YONSEI UNIVERSITY
Department of Computer Science

CSI3109 Automata and Formal Languages, SPRING 2017

Homework No.2

DUE: 2017.04.17 13:00:00pm

Student ID: 20xxxxxx

Student Name: Hong, Gil-Dong

1. Given a DFA $A = (Q, \Sigma, \delta, s, F)$, where

   $$Q = \{1, 2, 3, 4, 5, 6\}, \Sigma = \{a, b\}, s = 1, F = \{2, 5, 6\}$$

   and δ is defined as follows:

	a	b
1	2	3
2	2	4
3	4	5
4	2	6
5	5	4
6	6	6

   Run the TF (Table Filling) algorithm and draw a minimal DFA for $L(A)$.

2. Prove that the following languages are not regular. You may use the pumping lemma or the closure properties of regular languages under union, intersection and complement.
   a) $L = \{0^n 1^m 0^n \mid m, n \geq 0\}$.

b) $L = \{w \mid w \in \{0,1\}^*$ is not a palindrome$\}$.
(A palindrome is a string that reads the same forward and backward. e.g.: racecar)

3. Show that the regular languages are closed under the following operations:

   a)

$$\mathbb{DROPOUT}(L) = \{xz \mid xyz \in L, \text{ where } x, z \in \Sigma^*, y \in \Sigma\}.$$

   Namely, $\mathbb{DROPOUT}(L)$ is the language containing all strings that can be obtained by removing one symbol from a string in L.
   For example, if $L = \{012\}$, then $\mathbb{DROPOUT}(L) = \{12, 02, 01\}$.

b)

$$\mathbb{INIT}(L) = \{w \mid w \text{ for some } x, wx \in L\}.$$

For example, if $L = \{01, 110\}$, then $\mathbb{INIT}(L) = \{0, 01, 1, 11, 110\}$.
(*HINT:* Start with a DFA A for L and describe how to construct an FA for $\mathbb{INIT}(L)$ using A. We assume that A has no sink states.)

4. Given two NFAs $A_1 = (Q_1, \Sigma, \delta_1, s_1, F_1)$ and $A_2 = (Q_2, \Sigma, \delta_2, s_2, F_2)$, suggest an NFA construction for $L(A_1) \cap L(A_2)$ and justify the construction (in other words, prove the correctness of your construction.)

5. Consider the following two languages:
   - $L_1 = \{w \mid w$ has the same number of a's and b's$\}$.
   - $L_2 = \{w \mid w$ has the same number of the substrings ab and $ba.\}$.

   a) Is L_1 regular? Justify your answer—If L_1 is regular, show an regular expression or an FA. If not, prove it.

b) Is L_2 regular? Justify your answer.