Solve $Z_1 = Z_2$ for x_1

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1 Given:

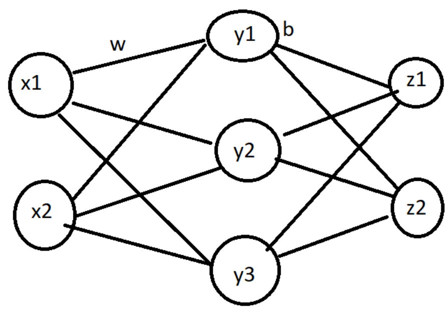

Figure 1: Given Neural Net

Figure 1 shows the given Neural Net that will be analysed.

x_1, x_2 are the input neurons
y_1, y_2, y_3 are the hidden layers neurons
z_1, z_2 are the output neurons

w denotes a weight
$w_{y_1->z_2}$ denotes the weight from y_1 to z_2

b deontes a bias
b_{y_3} deontes the bias associated with neuron y_3

The input to a y_i neuron will be denoted as:
$y_i = \sigma(w_{x_1->y_i} * x_1 + w_{x_2->y_i} * x_2 + b_{y_i})$
Where $\sigma(x) = \frac{1}{1+e^{-x}} = \frac{1}{1+\exp[-x]}$

The input to a z_i neuron will be denoted as:
$z_i = \sigma(w_{y_1->z_i} * y_1 + w_{y_2->z_i} * y_2 + w_{y_3->z_i} * y_3 + b_{z_i})$

2 Solve $z_1 = z_2$ for x_1

$$z_1 = z_2 \tag{1}$$

$$z_1 = \sigma(w_{y_1->z_1} * y_1 + w_{y_2->z_1} * y_2 + w_{y_3->z_1} * y_3 + b_{z_1}) \tag{2}$$

$$\begin{aligned} z_1 = \sigma(w_{y_1->z_1} * \sigma(w_{x_1->y_1} * x_1 + w_{x_2->y_1} * x_2 + b_{y_1}) \\ + w_{y_2->z_1} * \sigma(w_{x_1->y_2} * x_1 + w_{x_2->y_2} * x_2 + b_{y_2}) \\ + w_{y_3->z_1} * \sigma(w_{x_1->y_3} * x_1 + w_{x_2->y_3} * x_2 + b_{y_3}) \\ + b_{z_1}) \end{aligned} \tag{3}$$

$$\begin{aligned} z_2 = \sigma(w_{y_1->z_2} * \sigma(w_{x_1->y_1} * x_1 + w_{x_2->y_1} * x_2 + b_{y_1}) \\ + w_{y_2->z_2} * \sigma(w_{x_1->y_2} * x_1 + w_{x_2->y_2} * x_2 + b_{y_2}) \\ + w_{y_3->z_2} * \sigma(w_{x_1->y_3} * x_1 + w_{x_2->y_3} * x_2 + b_{y_3}) \\ + b_{z_2}) \end{aligned} \tag{4}$$

$$\begin{gathered} (1 + \exp[-(w_{y_1->z_1} * \frac{1}{1 + \exp[w_{x_1->y_1} * x_1 + w_{x_2->y_1} * x_2 + b_{y_1}]} \\ + w_{y_2->z_1} * \frac{1}{1 + \exp[w_{x_1->y_2} * x_1 + w_{x_2->y_2} * x_2 + b_{y_2}]} \\ + w_{y_3->z_1} * \frac{1}{1 + \exp[w_{x_1->y_3} * x_1 + w_{x_2->y_3} * x_2 + b_{y_3}]} \\ + b_{z_1})])^{-1} \\ = \\ (1 + \exp[-(w_{y_1->z_2} * \frac{1}{1 + \exp[w_{x_1->y_1} * x_1 + w_{x_2->y_1} * x_2 + b_{y_1}]} \\ + w_{y_2->z_2} * \frac{1}{1 + \exp[w_{x_1->y_2} * x_1 + w_{x_2->y_2} * x_2 + b_{y_2}]} \\ + w_{y_3->z_2} * \frac{1}{1 + \exp[w_{x_1->y_3} * x_1 + w_{x_2->y_3} * x_2 + b_{y_3}]} \\ + b_{z_2})])^{-1} \end{gathered} \tag{5}$$

NOTE: this equation is too big. Lets scope it down.
Define:

$$B_1 = -w_{x_1->y_1} * x_1 + w_{x_2->y_1} * x_2 + b_{y_1} \tag{6}$$
$$B_2 = -w_{x_1->y_2} * x_1 + w_{x_2->y_2} * x_2 + b_{y_2} \tag{7}$$
$$B_3 = -w_{x_1->y_3} * x_1 + w_{x_2->y_3} * x_2 + b_{y_3} \tag{8}$$
$$\tag{9}$$

NOTE: Substatuting B in.

$$(1 + \exp[-(w_{y_1->z_1} * \frac{1}{1 + \exp[B_1]} \tag{10}$$
$$+w_{y_2->z_1} * \frac{1}{1 + \exp[B_2]} \tag{11}$$
$$+w_{y_3->z_1} * \frac{1}{1 + \exp[B_3]} \tag{12}$$
$$+b_{z_1})])^{-1} \tag{13}$$
$$= \tag{14}$$
$$(1 + \exp[-(w_{y_1->z_2} * \frac{1}{1 + \exp[B_1]} \tag{15}$$
$$+w_{y_2->z_2} * \frac{1}{1 + \exp[B_2]} \tag{16}$$
$$+w_{y_3->z_2} * \frac{1}{1 + \exp[B_3]} \tag{17}$$
$$+b_{z_2})])^{-1} \tag{18}$$

Define A

$$A_1 = w_{y_1->z_1} * \frac{1}{1 + \exp[B_1]} \tag{19}$$
$$+w_{y_2->z_1} * \frac{1}{1 + \exp[B_2]} \tag{20}$$
$$+w_{y_3->z_1} * \frac{1}{1 + \exp[B_3]} \tag{21}$$
$$+b_{z_1} \tag{22}$$

$$A_2 = w_{y_1->z_2} * \frac{1}{1 + \exp[B_1]} \tag{23}$$
$$+w_{y_2->z_2} * \frac{1}{1 + \exp[B_2]} \tag{24}$$
$$+w_{y_3->z_2} * \frac{1}{1 + \exp[B_3]} \tag{25}$$
$$+b_{z_2} \tag{26}$$

NOTE: Substatuting A in.

$$(1+\exp[-(A_1)])^{-1} = (1+\exp[-(A_2)])^{-1} \tag{27}$$

$$(1+\exp[-(A_1)]) = (1+\exp[-(A_2)]) \tag{28}$$

$$1+\exp[-(A_1)] = 1+\exp[-(A_2)] \tag{29}$$

$$\exp[-(A_1)] = \exp[-(A_2)] \tag{30}$$

$$(A_1) = (A_2) \tag{31}$$

$$A_1 = A_2 \tag{32}$$

Sub in the values of A_1 and A_2

$$\begin{gathered} w_{y_1->z_1} * \frac{1}{1+\exp[B_1]} + w_{y_2->z_1} * \frac{1}{1+\exp[B_2]} + w_{y_3->z_1} * \frac{1}{1+\exp[B_3]} \\ + b_{z_1} \\ = \\ w_{y_1->z_2} * \frac{1}{1+\exp[B_1]} + w_{y_2->z_2} * \frac{1}{1+\exp[B_2]} + w_{y_3->z_2} * \frac{1}{1+\exp[B_3]} \\ + b_{z_2} \end{gathered} \tag{33}$$

$$\begin{gathered} \frac{w_{y_1->z_1}}{1+\exp[B_1]} + \frac{w_{y_2->z_1}}{1+\exp[B_2]} + \frac{w_{y_3->z_1}}{1+\exp[B_3]} + b_{z_1} \\ = \\ \frac{w_{y_1->z_2}}{1+\exp[B_1]} + \frac{w_{y_2->z_2}}{1+\exp[B_2]} + \frac{w_{y_3->z_2}}{1+\exp[B_3]} + b_{z_2} \end{gathered} \tag{34}$$

Multiply by botoms to create common denominators.

$$
\begin{gathered}
\frac{w_{y_1->z_1} * (1+\exp[B_2]) * (1+\exp[B_3])}{(1+\exp[B_1]) * (1+\exp[B_2]) * (1+\exp[B_3])} + \\
\frac{w_{y_2->z_1} * (1+\exp[B_1]) * (1+\exp[B_3])}{(1+\exp[B_2]) * (1+\exp[B_1]) * (1+\exp[B_3])} + \\
\frac{w_{y_3->z_1} * (1+\exp[B_1]) * (1+\exp[B_2])}{(1+\exp[B_3]) * (1+\exp[B_1]) * (1+\exp[B_2])} + \\
b_{z_1} \\
= \\
\frac{w_{y_1->z_2} * (1+\exp[B_2]) * (1+\exp[B_3])}{(1+\exp[B_1]) * (1+\exp[B_2]) * (1+\exp[B_3])} + \\
\frac{w_{y_2->z_2} * (1+\exp[B_1]) * (1+\exp[B_3])}{(1+\exp[B_2]) * (1+\exp[B_1]) * (1+\exp[B_3])} + \\
\frac{w_{y_3->z_2} * (1+\exp[B_2]) * (1+\exp[B_1])}{(1+\exp[B_3]) * (1+\exp[B_1]) * (1+\exp[B_2])} + \\
b_{z_2}
\end{gathered}
\tag{35}
$$

REORDRER DENOMINATORS

$$
\begin{gathered}
\frac{w_{y_1->z_1} * (1+\exp[B_2]) * (1+\exp[B_3])}{(1+\exp[B_1]) * (1+\exp[B_2]) * (1+\exp[B_3])} + \\
\frac{w_{y_2->z_1} * (1+\exp[B_1]) * (1+\exp[B_3])}{(1+\exp[B_1]) * (1+\exp[B_2]) * (1+\exp[B_3])} + \\
\frac{w_{y_3->z_1} * (1+\exp[B_1]) * (1+\exp[B_2])}{(1+\exp[B_1]) * (1+\exp[B_2]) * (1+\exp[B_3])} + \\
b_{z_1} \\
= \\
\frac{w_{y_1->z_2} * (1+\exp[B_2]) * (1+\exp[B_3])}{(1+\exp[B_1]) * (1+\exp[B_2]) * (1+\exp[B_3])} + \\
\frac{w_{y_2->z_2} * (1+\exp[B_1]) * (1+\exp[B_3])}{(1+\exp[B_1]) * (1+\exp[B_2]) * (1+\exp[B_3])} + \\
\frac{w_{y_3->z_2} * (1+\exp[B_2]) * (1+\exp[B_1])}{(1+\exp[B_1]) * (1+\exp[B_2]) * (1+\exp[B_3])} + \\
b_{z_2}
\end{gathered}
\tag{36}
$$

MOVE b_{z_1}

$$\begin{gathered}
\frac{w_{y_1->z_1}*(1+\exp[B_2])*(1+\exp[B_3])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}+ \\
\frac{w_{y_2->z_1}*(1+\exp[B_1])*(1+\exp[B_3])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}+ \\
\frac{w_{y_3->z_1}*(1+\exp[B_1])*(1+\exp[B_2])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])} \\
= \\
\frac{w_{y_1->z_2}*(1+\exp[B_2])*(1+\exp[B_3])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}+ \\
\frac{w_{y_2->z_2}*(1+\exp[B_1])*(1+\exp[B_3])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}+ \\
\frac{w_{y_3->z_2}*(1+\exp[B_2])*(1+\exp[B_1])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}+ \\
b_{z_2}-b_{z_1}
\end{gathered} \tag{37}$$

MOVE THE BIG PIECE

$$\begin{gathered}
\frac{w_{y_1->z_1}*(1+\exp[B_2])*(1+\exp[B_3])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}+ \\
\frac{w_{y_2->z_1}*(1+\exp[B_1])*(1+\exp[B_3])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}+ \\
\frac{w_{y_3->z_1}*(1+\exp[B_1])*(1+\exp[B_2])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}- \\
\frac{w_{y_1->z_2}*(1+\exp[B_2])*(1+\exp[B_3])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}- \\
\frac{w_{y_2->z_2}*(1+\exp[B_1])*(1+\exp[B_3])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}- \\
\frac{w_{y_3->z_2}*(1+\exp[B_2])*(1+\exp[B_1])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])} \\
= b_{z_2}-b_{z_1}
\end{gathered} \tag{38}$$

REORDER FOR SIMMILAR TERMS

$$\frac{w_{y_1->z_1}*(1+\exp[B_2])*(1+\exp[B_3])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}-\frac{w_{y_1->z_2}*(1+\exp[B_2])*(1+\exp[B_3])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}+$$
$$\frac{w_{y_2->z_1}*(1+\exp[B_1])*(1+\exp[B_3])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}-\frac{w_{y_2->z_2}*(1+\exp[B_1])*(1+\exp[B_3])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}+$$
$$\frac{w_{y_3->z_1}*(1+\exp[B_1])*(1+\exp[B_2])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}-\frac{w_{y_3->z_2}*(1+\exp[B_2])*(1+\exp[B_1])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}$$
$$=b_{z_2}-b_{z_1} \tag{39}$$

COMBINE LIKE TERMS

$$\frac{w_{y_1->z_1}*(1+\exp[B_2])*(1+\exp[B_3])-w_{y_1->z_2}*(1+\exp[B_2])*(1+\exp[B_3])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}+$$
$$\frac{w_{y_2->z_1}*(1+\exp[B_1])*(1+\exp[B_3])-w_{y_2->z_2}*(1+\exp[B_1])*(1+\exp[B_3])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}+$$
$$\frac{w_{y_3->z_1}*(1+\exp[B_1])*(1+\exp[B_2])-w_{y_3->z_2}*(1+\exp[B_2])*(1+\exp[B_1])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}$$
$$=b_{z_2}-b_{z_1} \tag{40}$$

fACTOR COMMON NUMERATORS

$$\frac{(w_{y_1->z_1}-w_{y_1->z_2})*(1+\exp[B_2])*(1+\exp[B_3])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}+$$
$$\frac{(w_{y_2->z_1}-w_{y_2->z_2})*(1+\exp[B_1])*(1+\exp[B_3])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}+$$
$$\frac{(w_{y_3->z_1}-w_{y_3->z_2})*(1+\exp[B_1])*(1+\exp[B_2])}{(1+\exp[B_1])*(1+\exp[B_2])*(1+\exp[B_3])}$$
$$=b_{z_2}-b_{z_1} \tag{41}$$

SANITY CHECK

$$aAB-bAB=(a-b)AB \tag{42}$$

MULTIPLY BY COMMON DENOMINATOR

$$
\begin{gathered}
(w_{y_1->z_1} - w_{y_1->z_2}) * (1 + \exp[B_2]) * (1 + \exp[B_3]) + \\
(w_{y_2->z_1} - w_{y_2->z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_3]) + \\
(w_{y_3->z_1} - w_{y_3->z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_2]) \\
= (b_{z_2} - b_{z_1}) * ((1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3]))
\end{gathered}
\tag{43}
$$

The next step is to multiply this out and emilimate terms. I do not want to do that rn :/

GO BACK TO EQ.34

$$\begin{gathered}\frac{w_{y_1->z_1}}{1+\exp[B_1]}+\frac{w_{y_2->z_1}}{1+\exp[B_2]}+\frac{w_{y_3->z_1}}{1+\exp[B_3]}+b_{z_1}\\ =\\ \frac{w_{y_1->z_2}}{1+\exp[B_1]}+\frac{w_{y_2->z_2}}{1+\exp[B_2]}+\frac{w_{y_3->z_2}}{1+\exp[B_3]}+b_{z_2}\end{gathered}\tag{44}$$

SUBRTRACT b

$$\begin{gathered}\frac{w_{y_1->z_1}}{1+\exp[B_1]}+\frac{w_{y_2->z_1}}{1+\exp[B_2]}+\frac{w_{y_3->z_1}}{1+\exp[B_3]}\\ =\\ \frac{w_{y_1->z_2}}{1+\exp[B_1]}+\frac{w_{y_2->z_2}}{1+\exp[B_2]}+\frac{w_{y_3->z_2}}{1+\exp[B_3]}+b_{z_2}-b_{z_1}\end{gathered}\tag{45}$$

SUBRTRACT REST OF THE STUFF

$$\begin{gathered}\frac{w_{y_1->z_1}}{1+\exp[B_1]}-\frac{w_{y_1->z_2}}{1+\exp[B_1]}+\frac{w_{y_2->z_1}}{1+\exp[B_2]}-\frac{w_{y_2->z_2}}{1+\exp[B_2]}+\frac{w_{y_3->z_1}}{1+\exp[B_3]}-\frac{w_{y_3->z_2}}{1+\exp[B_3]}\\ =\\ b_{z_2}-b_{z_1}\end{gathered}\tag{46}$$

COMBINE LIKE TERMS

$$\begin{gathered}\frac{w_{y_1->z_1}-w_{y_1->z_2}}{1+\exp[B_1]}+\frac{w_{y_2->z_1}-w_{y_2->z_2}}{1+\exp[B_2]}+\frac{w_{y_3->z_1}-w_{y_3->z_2}}{1+\exp[B_3]}\\ =\\ b_{z_2}-b_{z_1}\end{gathered}\tag{47}$$

This will got to EQ.43. This is a dead end.

NOTE: continue wiht eq.43
MULTIPLY BY COMMON DENOMINATOR

$$
\begin{gathered}
(w_{y_1->z_1} - w_{y_1->z_2}) * (1 + \exp[B_2]) * (1 + \exp[B_3]) + \\
(w_{y_2->z_1} - w_{y_2->z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_3]) + \\
(w_{y_3->z_1} - w_{y_3->z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_2]) \\
= (b_{z_2} - b_{z_1}) * ((1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3]))
\end{gathered}
\tag{48}
$$

define:

$$
\begin{gathered}
A_1 = 1 + \exp[B_1] \\
A_2 = 1 + \exp[B_2] \\
A_3 = 1 + \exp[B_3]
\end{gathered}
\tag{49}
$$

substatute in

$$
\begin{gathered}
(w_{y_1->z_1} - w_{y_1->z_2}) A_2 A_3 + (w_{y_2->z_1} - w_{y_2->z_2}) A_1 A_3 + (w_{y_3->z_1} - w_{y_3->z_2}) A_1 A_2 \\
= (b_{z_2} - b_{z_1}) A_1 A_2 A_3
\end{gathered}
\tag{50}
$$

simplify $A_1 * A_2$

$$
\begin{gathered}
A_1 A_2 = (1 + \exp[B_1])(1 + \exp[B_2]) \\
A_1 A_2 = 1 + \exp[B_2] + \exp[B_1] + \exp[B_1] \exp[B_2] \\
A_1 A_2 = 1 + \exp[B_2] + \exp[B_1] + \exp[B_1 + B_2]
\end{gathered}
\tag{51}
$$

simplify $A_1 * A_3$

$$
\begin{gathered}
A_1A_3 = (1+\exp[B_1])(1+\exp[B_3]) \\
A_1A_3 = 1+\exp[B_3]+\exp[B_1]+\exp[B_1]\exp[B_3] \\
A_1A_3 = 1+\exp[B_3]+\exp[B_1]+\exp[B_1+B_3]
\end{gathered}
\tag{52}
$$

simplify $A_2 * A_3$

$$
\begin{gathered}
A_2A_3 = (1+\exp[B_2])(1+\exp[B_3]) \\
A_2A_3 = 1+\exp[B_3]+\exp[B_2]+\exp[B_2]\exp[B_3] \\
A_2A_3 = 1+\exp[B_3]+\exp[B_2]+\exp[B_2+B_3]
\end{gathered}
\tag{53}
$$

simplify $A_1 * A_2 * A_3$

$$
\begin{gathered}
A_1A_2A_3 = (1+\exp[B_2])(1+\exp[B_3]) \\
A_1A_2A_3 = (1+\exp[B_1])(1+\exp[B_3]+\exp[B_2]+\exp[B_2]\exp[B_3]) \\
A_1A_2A_3 = 1+\exp[B_3]+\exp[B_2]+\exp[B_1]+ \\
\exp[B_2]\exp[B_3]+\exp[B_1]\exp[B_3]+\exp[B_1]\exp[B_2]+ \\
\exp[B_1]\exp[B_2]\exp[B_3] \\
A_1A_2A_3 = 1+\exp[B_3]+\exp[B_2]+\exp[B_1]+\exp[B_2+B_3]+\exp[B_1+B_3]+\exp[B_1+B_2]+\exp[B_1+B_2+B_3]
\end{gathered}
\tag{54}
$$

substatute into eq.50

$$
\begin{gathered}
(w_{y_1->z_1}-w_{y_1->z_2})(1+\exp[B_3]+\exp[B_2]+\exp[B_2+B_3])+ \\
(w_{y_2->z_1}-w_{y_2->z_2})(1+\exp[B_3]+\exp[B_1]+\exp[B_1+B_3])+ \\
(w_{y_3->z_1}-w_{y_3->z_2})(1+\exp[B_2]+\exp[B_1]+\exp[B_1+B_2]) \\
=(b_{z_2}-b_{z_1})(1+\exp[B_3]+\exp[B_2]+\exp[B_1]+\exp[B_2+B_3]+\exp[B_1+B_3]+\exp[B_1+B_2]+\exp[B_1+B_2+B_3])
\end{gathered}
\tag{55}
$$

expand each line of eq.55:

$$
\begin{gathered}
w_{y_1->z_1} - w_{y_1->z_2} + \\
(w_{y_1->z_1} - w_{y_1->z_2}) \exp[B_3] + \\
(w_{y_1->z_1} - w_{y_1->z_2}) \exp[B_2] + \\
(w_{y_1->z_1} - w_{y_1->z_2}) \exp[B_2 + B_3]
\end{gathered} \tag{56}
$$

$$
\begin{gathered}
w_{y_2->z_1} - w_{y_2->z_2} + \\
(w_{y_2->z_1} - w_{y_2->z_2}) \exp[B_3] + \\
(w_{y_2->z_1} - w_{y_2->z_2}) \exp[B_1] + \\
(w_{y_2->z_1} - w_{y_2->z_2}) \exp[B_1 + B_3]
\end{gathered} \tag{57}
$$

$$
\begin{gathered}
w_{y_3->z_1} - w_{y_3->z_2} + \\
(w_{y_3->z_1} - w_{y_3->z_2}) \exp[B_2] + \\
(w_{y_3->z_1} - w_{y_3->z_2}) \exp[B_1] + \\
(w_{y_3->z_1} - w_{y_3->z_2}) \exp[B_1 + B_2]
\end{gathered} \tag{58}
$$

$$
\begin{gathered}
b_{z_2} - b_{z_1} + \\
(b_{z_2} - b_{z_1}) \exp[B_3] + \\
(b_{z_2} - b_{z_1}) \exp[B_2] + \\
(b_{z_2} - b_{z_1}) \exp[B_1] + \\
(b_{z_2} - b_{z_1}) \exp[B_2 + B_3] + \\
(b_{z_2} - b_{z_1}) \exp[B_1 + B_3] + \\
(b_{z_2} - b_{z_1}) \exp[B_1 + B_2] + \\
(b_{z_2} - b_{z_1}) \exp[B_1 + B_2 + B_3]
\end{gathered} \tag{59}
$$

combine everything

$$\begin{gathered}
w_{y_1->z_1} - w_{y_1->z_2} + \\
(w_{y_1->z_1} - w_{y_1->z_2}) \exp[B_3] + \\
(w_{y_1->z_1} - w_{y_1->z_2}) \exp[B_2] + \\
(w_{y_1->z_1} - w_{y_1->z_2}) \exp[B_2 + B_3] + \\
w_{y_2->z_1} - w_{y_2->z_2} + \\
(w_{y_2->z_1} - w_{y_2->z_2}) \exp[B_3] + \\
(w_{y_2->z_1} - w_{y_2->z_2}) \exp[B_1] + \\
(w_{y_2->z_1} - w_{y_2->z_2}) \exp[B_1 + B_3] + \\
w_{y_3->z_1} - w_{y_3->z_2} + \\
(w_{y_3->z_1} - w_{y_3->z_2}) \exp[B_2] + \\
(w_{y_3->z_1} - w_{y_3->z_2}) \exp[B_1] + \\
(w_{y_3->z_1} - w_{y_3->z_2}) \exp[B_1 + B_2] = \\
b_{z_2} - b_{z_1} + \\
(b_{z_2} - b_{z_1}) \exp[B_3] + \\
(b_{z_2} - b_{z_1}) \exp[B_2] + \\
(b_{z_2} - b_{z_1}) \exp[B_1] + \\
(b_{z_2} - b_{z_1}) \exp[B_2 + B_3] + \\
(b_{z_2} - b_{z_1}) \exp[B_1 + B_3] + \\
(b_{z_2} - b_{z_1}) \exp[B_1 + B_2] + \\
(b_{z_2} - b_{z_1}) \exp[B_1 + B_2 + B_3]
\end{gathered} \tag{60}$$

move everything to one side

$$
\begin{gathered}
w_{y_1->z_1} - w_{y_1->z_2} + \\
(w_{y_1->z_1} - w_{y_1->z_2})\exp[B_3] + \\
(w_{y_1->z_1} - w_{y_1->z_2})\exp[B_2] + \\
(w_{y_1->z_1} - w_{y_1->z_2})\exp[B_2 + B_3] + \\
w_{y_2->z_1} - w_{y_2->z_2} + \\
(w_{y_2->z_1} - w_{y_2->z_2})\exp[B_3] + \\
(w_{y_2->z_1} - w_{y_2->z_2})\exp[B_1] + \\
(w_{y_2->z_1} - w_{y_2->z_2})\exp[B_1 + B_3] + \\
w_{y_3->z_1} - w_{y_3->z_2} + \\
(w_{y_3->z_1} - w_{y_3->z_2})\exp[B_2] + \\
(w_{y_3->z_1} - w_{y_3->z_2})\exp[B_1] + \\
(w_{y_3->z_1} - w_{y_3->z_2})\exp[B_1 + B_2] - \\
b_{z_2} + b_{z_1} - \\
(b_{z_2} - b_{z_1})\exp[B_3] - \\
(b_{z_2} - b_{z_1})\exp[B_2] - \\
(b_{z_2} - b_{z_1})\exp[B_1] - \\
(b_{z_2} - b_{z_1})\exp[B_2 + B_3] - \\
(b_{z_2} - b_{z_1})\exp[B_1 + B_3] - \\
(b_{z_2} - b_{z_1})\exp[B_1 + B_2] - \\
(b_{z_2} - b_{z_1})\exp[B_1 + B_2 + B_3] = 0
\end{gathered} \tag{61}
$$

Reorder based on exponent

$$
\begin{gathered}
w_{y_1->z_1} - w_{y_1->z_2} + w_{y_2->z_1} - w_{y_2->z_2} + w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1} \\
(w_{y_1->z_1} - w_{y_1->z_2})\exp[B_3] + (w_{y_2->z_1} - w_{y_2->z_2})\exp[B_3] - (b_{z_2} - b_{z_1})\exp[B_3] \\
(w_{y_1->z_1} - w_{y_1->z_2})\exp[B_2] + (w_{y_3->z_1} - w_{y_3->z_2})\exp[B_2] - (b_{z_2} - b_{z_1})\exp[B_2] \\
(w_{y_2->z_1} - w_{y_2->z_2})\exp[B_1] + (w_{y_3->z_1} - w_{y_3->z_2})\exp[B_1] - (b_{z_2} - b_{z_1})\exp[B_1] \\
(w_{y_1->z_1} - w_{y_1->z_2})\exp[B_2 + B_3] - (b_{z_2} - b_{z_1})\exp[B_2 + B_3] + \\
(w_{y_2->z_1} - w_{y_2->z_2})\exp[B_1 + B_3] - (b_{z_2} - b_{z_1})\exp[B_1 + B_3] \\
(w_{y_3->z_1} - w_{y_3->z_2})\exp[B_1 + B_2] - (b_{z_2} - b_{z_1})\exp[B_1 + B_2] \\
-(b_{z_2} - b_{z_1})\exp[B_1 + B_2 + B_3] = 0
\end{gathered} \tag{62}
$$

factor

$$
\begin{gathered}
w_{y_1->z_1} - w_{y_1->z_2} + w_{y_2->z_1} - w_{y_2->z_2} + w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1} + \\
(w_{y_1->z_1} - w_{y_1->z_2} + w_{y_2->z_1} - w_{y_2->z_2} - b_{z_2} + b_{z_1}) \exp[B_3] + \\
(w_{y_1->z_1} - w_{y_1->z_2} + w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1}) \exp[B_2] + \\
(w_{y_2->z_1} - w_{y_2->z_2} + w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1}) \exp[B_1] + \\
(w_{y_1->z_1} - w_{y_1->z_2} - b_{z_2} + b_{z_1}) \exp[B_2 + B_3] + \\
(w_{y_2->z_1} - w_{y_2->z_2} - b_{z_2} + b_{z_1}) \exp[B_1 + B_3] + \\
(w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1}) \exp[B_1 + B_2] + \\
(-b_{z_2} + b_{z_1}) \exp[B_1 + B_2 + B_3] = 0
\end{gathered}
\tag{63}
$$

Define Constants

$$
\begin{gathered}
C_0 = w_{y_1->z_1} - w_{y_1->z_2} + w_{y_2->z_1} - w_{y_2->z_2} + w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1} \\
C_3 = (w_{y_1->z_1} - w_{y_1->z_2} + w_{y_2->z_1} - w_{y_2->z_2} - b_{z_2} + b_{z_1}) \\
C_2 = (w_{y_1->z_1} - w_{y_1->z_2} + w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1}) \\
C_1 = (w_{y_2->z_1} - w_{y_2->z_2} + w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1}) \\
C_4 = (w_{y_1->z_1} - w_{y_1->z_2} - b_{z_2} + b_{z_1}) \\
C_5 = (w_{y_2->z_1} - w_{y_2->z_2} - b_{z_2} + b_{z_1}) \\
C_6 = (w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1}) \\
C_7 = (-b_{z_2} + b_{z_1})
\end{gathered}
\tag{64}
$$

substatute constants:

$$
\begin{gathered}
C_0 + \\
C_3 * \exp[B_3] + \\
C_2 * \exp[B_2] + \\
C_1 * \exp[B_1] + \\
C_4 * \exp[B_2 + B_3] + \\
C_5 * \exp[B_1 + B_3] + \\
C_6 * \exp[B_1 + B_2] + \\
C_7 * \exp[B_1 + B_2 + B_3] = 0
\end{gathered}
\tag{65}
$$

Reorder:

$$
\begin{gathered}
C_0 + C_1 * \exp[B_1] + C_2 * \exp[B_2] + C_3 * \exp[B_3] + \\
C_4 * \exp[B_2 + B_3] + C_5 * \exp[B_1 + B_3] + C_6 * \exp[B_1 + B_2] + \\
C_7 * \exp[B_1 + B_2 + B_3] = 0
\end{gathered}
\tag{66}
$$

Observe the Basics of Diff Eq:
Given:

$$\frac{dy}{dx} = ky \tag{67}$$

The genral solution is:

$$y = Ce^{kx} = C * \exp[kx] \tag{68}$$

where C and k are constants (Remmeber that this C different than our C_n)

Let me reorganize our equasion:

$$
\begin{gathered}
C_0 + C_1 e^{B_1} + C_2 e^{B_2} + C_3 e^{B_3} + \\
C_4 e^{B_2 + B_3} + C_5 e^{B_1 + B_3} + C_6 e^{B_1 + B_2} + \\
C_7 e^{B_1 + B_2 + B_3} = 0
\end{gathered}
\tag{69}
$$

Explicitly, the idea going foward is to convert this into a system of differential equasions. Then use numerical meathods to get a "close enough" solution that can be computed in "reasonable" time. Equ. 69 is directly related to the variable B. Whatever method used to aproxomate B can be used again for the components of B. Since B is not the variables we care about, this proccess will be generalizable, and SHOULD scale to "deaper" equasions (This translates to allowing one to solve arbitrarily deap NN). It is also interesting to note that x_1 and x_2 are not dependent variables. They are independent since the value of input neuron 1 is not related to the input neuron 2 value. This impleis that for tasks such as image detection and image processing where 1 pixel is not related to the value of another pixel will be inherintly different than other tasks such as forcast predictions (where sunlight, windspeed, terrane, etc. are all dependent on each other). For our purposes, having x_1 and x_2 be indepenent means that

they can be decoupled allsowing for simpler ODE meathods. Being able to use simplar ODE methods means this will be be scaleable to x_n.

Remmeber B

TLDR: By solving for B instead of directly for x_1, x_2 than there is a method for solving Deep NN's. Having x_1, x_2 be independent variables than we can solve for an arbitrary number of inputs.

$$\begin{aligned} B_1 = -w_{x_1->y_1} * x_1 + w_{x_2->y_1} * x_2 + b_{y_1} \\ B_2 = -w_{x_1->y_2} * x_1 + w_{x_2->y_2} * x_2 + b_{y_2} \\ B_3 = -w_{x_1->y_3} * x_1 + w_{x_2->y_3} * x_2 + b_{y_3} \end{aligned} \tag{70}$$

MESSING AROUND -¿ Sub in B:
Sub in B and expand powers of e:

$$\begin{gathered} C_0 + C_1 e^{-w_{x_1->y_1} * x_1} * e^{w_{x_2->y_1} * x_2} * e^{b_{y_1}} + \\ C_2 e^{-w_{x_1->y_2} * x_1} * e^{w_{x_2->y_2} * x_2} * e^{b_{y_2}} + \\ C_3 e^{-w_{x_1->y_3} * x_1} * e^{w_{x_2->y_3} * x_2} * e^{b_{y_3}} + \\ C_4 e^{-w_{x_1->y_1} * x_1} * e^{w_{x_2->y_1} * x_2} * e^{b_{y_1}} e^{-w_{x_1->y_3} * x_1} * e^{w_{x_2->y_3} * x_2} * e^{b_{y_3}} + \\ C_5 e^{-w_{x_1->y_1} * x_1} * e^{w_{x_2->y_1} * x_2} * e^{b_{y_1}} e^{-w_{x_1->y_2} * x_1} * e^{w_{x_2->y_2} * x_2} * e^{b_{y_2}} + \\ C_6 e^{-w_{x_1->y_2} * x_1} * e^{w_{x_2->y_2} * x_2} * e^{b_{y_2}} e^{-w_{x_1->y_3} * x_1} * e^{w_{x_2->y_3} * x_2} * e^{b_{y_3}} + \\ C_7 e^{-w_{x_1->y_1} * x_1} * e^{w_{x_2->y_1} * x_2} * e^{b_{y_1}} e^{-w_{x_1->y_2} * x_1} * e^{w_{x_2->y_2} * x_2} * e^{b_{y_2}} e^{-w_{x_1->y_3} * x_1} * e^{w_{x_2->y_3} * x_2} * e^{b_{y_3}} \\ = 0 \end{gathered} \tag{71}$$

Define:

$$\begin{gathered} \alpha = e_1^x \\ \beta = e_2^x \end{gathered} \tag{72}$$

sub in α and β

$$
\begin{gathered}
C_0 + C_1\alpha^{-w_{x_1->y_1}} * \beta^{w_{x_2->y_1}} * e^{b_{y_1}} + \\
C_2\alpha^{-w_{x_1->y_2}} * \beta^{w_{x_2->y_2}} * e^{b_{y_2}} + \\
C_3\alpha^{-w_{x_1->y_3}} * \beta^{w_{x_2->y_3}} * e^{b_{y_3}} + \\
C_4\alpha^{-w_{x_1->y_1}} * \beta^{w_{x_2->y_1}} * e^{b_{y_1}} \alpha^{-w_{x_1->y_3}} * \beta^{w_{x_2->y_3}} * e^{b_{y_3}} + \\
C_5\alpha^{-w_{x_1->y_1}} * \beta^{w_{x_2->y_1}} * e^{b_{y_1}} \alpha^{-w_{x_1->y_2}} * \beta^{w_{x_2->y_2}} * e^{b_{y_2}} + \\
C_6\alpha^{-w_{x_1->y_2}} * \beta^{w_{x_2->y_2}} * e^{b_{y_2}} \alpha^{-w_{x_1->y_3}} * \beta^{w_{x_2->y_3}} * e^{b_{y_3}} + \\
C_7\alpha^{-w_{x_1->y_1}} * \beta^{w_{x_2->y_1}} * e^{b_{y_1}} \alpha^{-w_{x_1->y_2}} * \beta^{w_{x_2->y_2}} * e^{b_{y_2}} \alpha^{-w_{x_1->y_3}} * \beta^{w_{x_2->y_3}} * e^{b_{y_3}} \\
= 0 \qquad (73)
\end{gathered}
$$

subtract C_0

$$
\begin{gathered}
C_1\alpha^{-w_{x_1->y_1}} * \beta^{w_{x_2->y_1}} * e^{b_{y_1}} + \\
C_2\alpha^{-w_{x_1->y_2}} * \beta^{w_{x_2->y_2}} * e^{b_{y_2}} + \\
C_3\alpha^{-w_{x_1->y_3}} * \beta^{w_{x_2->y_3}} * e^{b_{y_3}} + \\
C_4\alpha^{-w_{x_1->y_1}} * \beta^{w_{x_2->y_1}} * e^{b_{y_1}} \alpha^{-w_{x_1->y_3}} * \beta^{w_{x_2->y_3}} * e^{b_{y_3}} + \\
C_5\alpha^{-w_{x_1->y_1}} * \beta^{w_{x_2->y_1}} * e^{b_{y_1}} \alpha^{-w_{x_1->y_2}} * \beta^{w_{x_2->y_2}} * e^{b_{y_2}} + \\
C_6\alpha^{-w_{x_1->y_2}} * \beta^{w_{x_2->y_2}} * e^{b_{y_2}} \alpha^{-w_{x_1->y_3}} * \beta^{w_{x_2->y_3}} * e^{b_{y_3}} + \\
C_7\alpha^{-w_{x_1->y_1}} * \beta^{w_{x_2->y_1}} * e^{b_{y_1}} \alpha^{-w_{x_1->y_2}} * \beta^{w_{x_2->y_2}} * e^{b_{y_2}} \alpha^{-w_{x_1->y_3}} * \beta^{w_{x_2->y_3}} * e^{b_{y_3}} \\
= -C_0 \qquad (74)
\end{gathered}
$$

deivide by $-C_0$

$$
\begin{aligned}
&- \frac{C_1}{C_0}\alpha^{-w_{x_1->y_1}} * \beta^{w_{x_2->y_1}} * e^{b_{y_1}} \\
&- \frac{C_2}{C_0}\alpha^{-w_{x_1->y_2}} * \beta^{w_{x_2->y_2}} * e^{b_{y_2}} \\
&- \frac{C_3}{C_0}\alpha^{-w_{x_1->y_3}} * \beta^{w_{x_2->y_3}} * e^{b_{y_3}} \\
&- \frac{C_4}{C_0}\alpha^{-w_{x_1->y_1}} * \beta^{w_{x_2->y_1}} * e^{b_{y_1}}\alpha^{-w_{x_1->y_3}} * \beta^{w_{x_2->y_3}} * e^{b_{y_3}} \\
&- \frac{C_5}{C_0}\alpha^{-w_{x_1->y_1}} * \beta^{w_{x_2->y_1}} * e^{b_{y_1}}\alpha^{-w_{x_1->y_2}} * \beta^{w_{x_2->y_2}} * e^{b_{y_2}} \\
&- \frac{C_6}{C_0}\alpha^{-w_{x_1->y_2}} * \beta^{w_{x_2->y_2}} * e^{b_{y_2}}\alpha^{-w_{x_1->y_3}} * \beta^{w_{x_2->y_3}} * e^{b_{y_3}} \\
&- \frac{C_7}{C_0}\alpha^{-w_{x_1->y_1}} * \beta^{w_{x_2->y_1}} * e^{b_{y_1}}\alpha^{-w_{x_1->y_2}} * \beta^{w_{x_2->y_2}} * e^{b_{y_2}}\alpha^{-w_{x_1->y_3}} * \beta^{w_{x_2->y_3}} * e^{b_{y_3}} \\
&= 1 \qquad (75)
\end{aligned}
$$

3 Conclusion

x^n
$n =$ number of input parameters
$x =$ number of different values n can take
heelow i owuld oinaga;ogina

$(256 * 3)^{1280*720} \approx 10^{2659148}$
Note: There are $\approx 10^{82}$ atoms in the observable universe

Not much of a paper, but it's a start.