

Solve  $Z_1 = Z_2$  for  $x_1$

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## 1 Given:

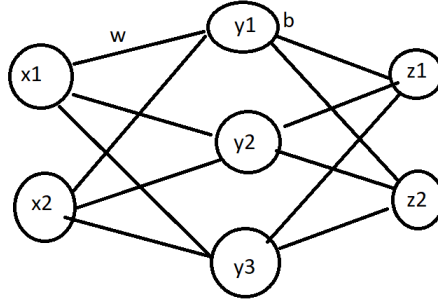


Figure 1: Given Neural Net

Figure 1 shows the given Neural Net that will be analysed.

$x_1, x_2$  are the input neurons

$y_1, y_2, y_3$  are the hidden layers neurons

$z_1, z_2$  are the output neurons

$w$  denotes a weight

$w_{y_1 \rightarrow z_2}$  denotes the weight from  $y_1$  to  $z_2$

$b$  denotes a bias

$b_{y_3}$  denotes the bias associated with neuron  $y_3$

The input to a  $y_i$  neuron will be denoted as:

$$y_i = \sigma(w_{x_1 \rightarrow y_i} * x_1 + w_{x_2 \rightarrow y_i} * x_2 + b_{y_i})$$

$$\text{Where } \sigma(x) = \frac{1}{1+e^{-x}} = \frac{1}{1+\exp[-x]}$$

The input to a  $z_i$  neuron will be denoted as:

$$z_i = \sigma(w_{y_1 \rightarrow z_i} * y_1 + w_{y_2 \rightarrow z_i} * y_2 + w_{y_3 \rightarrow z_i} * y_3 + b_{z_i})$$

## 2 Solve $z_1 = z_2$ for $x_1$

$$z_1 = z_2 \quad (1)$$

$$z_1 = \sigma(w_{y_1 \rightarrow z_1} * y_1 + w_{y_2 \rightarrow z_1} * y_2 + w_{y_3 \rightarrow z_1} * y_3 + b_{z_1}) \quad (2)$$

$$\begin{aligned} z_1 = & \sigma(w_{y_1 \rightarrow z_1} * \sigma(w_{x_1 \rightarrow y_1} * x_1 + w_{x_2 \rightarrow y_1} * x_2 + b_{y_1}) \\ & + w_{y_2 \rightarrow z_1} * \sigma(w_{x_1 \rightarrow y_2} * x_1 + w_{x_2 \rightarrow y_2} * x_2 + b_{y_2}) \\ & + w_{y_3 \rightarrow z_1} * \sigma(w_{x_1 \rightarrow y_3} * x_1 + w_{x_2 \rightarrow y_3} * x_2 + b_{y_3}) \\ & + b_{z_1}) \end{aligned} \quad (3)$$

$$\begin{aligned} z_2 = & \sigma(w_{y_1 \rightarrow z_2} * \sigma(w_{x_1 \rightarrow y_1} * x_1 + w_{x_2 \rightarrow y_1} * x_2 + b_{y_1}) \\ & + w_{y_2 \rightarrow z_2} * \sigma(w_{x_1 \rightarrow y_2} * x_1 + w_{x_2 \rightarrow y_2} * x_2 + b_{y_2}) \\ & + w_{y_3 \rightarrow z_2} * \sigma(w_{x_1 \rightarrow y_3} * x_1 + w_{x_2 \rightarrow y_3} * x_2 + b_{y_3}) \\ & + b_{z_2}) \end{aligned} \quad (4)$$

$$\begin{aligned} & (1 + \exp[-(w_{y_1 \rightarrow z_1} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_1} * x_1 + w_{x_2 \rightarrow y_1} * x_2 + b_{y_1}]} \\ & + w_{y_2 \rightarrow z_1} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_2} * x_1 + w_{x_2 \rightarrow y_2} * x_2 + b_{y_2}]} \\ & + w_{y_3 \rightarrow z_1} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_3} * x_1 + w_{x_2 \rightarrow y_3} * x_2 + b_{y_3}]} \\ & + b_{z_1}))^{-1} \\ & = \\ & (1 + \exp[-(w_{y_1 \rightarrow z_2} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_1} * x_1 + w_{x_2 \rightarrow y_1} * x_2 + b_{y_1}]} \\ & + w_{y_2 \rightarrow z_2} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_2} * x_1 + w_{x_2 \rightarrow y_2} * x_2 + b_{y_2}]} \\ & + w_{y_3 \rightarrow z_2} * \frac{1}{1 + \exp[w_{x_1 \rightarrow y_3} * x_1 + w_{x_2 \rightarrow y_3} * x_2 + b_{y_3}]} \\ & + b_{z_2}))^{-1} \end{aligned} \quad (5)$$

NOTE: this equation is too big. Lets scope it down.

Define:

$$B_1 = -w_{x_1->y_1} * x_1 + w_{x_2->y_1} * x_2 + b_{y_1} \quad (6)$$

$$B_2 = -w_{x_1->y_2} * x_1 + w_{x_2->y_2} * x_2 + b_{y_2} \quad (7)$$

$$B_3 = -w_{x_1->y_3} * x_1 + w_{x_2->y_3} * x_2 + b_{y_3} \quad (8)$$

$$(9)$$

NOTE: Substatuting B in.

$$(1 + \exp[-(w_{y_1->z_1} * \frac{1}{1 + \exp[B_1]} \quad (10)$$

$$+ w_{y_2->z_1} * \frac{1}{1 + \exp[B_2]} \quad (11)$$

$$+ w_{y_3->z_1} * \frac{1}{1 + \exp[B_3]} \quad (12)$$

$$+ b_{z_1}))^{-1} \quad (13)$$

$$= \quad (14)$$

$$(1 + \exp[-(w_{y_1->z_2} * \frac{1}{1 + \exp[B_1]} \quad (15)$$

$$+ w_{y_2->z_2} * \frac{1}{1 + \exp[B_2]} \quad (16)$$

$$+ w_{y_3->z_2} * \frac{1}{1 + \exp[B_3]} \quad (17)$$

$$+ b_{z_2}))^{-1} \quad (18)$$

Define A

$$A_1 = w_{y_1->z_1} * \frac{1}{1 + \exp[B_1]} \quad (19)$$

$$+ w_{y_2->z_1} * \frac{1}{1 + \exp[B_2]} \quad (20)$$

$$+ w_{y_3->z_1} * \frac{1}{1 + \exp[B_3]} \quad (21)$$

$$+ b_{z_1} \quad (22)$$

$$A_2 = w_{y_1->z_2} * \frac{1}{1 + \exp[B_1]} \quad (23)$$

$$+ w_{y_2->z_2} * \frac{1}{1 + \exp[B_2]} \quad (24)$$

$$+ w_{y_3->z_2} * \frac{1}{1 + \exp[B_3]} \quad (25)$$

$$+ b_{z_2} \quad (26)$$

NOTE: Substatuting A in.

$$(1 + \exp[-(A_1)])^{-1} = (1 + \exp[-(A_2)])^{-1} \quad (27)$$

$$(1 + \exp[-(A_1)]) = (1 + \exp[-(A_2)]) \quad (28)$$

$$1 + \exp[-(A_1)] = 1 + \exp[-(A_2)] \quad (29)$$

$$\exp[-(A_1)] = \exp[-(A_2)] \quad (30)$$

$$(A_1) = (A_2) \quad (31)$$

$$A_1 = A_2 \quad (32)$$

Sub in the values of  $A_1$  and  $A_2$

$$\begin{aligned} & w_{y_1 \rightarrow z_1} * \frac{1}{1 + \exp[B_1]} + w_{y_2 \rightarrow z_1} * \frac{1}{1 + \exp[B_2]} + w_{y_3 \rightarrow z_1} * \frac{1}{1 + \exp[B_3]} \\ & \quad + b_{z_1} \\ & = \\ & w_{y_1 \rightarrow z_2} * \frac{1}{1 + \exp[B_1]} + w_{y_2 \rightarrow z_2} * \frac{1}{1 + \exp[B_2]} + w_{y_3 \rightarrow z_2} * \frac{1}{1 + \exp[B_3]} \\ & \quad + b_{z_2} \end{aligned} \quad (33)$$

$$\begin{aligned} & \frac{w_{y_1 \rightarrow z_1}}{1 + \exp[B_1]} + \frac{w_{y_2 \rightarrow z_1}}{1 + \exp[B_2]} + \frac{w_{y_3 \rightarrow z_1}}{1 + \exp[B_3]} + b_{z_1} \\ & = \\ & \frac{w_{y_1 \rightarrow z_2}}{1 + \exp[B_1]} + \frac{w_{y_2 \rightarrow z_2}}{1 + \exp[B_2]} + \frac{w_{y_3 \rightarrow z_2}}{1 + \exp[B_3]} + b_{z_2} \end{aligned} \quad (34)$$

Multiply by botoms to create common denominators.

$$\begin{aligned}
& \frac{w_{y_1->z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_2]) * (1 + \exp[B_1]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_3]) * (1 + \exp[B_1]) * (1 + \exp[B_2])} + \\
& \quad b_{z_1} \\
& = \\
& \frac{w_{y_1->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_2]) * (1 + \exp[B_1]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_3]) * (1 + \exp[B_1]) * (1 + \exp[B_2])} + \\
& \quad b_{z_2}
\end{aligned} \tag{35}$$

REORDERER DENOMINATORS

$$\begin{aligned}
& \frac{w_{y_1->z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \quad b_{z_1} \\
& = \\
& \frac{w_{y_1->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \quad b_{z_2}
\end{aligned} \tag{36}$$

MOVE  $b_{z_1}$

$$\begin{aligned}
& \frac{w_{y_1 \rightarrow z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2 \rightarrow z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3 \rightarrow z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} \\
& = \\
& \frac{w_{y_1 \rightarrow z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2 \rightarrow z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3 \rightarrow z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \quad b_{z_2} - b_{z_1}
\end{aligned} \tag{37}$$

MOVE THE BIG PIECE

$$\begin{aligned}
& \frac{w_{y_1 \rightarrow z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2 \rightarrow z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3 \rightarrow z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \\
& \frac{w_{y_1 \rightarrow z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \\
& \frac{w_{y_2 \rightarrow z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \\
& \frac{w_{y_3 \rightarrow z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} \\
& = b_{z_2} - b_{z_1}
\end{aligned} \tag{38}$$

REORDER FOR SIMILAR TERMS

$$\begin{aligned}
& \frac{w_{y_1->z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \frac{w_{y_1->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \frac{w_{y_2->z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} - \frac{w_{y_3->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} \\
& = b_{z_2} - b_{z_1}
\end{aligned} \tag{39}$$

COMBINE LIKE TERMS

$$\begin{aligned}
& \frac{w_{y_1->z_1} * (1 + \exp[B_2]) * (1 + \exp[B_3]) - w_{y_1->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_2->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_3]) - w_{y_2->z_2} * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{w_{y_3->z_1} * (1 + \exp[B_1]) * (1 + \exp[B_2]) - w_{y_3->z_2} * (1 + \exp[B_2]) * (1 + \exp[B_1])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} \\
& = b_{z_2} - b_{z_1}
\end{aligned} \tag{40}$$

FACTOR COMMON NUMERATORS

$$\begin{aligned}
& \frac{(w_{y_1->z_1} - w_{y_1->z_2}) * (1 + \exp[B_2]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{(w_{y_2->z_1} - w_{y_2->z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_3])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} + \\
& \frac{(w_{y_3->z_1} - w_{y_3->z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_2])}{(1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3])} \\
& = b_{z_2} - b_{z_1}
\end{aligned} \tag{41}$$

SANITY CHECK

$$aAB - bAB = (a - b)AB \tag{42}$$

MULTIPLY BY COMMON DENOMINATOR

$$\begin{aligned}
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) * (1 + \exp[B_2]) * (1 + \exp[B_3]) + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_3]) + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_2]) \\
& = (b_{z_2} - b_{z_1}) * ((1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3]))
\end{aligned} \tag{43}$$

The next step is to multiply this out and eliminate terms. I do not want to do that rn :/



GO BACK TO EQ.34

$$\begin{aligned}
& \frac{w_{y_1->z_1}}{1+\exp[B_1]} + \frac{w_{y_2->z_1}}{1+\exp[B_2]} + \frac{w_{y_3->z_1}}{1+\exp[B_3]} + b_{z_1} \\
& = \\
& \frac{w_{y_1->z_2}}{1+\exp[B_1]} + \frac{w_{y_2->z_2}}{1+\exp[B_2]} + \frac{w_{y_3->z_2}}{1+\exp[B_3]} + b_{z_2}
\end{aligned} \tag{44}$$

SUBTRACT b

$$\begin{aligned}
& \frac{w_{y_1->z_1}}{1+\exp[B_1]} + \frac{w_{y_2->z_1}}{1+\exp[B_2]} + \frac{w_{y_3->z_1}}{1+\exp[B_3]} \\
& = \\
& \frac{w_{y_1->z_2}}{1+\exp[B_1]} + \frac{w_{y_2->z_2}}{1+\exp[B_2]} + \frac{w_{y_3->z_2}}{1+\exp[B_3]} + b_{z_2} - b_{z_1}
\end{aligned} \tag{45}$$

SUBTRACT REST OF THE STUFF

$$\begin{aligned}
& \frac{w_{y_1->z_1}}{1+\exp[B_1]} - \frac{w_{y_1->z_2}}{1+\exp[B_1]} + \frac{w_{y_2->z_1}}{1+\exp[B_2]} - \frac{w_{y_2->z_2}}{1+\exp[B_2]} + \frac{w_{y_3->z_1}}{1+\exp[B_3]} - \frac{w_{y_3->z_2}}{1+\exp[B_3]} \\
& = \\
& b_{z_2} - b_{z_1}
\end{aligned} \tag{46}$$

COMBINE LIKE TERMS

$$\begin{aligned}
& \frac{w_{y_1->z_1} - w_{y_1->z_2}}{1+\exp[B_1]} + \frac{w_{y_2->z_1} - w_{y_2->z_2}}{1+\exp[B_2]} + \frac{w_{y_3->z_1} - w_{y_3->z_2}}{1+\exp[B_3]} \\
& = \\
& b_{z_2} - b_{z_1}
\end{aligned} \tag{47}$$

This will got to EQ.43. This is a dead end.

NOTE: continue wiht eq.43  
MULTIPLY BY COMMON DENOMINATOR

$$\begin{aligned}
& (w_{y_1->z_1} - w_{y_1->z_2}) * (1 + \exp[B_2]) * (1 + \exp[B_3]) + \\
& (w_{y_2->z_1} - w_{y_2->z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_3]) + \\
& (w_{y_3->z_1} - w_{y_3->z_2}) * (1 + \exp[B_1]) * (1 + \exp[B_2]) \\
& = (b_{z_2} - b_{z_1}) * ((1 + \exp[B_1]) * (1 + \exp[B_2]) * (1 + \exp[B_3]))
\end{aligned} \tag{48}$$

define:

$$\begin{aligned}
A_1 &= 1 + \exp[B_1] \\
A_2 &= 1 + \exp[B_2] \\
A_3 &= 1 + \exp[B_3]
\end{aligned} \tag{49}$$

substatute in

$$\begin{aligned}
& (w_{y_1->z_1} - w_{y_1->z_2})A_2A_3 + (w_{y_2->z_1} - w_{y_2->z_2})A_1A_3 + (w_{y_3->z_1} - w_{y_3->z_2})A_1A_2 \\
& = (b_{z_2} - b_{z_1})A_1A_2A_3
\end{aligned} \tag{50}$$

simplify  $A_1 * A_2$

$$\begin{aligned}
A_1A_2 &= (1 + \exp[B_1])(1 + \exp[B_2]) \\
A_1A_2 &= 1 + \exp[B_2] + \exp[B_1] + \exp[B_1]\exp[B_2] \\
A_1A_2 &= 1 + \exp[B_2] + \exp[B_1] + \exp[B_1 + B_2]
\end{aligned} \tag{51}$$

simplify  $A_1 * A_3$

$$\begin{aligned}
A_1 A_3 &= (1 + \exp[B_1])(1 + \exp[B_3]) \\
A_1 A_3 &= 1 + \exp[B_3] + \exp[B_1] + \exp[B_1] \exp[B_3] \\
A_1 A_3 &= 1 + \exp[B_3] + \exp[B_1] + \exp[B_1 + B_3]
\end{aligned} \tag{52}$$

simplify  $A_2 * A_3$

$$\begin{aligned}
A_2 A_3 &= (1 + \exp[B_2])(1 + \exp[B_3]) \\
A_2 A_3 &= 1 + \exp[B_3] + \exp[B_2] + \exp[B_2] \exp[B_3] \\
A_2 A_3 &= 1 + \exp[B_3] + \exp[B_2] + \exp[B_2 + B_3]
\end{aligned} \tag{53}$$

simplify  $A_1 * A_2 * A_3$

$$\begin{aligned}
A_1 A_2 A_3 &= (1 + \exp[B_2])(1 + \exp[B_3]) \\
A_1 A_2 A_3 &= (1 + \exp[B_1])(1 + \exp[B_3] + \exp[B_2] + \exp[B_2] \exp[B_3]) \\
A_1 A_2 A_3 &= 1 + \exp[B_3] + \exp[B_2] + \exp[B_1] + \\
&\quad \exp[B_2] \exp[B_3] + \exp[B_1] \exp[B_3] + \exp[B_1] \exp[B_2] + \\
&\quad \exp[B_1] \exp[B_2] \exp[B_3] \\
A_1 A_2 A_3 &= 1 + \exp[B_3] + \exp[B_2] + \exp[B_1] + \exp[B_2 + B_3] + \exp[B_1 + B_3] + \exp[B_1 + B_2] + \exp[B_1 + B_2 + B_3]
\end{aligned} \tag{54}$$

substatute into eq.50

$$\begin{aligned}
&(w_{y_1 - > z_1} - w_{y_1 - > z_2})(1 + \exp[B_3] + \exp[B_2] + \exp[B_2 + B_3]) + \\
&(w_{y_2 - > z_1} - w_{y_2 - > z_2})(1 + \exp[B_3] + \exp[B_1] + \exp[B_1 + B_3]) + \\
&(w_{y_3 - > z_1} - w_{y_3 - > z_2})(1 + \exp[B_2] + \exp[B_1] + \exp[B_1 + B_2]) \\
&= (b_{z_2} - b_{z_1})(1 + \exp[B_3] + \exp[B_2] + \exp[B_1] + \exp[B_2 + B_3] + \exp[B_1 + B_3] + \exp[B_1 + B_2] + \exp[B_1 + B_2 + B_3])
\end{aligned} \tag{55}$$

expand each line of eq.55:

$$\begin{aligned}
& w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2 + B_3]
\end{aligned} \tag{56}$$

$$\begin{aligned}
& w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1 + B_3]
\end{aligned} \tag{57}$$

$$\begin{aligned}
& w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1 + B_2]
\end{aligned} \tag{58}$$

$$\begin{aligned}
& b_{z_2} - b_{z_1} + \\
& (b_{z_2} - b_{z_1}) \exp[B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_2] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1] + \\
& (b_{z_2} - b_{z_1}) \exp[B_2 + B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2 + B_3]
\end{aligned} \tag{59}$$

combine everything

$$\begin{aligned}
& w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2 + B_3] + \\
& w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1 + B_3] + \\
& w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1 + B_2] = \\
& b_{z_2} - b_{z_1} + \\
& (b_{z_2} - b_{z_1}) \exp[B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_2] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1] + \\
& (b_{z_2} - b_{z_1}) \exp[B_2 + B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_3] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2] + \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2 + B_3]
\end{aligned} \tag{60}$$

move everything to one side

$$\begin{aligned}
& w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2 + B_3] + \\
& w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_3] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1 + B_3] + \\
& w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_2] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1] + \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1 + B_2] - \\
& b_{z_2} + b_{z_1} - \\
& (b_{z_2} - b_{z_1}) \exp[B_3] - \\
& (b_{z_2} - b_{z_1}) \exp[B_2] - \\
& (b_{z_2} - b_{z_1}) \exp[B_1] - \\
& (b_{z_2} - b_{z_1}) \exp[B_2 + B_3] - \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_3] - \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2] - \\
& (b_{z_2} - b_{z_1}) \exp[B_1 + B_2 + B_3] = 0
\end{aligned} \tag{61}$$

Reorder based on exponent

$$\begin{aligned}
& w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2} + w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2} + w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2} - b_{z_2} + b_{z_1} \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_3] + (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_3] - (b_{z_2} - b_{z_1}) \exp[B_3] \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2] + (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_2] - (b_{z_2} - b_{z_1}) \exp[B_2] \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1] + (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1] - (b_{z_2} - b_{z_1}) \exp[B_1] \\
& (w_{y_1 \rightarrow z_1} - w_{y_1 \rightarrow z_2}) \exp[B_2 + B_3] - (b_{z_2} - b_{z_1}) \exp[B_2 + B_3] + \\
& (w_{y_2 \rightarrow z_1} - w_{y_2 \rightarrow z_2}) \exp[B_1 + B_3] - (b_{z_2} - b_{z_1}) \exp[B_1 + B_3] \\
& (w_{y_3 \rightarrow z_1} - w_{y_3 \rightarrow z_2}) \exp[B_1 + B_2] - (b_{z_2} - b_{z_1}) \exp[B_1 + B_2] \\
& - (b_{z_2} - b_{z_1}) \exp[B_1 + B_2 + B_3] = 0
\end{aligned} \tag{62}$$

factor

$$\begin{aligned}
& w_{y_1->z_1} - w_{y_1->z_2} + w_{y_2->z_1} - w_{y_2->z_2} + w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1} + \\
& (w_{y_1->z_1} - w_{y_1->z_2} + w_{y_2->z_1} - w_{y_2->z_2} - b_{z_2} + b_{z_1}) \exp[B_3] + \\
& (w_{y_1->z_1} - w_{y_1->z_2} + w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1}) \exp[B_2] + \\
& (w_{y_2->z_1} - w_{y_2->z_2} + w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1}) \exp[B_1] + \\
& (w_{y_1->z_1} - w_{y_1->z_2} - b_{z_2} + b_{z_1}) \exp[B_2 + B_3] + \\
& (w_{y_2->z_1} - w_{y_2->z_2} - b_{z_2} + b_{z_1}) \exp[B_1 + B_3] + \\
& (w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1}) \exp[B_1 + B_2] + \\
& (-b_{z_2} + b_{z_1}) \exp[B_1 + B_2 + B_3] = 0
\end{aligned} \tag{63}$$

Define Constants

$$\begin{aligned}
C_0 &= w_{y_1->z_1} - w_{y_1->z_2} + w_{y_2->z_1} - w_{y_2->z_2} + w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1} \\
C_3 &= (w_{y_1->z_1} - w_{y_1->z_2} + w_{y_2->z_1} - w_{y_2->z_2} - b_{z_2} + b_{z_1}) \\
C_2 &= (w_{y_1->z_1} - w_{y_1->z_2} + w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1}) \\
C_1 &= (w_{y_2->z_1} - w_{y_2->z_2} + w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1}) \\
C_4 &= (w_{y_1->z_1} - w_{y_1->z_2} - b_{z_2} + b_{z_1}) \\
C_5 &= (w_{y_2->z_1} - w_{y_2->z_2} - b_{z_2} + b_{z_1}) \\
C_6 &= (w_{y_3->z_1} - w_{y_3->z_2} - b_{z_2} + b_{z_1}) \\
C_7 &= (-b_{z_2} + b_{z_1})
\end{aligned} \tag{64}$$

substatute constants:

$$\begin{aligned}
& C_0 + \\
& C_3 * \exp[B_3] + \\
& C_2 * \exp[B_2] + \\
& C_1 * \exp[B_1] + \\
& C_4 * \exp[B_2 + B_3] + \\
& C_5 * \exp[B_1 + B_3] + \\
& C_6 * \exp[B_1 + B_2] + \\
& C_7 * \exp[B_1 + B_2 + B_3] = 0
\end{aligned} \tag{65}$$

Reorder:

$$\begin{aligned}
& C_0 + C_1 * \exp[B_1] + C_2 * \exp[B_2] + C_3 * \exp[B_3] + \\
& C_4 * \exp[B_2 + B_3] + C_5 * \exp[B_1 + B_3] + C_6 * \exp[B_1 + B_2] + \\
& C_7 * \exp[B_1 + B_2 + B_3] = 0
\end{aligned} \tag{66}$$

Observe the Basics of Diff Eq:

Given:

$$\frac{dy}{dx} = ky \tag{67}$$

The genral solution is:

$$y = Ce^{kx} = C * \exp[kx] \tag{68}$$

where  $C$  and  $k$  are constants (Remmeber that this C different than our  $C_n$ )

Let me reorganize our equasion:

$$\begin{aligned}
& C_0 + C_1 e^{B_1} + C_2 e^{B_2} + C_3 e^{B_3} + \\
& C_4 e^{B_2+B_3} + C_5 e^{B_1+B_3} + C_6 e^{B_1+B_2} + \\
& C_7 e^{B_1+B_2+B_3} = 0
\end{aligned} \tag{69}$$

Explicitly, the idea going foward is to convert this into a system of differential equations. Then use numerical meathods to get a "close enough" solution that can be computed in "reasonable" time. Equ. 69 is directly related to the variable  $B$ . Whatever method used to aproximate  $B$  can be used again for the components of  $B$ . Since  $B$  is not the variables we care about, this proccess will be generalizable, and SHOULD scale to "deaper" equations (This translates to allowing one to solve arbitrarily deap NN). It is also interesting to note that  $x_1$  and  $x_2$  are not dependent variables. They are independent since the value of input neuron 1 is not related to the input neuron 2 value. This impleis that for tasks such as image detection and image processing where 1 pixel is not related to the value of another pixel will be inherently different than other tasks such as forcast predictions (where sunlight, windspeed, terrane, etc. are all dependent on each other). For our purposes, having  $x_1$  and  $x_2$  be indepenent means that



they can be decoupled allsowing for simpler ODE meathods. Being able to use simplar ODE methods means this will be be scaleable to  $x_n$ .

Remmeber B

**TLDR:** By solving for B instead of directly for  $x_1, x_2$  than there is a method for solving Deep NN's. Having  $x_1, x_2$  be independent variables than we can solve for an arbitrary number of inputs.

$$\begin{aligned} B_1 &= -w_{x_1 \rightarrow y_1} * x_1 + w_{x_2 \rightarrow y_1} * x_2 + b_{y_1} \\ B_2 &= -w_{x_1 \rightarrow y_2} * x_1 + w_{x_2 \rightarrow y_2} * x_2 + b_{y_2} \\ B_3 &= -w_{x_1 \rightarrow y_3} * x_1 + w_{x_2 \rightarrow y_3} * x_2 + b_{y_3} \end{aligned} \quad (70)$$

MESSING AROUND -i Sub in B:

Sub in B and expand powers of e:

$$\begin{aligned} &C_0 + C_1 e^{-w_{x_1 \rightarrow y_1} * x_1} * e^{w_{x_2 \rightarrow y_1} * x_2} * e^{b_{y_1}} + \\ &C_2 e^{-w_{x_1 \rightarrow y_2} * x_1} * e^{w_{x_2 \rightarrow y_2} * x_2} * e^{b_{y_2}} + \\ &C_3 e^{-w_{x_1 \rightarrow y_3} * x_1} * e^{w_{x_2 \rightarrow y_3} * x_2} * e^{b_{y_3}} + \\ &C_4 e^{-w_{x_1 \rightarrow y_1} * x_1} * e^{w_{x_2 \rightarrow y_1} * x_2} * e^{b_{y_1}} e^{-w_{x_1 \rightarrow y_3} * x_1} * e^{w_{x_2 \rightarrow y_3} * x_2} * e^{b_{y_3}} + \\ &C_5 e^{-w_{x_1 \rightarrow y_1} * x_1} * e^{w_{x_2 \rightarrow y_1} * x_2} * e^{b_{y_1}} e^{-w_{x_1 \rightarrow y_2} * x_1} * e^{w_{x_2 \rightarrow y_2} * x_2} * e^{b_{y_2}} + \\ &C_6 e^{-w_{x_1 \rightarrow y_2} * x_1} * e^{w_{x_2 \rightarrow y_2} * x_2} * e^{b_{y_2}} e^{-w_{x_1 \rightarrow y_3} * x_1} * e^{w_{x_2 \rightarrow y_3} * x_2} * e^{b_{y_3}} + \\ &C_7 e^{-w_{x_1 \rightarrow y_1} * x_1} * e^{w_{x_2 \rightarrow y_1} * x_2} * e^{b_{y_1}} e^{-w_{x_1 \rightarrow y_2} * x_1} * e^{w_{x_2 \rightarrow y_2} * x_2} * e^{b_{y_2}} e^{-w_{x_1 \rightarrow y_3} * x_1} * e^{w_{x_2 \rightarrow y_3} * x_2} * e^{b_{y_3}} \\ &= 0 \quad (71) \end{aligned}$$

Define:

$$\begin{aligned} \alpha &= e_1^x \\ \beta &= e_2^x \quad (72) \end{aligned}$$

sub in  $\alpha$  and  $\beta$

$$\begin{aligned}
& C_0 + C_1 \alpha^{-w_{x_1} \rightarrow y_1} * \beta^{w_{x_2} \rightarrow y_1} * e^{b_{y_1}} + \\
& C_2 \alpha^{-w_{x_1} \rightarrow y_2} * \beta^{w_{x_2} \rightarrow y_2} * e^{b_{y_2}} + \\
& C_3 \alpha^{-w_{x_1} \rightarrow y_3} * \beta^{w_{x_2} \rightarrow y_3} * e^{b_{y_3}} + \\
& C_4 \alpha^{-w_{x_1} \rightarrow y_1} * \beta^{w_{x_2} \rightarrow y_1} * e^{b_{y_1}} \alpha^{-w_{x_1} \rightarrow y_3} * \beta^{w_{x_2} \rightarrow y_3} * e^{b_{y_3}} + \\
& C_5 \alpha^{-w_{x_1} \rightarrow y_1} * \beta^{w_{x_2} \rightarrow y_1} * e^{b_{y_1}} \alpha^{-w_{x_1} \rightarrow y_2} * \beta^{w_{x_2} \rightarrow y_2} * e^{b_{y_2}} + \\
& C_6 \alpha^{-w_{x_1} \rightarrow y_2} * \beta^{w_{x_2} \rightarrow y_2} * e^{b_{y_2}} \alpha^{-w_{x_1} \rightarrow y_3} * \beta^{w_{x_2} \rightarrow y_3} * e^{b_{y_3}} + \\
& C_7 \alpha^{-w_{x_1} \rightarrow y_1} * \beta^{w_{x_2} \rightarrow y_1} * e^{b_{y_1}} \alpha^{-w_{x_1} \rightarrow y_2} * \beta^{w_{x_2} \rightarrow y_2} * e^{b_{y_2}} \alpha^{-w_{x_1} \rightarrow y_3} * \beta^{w_{x_2} \rightarrow y_3} * e^{b_{y_3}} \\
& \hspace{15em} = 0 \quad (73)
\end{aligned}$$

subtract  $C_0$

$$\begin{aligned}
& C_1 \alpha^{-w_{x_1} \rightarrow y_1} * \beta^{w_{x_2} \rightarrow y_1} * e^{b_{y_1}} + \\
& C_2 \alpha^{-w_{x_1} \rightarrow y_2} * \beta^{w_{x_2} \rightarrow y_2} * e^{b_{y_2}} + \\
& C_3 \alpha^{-w_{x_1} \rightarrow y_3} * \beta^{w_{x_2} \rightarrow y_3} * e^{b_{y_3}} + \\
& C_4 \alpha^{-w_{x_1} \rightarrow y_1} * \beta^{w_{x_2} \rightarrow y_1} * e^{b_{y_1}} \alpha^{-w_{x_1} \rightarrow y_3} * \beta^{w_{x_2} \rightarrow y_3} * e^{b_{y_3}} + \\
& C_5 \alpha^{-w_{x_1} \rightarrow y_1} * \beta^{w_{x_2} \rightarrow y_1} * e^{b_{y_1}} \alpha^{-w_{x_1} \rightarrow y_2} * \beta^{w_{x_2} \rightarrow y_2} * e^{b_{y_2}} + \\
& C_6 \alpha^{-w_{x_1} \rightarrow y_2} * \beta^{w_{x_2} \rightarrow y_2} * e^{b_{y_2}} \alpha^{-w_{x_1} \rightarrow y_3} * \beta^{w_{x_2} \rightarrow y_3} * e^{b_{y_3}} + \\
& C_7 \alpha^{-w_{x_1} \rightarrow y_1} * \beta^{w_{x_2} \rightarrow y_1} * e^{b_{y_1}} \alpha^{-w_{x_1} \rightarrow y_2} * \beta^{w_{x_2} \rightarrow y_2} * e^{b_{y_2}} \alpha^{-w_{x_1} \rightarrow y_3} * \beta^{w_{x_2} \rightarrow y_3} * e^{b_{y_3}} \\
& \hspace{15em} = -C_0 \quad (74)
\end{aligned}$$

divide by  $-C_0$

$$\begin{aligned}
& -\frac{C_1}{C_0}\alpha^{-w_{x_1->y_1}}*\beta^{w_{x_2->y_1}}*e^{b_{y_1}} \\
& -\frac{C_2}{C_0}\alpha^{-w_{x_1->y_2}}*\beta^{w_{x_2->y_2}}*e^{b_{y_2}} \\
& -\frac{C_3}{C_0}\alpha^{-w_{x_1->y_3}}*\beta^{w_{x_2->y_3}}*e^{b_{y_3}} \\
& -\frac{C_4}{C_0}\alpha^{-w_{x_1->y_1}}*\beta^{w_{x_2->y_1}}*e^{b_{y_1}}\alpha^{-w_{x_1->y_3}}*\beta^{w_{x_2->y_3}}*e^{b_{y_3}} \\
& -\frac{C_5}{C_0}\alpha^{-w_{x_1->y_1}}*\beta^{w_{x_2->y_1}}*e^{b_{y_1}}\alpha^{-w_{x_1->y_2}}*\beta^{w_{x_2->y_2}}*e^{b_{y_2}} \\
& -\frac{C_6}{C_0}\alpha^{-w_{x_1->y_2}}*\beta^{w_{x_2->y_2}}*e^{b_{y_2}}\alpha^{-w_{x_1->y_3}}*\beta^{w_{x_2->y_3}}*e^{b_{y_3}} \\
& -\frac{C_7}{C_0}\alpha^{-w_{x_1->y_1}}*\beta^{w_{x_2->y_1}}*e^{b_{y_1}}\alpha^{-w_{x_1->y_2}}*\beta^{w_{x_2->y_2}}*e^{b_{y_2}}\alpha^{-w_{x_1->y_3}}*\beta^{w_{x_2->y_3}}*e^{b_{y_3}} \\
& = 1 \quad (75)
\end{aligned}$$

### 3 Conclusion

$x^n$

$n$  = number of input parameters

$x$  = number of different values  $n$  can take

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$$S1(x) = \begin{cases} 'white' & x_1 + x_2 < 255 - n \\ 'Black' & x_1 + x_2 > 255 + n \\ 'Grey' & otherwise \end{cases}$$

$$(256 * 3)^{1280 * 720} \approx 10^{2659148}$$

Note: There are  $\approx 10^{82}$  atoms in the observable universe

Not much of a paper, but it's a start.