

Solve $Z_1 = Z_2$ for x_1

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1 Given:

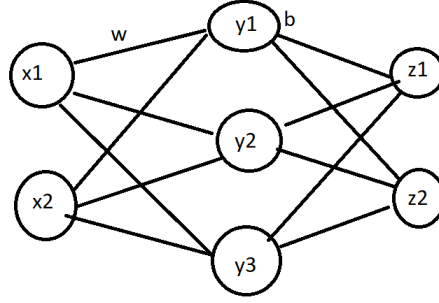


Figure 1: Given Neural Net

Figure 1 shows the given Neural Net that will be analysed.

x_1, x_2 are the input neurons

$[x]$ is the columnised vector notation of all "x" neurons of dimensions $1 \times x$

y_1, y_2, y_3 are the hidden layers neurons

$[y]$ is the columnised vector notation of all "y" neurons of dimensions $1 \times y$

z_1, z_2 are the output neurons

$[z]$ is the columnised vector notation of all "z" neurons of dimensions $1 \times z$

w denotes a weight

$w_{y_1 \rightarrow z_2}$ denotes the weight from y_1 to z_2

$[w_1]$ is the columnised vector notation of all " w_1 " weights of dimensions $y \times x$

$$[w_1] = \begin{bmatrix} w_{11} & w_{12} & w_{13} & \dots & w_{1x} \\ \vdots & & & & \\ w_{y1} & w_{y2} & w_{y3} & \dots & w_{yx} \end{bmatrix}$$

$[w_2]$ is the columnised vector notation of all " w_2 " weights of dimensions $z \times y$

$$[w_2] = \begin{bmatrix} w_{11}w_{12}w_{13}\dots w_{1y} \\ \vdots \\ w_{ij} \\ \vdots \\ w_{z1}w_{z2}w_{z3}\dots w_{zy} \end{bmatrix}$$

b deontes a bias

b_{y_3} deontes the bias associated with neuron y_3

$[b_1]$ is the columnised vector notation of all " b_1 " biases of dimentions $1 \times y$

$$[b_1] = \begin{bmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{1y} \end{bmatrix}$$

$[b_2]$ is the columnised vector notation of all " b_2 " biases of dimentions $1 \times z$

$$[b_2] = \begin{bmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{1z} \end{bmatrix}$$

The input to a y_i neuron will be denoted as:

$$y_i = \sigma(w_{x_1 \rightarrow y_i} * x_1 + w_{x_2 \rightarrow y_i} * x_2 + b_{y_i})$$

OR in vector notation:

$$[y] = \sigma([x][w_1] + [b_1])$$

$$\text{Where } \sigma(x) = \frac{1}{1+e^{-x}} = \frac{1}{1+\exp[-x]}$$

The input to a z_i neuron will be denoted as:

$$z_i = \sigma(w_{y_1 \rightarrow z_i} * y_1 + w_{y_2 \rightarrow z_i} * y_2 + w_{y_3 \rightarrow z_i} * y_3 + b_{z_i})$$

OR in vector notation:

$$[z] = \sigma([y][w_2] + [b_2])$$

2 Solve $z_1 = z_2$ for x_1

$$z_1 = z_2 \tag{1}$$

3 Conclusion

Not much of a paper, but it's a start.