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# Data-driven attitude control law of a variable-pitch quadrotor: a comparison study

Davide Invernizzi \* Pietro Panizza \* Fabio Riccardi \*  
 Simone Formentin \*\* Marco Lovera \*

\* Dipartimento di Scienze e Tecnologie Aerospaziali, Politecnico di Milano,  
 Via La Masa 34, 20156 Milano, Italy  
 (e-mail: {davide.invernizzi, pietro.panizza, fabio.riccardi,  
 marco.lovera}@polimi.it)

\*\* Dipartimento di Elettronica, Informazione e Bioingegneria, Politecnico di  
 Milano, Piazza Leonardo da Vinci 32, 20133 Milano, Italy  
 (e-mail: simone.formentin@polimi.it)

**Abstract:** In this paper, the problem of tuning a cascade attitude control system of a variable-pitch quadrotor UAV is tackled, comparing two non-iterative data-driven approaches. The first method is the Virtual Reference Feedback Tuning (VRFT) while the second one is the Correlation based Tuning (CbT), both modified in order to tune both the inner and the outer loops by means of a single set of experimental data. These methods allow a fast tuning of controller parameters directly from data, without relying on an accurate knowledge of the plant dynamics. The experimental tests are performed on a variable-pitch quadrotor which operates indoor on a dedicated test bench that allows only the pitch attitude degree of freedom of the vehicle.

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## 1. INTRODUCTION

In recent years, small-scale Unmanned Aerial Vehicles (UAVs), and in particular the quadrotor platform, have been the focus of several research efforts. The quadrotor high payload capability combined with its great maneuverability in different flight conditions have also attracted the commercial interest for several applications. Depending on the specific application, requirements in terms of pointing and positioning accuracy demand a careful modelling and an appropriate choice of the control law design tools. While nonlinear modeling and control design approach have been considered in the literature (see, e.g., Mahony et al. (2012) for a recent survey), for civil applications such as surveillance, mapping, video and photography a linear approach is usually adopted. In these settings, for which hover and near hover operations are representative conditions, PID laws are usually employed for attitude control thanks to their inherent reliability and ease of implementation. Broadly speaking, the tuning of the PID parameters may be performed according to a manual tuning, when fast deployment has the priority over performance, or to advanced model-based methods (see, for example, Riccardi and Lovera (2014)), which should be applied when high performance is required. The latter approach may be specifically advantageous when considering variable-pitch quadrotors whose inherent wider bandwidths allows to reach a much tighter attitude regulation (see, e.g., Cutler et al. (2011); Riccardi et al. (2013)). The above mentioned model-based methods suffer from the fact that the mathematical modelling of quadrotors is particularly challenging due to the non trivial characterization of the aerodynamic effects and of the actuators and sensors dynamics (see Riccardi et al. (2014)). For this reason data-driven tuning methods have been developed in the last two decades. These control design tools are especially appealing when *a priori* knowledge about the plant model is limited, when an accurate modeling of the system is too expensive or when fast deployment of the control system is an impor-

tant requirement, since they allow the direct tuning of the controller parameters from experimental input-output data. Among the different data-driven methods available in the reference literature, a coarse classification can be made between iterative (e.g., the Iterative Feedback Tuning (IFT) Hjalmarsson et al. (1998)) and single-shot (non-iterative) methods (e.g., the Virtual Reference Feedback Tuning (VRFT) Campi et al. (2002), the Correlation-Based Tuning (CbT) Van Heusden et al. (2011); Formentin et al. (2014b)). Non-iterative methods are particularly attractive for a fast re-tuning of the controller when the plant performance is reduced (e.g., components aging) and/or operating conditions change (e.g., different payloads, environment). Recently (Panizza et al. (2016)) the VRFT algorithm has been employed to tune the controller parameters of a variable-pitch quadrotor exploiting experiments conducted indoor on a dedicated test-bed. The results have shown improvements in the tracking and load-rejection capabilities compared to those obtained with a manual tuning. Furthermore, comparable results with respect to a model-based structured  $H_\infty$  synthesis (Riccardi and Lovera (2014)), made data-driven methods a promising tool for this kind of applications. Nonetheless, a keypoint was raised: special care is required when a low signal-to-noise ratio (SNR) is unavoidable. In this case the VRFT algorithm employs an instrumental variables approach to counteract the effect of noise, which is constructed by means of repeated experiments or through an identification procedure. Repeated experiments are statistically inefficient, while the estimation of the plant, although it makes the procedure single-shot, is a difficult task due to a limited knowledge of the plant. Furthermore, an incorrect selection of the model order can lead to unsatisfactory performance. In the present work, the attitude controller tuning of the same variable-pitch quadrotor has been carried out to compare the VRFT and the CbT approach, both in terms of performance and robustness. The latter algorithm has been adapted to allow the direct tuning of a cascade controller configuration with a

single set of input-output data, following the procedure outlined for the VRFT (see Formentin et al. (2011a)). While *a-priori* knowledge of the main platform dynamics is still required as for the VRFT, the correlation-based approach is expected to deal better with a low SNR.

The paper is organized as follows. In Section 2 the considered quadrotor platform and its controller architecture are introduced in detail. The data-driven framework is presented in Section 3; subsequently, in Section 3.1 a short introduction to the standard VRFT approach is given, while the CbT method and its extension to the cascade control architecture are outlined in section 3.2. Finally, results and performance comparison are presented in Section 4. The last Section contains some concluding remarks about the proposed methodologies and their applicability.

## 2. QUADROTOR PLATFORM

The quadrotor studied in this paper has rotors operating at a fixed angular rate and uses variable collective pitch as control variables, unlike most quadrotors, which use variable rotor angular rates as control inputs with fixed rotors blade pitch. In this work the pitch attitude controller of the Aermatica P2-A1 prototype is tuned (see Figure 1): a platform having a maximum take-off weight of about 5 kg and an arm length of 0.415 m. The four rotors have a radius of 0.27 m and a teetering articulation with flapping motion partially restrained by rubber elastic elements.

The quadrotor is placed on a test-bed that constrains all translational and rotational degree of freedom except for pitch rotation, as shown in Figure 1. All the experiments considered in this work have been conducted exploiting this laboratory set-up since it is representative of the pitch attitude dynamics in flight near hovering condition, as discussed in Riccardi (2015). This indoor setup provokes a recirculation of rotor wakes, as the tests occur in a closed volume with limited dimensions. While this represents a discrepancy with respect to outdoor flight, where the rotor-induced wakes develop free from obstacles, results in Riccardi (2015) show that for parameter estimation purposes the test bed is representative of actual attitude dynamics in flight.



Fig. 1. Aermatica P2-A1 on laboratory test-bed.

Concerning the control architecture, the P2-A1 platform adopts a classical attitude control scheme based on decoupled cascaded PID loops for the pitch, roll and yaw axes (see the block diagram in Figure 2, where the pitch control loop is represented).

More precisely, an outer PD loop based on attitude feedback (measured angle  $\theta$ , set-point  $\theta^o$ ) and an inner PID loop on angular rate feedback (measured angular velocity  $q$ , set-point  $q^o$ , control variable  $u$ ). The overall delay of the control loop, from IMU measurements, through acquisition and processing, to servo actuation of blade collective pitch, is estimated to be 0.06 s.

## 3. DATA-DRIVEN CONTROL LAW DESIGN

Consider a linear time-invariant discrete-time system  $G(z)$ , where  $z$  denotes the forward time-shift unit (*i.e.*,  $zx(t) = x(t+1)$ ), a class of controllers  $\mathcal{C}(\theta) = \{C(z, \theta), \theta \in R^n\}$ , and a given target closed-loop behaviour  $M(z)$ . The control aim of the data-driven methods is the minimization of the  $\mathcal{L}_2$ -norm of the mismatch between  $M$  and the actual closed-loop system:

$$J_{MR}(\theta) = \left\| \left( \frac{G(z)C(z, \theta)}{1 + G(z)C(z, \theta)} - M(z) \right) W(z) \right\|_2^2 \quad (1)$$

where  $W(z)$  is a weighting function chosen by the user. The main features of the data-driven approaches are that the model-reference problem (1) is solved with limited knowledge of the system (4.2) and using only a set of available open-loop measurements  $D_N = \{u(t), y(t)\}_{t=1..N}$ , where  $N$  is the length of the data-set.

In the cascade control framework, it has been shown in Formentin et al. (2011a) that the VRFT rationale can be extended to multiple nested loops, by still relying on a single experiment. The idea is exploited herein to modify also the CbT algorithm to deal with multiple nested loop control architecture.

Consider the cascade control scheme in Figure 3 (where only two loops are shown without loss of generality). Given two reference models  $M_i(z)$  and  $M_o(z)$ , for the inner loop and the outer loop respectively, consider two families of linear proper controllers  $\mathcal{C}_i(\theta_i) = \{C_i(z, \theta_i), \theta_i \in R_i^n\}$  and  $\mathcal{C}_o(\theta_o) = \{C_o(z, \theta_o), \theta_o \in R_o^n\}$  and the set of data  $D_N = \{u(t), y_i(t), y_o(t)\}_{t=1..N}$  being  $u(t)$  the control variable,  $y_i(t)$  the output of the inner loop,  $y_o(t)$  the output of the outer loop. The inner controller can be tuned by applying the standard VRFT or the standard CbT since all signals are available. For the outer controller, on the other hand, the approach needs to be different, as the input of the system to control is the reference  $r_i(t)$  (see again Figure 3), that is not available in the dataset, since measurements are collected during open-loop operation.

Nevertheless, in Formentin et al. (2011a) it has been shown that the reference signal  $r_i(t)$  can be derived from the available data by exploiting the fact that the inner controller is designed independently of the outer one. In detail, once  $C_i(z, \theta_i)$  is fixed, the input of the inner loop can be calculated as

$$r_i(t) = e_i(t) + y_i(t), \quad (2)$$

where the tracking error comes from the result of the inner design as

$$e_i(t) = C_i^{-1}(z, \theta_i)u(t).$$

With such a choice,  $r_i(t)$  is exactly the signal that would feed the inner loop in closed-loop working conditions when the output is  $y_i(t)$ . Then, the outer controller can be easily found by using the set of I/O data  $D_N^o = \{r_i(t), y_o(t)\}_{t=1..N}$ . It is evident that this procedure is not feasible whenever the inner controller is non-minimum phase, since it would produce a non-stationary reference signal for the outer-loop design. In the next

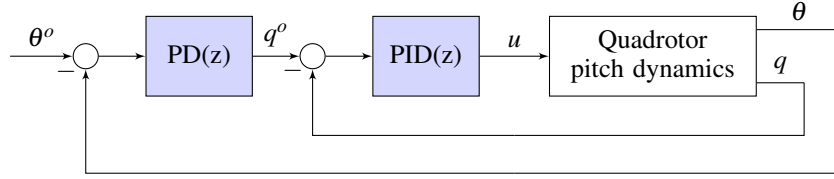


Fig. 2. Aermatica P2-A1 pitch attitude controller structure.

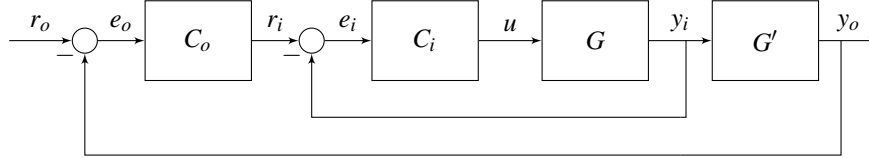


Fig. 3. Cascade control scheme with two nested loops.

two sections the VRFT and the CbT methods are presented in details.

### 3.1 VRFT tuning of cascade control system

The main idea of VRFT tuning can be described as follows. Consider the reference signal  $r(t)$  that would feed the system in closed-loop operation when the closed-loop model is  $M(z)$  and the output is the measured  $y(t)$ . Such a signal is called *virtual reference* and can be computed from the output data (offline) as

$$r(t) = M^{-1}(z)y(t).$$

A good controller (making the closed-loop as close as possible to  $M(z)$ ) is then the one that produces the input sequence of the experiment  $u(t)$  when it is fed by the error signal  $e(t) = r(t) - y(t)$ .

Formally, the cost criterion minimized by the VRFT algorithm is the following:

$$J_{VR}^N(\theta) = \frac{1}{N} \sum_{t=1}^N (u_L(t) - C(z, \theta)e_L(t))^2, \quad (3)$$

where  $u_L(t)$  and  $e_L(t)$  are suitably filtered versions of  $u(t)$  and  $e(t)$ , such that the cost function (3) is a local approximation of the criterion (1) in the neighborhood of the minimum point (Campi et al. (2002)). Recent advances on the VRFT method can be found, e.g., in Campestrini et al. (2009); Formentin et al. (2011b); Formentin and Karimi (2014), while application studies are available, e.g., in Previdi et al. (2004); Formentin et al. (2014a).

Considering the inner loop,  $\theta_i$  can be obtained with the standard VRFT minimizing

$$J_{VR}^N(\theta_i) = \frac{1}{N} \sum_{t=1}^N (u_L(t) - C_i(z, \theta_i)e_{iL}(t))^2,$$

where  $e_{iL}(t)$  is a suitably filtered version of  $e_i(t) = r_i(t) - y_i(t)$ . As declared in the previous section, the input of the system to control, when considering the outer controller, has to be computed with (2). More specifically,  $\theta_o$  comes as the minimizer of

$$J_{VR}^N(\theta_o) = \frac{1}{N} \sum_{t=1}^N (r_{iL}(t) - C_o(z, \theta_o)e_{oL}(t))^2 \quad (4)$$

where  $r_{iL}(t)$  and  $e_{oL}(t)$  are suitably filtered versions of  $r_i(t)$  and  $e_o(t)$ , the latter being the virtual error of the outer loop:

$$e_{oV}(t) = (M_o^{-1}(z) - 1)y_o(t).$$

The optimal filters for the inner and outer loop are discussed in Formentin et al. (2011a), following the rationale of Campi et al. (2002).

To counteract the effect of noise, an instrumental variable method is implemented in this work as discussed in Campi et al. (2002). The instrumental variable is constructed through the identification of simple ARX( $p, p$ ) models for the inner and the outer loops. It is important to note that the estimated plant is used only to generate the instrumental variable, thus a large value of  $p$  can be employed as suggested in Campi et al. (2002).

### 3.2 CbT tuning of cascade control system

In the CbT approach, the optimal controller is computed exploiting the error  $\varepsilon(t, \theta)$  as depicted in Figure 4, that depends on the exogenous signals  $r(t)$  and  $v(t)$ :

$$\begin{aligned} \varepsilon(t, \theta) &= M(z)r(t) - C(z, \theta)(1 - M(z))y(t) \\ &= (M(z) - C(z, \theta)(1 - M(z))G(z))r(t) \\ &\quad - C(z, \theta)(1 - M(z))v(t). \end{aligned}$$

Prediction-error methods are not consistent for this identification problem since the input is affected by noise but it can be solved using the correlation approach. The goal is to find the optimal controller parameter  $\theta$  such that the error  $\varepsilon(t, \theta)$  is uncorrelated with  $r(t)$ . To decorrelate  $\varepsilon(t, \theta)$  and  $r(t)$ , an extended instrumental variable  $\zeta(t)$  correlated with  $r(t)$  is introduced:

$$\zeta(t) = [r_F(t+L) \ \dots \ r_F(t) \ \dots \ r_F(t-L)]^T$$

where  $L$  is a sufficiently large integer and  $r_F(t)$  is suitably filtered versions of  $r(t)$  (see Van Heusden et al. (2011) for discussion on the optimal filter). The correlation function is defined as

$$f_{N,L}(\theta) = \frac{1}{N} \sum_{t=1}^N \zeta(t)\varepsilon(t, \theta)$$

and the correlation criterion to minimize is

$$J_{N,L}(\theta) = f_{N,L}^T(\theta)f_{N,L}(\theta). \quad (5)$$

The optimal parameters for (5) asymptotically converge to the optimizer of (1) (the proof is provided in Van Heusden et al. (2011)). For a cascade control architecture, the inner loop is tuned with  $y(t) = y_i(t)$  and  $r(t) = u(t)$ , whereas the outer loop considers  $y(t) = y_o(t)$  and  $r(t) = r_i(t)$  computed as in (2).



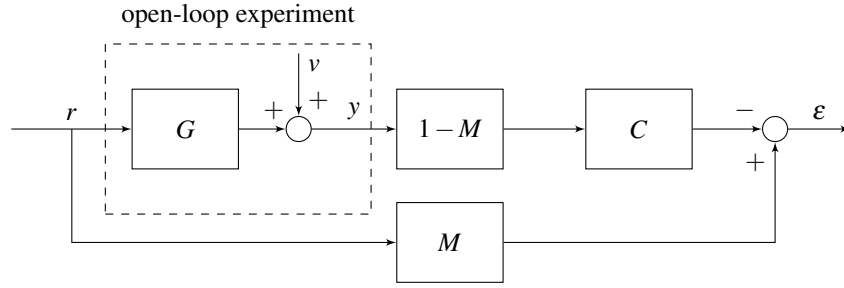


Fig. 4. Tuning scheme for Correlation-based Tuning.

#### 4. EXPERIMENTAL RESULTS

##### 4.1 Tuning experiments

The data collection experiments have been carried out indoor, operating the quadrotor on the test-bed shown in Figure 1, which fixes all but the pitch degree of freedom. A PRBS (Pseudo Random Binary Sequence) signal was applied in quasi open-loop conditions: while keeping off the nominal attitude and position controllers, the supervision task enforcing attitude limits was active throughout the collection campaign (maximum attitude excursion guaranteed from the adopted test-bed is  $\pm 20^\circ$ ). The parameters of the PRBS sequence (amplitude and min/max switching interval) were chosen as to obtain an excitation spectrum large enough to influence the dominant attitude dynamics. The on-board IMU recorded the pitch angular velocity and the pitch angle, which were logged with sampling a time equal to 0.02 s (see Figure 5). The control variable is expressed as the collective command difference (in %) among the front and back rotor.

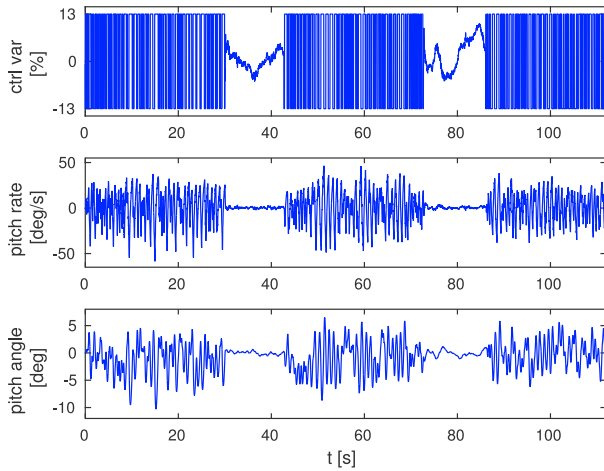


Fig. 5. Open-loop experimental dataset used for controller tuning.

##### 4.2 Controller design

The input-output data collected in the experiment presented above have been used to tune the cascade pitch attitude control loop with the two data-driven methods. As mentioned in Section 3, the user is left in charge of the reference model selection, whose design procedure requires some prior knowledge about the system dynamics. In particular the choice of  $M_i(z)$  and  $M_o(z)$  is feasible when preliminary information is available,

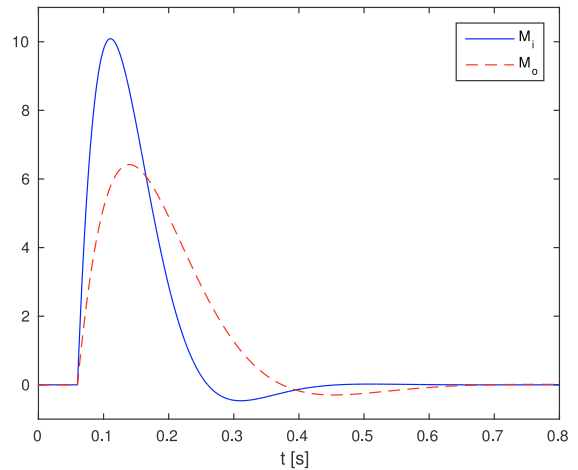
such as, *e.g.*, the achievable closed-loop bandwidth, dominant dynamics, presence of time-delays. Satisfactory results cannot be achieved when an unattainable closed-loop reference model is selected, which is not unlike an erroneous structure selection in model identification problems. As already underlined in Panizza et al. (2016), the preliminary information needed to apply model reference data-driven approaches is significantly smaller than the one required by a model-based design and may be retrieved from the plant manufacturer or can be obtained with simple open-loop or closed-loop tests. In particular, the reference models  $M_i(z)$  and  $M_o(z)$  for, respectively, the inner and the outer control loop, have been defined on the basis of available requirements for the desired bandwidth and damping factor of the inner and outer complementary sensitivity functions (see Riccardi et al. (2014)). The reference models of the two control loops have been defined as second order systems with a damping ratio of 0.7, a time delay of 3 samples and a desired bandwidth of 24 rad/s (inner) and 16 rad/s (outer):

$$M_i(z) = \frac{0.09151z + 0.07308}{z^2 - 1.346z + 0.5107} \frac{1}{z^3}$$

$$M_o(z) = \frac{0.04397z + 0.03786}{z^2 - 1.557z + 0.6389} \frac{1}{z^3}.$$

For simplicity the weighting functions defined in Section 3 are  $W_i(z) = 1$  and  $W_o(z) = 1$ .

In order to apply the CbT approach, the parameter  $l$  has to be selected. As explained in Section 4.5, this parameter should be close to the length of the impulse response of  $M(z)$  (Figure 6) ( $l_i = 20, l_o = 35$ ).

Fig. 6. Impulse response of  $M_i$  and  $M_o$ .

For what concerns the VRFT approach, the order of the ARX model, which provided satisfactory results, is  $p = 15$ , both



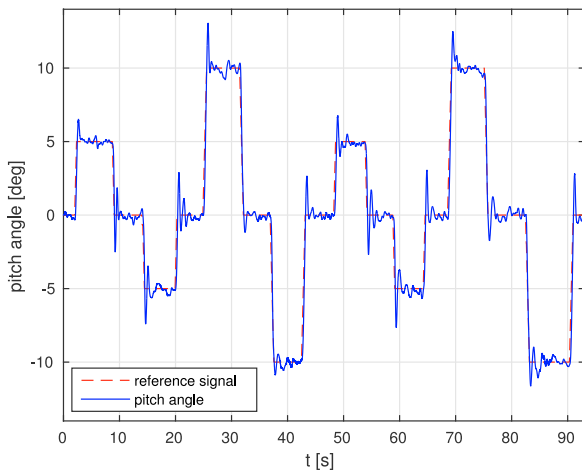


Fig. 8. Set-point tracking with VRFT tuning.

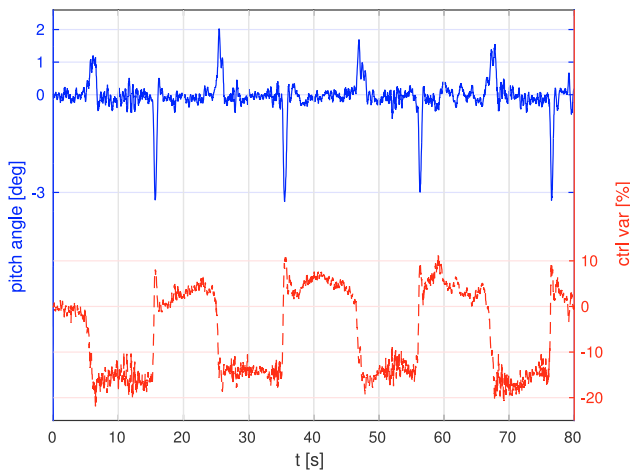


Fig. 9. Load disturbance rejection with CbT tuning. Blue solid line: pitch angle, red dashed line: control variable.

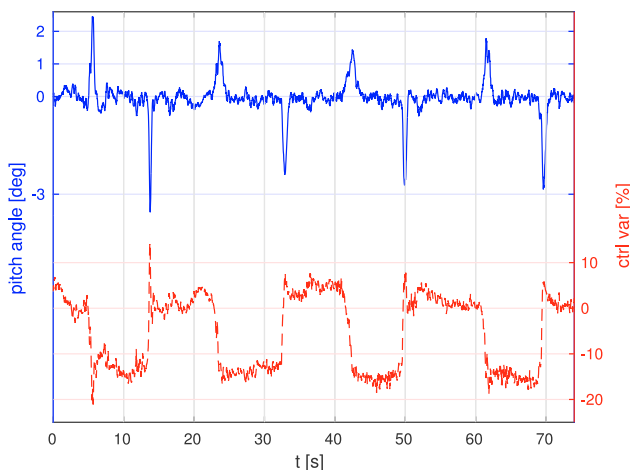


Fig. 10. Load disturbance rejection with VRFT tuning. Blue solid line: pitch angle, red dashed line: control variable.

cascade control architecture with a single set of experimental data. The two methods lead to similar results in terms of set-point tracking and disturbance rejection but the CbT is more robust than the VRFT algorithm in the presence of a low level of SNR. The ability to provide satisfactory controller tunings

in the presence of low SNR is vital for the application of data-driven methods to low-cost quadrotor platforms.

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