

Solve $Z_1 = Z_2$ for x_1

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1 Given:

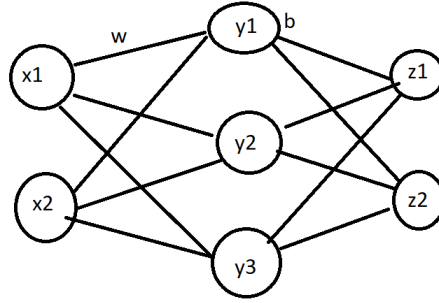


Figure 1: Given Neural Net

Figure 1 shows the given Neural Net that will be analysed.

x_1, x_2 are the input neurons

y_1, y_2, y_3 are the hidden layers neurons

z_1, z_2 are the output neurons

w denotes a weight

$w_{y_1 \rightarrow z_2}$ denotes the weight from y_1 to z_2

b denotes a bias

b_{y_3} denotes the bias associated with neuron y_3

The input to a y_i neuron will be denoted as:

$$y_i = \sigma(w_{x_1 \rightarrow y_i} * x_1 + w_{x_2 \rightarrow y_i} * x_2 + b_{y_i})$$

$$\text{Where } \sigma(x) = \frac{1}{1+e^{-x}} = \frac{1}{1+\exp[-x]}$$

The input to a z_i neuron will be denoted as:

$$z_i = \sigma(w_{y_1 \rightarrow z_i} * y_1 + w_{y_2 \rightarrow z_i} * y_2 + w_{y_3 \rightarrow z_i} * y_3 + b_{z_i})$$

2 Solve $z_1 = z_2$ for x_1

$$z_1 = z_2 \quad (1)$$

$$z_1 = \sigma(w_{y_1 \rightarrow z_1} * y_1 + w_{y_2 \rightarrow z_1} * y_2 + w_{y_3 \rightarrow z_1} * y_3 + b_{z_1}) \quad (2)$$

$$\begin{aligned} z_1 = & \sigma(w_{y_1 \rightarrow z_1} * \sigma(w_{x_1 \rightarrow y_1} * x_1 + w_{x_2 \rightarrow y_1} * x_2 + b_{y_1}) \\ & + w_{y_2 \rightarrow z_1} * \sigma(w_{x_1 \rightarrow y_2} * x_1 + w_{x_2 \rightarrow y_2} * x_2 + b_{y_2}) \\ & + w_{y_3 \rightarrow z_1} * \sigma(w_{x_1 \rightarrow y_3} * x_1 + w_{x_2 \rightarrow y_3} * x_2 + b_{y_3}) \\ & + b_{z_1}) \end{aligned} \quad (3)$$

$$\begin{aligned} z_2 = & \sigma(w_{y_1 \rightarrow z_2} * \sigma(w_{x_1 \rightarrow y_1} * x_1 + w_{x_2 \rightarrow y_1} * x_2 + b_{y_1}) \\ & + w_{y_2 \rightarrow z_2} * \sigma(w_{x_1 \rightarrow y_2} * x_1 + w_{x_2 \rightarrow y_2} * x_2 + b_{y_2}) \\ & + w_{y_3 \rightarrow z_2} * \sigma(w_{x_1 \rightarrow y_3} * x_1 + w_{x_2 \rightarrow y_3} * x_2 + b_{y_3}) \\ & + b_{z_2}) \end{aligned} \quad (4)$$

$$\begin{aligned} & (1 + \exp[-(w_{y_1 \rightarrow z_1} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_1} * x_1 + w_{x_2 \rightarrow y_1} * x_2 + b_{y_1}]} \\ & + w_{y_2 \rightarrow z_1} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_2} * x_1 + w_{x_2 \rightarrow y_2} * x_2 + b_{y_2}]} \\ & + w_{y_3 \rightarrow z_1} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_3} * x_1 + w_{x_2 \rightarrow y_3} * x_2 + b_{y_3}]} \\ & + b_{z_1}))^{-1} \\ & = \\ & (1 + \exp[-(w_{y_1 \rightarrow z_2} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_1} * x_1 + w_{x_2 \rightarrow y_1} * x_2 + b_{y_1}]} \\ & + w_{y_2 \rightarrow z_2} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_2} * x_1 + w_{x_2 \rightarrow y_2} * x_2 + b_{y_2}]} \\ & + w_{y_3 \rightarrow z_2} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_3} * x_1 + w_{x_2 \rightarrow y_3} * x_2 + b_{y_3}]} \\ & + b_{z_2}))^{-1} \end{aligned} \quad (5)$$

NOTE: this equation is too big. Lets scope it down.

$$B_1 = -w_{x_1->y_1} * x_1 + w_{x_2->y_1} * x_2 + b_{y_1} \quad (6)$$

$$B_2 = -w_{x_1->y_2} * x_1 + w_{x_2->y_2} * x_2 + b_{y_2} \quad (7)$$

$$B_3 = -w_{x_1->y_3} * x_1 + w_{x_2->y_3} * x_2 + b_{y_3} \quad (8)$$

$$(9)$$

NOTE: Substatuting B in.

$$(1 + \exp[-(w_{y_1->z_1} * \frac{1}{1 + \exp[B_1]} \quad (10)$$

$$+ w_{y_2->z_1} * \frac{1}{1 + \exp[B_2]} \quad (11)$$

$$+ w_{y_3->z_1} * \frac{1}{1 + \exp[B_3]} \quad (12)$$

$$+ b_{z_1}))^{-1} \quad (13)$$

$$= \quad (14)$$

$$(1 + \exp[-(w_{y_1->z_2} * \frac{1}{1 + \exp[B_1]} \quad (15)$$

$$+ w_{y_2->z_2} * \frac{1}{1 + \exp[B_2]} \quad (16)$$

$$+ w_{y_3->z_2} * \frac{1}{1 + \exp[B_3]} \quad (17)$$

$$+ b_{z_2}))^{-1} \quad (18)$$

Define A

$$A_1 = w_{y_1->z_1} * \frac{1}{1 + \exp[B_1]} \quad (19)$$

$$+ w_{y_2->z_1} * \frac{1}{1 + \exp[B_2]} \quad (20)$$

$$+ w_{y_3->z_1} * \frac{1}{1 + \exp[B_3]} \quad (21)$$

$$+ b_{z_1} \quad (22)$$

$$A_2 = w_{y_1->z_2} * \frac{1}{1 + \exp[B_1]} \quad (23)$$

$$+ w_{y_2->z_2} * \frac{1}{1 + \exp[B_2]} \quad (24)$$

$$+ w_{y_3->z_2} * \frac{1}{1 + \exp[B_3]} \quad (25)$$

$$+ b_{z_2} \quad (26)$$

NOTE: Substatuting A in.

$$(1 + \exp[-(A_1)])^{-1} = (1 + \exp[-(A_2)])^{-1} \quad (27)$$

$$(1 + \exp[-(A_1)]) = (1 + \exp[-(A_2)]) \quad (28)$$

$$1 + \exp[-(A_1)] = 1 + \exp[-(A_2)] \quad (29)$$

$$\exp[-(A_1)] = \exp[-(A_2)] \quad (30)$$

$$(A_1) = (A_2) \quad (31)$$

$$A_1 = A_2 \quad (32)$$

Sub in the values of A_1 and A_2

$$\begin{aligned} & w_{y_1 \rightarrow z_1} * \frac{1}{1 + \exp[B_1]} + w_{y_2 \rightarrow z_1} * \frac{1}{1 + \exp[B_2]} + w_{y_3 \rightarrow z_1} * \frac{1}{1 + \exp[B_3]} \\ & \quad + b_{z_1} \\ & = \\ & w_{y_1 \rightarrow z_2} * \frac{1}{1 + \exp[B_1]} + w_{y_2 \rightarrow z_2} * \frac{1}{1 + \exp[B_2]} + w_{y_3 \rightarrow z_2} * \frac{1}{1 + \exp[B_3]} \\ & \quad + b_{z_2} \end{aligned} \quad (33)$$

$$\begin{aligned} & \frac{w_{y_1 \rightarrow z_1}}{1 + \exp[B_1]} + \frac{w_{y_2 \rightarrow z_1}}{1 + \exp[B_2]} + \frac{w_{y_3 \rightarrow z_1}}{1 + \exp[B_3]} + b_{z_1} \\ & = \\ & \frac{w_{y_1 \rightarrow z_2}}{1 + \exp[B_1]} + \frac{w_{y_2 \rightarrow z_2}}{1 + \exp[B_2]} + \frac{w_{y_3 \rightarrow z_2}}{1 + \exp[B_3]} + b_{z_2} \end{aligned} \quad (34)$$

Multiply by botoms to create common denominators.

$$\begin{aligned}
& \frac{w_{y_1->z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_2]) * (1 + \exp[B_1]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_3]) * (1 + \exp[B_1]) * (1 + \exp[B_2])} + \\
& \quad b_{z_1} \\
& = \\
& \frac{w_{y_1->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_2]) * (1 + \exp[B_1]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_3]) * (1 + \exp[B_1]) * (1 + \exp[B_2])} + \\
& \quad b_{z_2}
\end{aligned} \tag{35}$$

REORDERER DENOMINATORS

$$\begin{aligned}
& \frac{w_{y_1->z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \quad b_{z_1} \\
& = \\
& \frac{w_{y_1->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \quad b_{z_2}
\end{aligned} \tag{36}$$

MOVE b_{z_1}

$$\begin{aligned}
& \frac{w_{y_1 \rightarrow z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2 \rightarrow z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3 \rightarrow z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} \\
& = \\
& \frac{w_{y_1 \rightarrow z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2 \rightarrow z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3 \rightarrow z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \quad b_{z_2} - b_{z_1}
\end{aligned} \tag{37}$$

MOVE THE BIG PIECE

$$\begin{aligned}
& \frac{w_{y_1 \rightarrow z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2 \rightarrow z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3 \rightarrow z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \\
& \frac{w_{y_1 \rightarrow z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \\
& \frac{w_{y_2 \rightarrow z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \\
& \frac{w_{y_3 \rightarrow z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} \\
& = b_{z_2} - b_{z_1}
\end{aligned} \tag{38}$$

REORDER FOR SIMILAR TERMS

$$\begin{aligned}
& \frac{w_{y_1->z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \frac{w_{y_1->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \frac{w_{y_2->z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \frac{w_{y_3->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} \\
& = b_{z_2} - b_{z_1}
\end{aligned} \tag{39}$$

COMBINE LIKE TERMS

$$\begin{aligned}
& \frac{w_{y_1->z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3]) - w_{y_1->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3]) - w_{y_2->z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2]) - w_{y_3->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} \\
& = b_{z_2} - b_{z_1}
\end{aligned} \tag{40}$$

FACTOR COMMON NUMERATORS

$$\begin{aligned}
& \frac{(w_{y_1->z_1} - w_{y_1->z_2}) * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{(w_{y_2->z_1} - w_{y_2->z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{(w_{y_3->z_1} - w_{y_3->z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} \\
& = b_{z_2} - b_{z_1}
\end{aligned} \tag{41}$$

SANITY CHECK

$$aAB - bAB = (a - b)AB \tag{42}$$

MULTIPLY BY COMMON DENOMINATOR

$$\begin{aligned}
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) * (1 + \exp[B_2]) * (1 + \exp[B_3]) + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_3]) + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_2]) \\
& = (b_{z_2} - b_{z_1}) * ((1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3]))
\end{aligned} \tag{43}$$

The next step is to multiply this out and eliminate terms. I do not want to do that rn :/

GO BACK TO EQ.34

$$\begin{aligned}
& \frac{w_{y_1->z_1}}{1+\exp[B_1]} + \frac{w_{y_2->z_1}}{1+\exp[B_2]} + \frac{w_{y_3->z_1}}{1+\exp[B_3]} + b_{z_1} \\
& = \\
& \frac{w_{y_1->z_2}}{1+\exp[B_1]} + \frac{w_{y_2->z_2}}{1+\exp[B_2]} + \frac{w_{y_3->z_2}}{1+\exp[B_3]} + b_{z_2}
\end{aligned} \tag{44}$$

SUBTRACT b

$$\begin{aligned}
& \frac{w_{y_1->z_1}}{1+\exp[B_1]} + \frac{w_{y_2->z_1}}{1+\exp[B_2]} + \frac{w_{y_3->z_1}}{1+\exp[B_3]} \\
& = \\
& \frac{w_{y_1->z_2}}{1+\exp[B_1]} + \frac{w_{y_2->z_2}}{1+\exp[B_2]} + \frac{w_{y_3->z_2}}{1+\exp[B_3]} + b_{z_2} - b_{z_1}
\end{aligned} \tag{45}$$

SUBTRACT REST OF THE STUFF

$$\begin{aligned}
& \frac{w_{y_1->z_1}}{1+\exp[B_1]} - \frac{w_{y_1->z_2}}{1+\exp[B_1]} + \frac{w_{y_2->z_1}}{1+\exp[B_2]} - \frac{w_{y_2->z_2}}{1+\exp[B_2]} + \frac{w_{y_3->z_1}}{1+\exp[B_3]} - \frac{w_{y_3->z_2}}{1+\exp[B_3]} \\
& = \\
& b_{z_2} - b_{z_1}
\end{aligned} \tag{46}$$

COMBINE LIKE TERMS

$$\begin{aligned}
& \frac{w_{y_1->z_1} - w_{y_1->z_2}}{1+\exp[B_1]} + \frac{w_{y_2->z_1} - w_{y_2->z_2}}{1+\exp[B_2]} + \frac{w_{y_3->z_1} - w_{y_3->z_2}}{1+\exp[B_3]} \\
& = \\
& b_{z_2} - b_{z_1}
\end{aligned} \tag{47}$$

This will got to EQ.43. This is a dead end.

NOTE: continue wiht eq.43
MULTIPLY BY COMMON DENOMINATOR

$$\begin{aligned}
& (w_{y_1->z_1} - w_{y_1->z_2}) * (1 + \exp[B_2]) * (1 + \exp[B_3]) + \\
& (w_{y_2->z_1} - w_{y_2->z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_3]) + \\
& (w_{y_3->z_1} - w_{y_3->z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_2]) \\
& = (b_{z_2} - b_{z_1}) * ((1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3]))
\end{aligned} \tag{48}$$

define:

$$\begin{aligned}
A_1 &= 1 + \exp[B_1] \\
A_2 &= 1 + \exp[B_2] \\
A_3 &= 1 + \exp[B_3]
\end{aligned} \tag{49}$$

substatute in

$$\begin{aligned}
& (w_{y_1->z_1} - w_{y_1->z_2})A_2A_3 + (w_{y_2->z_1} - w_{y_2->z_2})A_1A_3 + (w_{y_3->z_1} - w_{y_3->z_2})A_1A_2 \\
& = (b_{z_2} - b_{z_1})A_1A_2A_3
\end{aligned} \tag{50}$$

simplify $A_1 * A_2$

$$\begin{aligned}
A_1A_2 &= (1 + \exp[B_1])(1 + \exp[B_2]) \\
A_1A_2 &= 1 + \exp[B_2] + \exp[B_1] + \exp[B_1]\exp[B_2] \\
A_1A_2 &= 1 + \exp[B_2] + \exp[B_1] + \exp[B_1 + B_2]
\end{aligned} \tag{51}$$

simplify $A_1 * A_3$

$$\begin{aligned}
A_1 A_3 &= (1 + \exp[B_1])(1 + \exp[B_3]) \\
A_1 A_3 &= 1 + \exp[B_3] + \exp[B_1] + \exp[B_1] \exp[B_3] \\
A_1 A_3 &= 1 + \exp[B_3] + \exp[B_1] + \exp[B_1 + B_3]
\end{aligned} \tag{52}$$

simplify $A_2 * A_3$

$$\begin{aligned}
A_2 A_3 &= (1 + \exp[B_2])(1 + \exp[B_3]) \\
A_2 A_3 &= 1 + \exp[B_3] + \exp[B_2] + \exp[B_2] \exp[B_3] \\
A_2 A_3 &= 1 + \exp[B_3] + \exp[B_2] + \exp[B_2 + B_3]
\end{aligned} \tag{53}$$

simplify $A_1 * A_2 * A_3$

$$\begin{aligned}
A_1 A_2 A_3 &= (1 + \exp[B_2])(1 + \exp[B_3]) \\
A_1 A_2 A_3 &= (1 + \exp[B_1])(1 + \exp[B_3] + \exp[B_2] + \exp[B_2] \exp[B_3]) \\
A_1 A_2 A_3 &= 1 + \exp[B_3] + \exp[B_2] + \exp[B_1] + \\
&\quad \exp[B_2] \exp[B_3] + \exp[B_1] \exp[B_3] + \exp[B_1] \exp[B_2] + \\
&\quad \exp[B_1] \exp[B_2] \exp[B_3] \\
A_1 A_2 A_3 &= 1 + \exp[B_3] + \exp[B_2] + \exp[B_1] + \exp[B_2 + B_3] + \exp[B_1 + B_3] + \exp[B_1 + B_2] + \exp[B_1 + B_2 + B_3]
\end{aligned} \tag{54}$$

substatute into eq.50

$$\begin{aligned}
&(w_{y_1 - > z_1} - w_{y_1 - > z_2})(1 + \exp[B_3] + \exp[B_2] + \exp[B_2 + B_3]) + \\
&(w_{y_2 - > z_1} - w_{y_2 - > z_2})(1 + \exp[B_3] + \exp[B_1] + \exp[B_1 + B_3]) + \\
&(w_{y_3 - > z_1} - w_{y_3 - > z_2})(1 + \exp[B_2] + \exp[B_1] + \exp[B_1 + B_2]) \\
&= (b_{z_2} - b_{z_1})(1 + \exp[B_3] + \exp[B_2] + \exp[B_1] + \exp[B_2 + B_3] + \exp[B_1 + B_3] + \exp[B_1 + B_2] + \exp[B_1 + B_2 + B_3])
\end{aligned} \tag{55}$$

expand each line of eq.55:

$$\begin{aligned}
& w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2 + B_3]
\end{aligned} \tag{56}$$

$$\begin{aligned}
& w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1 + B_3]
\end{aligned} \tag{57}$$

$$\begin{aligned}
& w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1 + B_2]
\end{aligned} \tag{58}$$

$$\begin{aligned}
& b_{z_2} - b_{z_1} + \\
& (b_{z_2} - b_{z_1}) \exp[B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_2] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1] + \\
& (b_{z_2} - b_{z_1}) \exp[B_2 + B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2 + B_3]
\end{aligned} \tag{59}$$

combine everything

$$\begin{aligned}
& w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2 + B_3] + \\
& w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1 + B_3] + \\
& w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1 + B_2] = \\
& b_{z_2} - b_{z_1} + \\
& (b_{z_2} - b_{z_1}) \exp[B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_2] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1] + \\
& (b_{z_2} - b_{z_1}) \exp[B_2 + B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2 + B_3]
\end{aligned} \tag{60}$$

move everything to one side

$$\begin{aligned}
& w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2 + B_3] + \\
& w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1 + B_3] + \\
& w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1 + B_2] - \\
& b_{z_2} + b_{z_1} - \\
& (b_{z_2} - b_{z_1}) \exp[B_3] - \\
& (b_{z_2} - b_{z_1}) \exp[B_2] - \\
& (b_{z_2} - b_{z_1}) \exp[B_1] - \\
& (b_{z_2} - b_{z_1}) \exp[B_2 + B_3] - \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_3] - \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2] - \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2 + B_3] = 0
\end{aligned} \tag{61}$$

Reorder based on exponent

$$\begin{aligned}
& w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1} \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_3] + (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_3] - (b_{z_2} - b_{z_1}) \exp[B_3] \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2] + (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_2] - (b_{z_2} - b_{z_1}) \exp[B_2] \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1] + (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1] - (b_{z_2} - b_{z_1}) \exp[B_1] \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2 + B_3] - (b_{z_2} - b_{z_1}) \exp[B_2 + B_3] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1 + B_3] - (b_{z_2} - b_{z_1}) \exp[B_1 + B_3] \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1 + B_2] - (b_{z_2} - b_{z_1}) \exp[B_1 + B_2] \\
& - (b_{z_2} - b_{z_1}) \exp[B_1 + B_2 + B_3] = 0
\end{aligned} \tag{62}$$

factor

$$\begin{aligned}
& w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1} + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} - b_{z_2} + b_{z_1}) \exp[B_3] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1}) \exp[B_2] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1}) \exp[B_1] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} - b_{z_2} + b_{z_1}) \exp[B_2 + B_3] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} - b_{z_2} + b_{z_1}) \exp[B_1 + B_3] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1}) \exp[B_1 + B_2] + \\
& (-b_{z_2} + b_{z_1}) \exp[B_1 + B_2 + B_3] = 0
\end{aligned} \tag{63}$$

Define Constants

$$\begin{aligned}
C_0 &= w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1} \\
C_3 &= (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} - b_{z_2} + b_{z_1}) \\
C_2 &= (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1}) \\
C_1 &= (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1}) \\
C_4 &= (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} - b_{z_2} + b_{z_1}) \\
C_5 &= (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} - b_{z_2} + b_{z_1}) \\
C_6 &= (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1}) \\
C_7 &= (-b_{z_2} + b_{z_1})
\end{aligned} \tag{64}$$

substatute constants:

$$\begin{aligned}
& C_0 + \\
& C_3 * \exp[B_3] + \\
& C_2 * \exp[B_2] + \\
& C_1 * \exp[B_1] + \\
& C_4 * \exp[B_2 + B_3] + \\
& C_5 * \exp[B_1 + B_3] + \\
& C_6 * \exp[B_1 + B_2] + \\
& C_7 * \exp[B_1 + B_2 + B_3] = 0
\end{aligned} \tag{65}$$

Reorder:

$$\begin{aligned}
& C_0 + C_1 * \exp[B_1] + C_2 * \exp[B_2] + C_3 * \exp[B_3] + \\
& C_4 * \exp[B_2 + B_3] + C_5 * \exp[B_1 + B_3] + C_6 * \exp[B_1 + B_2] + \\
& C_7 * \exp[B_1 + B_2 + B_3] = 0
\end{aligned} \tag{66}$$

Move multipliers to exponents

$$\begin{aligned}
& C_0 + \exp[B_1^{C_1}] + \exp[B_2^{C_2}] + \exp[B_3^{C_3}] + \\
& \exp[(B_2 + B_3)^{C_4}] + \exp[(B_1 + B_3)^{C_5}] + \exp[(B_1 + B_2)^{C_6}] + \\
& \exp[(B_1 + B_2 + B_3)^{C_7}] = 0
\end{aligned} \tag{67}$$

Make non-zero

$$\begin{aligned}
& C_0 + \exp[B_1^{C_1}] + \exp[B_2^{C_2}] + \exp[B_3^{C_3}] + \\
& \exp[(B_2 + B_3)^{C_4}] + \exp[(B_1 + B_3)^{C_5}] + \exp[(B_1 + B_2)^{C_6}] + \\
& = -\exp[(B_1 + B_2 + B_3)^{C_7}]
\end{aligned} \tag{68}$$

Make non-zero

$$\begin{aligned}
& \exp[B_1^{C_1}] + \exp[B_2^{C_2}] + \exp[B_3^{C_3}] + \\
& \exp[(B_2 + B_3)^{C_4}] + \exp[(B_1 + B_3)^{C_5}] + \exp[(B_1 + B_2)^{C_6}] + \\
& \exp[(B_1 + B_2 + B_3)^{C_7}] = -C_0
\end{aligned} \tag{69}$$

Make non-zero

$$\begin{aligned}
& \exp[B_1^{C_1}] + \exp[B_2^{C_2}] + \exp[B_3^{C_3}] + \\
& \exp[(B_2 + B_3)^{C_4}] + \exp[(B_1 + B_3)^{C_5}] + \exp[(B_1 + B_2)^{C_6}] + \\
& \exp[(B_1 + B_2 + B_3)^{C_7}] = -C_0
\end{aligned} \tag{70}$$

3 Conclusion

Not much of a paper, but it's a start.