

Solve $Z_1 = Z_2$ for x_1

Daniel Fishbein

April 24, 2023

1 Given:

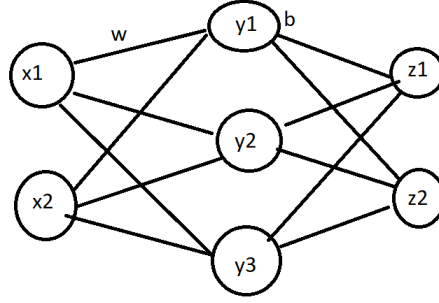


Figure 1: Given Neural Net

Figure 1 shows the given Neural Net that will be analysed.

x_1, x_2 are the input neurons

y_1, y_2, y_3 are the hidden layers neurons

z_1, z_2 are the output neurons

w denotes a weight

$w_{y_1 \rightarrow z_2}$ denotes the weight from y_1 to z_2

b denotes a bias

b_{y_3} denotes the bias associated with neuron y_3

The input to a y_i neuron will be denoted as:

$$y_i = \sigma(w_{x_1 \rightarrow y_i} * x_1 + w_{x_2 \rightarrow y_i} * x_2 + b_{y_i})$$

$$\text{Where } \sigma(x) = \frac{1}{1+e^{-x}} = \frac{1}{1+\exp[-x]}$$

The input to a z_i neuron will be denoted as:

$$z_i = \sigma(w_{y_1 \rightarrow z_i} * y_1 + w_{y_2 \rightarrow z_i} * y_2 + w_{y_3 \rightarrow z_i} * y_3 + b_{z_i})$$

2 Solve $z_1 = z_2$ for x_1

$$z_1 = z_2 \quad (1)$$

$$z_1 = \sigma(w_{y_1 \rightarrow z_1} * y_1 + w_{y_2 \rightarrow z_1} * y_2 + w_{y_3 \rightarrow z_1} * y_3 + b_{z_1}) \quad (2)$$

$$\begin{aligned} z_1 = & \sigma(w_{y_1 \rightarrow z_1} * \sigma(w_{x_1 \rightarrow y_1} * x_1 + w_{x_2 \rightarrow y_1} * x_2 + b_{y_1}) \\ & + w_{y_2 \rightarrow z_1} * \sigma(w_{x_1 \rightarrow y_2} * x_1 + w_{x_2 \rightarrow y_2} * x_2 + b_{y_2}) \\ & + w_{y_3 \rightarrow z_1} * \sigma(w_{x_1 \rightarrow y_3} * x_1 + w_{x_2 \rightarrow y_3} * x_2 + b_{y_3}) \\ & + b_{z_1}) \quad (3) \end{aligned}$$

$$\begin{aligned} z_2 = & \sigma(w_{y_1 \rightarrow z_2} * \sigma(w_{x_1 \rightarrow y_1} * x_1 + w_{x_2 \rightarrow y_1} * x_2 + b_{y_1}) \\ & + w_{y_2 \rightarrow z_2} * \sigma(w_{x_1 \rightarrow y_2} * x_1 + w_{x_2 \rightarrow y_2} * x_2 + b_{y_2}) \\ & + w_{y_3 \rightarrow z_2} * \sigma(w_{x_1 \rightarrow y_3} * x_1 + w_{x_2 \rightarrow y_3} * x_2 + b_{y_3}) \\ & + b_{z_2}) \quad (4) \end{aligned}$$

$$\begin{aligned} & (1 + \exp[-(w_{y_1 \rightarrow z_1} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_1} * x_1 + w_{x_2 \rightarrow y_1} * x_2 + b_{y_1}]} \\ & + w_{y_2 \rightarrow z_1} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_2} * x_1 + w_{x_2 \rightarrow y_2} * x_2 + b_{y_2}]} \\ & + w_{y_3 \rightarrow z_1} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_3} * x_1 + w_{x_2 \rightarrow y_3} * x_2 + b_{y_3}]} \\ & + b_{z_1}))^{-1} \\ & = \\ & (1 + \exp[-(w_{y_1 \rightarrow z_2} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_1} * x_1 + w_{x_2 \rightarrow y_1} * x_2 + b_{y_1}]} \\ & + w_{y_2 \rightarrow z_2} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_2} * x_1 + w_{x_2 \rightarrow y_2} * x_2 + b_{y_2}]} \\ & + w_{y_3 \rightarrow z_2} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_3} * x_1 + w_{x_2 \rightarrow y_3} * x_2 + b_{y_3}]} \\ & + b_{z_2}))^{-1} \quad (5) \end{aligned}$$

NOTE: this equation is too big. Lets scope it down.

Define:

$$B_1 = -w_{x_1->y_1} * x_1 + w_{x_2->y_1} * x_2 + b_{y_1} \quad (6)$$

$$B_2 = -w_{x_1->y_2} * x_1 + w_{x_2->y_2} * x_2 + b_{y_2} \quad (7)$$

$$B_3 = -w_{x_1->y_3} * x_1 + w_{x_2->y_3} * x_2 + b_{y_3} \quad (8)$$

$$(9)$$

NOTE: Substatuting B in.

$$(1 + \exp[-(w_{y_1->z_1} * \frac{1}{1 + \exp[B_1]} \quad (10)$$

$$+ w_{y_2->z_1} * \frac{1}{1 + \exp[B_2]} \quad (11)$$

$$+ w_{y_3->z_1} * \frac{1}{1 + \exp[B_3]} \quad (12)$$

$$+ b_{z_1}))^{-1} \quad (13)$$

$$= \quad (14)$$

$$(1 + \exp[-(w_{y_1->z_2} * \frac{1}{1 + \exp[B_1]} \quad (15)$$

$$+ w_{y_2->z_2} * \frac{1}{1 + \exp[B_2]} \quad (16)$$

$$+ w_{y_3->z_2} * \frac{1}{1 + \exp[B_3]} \quad (17)$$

$$+ b_{z_2}))^{-1} \quad (18)$$

Define A

$$A_1 = w_{y_1->z_1} * \frac{1}{1 + \exp[B_1]} \quad (19)$$

$$+ w_{y_2->z_1} * \frac{1}{1 + \exp[B_2]} \quad (20)$$

$$+ w_{y_3->z_1} * \frac{1}{1 + \exp[B_3]} \quad (21)$$

$$+ b_{z_1} \quad (22)$$

$$A_2 = w_{y_1->z_2} * \frac{1}{1 + \exp[B_1]} \quad (23)$$

$$+ w_{y_2->z_2} * \frac{1}{1 + \exp[B_2]} \quad (24)$$

$$+ w_{y_3->z_2} * \frac{1}{1 + \exp[B_3]} \quad (25)$$

$$+ b_{z_2} \quad (26)$$

NOTE: Substatuting A in.

$$(1 + \exp[-(A_1)])^{-1} = (1 + \exp[-(A_2)])^{-1} \quad (27)$$

$$(1 + \exp[-(A_1)]) = (1 + \exp[-(A_2)]) \quad (28)$$

$$1 + \exp[-(A_1)] = 1 + \exp[-(A_2)] \quad (29)$$

$$\exp[-(A_1)] = \exp[-(A_2)] \quad (30)$$

$$(A_1) = (A_2) \quad (31)$$

$$A_1 = A_2 \quad (32)$$

Sub in the values of A_1 and A_2

$$\begin{aligned} & w_{y_1 \rightarrow z_1} * \frac{1}{1 + \exp[B_1]} + w_{y_2 \rightarrow z_1} * \frac{1}{1 + \exp[B_2]} + w_{y_3 \rightarrow z_1} * \frac{1}{1 + \exp[B_3]} \\ & \quad + b_{z_1} \\ & = \\ & w_{y_1 \rightarrow z_2} * \frac{1}{1 + \exp[B_1]} + w_{y_2 \rightarrow z_2} * \frac{1}{1 + \exp[B_2]} + w_{y_3 \rightarrow z_2} * \frac{1}{1 + \exp[B_3]} \\ & \quad + b_{z_2} \end{aligned} \quad (33)$$

$$\begin{aligned} & \frac{w_{y_1 \rightarrow z_1}}{1 + \exp[B_1]} + \frac{w_{y_2 \rightarrow z_1}}{1 + \exp[B_2]} + \frac{w_{y_3 \rightarrow z_1}}{1 + \exp[B_3]} + b_{z_1} \\ & = \\ & \frac{w_{y_1 \rightarrow z_2}}{1 + \exp[B_1]} + \frac{w_{y_2 \rightarrow z_2}}{1 + \exp[B_2]} + \frac{w_{y_3 \rightarrow z_2}}{1 + \exp[B_3]} + b_{z_2} \end{aligned} \quad (34)$$

Multiply by botoms to create common denominators.

$$\begin{aligned}
& \frac{w_{y_1->z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_2]) * (1 + \exp[B_1]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_3]) * (1 + \exp[B_1]) * (1 + \exp[B_2])} + \\
& \quad b_{z_1} \\
& = \\
& \frac{w_{y_1->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_2]) * (1 + \exp[B_1]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_3]) * (1 + \exp[B_1]) * (1 + \exp[B_2])} + \\
& \quad b_{z_2}
\end{aligned} \tag{35}$$

REORDERER DENOMINATORS

$$\begin{aligned}
& \frac{w_{y_1->z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \quad b_{z_1} \\
& = \\
& \frac{w_{y_1->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \quad b_{z_2}
\end{aligned} \tag{36}$$

MOVE b_{z_1}

$$\begin{aligned}
& \frac{w_{y_1 \rightarrow z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2 \rightarrow z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3 \rightarrow z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} \\
& = \\
& \frac{w_{y_1 \rightarrow z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2 \rightarrow z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3 \rightarrow z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \quad b_{z_2} - b_{z_1}
\end{aligned} \tag{37}$$

MOVE THE BIG PIECE

$$\begin{aligned}
& \frac{w_{y_1 \rightarrow z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2 \rightarrow z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3 \rightarrow z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \\
& \frac{w_{y_1 \rightarrow z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \\
& \frac{w_{y_2 \rightarrow z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \\
& \frac{w_{y_3 \rightarrow z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} \\
& = b_{z_2} - b_{z_1}
\end{aligned} \tag{38}$$

REORDER FOR SIMILAR TERMS

$$\begin{aligned}
& \frac{w_{y_1->z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \frac{w_{y_1->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \frac{w_{y_2->z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \frac{w_{y_3->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} \\
& = b_{z_2} - b_{z_1}
\end{aligned} \tag{39}$$

COMBINE LIKE TERMS

$$\begin{aligned}
& \frac{w_{y_1->z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3]) - w_{y_1->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3]) - w_{y_2->z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2]) - w_{y_3->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} \\
& = b_{z_2} - b_{z_1}
\end{aligned} \tag{40}$$

FACTOR COMMON NUMERATORS

$$\begin{aligned}
& \frac{(w_{y_1->z_1} - w_{y_1->z_2}) * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{(w_{y_2->z_1} - w_{y_2->z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{(w_{y_3->z_1} - w_{y_3->z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} \\
& = b_{z_2} - b_{z_1}
\end{aligned} \tag{41}$$

SANITY CHECK

$$aAB - bAB = (a - b)AB \tag{42}$$

MULTIPLY BY COMMON DENOMINATOR

$$\begin{aligned}
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) * (1 + \exp[B_2]) * (1 + \exp[B_3]) + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_3]) + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_2]) \\
& = (b_{z_2} - b_{z_1}) * ((1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3]))
\end{aligned} \tag{43}$$

The next step is to multiply this out and eliminate terms. I do not want to do that rn :/

GO BACK TO EQ.34

$$\begin{aligned}
& \frac{w_{y_1->z_1}}{1+\exp[B_1]} + \frac{w_{y_2->z_1}}{1+\exp[B_2]} + \frac{w_{y_3->z_1}}{1+\exp[B_3]} + b_{z_1} \\
& = \\
& \frac{w_{y_1->z_2}}{1+\exp[B_1]} + \frac{w_{y_2->z_2}}{1+\exp[B_2]} + \frac{w_{y_3->z_2}}{1+\exp[B_3]} + b_{z_2}
\end{aligned} \tag{44}$$

SUBTRACT b

$$\begin{aligned}
& \frac{w_{y_1->z_1}}{1+\exp[B_1]} + \frac{w_{y_2->z_1}}{1+\exp[B_2]} + \frac{w_{y_3->z_1}}{1+\exp[B_3]} \\
& = \\
& \frac{w_{y_1->z_2}}{1+\exp[B_1]} + \frac{w_{y_2->z_2}}{1+\exp[B_2]} + \frac{w_{y_3->z_2}}{1+\exp[B_3]} + b_{z_2} - b_{z_1}
\end{aligned} \tag{45}$$

SUBTRACT REST OF THE STUFF

$$\begin{aligned}
& \frac{w_{y_1->z_1}}{1+\exp[B_1]} - \frac{w_{y_1->z_2}}{1+\exp[B_1]} + \frac{w_{y_2->z_1}}{1+\exp[B_2]} - \frac{w_{y_2->z_2}}{1+\exp[B_2]} + \frac{w_{y_3->z_1}}{1+\exp[B_3]} - \frac{w_{y_3->z_2}}{1+\exp[B_3]} \\
& = \\
& b_{z_2} - b_{z_1}
\end{aligned} \tag{46}$$

COMBINE LIKE TERMS

$$\begin{aligned}
& \frac{w_{y_1->z_1} - w_{y_1->z_2}}{1+\exp[B_1]} + \frac{w_{y_2->z_1} - w_{y_2->z_2}}{1+\exp[B_2]} + \frac{w_{y_3->z_1} - w_{y_3->z_2}}{1+\exp[B_3]} \\
& = \\
& b_{z_2} - b_{z_1}
\end{aligned} \tag{47}$$

This will got to EQ.43. This is a dead end.

NOTE: continue wiht eq.43
MULTIPLY BY COMMON DENOMINATOR

$$\begin{aligned}
& (w_{y_1->z_1} - w_{y_1->z_2}) * (1 + \exp[B_2]) * (1 + \exp[B_3]) + \\
& (w_{y_2->z_1} - w_{y_2->z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_3]) + \\
& (w_{y_3->z_1} - w_{y_3->z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_2]) \\
& = (b_{z_2} - b_{z_1}) * ((1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3]))
\end{aligned} \tag{48}$$

define:

$$\begin{aligned}
A_1 &= 1 + \exp[B_1] \\
A_2 &= 1 + \exp[B_2] \\
A_3 &= 1 + \exp[B_3]
\end{aligned} \tag{49}$$

substatute in

$$\begin{aligned}
& (w_{y_1->z_1} - w_{y_1->z_2})A_2A_3 + (w_{y_2->z_1} - w_{y_2->z_2})A_1A_3 + (w_{y_3->z_1} - w_{y_3->z_2})A_1A_2 \\
& = (b_{z_2} - b_{z_1})A_1A_2A_3
\end{aligned} \tag{50}$$

simplify $A_1 * A_2$

$$\begin{aligned}
A_1A_2 &= (1 + \exp[B_1])(1 + \exp[B_2]) \\
A_1A_2 &= 1 + \exp[B_2] + \exp[B_1] + \exp[B_1]\exp[B_2] \\
A_1A_2 &= 1 + \exp[B_2] + \exp[B_1] + \exp[B_1 + B_2]
\end{aligned} \tag{51}$$

simplify $A_1 * A_3$

$$\begin{aligned}
A_1 A_3 &= (1 + \exp[B_1])(1 + \exp[B_3]) \\
A_1 A_3 &= 1 + \exp[B_3] + \exp[B_1] + \exp[B_1] \exp[B_3] \\
A_1 A_3 &= 1 + \exp[B_3] + \exp[B_1] + \exp[B_1 + B_3]
\end{aligned} \tag{52}$$

simplify $A_2 * A_3$

$$\begin{aligned}
A_2 A_3 &= (1 + \exp[B_2])(1 + \exp[B_3]) \\
A_2 A_3 &= 1 + \exp[B_3] + \exp[B_2] + \exp[B_2] \exp[B_3] \\
A_2 A_3 &= 1 + \exp[B_3] + \exp[B_2] + \exp[B_2 + B_3]
\end{aligned} \tag{53}$$

simplify $A_1 * A_2 * A_3$

$$\begin{aligned}
A_1 A_2 A_3 &= (1 + \exp[B_2])(1 + \exp[B_3]) \\
A_1 A_2 A_3 &= (1 + \exp[B_1])(1 + \exp[B_3] + \exp[B_2] + \exp[B_2] \exp[B_3]) \\
A_1 A_2 A_3 &= 1 + \exp[B_3] + \exp[B_2] + \exp[B_1] + \\
&\quad \exp[B_2] \exp[B_3] + \exp[B_1] \exp[B_3] + \exp[B_1] \exp[B_2] + \\
&\quad \exp[B_1] \exp[B_2] \exp[B_3] \\
A_1 A_2 A_3 &= 1 + \exp[B_3] + \exp[B_2] + \exp[B_1] + \exp[B_2 + B_3] + \exp[B_1 + B_3] + \exp[B_1 + B_2] + \exp[B_1 + B_2 + B_3]
\end{aligned} \tag{54}$$

substatute into eq.50

$$\begin{aligned}
&(w_{y_1 - > z_1} - w_{y_1 - > z_2})(1 + \exp[B_3] + \exp[B_2] + \exp[B_2 + B_3]) + \\
&(w_{y_2 - > z_1} - w_{y_2 - > z_2})(1 + \exp[B_3] + \exp[B_1] + \exp[B_1 + B_3]) + \\
&(w_{y_3 - > z_1} - w_{y_3 - > z_2})(1 + \exp[B_2] + \exp[B_1] + \exp[B_1 + B_2]) \\
&= (b_{z_2} - b_{z_1})(1 + \exp[B_3] + \exp[B_2] + \exp[B_1] + \exp[B_2 + B_3] + \exp[B_1 + B_3] + \exp[B_1 + B_2] + \exp[B_1 + B_2 + B_3])
\end{aligned} \tag{55}$$

expand each line of eq.55:

$$\begin{aligned}
& w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2 + B_3]
\end{aligned} \tag{56}$$

$$\begin{aligned}
& w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1 + B_3]
\end{aligned} \tag{57}$$

$$\begin{aligned}
& w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1 + B_2]
\end{aligned} \tag{58}$$

$$\begin{aligned}
& b_{z_2} - b_{z_1} + \\
& (b_{z_2} - b_{z_1}) \exp[B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_2] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1] + \\
& (b_{z_2} - b_{z_1}) \exp[B_2 + B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2 + B_3]
\end{aligned} \tag{59}$$

combine everything

$$\begin{aligned}
& w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2 + B_3] + \\
& w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1 + B_3] + \\
& w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1 + B_2] = \\
& b_{z_2} - b_{z_1} + \\
& (b_{z_2} - b_{z_1}) \exp[B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_2] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1] + \\
& (b_{z_2} - b_{z_1}) \exp[B_2 + B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2 + B_3]
\end{aligned} \tag{60}$$

move everything to one side

$$\begin{aligned}
& w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2 + B_3] + \\
& w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1 + B_3] + \\
& w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1 + B_2] - \\
& b_{z_2} + b_{z_1} - \\
& (b_{z_2} - b_{z_1}) \exp[B_3] - \\
& (b_{z_2} - b_{z_1}) \exp[B_2] - \\
& (b_{z_2} - b_{z_1}) \exp[B_1] - \\
& (b_{z_2} - b_{z_1}) \exp[B_2 + B_3] - \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_3] - \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2] - \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2 + B_3] = 0
\end{aligned} \tag{61}$$

Reorder based on exponent

$$\begin{aligned}
& w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1} \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_3] + (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_3] - (b_{z_2} - b_{z_1}) \exp[B_3] \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2] + (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_2] - (b_{z_2} - b_{z_1}) \exp[B_2] \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1] + (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1] - (b_{z_2} - b_{z_1}) \exp[B_1] \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2 + B_3] - (b_{z_2} - b_{z_1}) \exp[B_2 + B_3] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1 + B_3] - (b_{z_2} - b_{z_1}) \exp[B_1 + B_3] \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1 + B_2] - (b_{z_2} - b_{z_1}) \exp[B_1 + B_2] \\
& - (b_{z_2} - b_{z_1}) \exp[B_1 + B_2 + B_3] = 0
\end{aligned} \tag{62}$$

factor

$$\begin{aligned}
& w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1} + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} - b_{z_2} + b_{z_1}) \exp[B_3] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1}) \exp[B_2] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1}) \exp[B_1] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} - b_{z_2} + b_{z_1}) \exp[B_2 + B_3] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} - b_{z_2} + b_{z_1}) \exp[B_1 + B_3] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1}) \exp[B_1 + B_2] + \\
& (-b_{z_2} + b_{z_1}) \exp[B_1 + B_2 + B_3] = 0
\end{aligned} \tag{63}$$

Define Constants

$$\begin{aligned}
C_0 &= w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1} \\
C_3 &= (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} - b_{z_2} + b_{z_1}) \\
C_2 &= (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1}) \\
C_1 &= (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1}) \\
C_4 &= (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} - b_{z_2} + b_{z_1}) \\
C_5 &= (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} - b_{z_2} + b_{z_1}) \\
C_6 &= (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1}) \\
C_7 &= (-b_{z_2} + b_{z_1})
\end{aligned} \tag{64}$$

substatute constants:

$$\begin{aligned}
& C_0 + \\
& C_3 * \exp[B_3] + \\
& C_2 * \exp[B_2] + \\
& C_1 * \exp[B_1] + \\
& C_4 * \exp[B_2 + B_3] + \\
& C_5 * \exp[B_1 + B_3] + \\
& C_6 * \exp[B_1 + B_2] + \\
& C_7 * \exp[B_1 + B_2 + B_3] = 0
\end{aligned} \tag{65}$$

Reorder:

$$\begin{aligned}
& C_0 + C_1 * \exp[B_1] + C_2 * \exp[B_2] + C_3 * \exp[B_3] + \\
& C_4 * \exp[B_2 + B_3] + C_5 * \exp[B_1 + B_3] + C_6 * \exp[B_1 + B_2] + \\
& C_7 * \exp[B_1 + B_2 + B_3] = 0
\end{aligned} \tag{66}$$

Observe the Basics of Diff Eq:

Given:

$$\frac{dy}{dx} = ky \tag{67}$$

The genral solution is:

$$y = Ce^{kx} = C * \exp[kx] \tag{68}$$

where C and k are constants (Remmeber that this C different than our C_n)

Let me reorganize our equasion:

$$\begin{aligned}
& C_0 + C_1 e^{B_1} + C_2 e^{B_2} + C_3 e^{B_3} + \\
& C_4 e^{B_2+B_3} + C_5 e^{B_1+B_3} + C_6 e^{B_1+B_2} + \\
& C_7 e^{B_1+B_2+B_3} = 0
\end{aligned} \tag{69}$$

Explicitly, the idea going foward is to convert this into a system of differential equations. Then use numerical meathods to get a "close enough" solution that can be computed in "reasonable" time. Equ. 69 is directly related to the variable B . Whatever method used to aproxomate B can be used again for the components of B . Since B is not the variables we care about, this proccess will be generalizable, and SHOULD scale to "deaper" equations (This translates to allowing one to solve arbitrarily deap NN). It is also interesting to note that x_1 and x_2 are not dependent variables. They are independent since the value of input neuron 1 is not related to the input neuron 2 value. This impleis that for tasks such as image detection and image processing where 1 pixel is not related to the value of another pixel will be inherently different than other tasks such as forcast predictions (where sunlight, windspeed, terrane, etc. are all dependent on each other). For our purposes, having x_1 and x_2 be indepenent means that

they can be decoupled allsowing for simpler ODE meathods. Being able to use simplar ODE methods means this will be be scaleable to x_n .

text

TLDR: By solving for B instead of directly for x_1, x_2 than there is a method for solving Deep NN's. Having x_1, x_2 be independent variables than we can solve for an arbitrary number of inputs.

Remmeber B

$$\begin{aligned} B_1 &= -w_{x_1->y_1} * x_1 + w_{x_2->y_1} * x_2 + b_{y_1} \\ B_2 &= -w_{x_1->y_2} * x_1 + w_{x_2->y_2} * x_2 + b_{y_2} \\ B_3 &= -w_{x_1->y_3} * x_1 + w_{x_2->y_3} * x_2 + b_{y_3} \end{aligned} \tag{70}$$

3 Conclusion

Not much of a paper, but it's a start.