$$
$$n = 10^{-9} \qquad M = 10^{6}$$
$$\mu = 10^{-6} \qquad G = 10^{9}$$
$$m = 10^{-3} \qquad T = 10^{12}$$

- ***The relationships between current and charge***

$$i(t) = \frac{dq(t)}{dt} \qquad \text{or} \qquad q(t) = \int_{-\infty}^{t} i(x)\,dx$$

- ***The relationships among power, energy, current, and voltage***

$$p = \frac{dw}{dt} = vi$$
$$\Delta w = \int_{t_1}^{t_2} p\,dt = \int_{t_1}^{t_2} vi\,dt$$

- ***The passive sign convention*** The passive sign convention states that if the voltage and current associated with an element are as shown in Fig. 1.11, the product of v and i, with their attendant signs, determines the magnitude and sign of the power. If the sign is positive, power is being absorbed by the element, and if the sign is negative, the element is supplying power.

- ***Independent and dependent sources*** An ideal independent voltage (current) source is a two-terminal element that maintains a specified voltage (current) between its terminals, regardless of the current (voltage) through (across) the element. Dependent or controlled sources generate a voltage or current that is determined by a voltage or current at a specified location in the circuit.

- ***Conservation of energy*** The electric circuits under investigation satisfy the conservation of energy.

- ***Tellegen's theorem*** The sum of the powers absorbed by all elements in an electrical network is zero.

PROBLEMS

1.1 If the current in an electric conductor is 2.4 A, how many coulombs of charge pass any point in a 30-second interval?

1.2 Determine the time interval required for a 12-A battery charger to deliver 4800 C.

1.3 A lightning bolt carrying 30,000 A lasts for 50 microseconds. If the lightning strikes an airplane flying at 20,000 feet, what is the charge deposited on the plane?

1.4 If a 12-V battery delivers 100 J in 5 s, find (a) the amount of charge delivered and (b) the current produced.

1.5 The current in a conductor is 1.5 A. How many coulombs of charge pass any point in a time interval of 1.5 minutes?

1.6 If 60 C of charge pass through an electric conductor in 30 seconds, determine the current in the conductor.

1.7 Determine the number of coulombs of charge produced by a 12-A battery charger in an hour.

1.8 Five coulombs of charge pass through the element in Fig. P1.8 from point A to point B. If the energy absorbed by the element is 120 J, determine the voltage across the element.

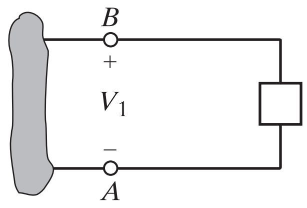


Figure P1.8

1.9 The current that enters an element is shown in Fig. P1.9. Find the charge that enters the element in the time interval $0 < t < 20$ s.

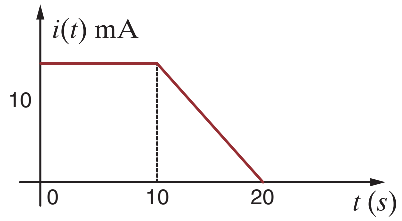


Figure P1.9

1.10 The charge entering the positive terminal of an element is $q(t) = -30e^{-4t}$ mC. If the voltage across the element is $120e^{-2t}$ V, determine the energy delivered to the element in the time interval $0 < t < 50$ ms.

1.11 The charge entering the positive terminal of an element is given by the expression $q(t) = -12e^{-2t}$ mC. The power delivered to the element is $p(t) = 2.4e^{-3t}$ W. Compute the current in the element, the voltage across the element, and the energy delivered to the element in the time interval $0 < t < 100$ ms.

1.12 The voltage across an element is $12e^{-2t}$ V. The current entering the positive terminal of the element is $2e^{-2t}$ A. Find the energy absorbed by the element in 1.5 s starting from $t = 0$.

1.13 The power absorbed by the BOX in Fig. P1.13 is $2e^{-2t}$ W. Calculate the amount of charge that enters the BOX between 0.1 and 0.4 seconds.

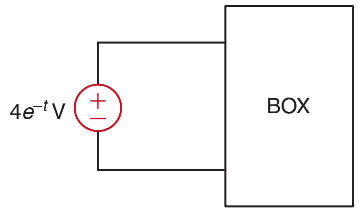


Figure P1.13

1.14 The power absorbed by the BOX in Fig. P1.14 is $0.1e^{-4t}$ W. Calculate the energy absorbed by the BOX during this same time interval.

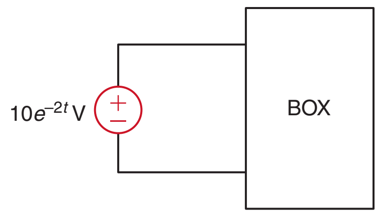


Figure P1.14

1.15 The energy absorbed by the BOX in Fig. P1.15 is shown below. How much charge enters the BOX between 0 and 10 milliseconds?

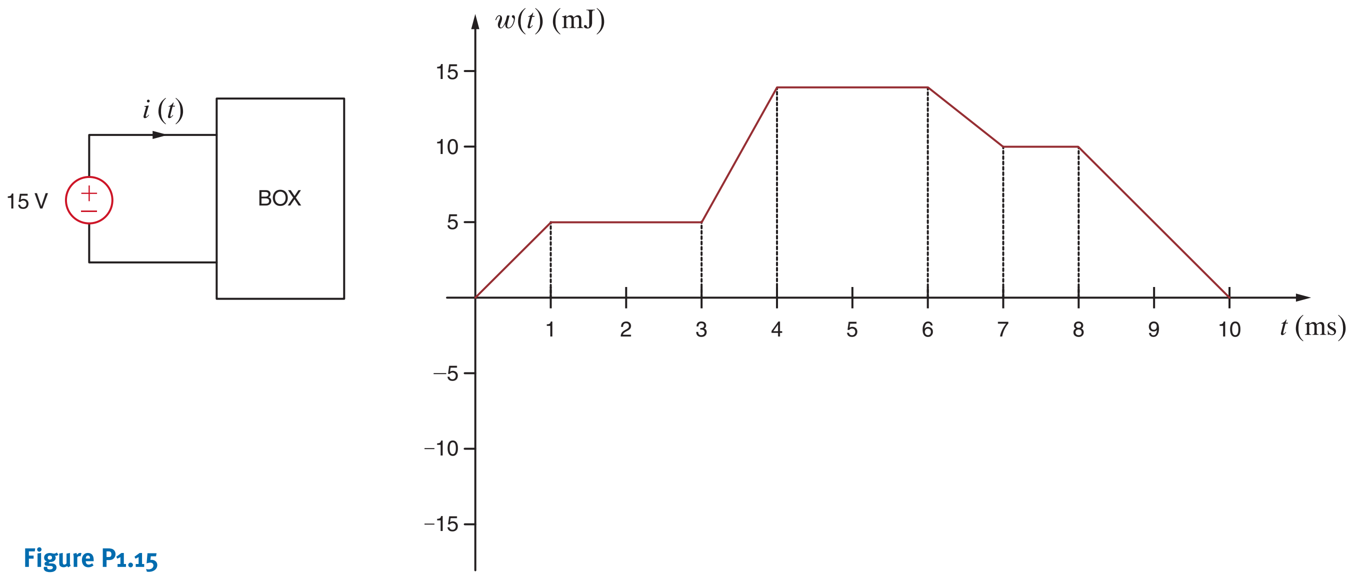


Figure P1.15

1.16 The charge that enters the BOX in Fig. P1.16 is shown in the graph below. Calculate and sketch the current flowing into and the power absorbed by the BOX between 0 and 10 milliseconds.

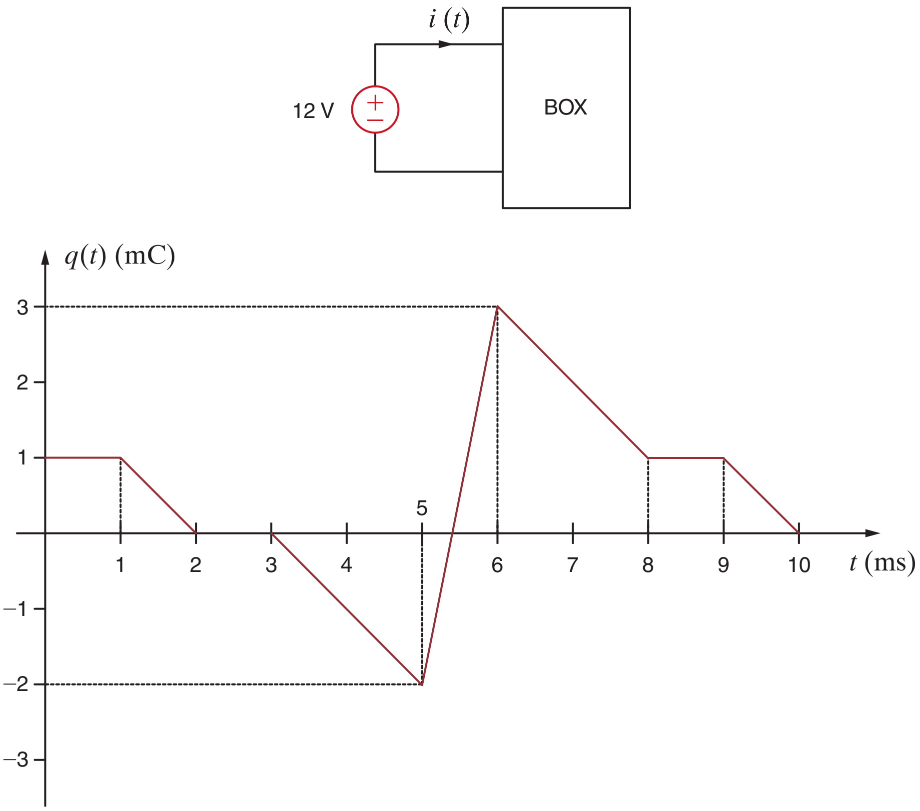


Figure P1.16

1.17 The energy absorbed by the BOX in Fig. P1.17 is given below. Calculate and sketch the current flowing into the BOX. Also calculate the charge which enters the BOX between 0 and 12 seconds.

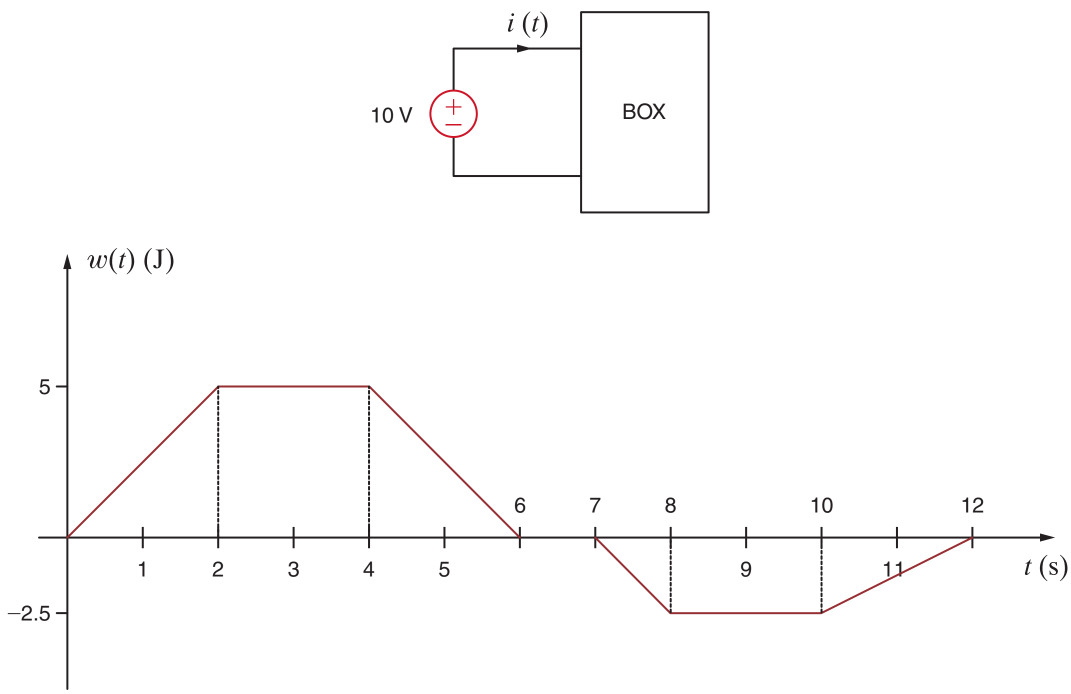


Figure P1.17

1.18 The charge entering the upper terminal of the BOX in Fig. P1.18 is shown below. How much energy is absorbed by the BOX between 0 and 9 seconds?

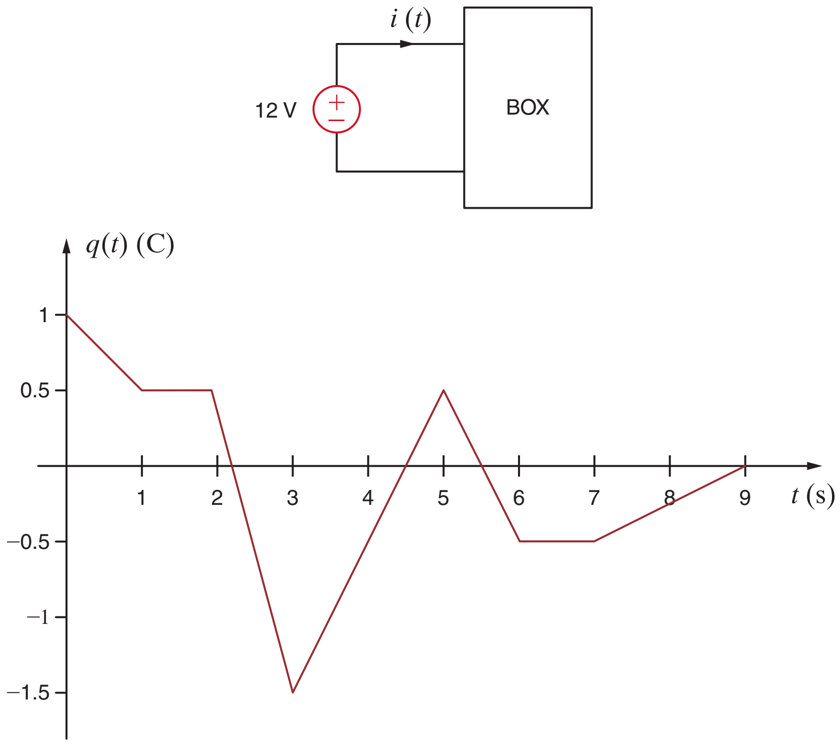


Figure P1.18

1.19 The energy absorbed by the BOX in Fig. P1.19 is shown in the graph below. Calculate and sketch the current flowing into the BOX between 0 and 10 milliseconds.

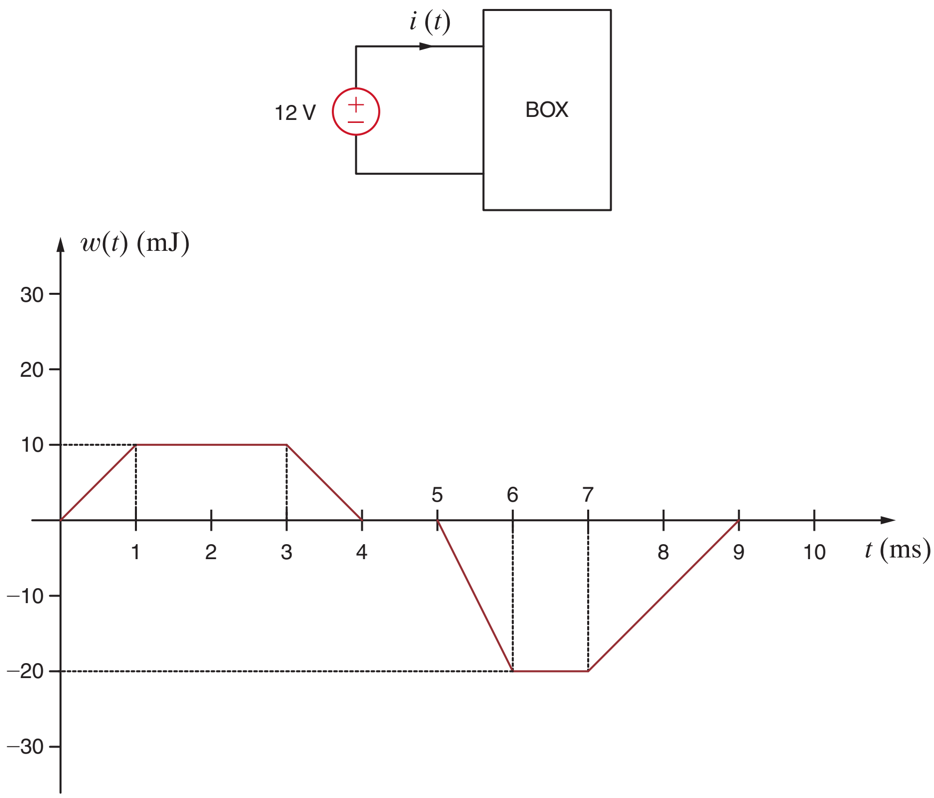


Figure P1.19

1.20 Determine the amount of power absorbed or supplied by the element in Fig. P1.20 if

(a) $V_1 = 9$ V and $I = 2$A

(b) $V_1 = 9$ V and $I = -3$A

(c) $V_1 = -12$ V and $I = 2$A

(d) $V_1 = -12$ V and $I = -3$A

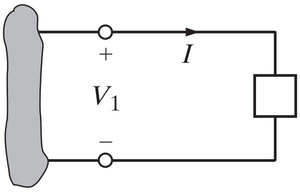


Figure P1.20

1.21 Calculate the power absorbed by element A in Fig. P1.21.

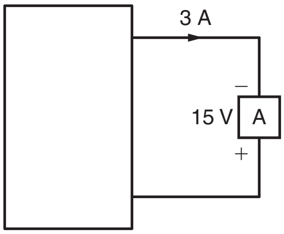


Figure P1.21

1.22 Calculate the power supplied by element A in Fig. P1.22.

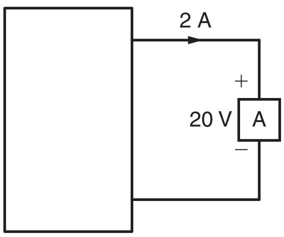


Figure P1.22

1.23 Element A in the diagram in Fig. P1.23 absorbs 30 W of power. Calculate V_x.

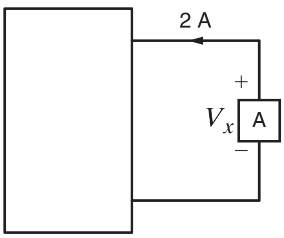


Figure P1.23

1.24 Element B in the diagram in Fig. P1.24 supplies 60 W of power. Calculate I_x.

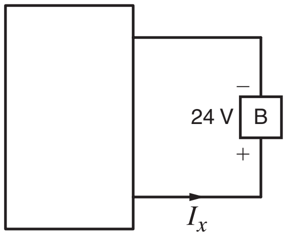


Figure P1.24

1.25 Element B in the diagram in Fig. P1.25 supplies 72 W of power. Calculate V_A.

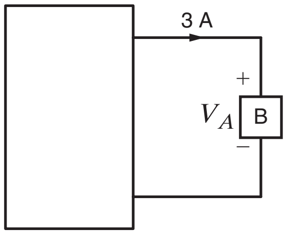


Figure P1.25

1.26 Element B in the diagram in Fig. P1.26 supplies 72 W of power. Calculate I_x.

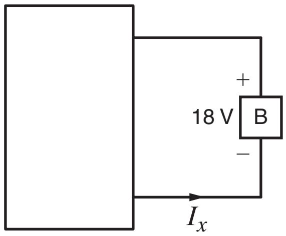


Figure P1.26

1.27 **(a)** In Fig. P1.27 (a), $P_1 = 36$ W. Is element 2 absorbing or supplying power, and how much?

(b) In Fig. P1.27 (b), $P_2 = -48$ W. Is element 1 absorbing or supplying power, and how much?

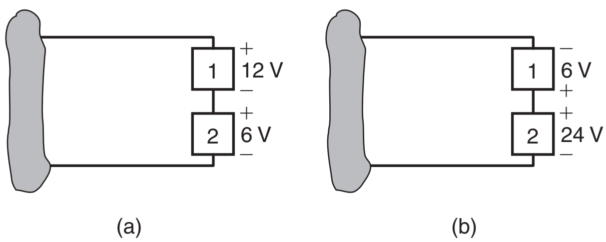


Figure P1.27

1.28 Two elements are connected in series, as shown in Fig. P1.28. Element 1 supplies 24 W of power. Is element 2 absorbing or supplying power, and how much?

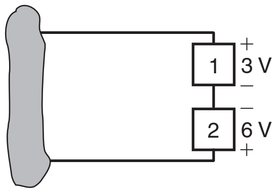


Figure P1.28

1.29 Element 2 in Fig. P1.29 absorbed 32 W. Find the power absorbed or supplied by elements 1 and 3.

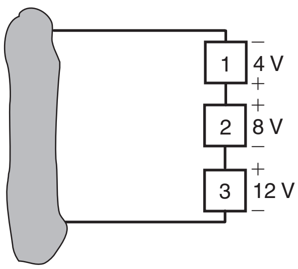


Figure P1.29

1.30 Choose I_s such that the power absorbed by element 2 in Fig. P1.30 is 7 W.

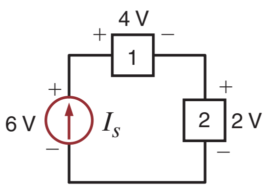


Figure P1.30

1.31 Find the power that is absorbed or supplied by the circuit elements in Fig. P1.31.

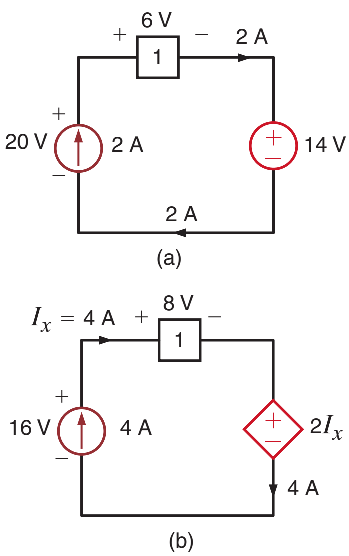


Figure P1.31

1.32 Find the power that is absorbed or supplied by the network elements in Fig. P1.32.

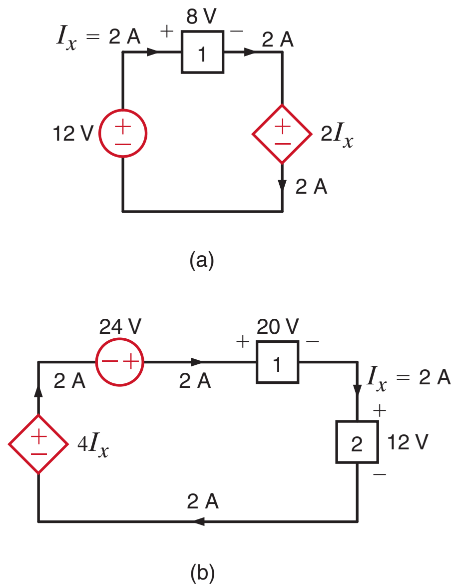


Figure P1.32

1.33 Compute the power that is absorbed or supplied by the elements in the network in Fig. P1.33.

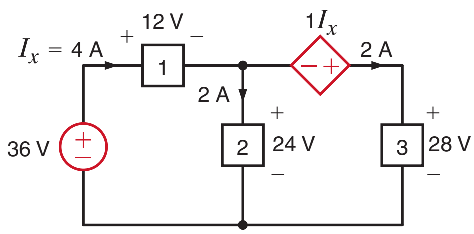


Figure P1.33

1.34 Find the power that is absorbed or supplied by element 2 in Fig. P1.34.

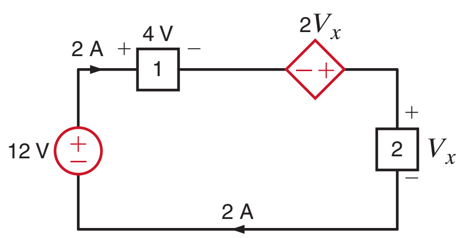


Figure P1.34

1.35 Find I_x in the network in Fig. P1.35.

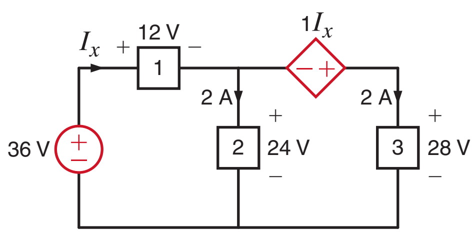


Figure P1.35

1.36 Determine the power absorbed by element 1 in Fig. P1.36.

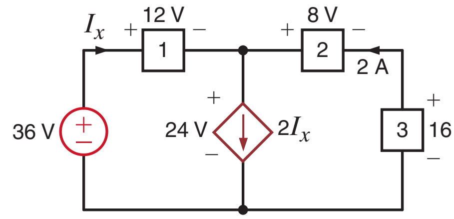


Figure P1.36

1.37 Find the power absorbed or supplied by element 1 in Fig. P1.37.

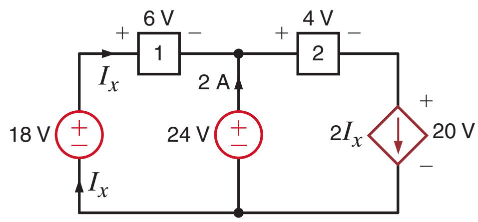


Figure P1.37

1.38 Find the power absorbed or supplied by element 3 in Fig. P1.38.

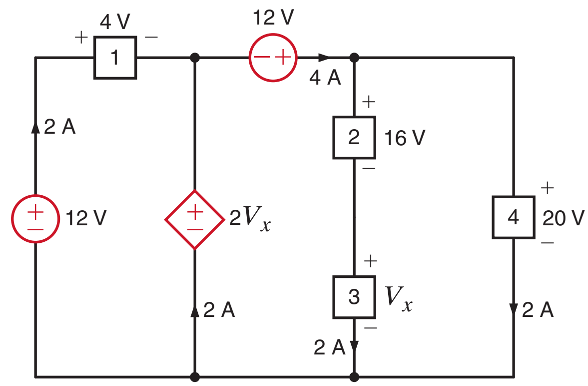


Figure P1.38

1.39 Find the power absorbed or supplied by element 1 in Fig. P1.39.

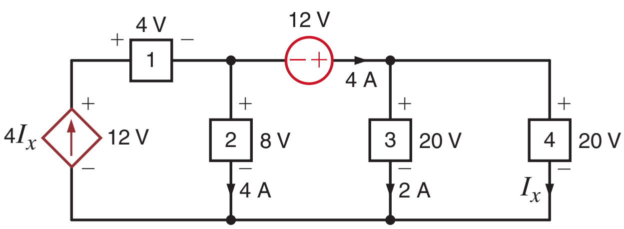


Figure P1.39

1.40 Find V_x in the network in Fig. P1.40 using Tellegen's theorem.

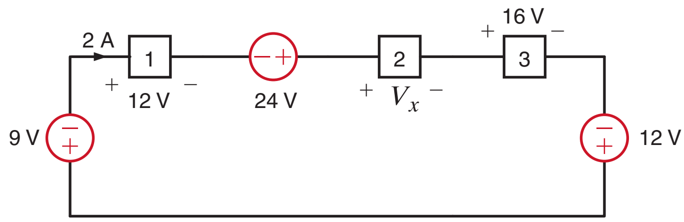


Figure P1.40

1.41 Find I_x in the circuit in Fig. P1.41 using Tellegen's theorem.

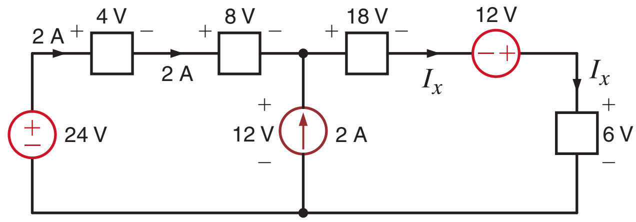


Figure P1.41

1.42 Is the source V_s in the network in Fig. P1.42 absorbing or supplying power, and how much?

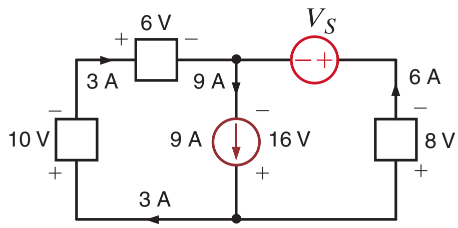


Figure P1.42

1.43 Find I_o in the network in Fig. P1.43 using Tellegen's theorem.

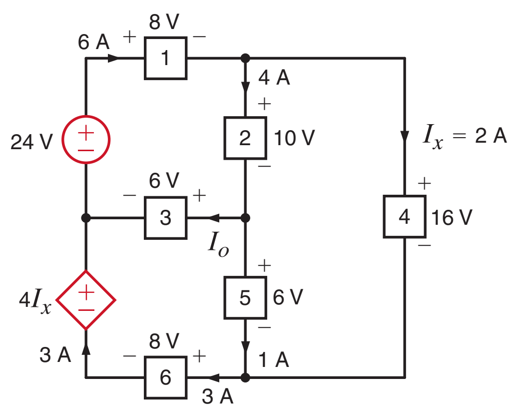


Figure P1.43

1.44 Calculate the power absorbed by each element in the circuit in Fig. P1.44. Also verify Tellegen's theorem is satisfied by this circuit.

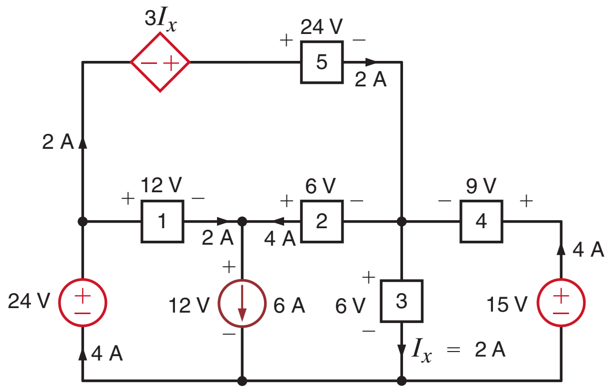


Figure P1.44

1.45 Calculate the power absorbed by each element in the circuit in Fig. P1.45. Also verify that Tellegen's theorem is satisfied by this circuit.

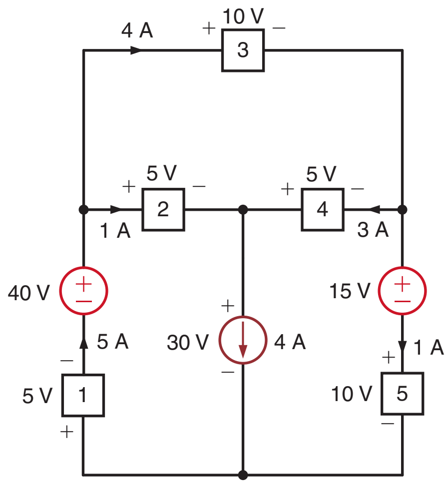


Figure P1.45

1.46 In the circuit in Fig. P1.46, element 1 absorbs 40 W, element 2 supplies 50 W, element 3 supplies 25 W, and element 4 absorbs 15 W. How much power is supplied by element 5?

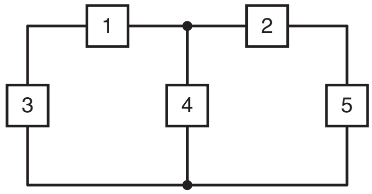


Figure P1.46

CHAPTER

2

RESISTIVE CIRCUITS

Courtesy of Tesla Motors

THE LEARNING GOALS FOR THIS CHAPTER ARE:

- **Be able to use Ohm's law to solve electric circuits**
- **Be able to apply Kirchhoff's current law and Kirchhoff's voltage law to solve electric circuits**
- **Know how to analyze single-loop and single-node-pair circuits**
- **Know how to combine resistors in series and parallel**
- **Be able to use voltage and current division to solve simple electric circuits**
- **Understand when and how to apply wye-delta transformations in the analysis of electric circuits**
- **Know how to analyze electric circuits containing dependent sources**

Tesla Roadster Green technologies come in many colors. The 2010 Tesla Roadster, for example, comes in Fusion Red, Arctic White, Racing Green and Electric Blue, to name a few. An environmentally friendly sports car that seats two, this convertible has rocket acceleration and hugs the road like a dream; it's the world's first high-performance electric car. The Roadster contains over 6,800 safe, rechargeable lithium-ion batteries that weigh about 1,000 pounds in total. It is twice as efficient as hybrid cars that combine a gasoline engine and an electric motor to provide propulsion, but its fantastic performance comes at a cost of over $100,000.

Choosing between an all-electric vehicle and a hybrid requires trade-offs on a wide range of criteria: performance, cost, efficiency, effects on the environment, safety, and reliability. Handling qualities may be highly important to some, cost and efficiency to others.

As a student of circuit analysis, you will make trade-offs in choosing between methods of analysis for different circuit topologies. This chapter describes fundamental laws that apply to all circuits regardless of their complexity. Ohm's law governs the most common relationship between voltage and current for circuits that are linear. Circuits having a single power source with resistances having the same currents and others having the same voltage will be analyzed using the series-parallel method. You'll learn more techniques in the chapters that follow, as you begin to master the same principles used by the designers of the Tesla Roadster.