EfficientML.ai Lecture 21
Basics of Quantum Computing

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Lecture Plan

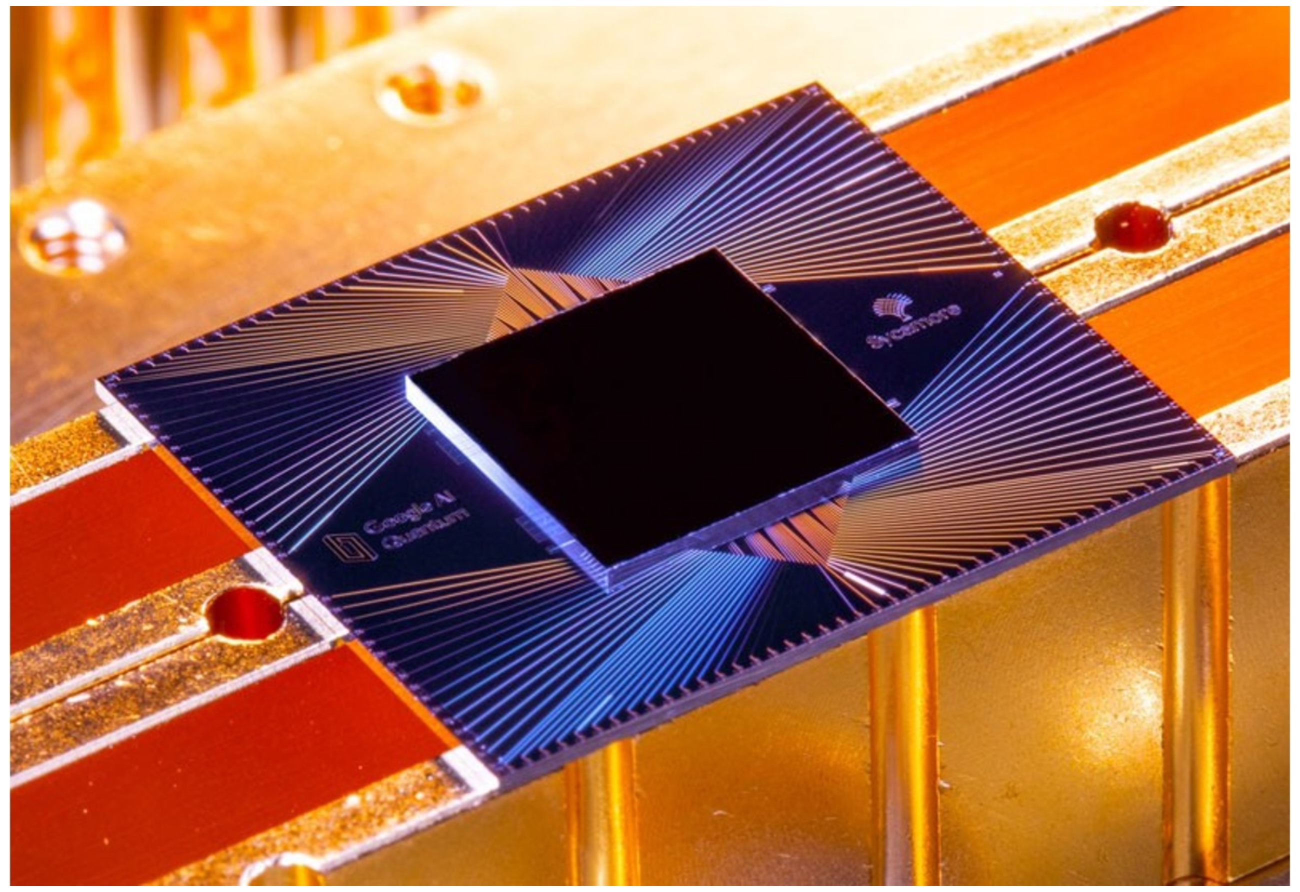
Today we will:

1. Introduce **single** qubit state and gates
2. Introduce **multiple**-qubit state and gates
3. Introduce quantum **circuit**
4. Introduce the **NISQ** era and compilation problems
5. Introduce the example **workflow on superconducting** quantum computer

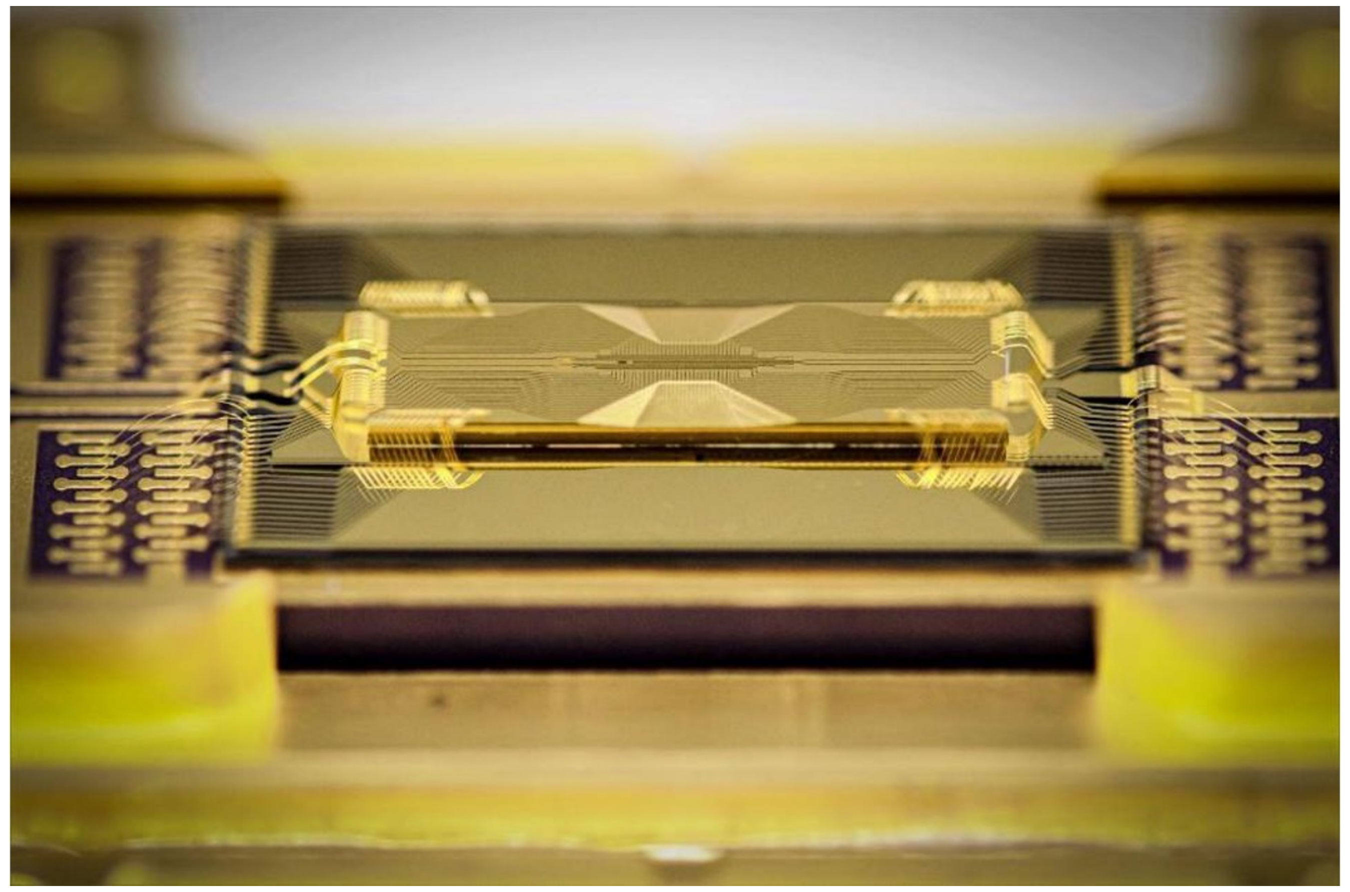
Quantum Computing

- Fast progress of quantum devices
- Different **technologies**
- Superconducting, trapped ion, neutral atom, photonics, etc.

433 Qubits



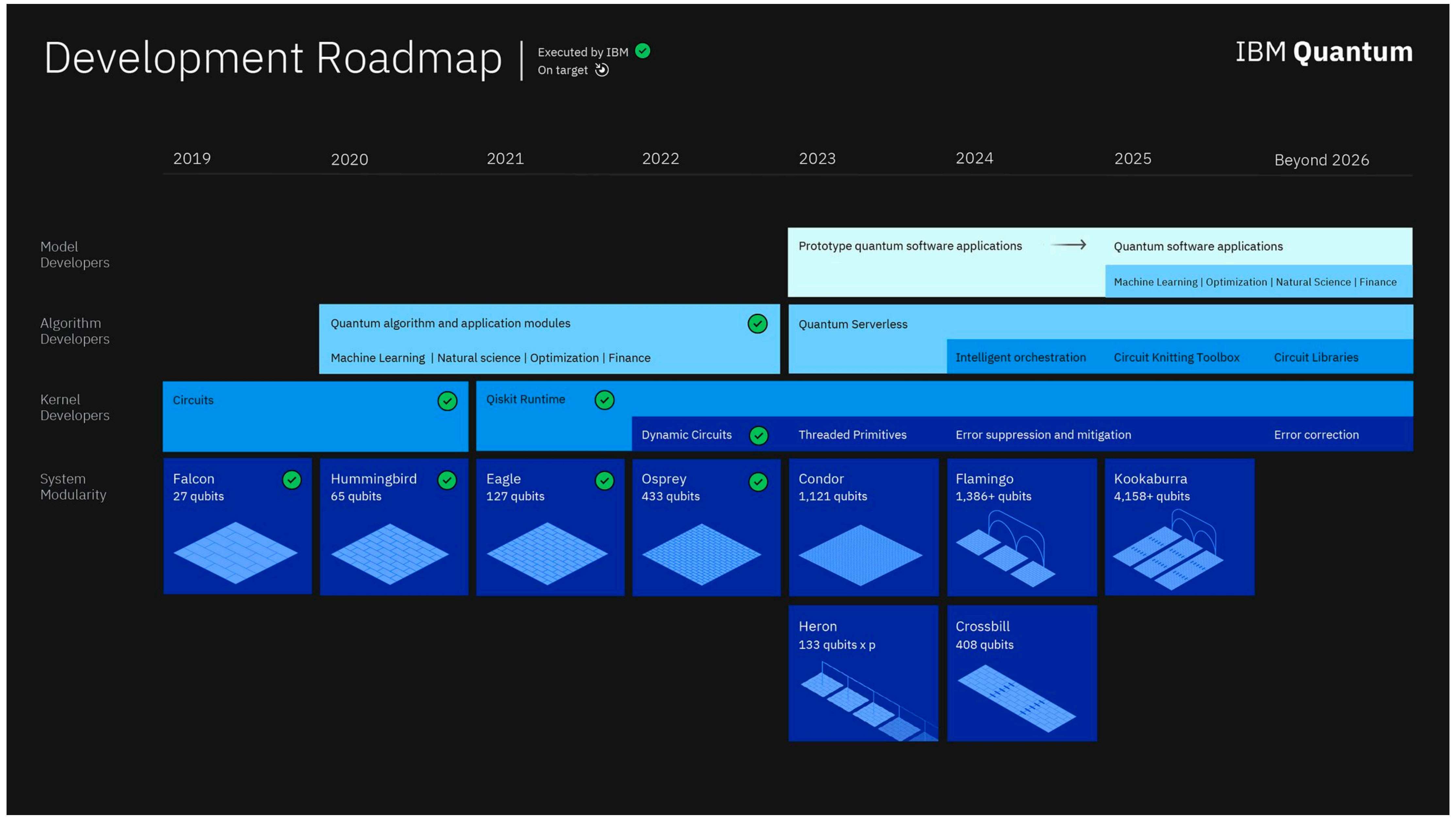
53 Qubits



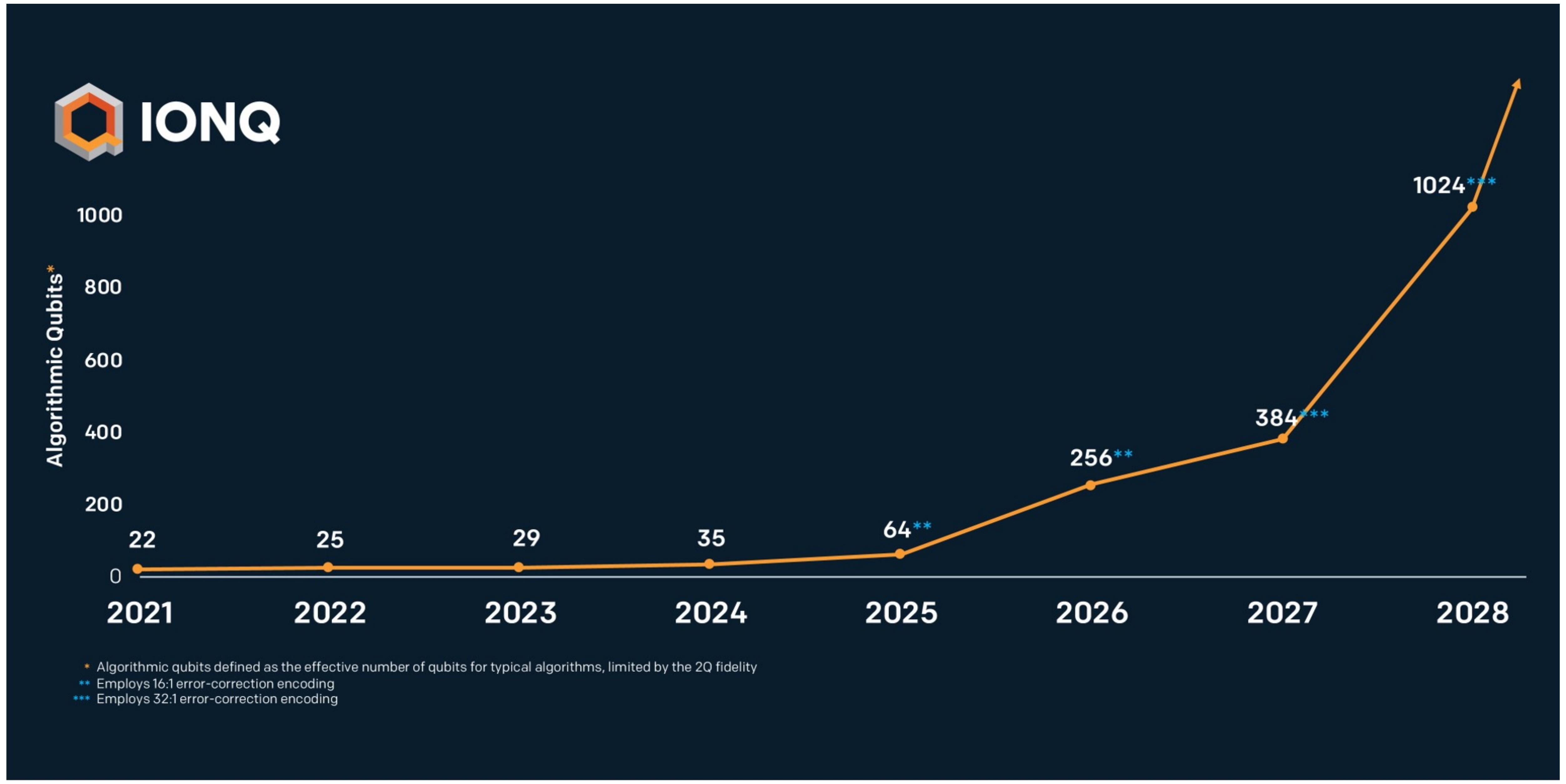
32+ Qubits

Quantum Computing

- The number of qubits increases **exponentially** over time
- The computing power increases **exponentially** with the number of qubits
- “**doubly exponential**” rate

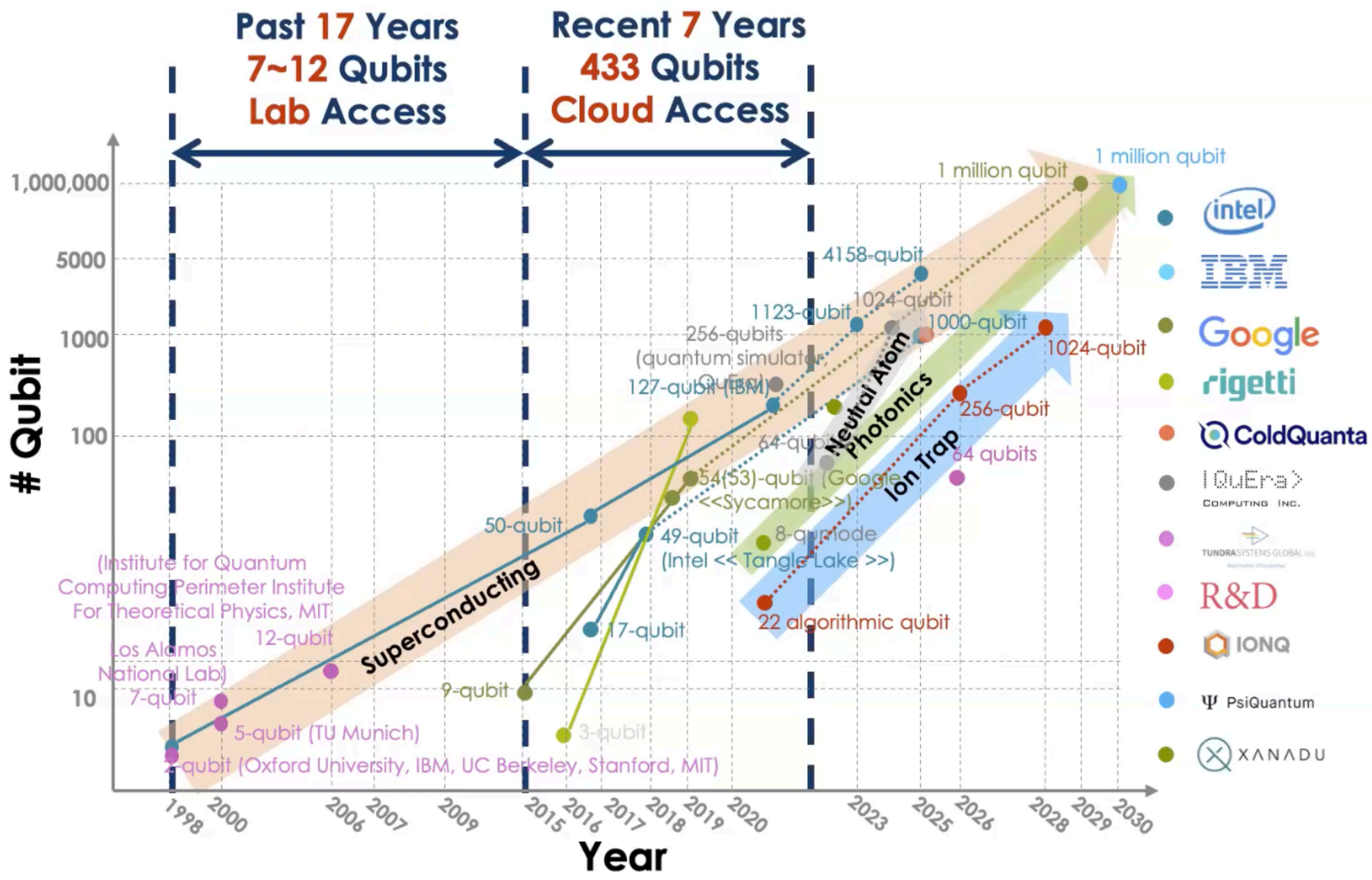


IBM Roadmap



IonQ Roadmap

Quick Device Improvements



source: Yufei Ding’s lecture

History

1980-90s

Algorithm that has **practical** quantum speedup on **no-noise large-scale** quantum computer

Quantum Simulation
Factoring algorithm

History

1980-90s

Algorithm that has **practical** quantum speedup on **no-noise large-scale** quantum computer

1995

Theoretically prove that practical quantum speedup on **noisy large-scale** quantum computer

Quantum Error Correction (QEC)

History

1980-90s

Algorithm that has **practical** quantum speedup on **no-noise large-scale** quantum computer

1995

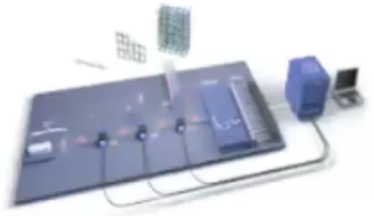
Theoretically prove that practical quantum speedup on **noisy large-scale** quantum computer

2019~

Experimentally prove that practical (?) quantum speedup on **noisy small (NISQ)** quantum computer

Google, 2019

USTC, 2020



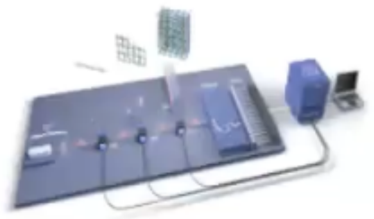
Xanadu, 2022

History

1980-90s	1995	2019~	Future
Algorithm that has **practical** quantum speedup on **no-noise large-scale** quantum computer	**Theoretically** prove that practical quantum speedup on **noisy large-scale** quantum computer	**Experimentally** prove that practical (?) quantum speedup on **noisy small (NISQ)** quantum computer	**NISQ computing** **Quantum error correction**

Google, 2019

USTC, 2020



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Section 1

Single Quantum Bit

Qubit States

- The basic **component** of quantum compute is a Quantum Bit (**Qubit**)
- Use **statevector** to describe the state of the system

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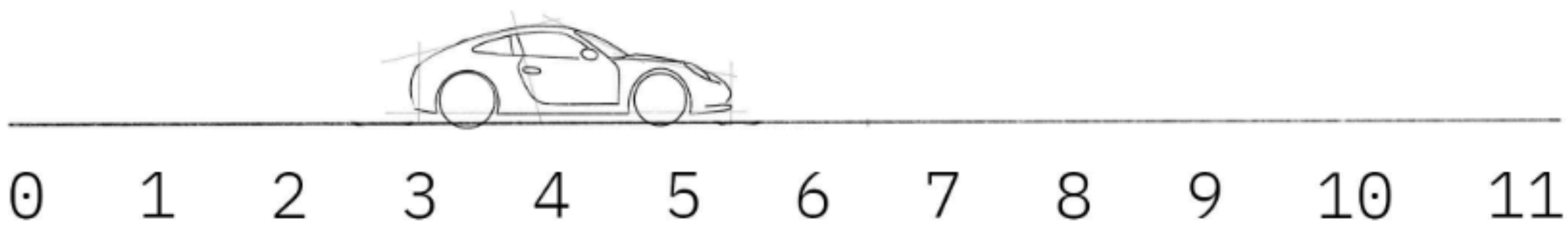
Qubit States

- Imagine a **classical** system, how to describe the state of the car?



$$x = 4$$

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Qubit States

- **Probability** of car being at a location
- **Inefficient** in classical but very effective for representing quantum states

0 0 0 0 1 0 0 0 0 0 0 0

$$|x\rangle = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \begin{array}{c} \\ \\ \\ \leftarrow \\ \\ \\ \\ \end{array}$$

Probability of car being at position 4

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Qubit Notation

- Classical: a = 0 or a = 1
- Quantum: use orthogonal vectors:
  - **Bra-ket** notation, or Dirac notation
  - |0> and |1> forms an orthonormal basis

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

- More complex states:

$$|q_0\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix}$$

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Superposition

- How to write this state in another form with the **combination** of two basis states?

$$|q_0\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$$

- This forms a superposition of two basis states: a **linear combination** of two states

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Measurement

- To find the probability of measuring a state $|\psi\rangle$ in the state $|x\rangle$ we do:

$$p(|x\rangle) = |\langle x|\psi\rangle|^2$$

- <x| is **row** vector and |x> is a **column** vectors
- Column vectors: **kets**
- Row vectors: **bras**
- Together bra-ket notation
- <x| is the conjugate transpose of |x>

$$|a\rangle = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix}$$

$$\langle a| = \begin{bmatrix} a_0^*, & a_1^*, & \dots & a_n^* \end{bmatrix}$$

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Measurement

$$p(|x\rangle) = \left|\langle x|\psi\rangle\right|^2$$

- |x> can be any states
- What is |q0> measured in |0>?

$$\begin{aligned} |q_0\rangle &= \frac{1}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle \\ \langle 0|q_0\rangle &= \frac{1}{\sqrt{2}}\langle 0|0\rangle + \frac{i}{\sqrt{2}}\langle 0|1\rangle \\ &= \frac{1}{\sqrt{2}} \cdot 1 + \frac{i}{\sqrt{2}} \cdot 0 \\ &= \frac{1}{\sqrt{2}} \\ \left|\langle 0|q_0\rangle\right|^2 &= \frac{1}{2} \end{aligned}$$

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Measurement

- The probability should **add up to 1**
- Measure a state in itself should be 1

$$\langle\psi|\psi\rangle = 1$$

Thus if:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Then:

$$|\alpha|^2 + |\beta|^2 = 1$$

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Alternative Basis

- We can measure the qubits in |0>, |1> states but also other basis
- There are **infinite** pairs of orthonormal basis
- When perform the measurement the state will choose one out of two basis states

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Global Phase

- Measure |1> will give 1 as output 100%
- What about i|1>?

$$\begin{bmatrix} 0 \\ i \end{bmatrix} = i|1\rangle$$

- i will **disappear** when taking magnitude

$$|\langle x|(i|1\rangle)|^2 = |i\langle x|1\rangle|^2 = |\langle x|1\rangle|^2$$

- i|1> and |1> are **equivalent** in all ways that are physically relevant

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Observer Effect

- Once we have **measured** the qubit, we know with certainty what the state of the qubit is

$$|q\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|q\rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \xrightarrow{\text{Measure } |0\rangle} |q\rangle = |0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

- The quantum state **collapses** to a classical state
- Typically measurement is performed at the **end** of computation
  - Otherwise the information will be lost
- When measure multiple qubits, we will only get a series of classical bits

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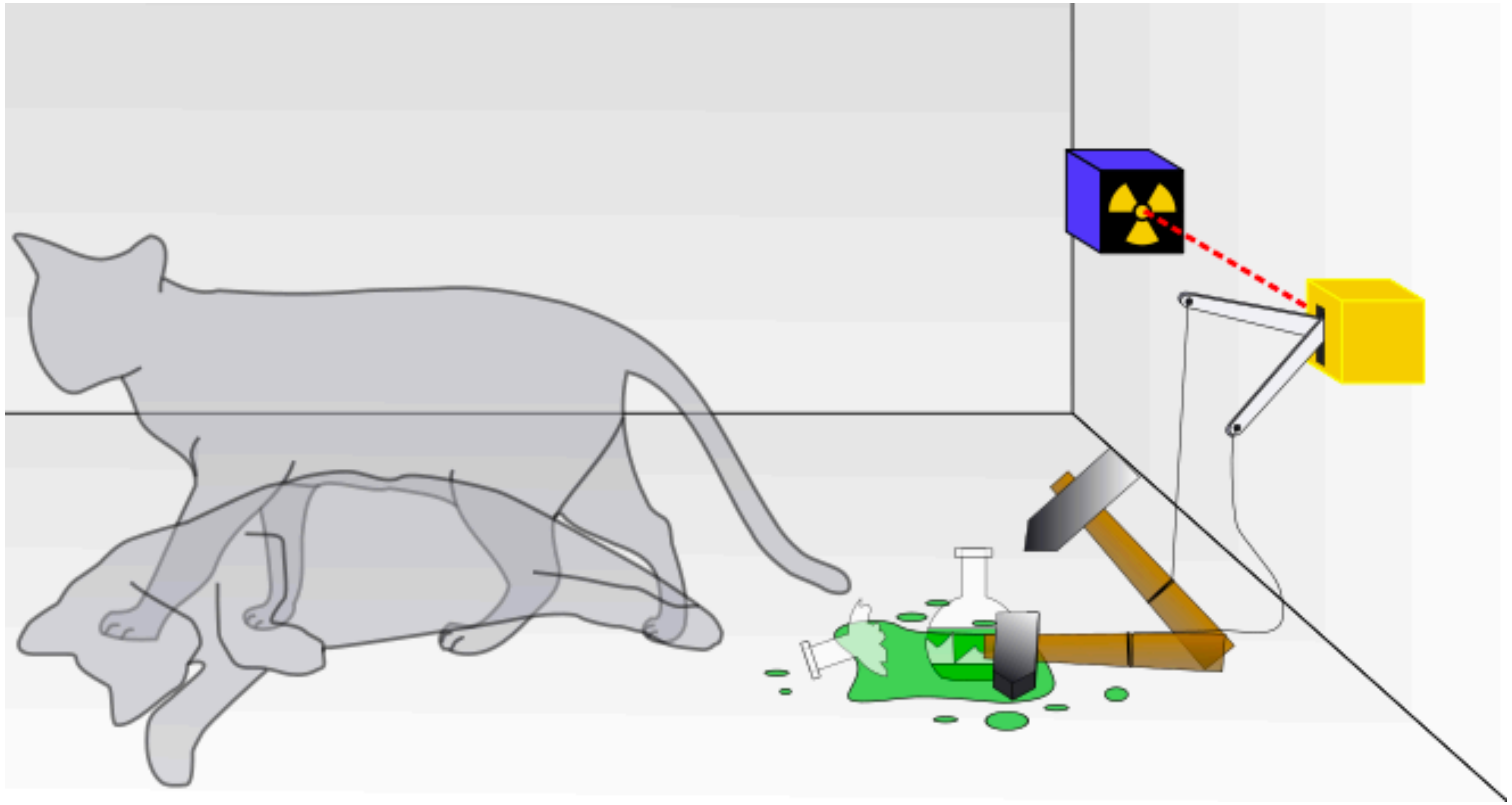
Schrodinger's cat

- In a superposition of live and die



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The Bloch Sphere

- How many free variables in a state?

$$|q\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\alpha, \beta \in \mathbb{C}$$

$$|q\rangle = \alpha|0\rangle + e^{i\phi}\beta|1\rangle$$

$$\alpha, \beta, \phi \in \mathbb{R}$$

$$\sqrt{\alpha^2 + \beta^2} = 1$$

$$|q\rangle = \cos\tfrac{\theta}{2}|0\rangle + e^{i\phi}\sin\tfrac{\theta}{2}|1\rangle$$

$$\theta, \phi \in \mathbb{R}$$

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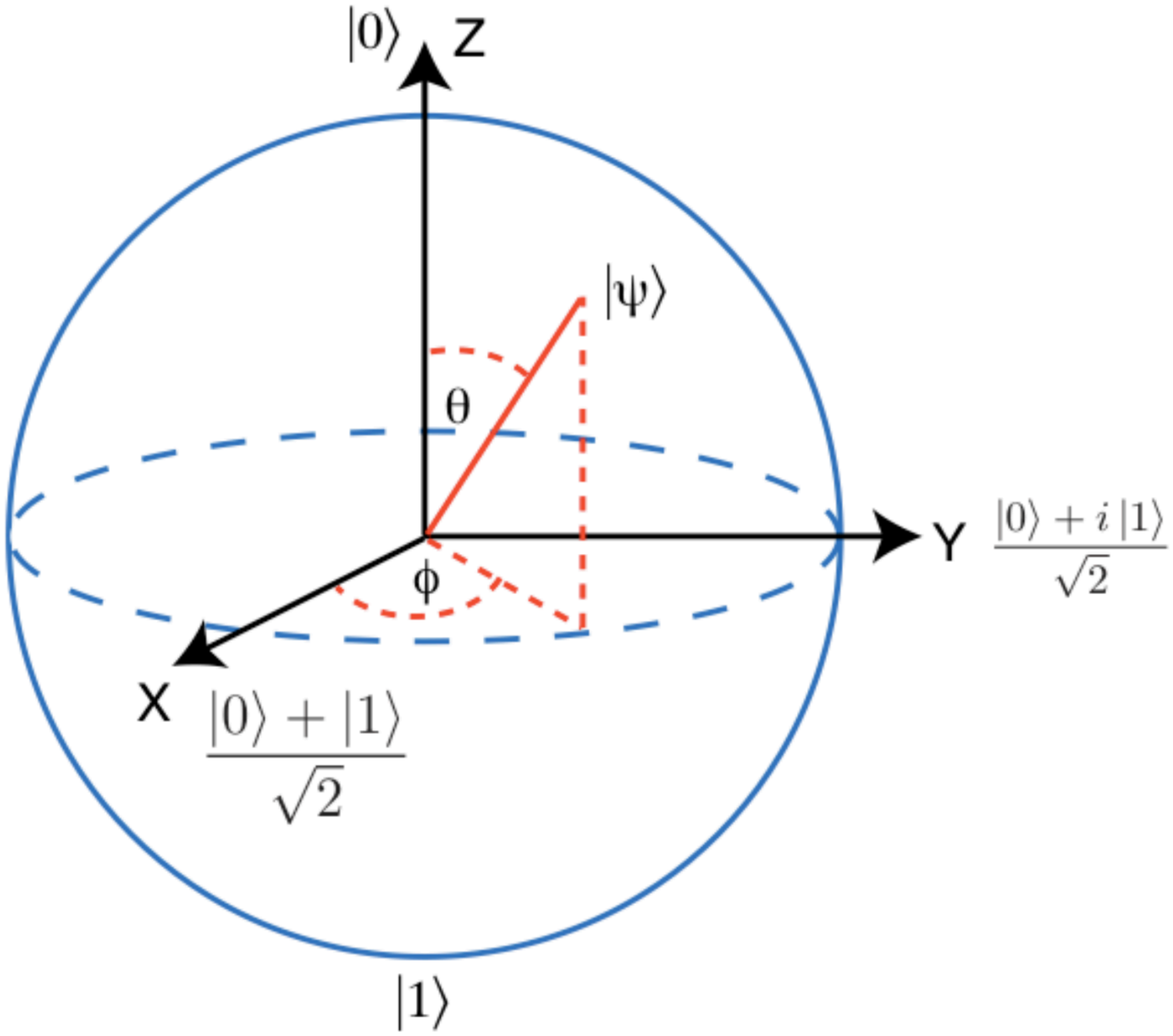
The Bloch Sphere

$$|q\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

$$\theta, \phi \in \mathbb{R}$$

- Consider theta and phi are **spherical** coordinates
- Any single qubit state is on the surface of a sphere — **Bloch sphere**



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Single Qubit Gates

- All the quantum gates are **reversible**
- What is the simplest reversible gate in classical computation?
- Reversible gates can be represented as matrices or rotations around the Bloch sphere

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

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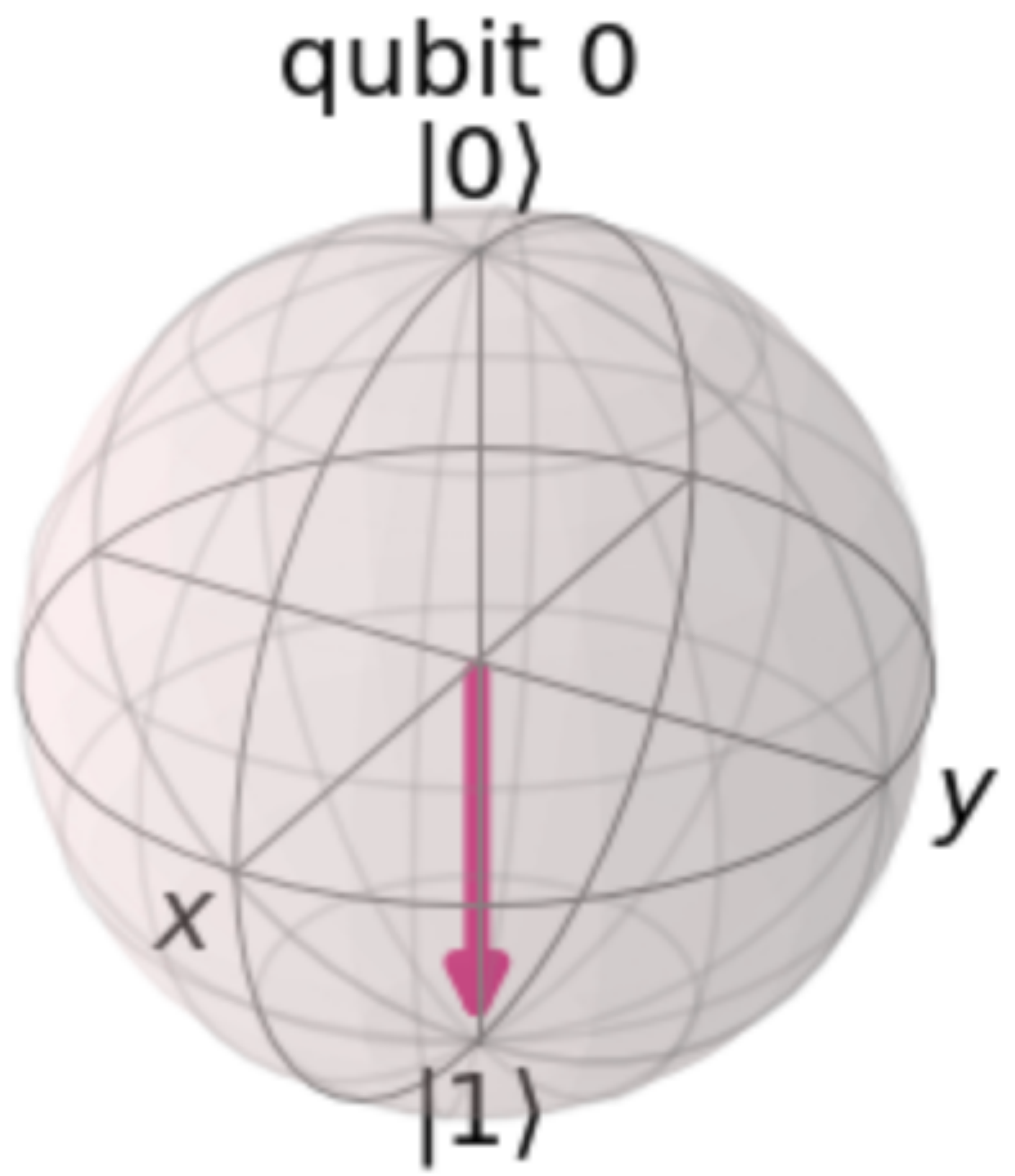
Pauli Gates

- X Gates

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = |0\rangle\langle 1| + |1\rangle\langle 0| \qquad X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = |1\rangle$$



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Pauli Gates

- Y and Z gates

$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$Y = -i|0\rangle\langle 1| + i|1\rangle\langle 0| \qquad Z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

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X, Y & Z-Bases

- What happens when we apply the Z gate to |0> state? still |0>
- What happens when we apply the Z gate to |1> state? still -|1> but physically **indistinguishable** to |1>

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$Z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

- |0> and |1> are the eigenstate of Z
- The computational basis formed by |0> and |1> is often called Z-basis

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X, Y & Z-Bases

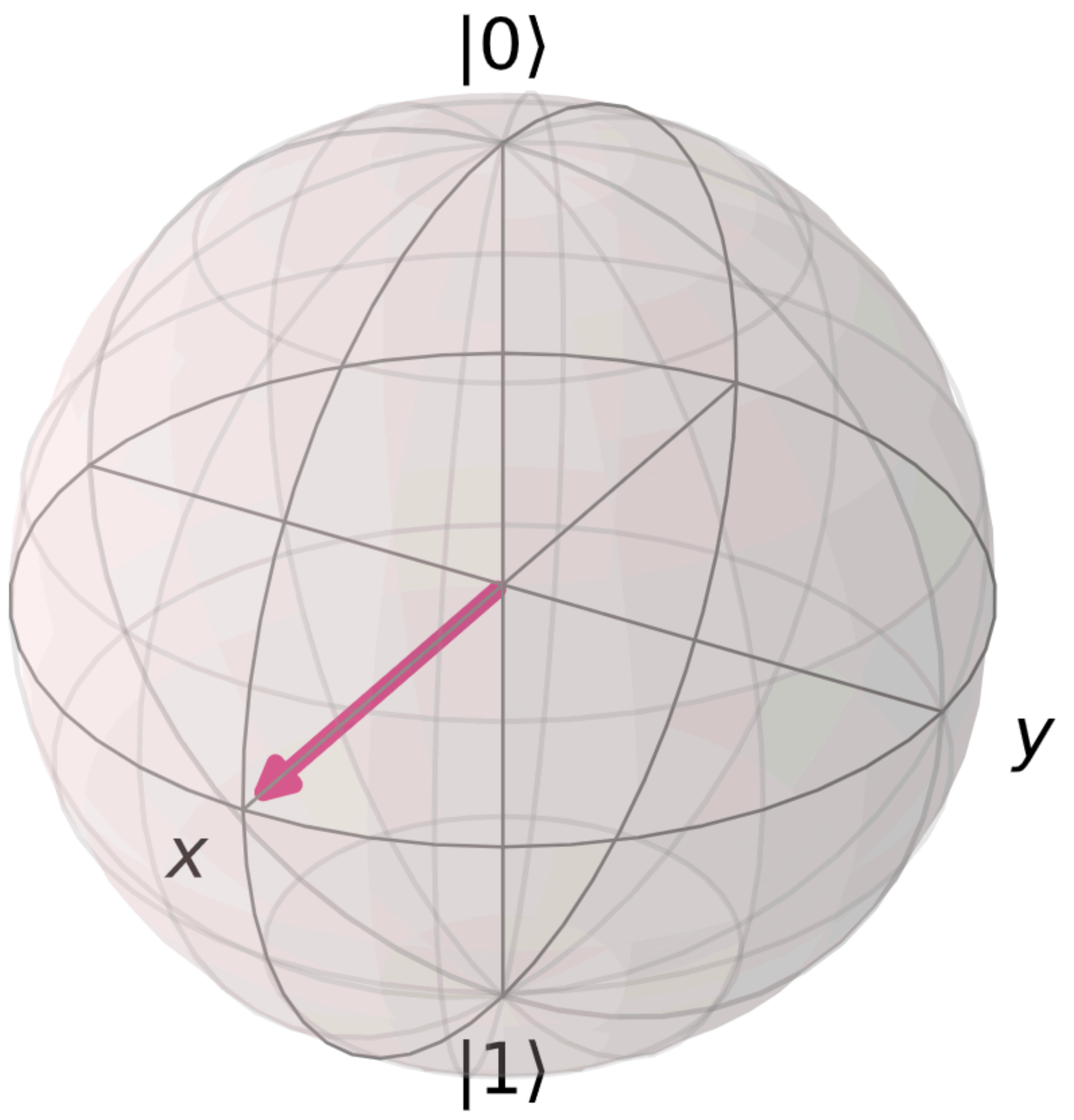
- X-Basis: the basis are |+> and |->:

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

- Y-Basis: the basis are |L> and |R>:

$$|R\rangle = \frac{|0\rangle + i|1\rangle}{\sqrt{2}} \quad |L\rangle = \frac{|0\rangle - i|1\rangle}{\sqrt{2}}$$



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Hadamard Gate

- Hadamard gate can create **superposition** of |0> and |1>

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$H|0\rangle = |+\rangle$$

$$H|1\rangle = |-\rangle$$

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Other Gates

- The single qubit gate that contains parameter

$$P(\phi) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{2}} \end{bmatrix}, \quad S^\dagger = \begin{bmatrix} 1 & 0 \\ 0 & e^{-\frac{i\pi}{2}} \end{bmatrix}$$

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U Gate

- Contains three parameters and can implement **all possible** gates

$$U(\theta, \phi, \lambda) = \begin{bmatrix} \cos(\frac{\theta}{2}) & -e^{i\lambda}\sin(\frac{\theta}{2}) \\ e^{i\phi}\sin(\frac{\theta}{2}) & e^{i(\phi+\lambda)}\cos(\frac{\theta}{2}) \end{bmatrix}$$

$$U(\tfrac{\pi}{2}, 0, \pi) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = H \qquad U(0, 0, \lambda) = \begin{bmatrix} 1 & 0 \\ 0 & e^{i\lambda} \end{bmatrix} = P$$

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Section 2

Multiple Quantum Bits

Multi-Qubit State

- 2 bits have 4 possible
- 00 01 10 11
- 2 quantum bits?

$$|a\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle = \begin{bmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{11} \end{bmatrix}$$

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Multi-Qubit Measurement

- The principle holds for multiple qubits

$$p(|00\rangle) = |\langle 00|a\rangle|^2 = |a_{00}|^2$$

- **Normalization** principle

$$|a_{00}|^2 + |a_{01}|^2 + |a_{10}|^2 + |a_{11}|^2 = 1$$

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Collective State

- Kronecker product
- **n** qubit: **2^n** complex amplitudes in the statevector
- Difficulty to simulate on classical machines

$$|a\rangle = \begin{bmatrix} a_0 \\ a_1 \end{bmatrix}, \quad |b\rangle = \begin{bmatrix} b_0 \\ b_1 \end{bmatrix}$$

$$|ba\rangle = |b\rangle \otimes |a\rangle = \begin{bmatrix} b_0 \times \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \\ b_1 \times \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} b_0 a_0 \\ b_0 a_1 \\ b_1 a_0 \\ b_1 a_1 \end{bmatrix}$$

$$|cba\rangle = \begin{bmatrix} c_0 b_0 a_0 \\ c_0 b_0 a_1 \\ c_0 b_1 a_0 \\ c_0 b_1 a_1 \\ c_1 b_0 a_0 \\ c_1 b_0 a_1 \\ c_1 b_1 a_0 \\ c_1 b_1 a_1 \end{bmatrix}$$

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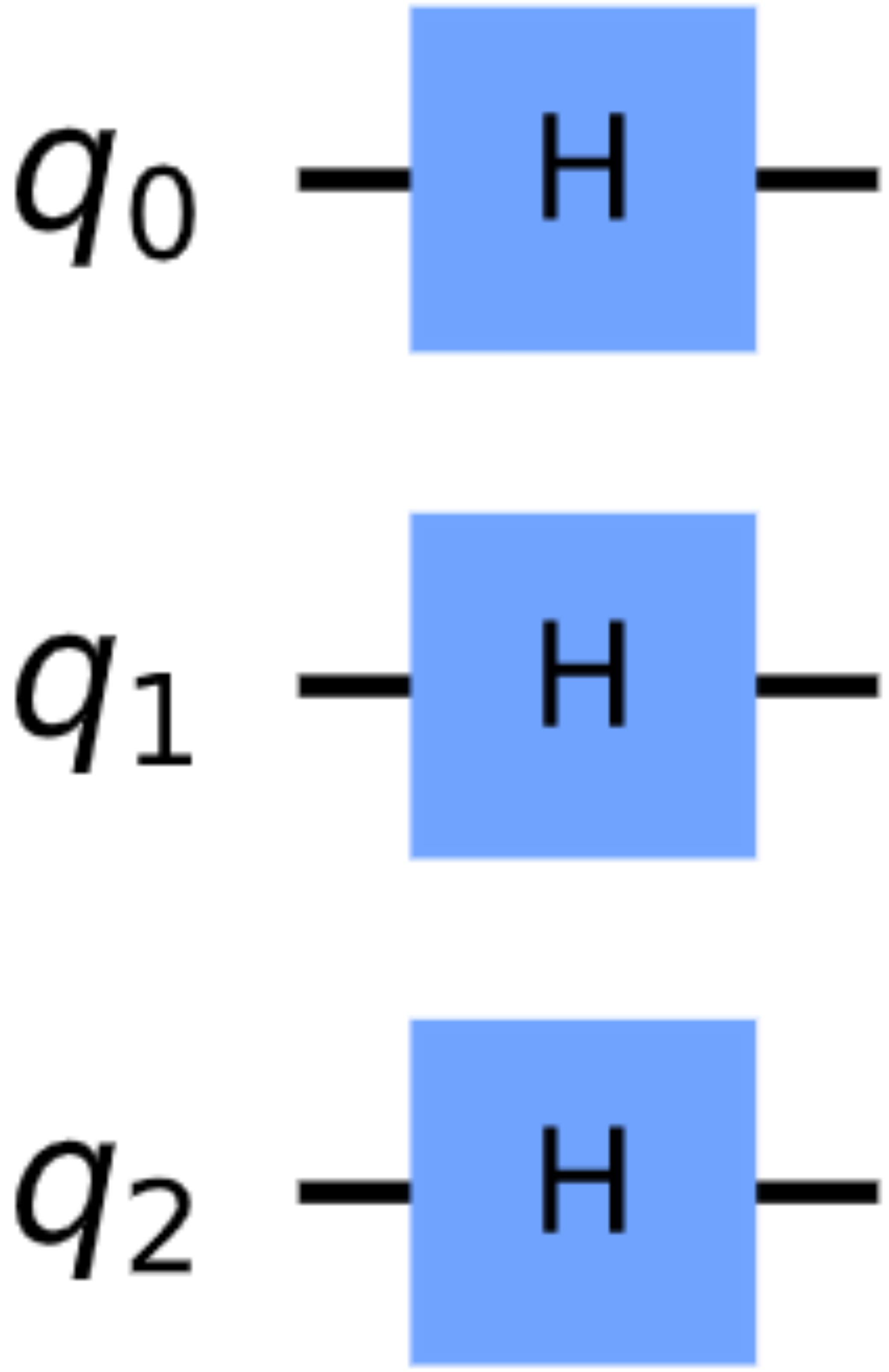
An collective state example

- Example of H gates on three qubits



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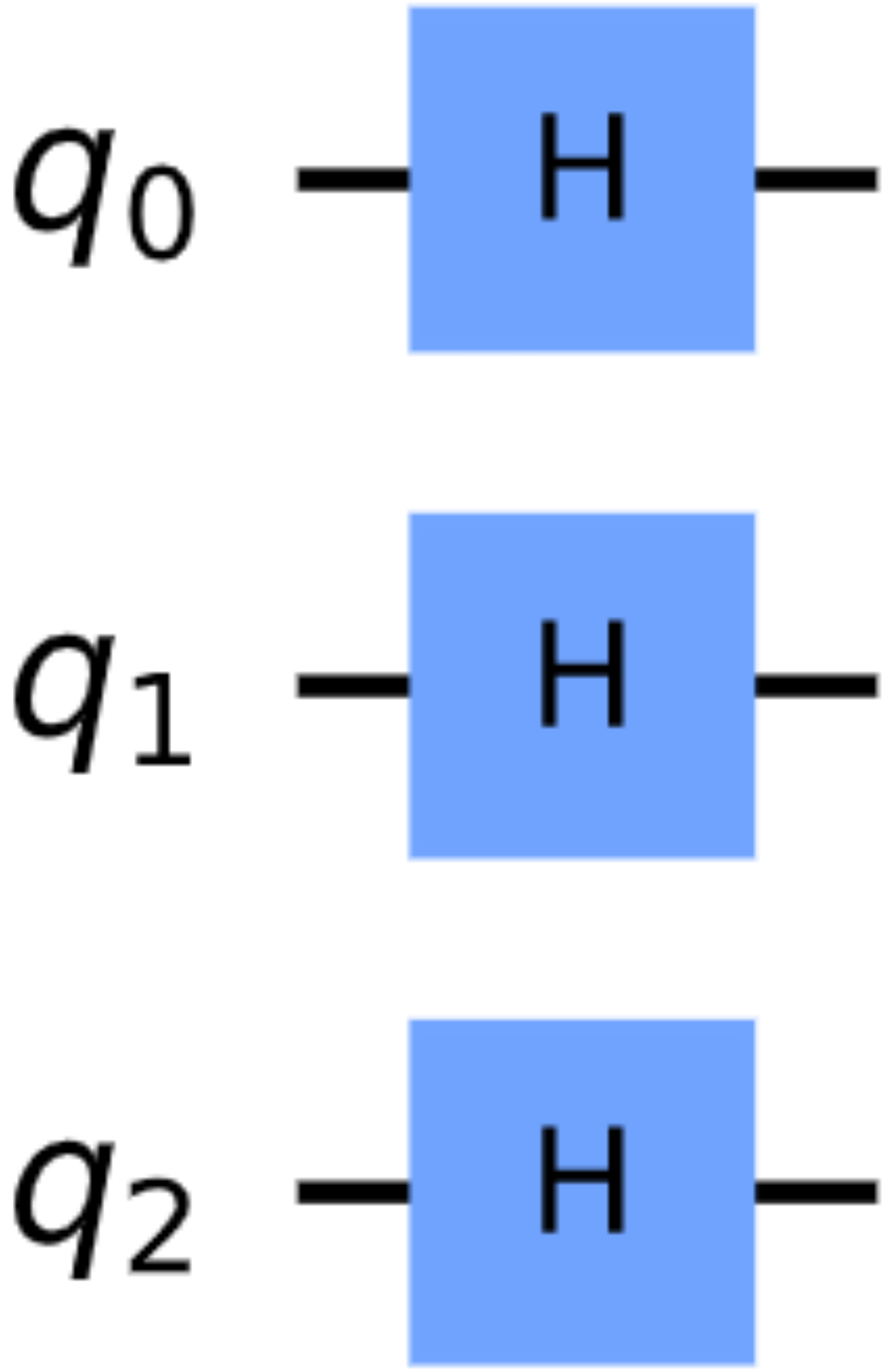
An collective state example

- Example of H gates on three qubits



$$|{+}{+}{+}\rangle = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

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Single Qubit Gate on Multi-Qubit system

- What happens when we apply single qubit gate to multt-qubit system?

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

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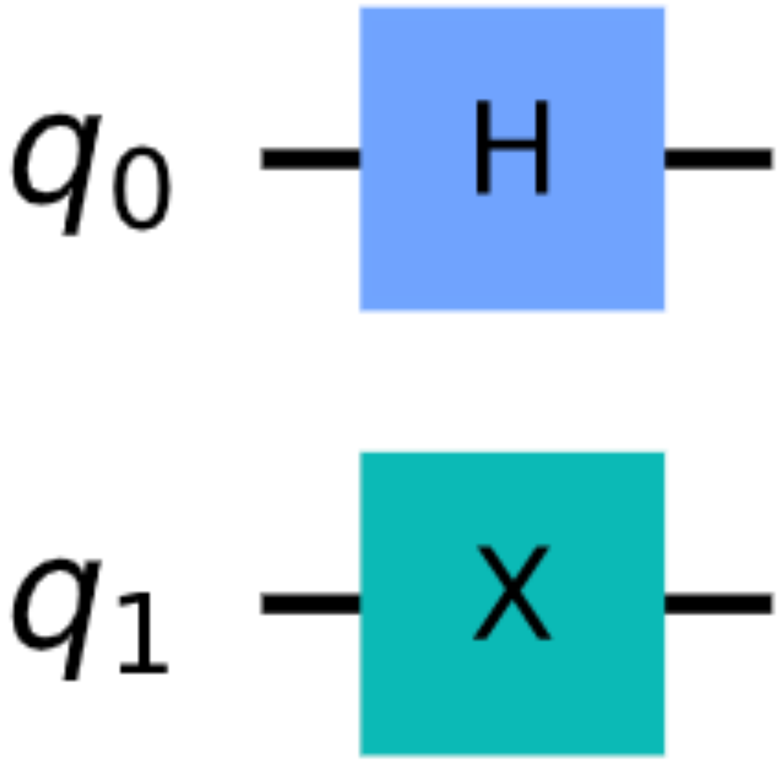
Single Qubit Gate on Multi-Qubit system

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad X|0\rangle = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$



$$X|q_1\rangle \otimes H|q_0\rangle = (X \otimes H)|q_1 q_0\rangle$$

$$X \otimes H = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \tfrac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \times \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} & 1 \times \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\ 1 \times \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} & 0 \times \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \qquad X \otimes H = \begin{bmatrix} 0 & H \\ H & 0 \end{bmatrix}$$

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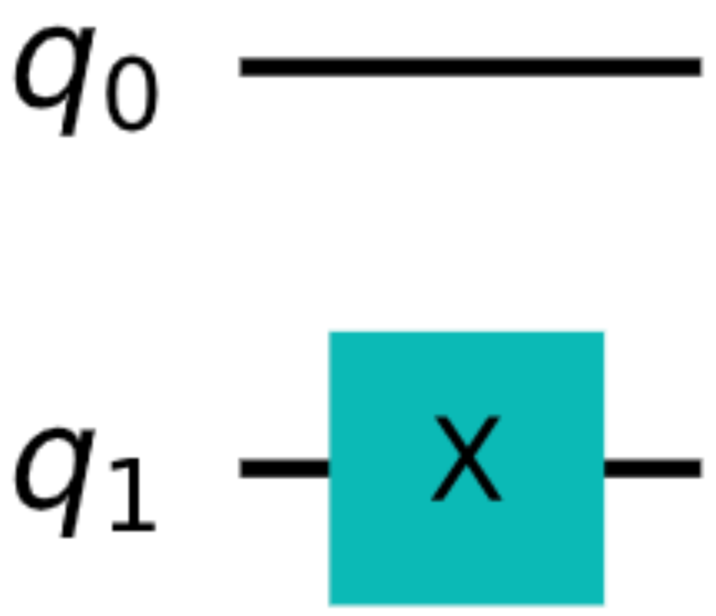
Single Qubit Gate on Multi-Qubit system

- Joint unitary matrix



$$X \otimes I = \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

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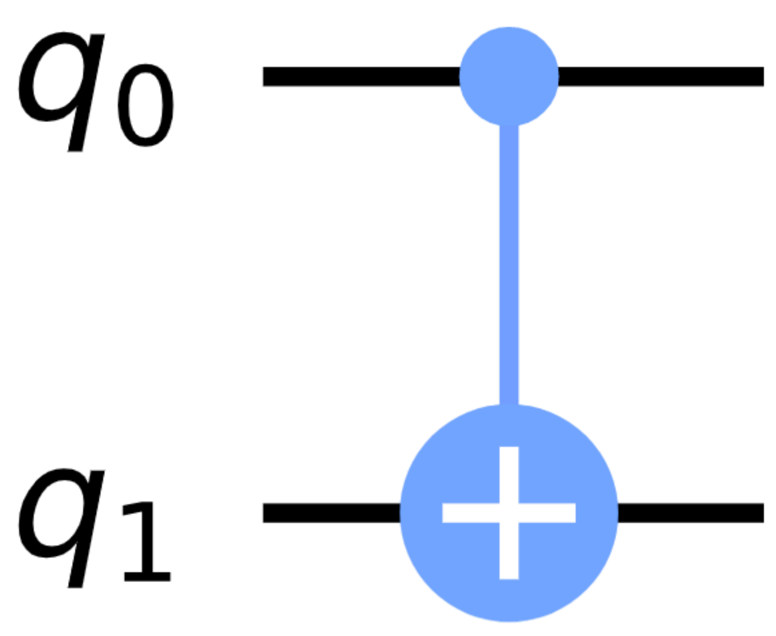
Multi-Qubit Gates

- CNOT gate



Input (t,c)	Output (t,c)
00	00
01	11
10	10
11	01

$$CNOT = \begin{array}{c|c} & \text{Output} \\ \text{Input} & \begin{array}{cccc} 00 & 01 & 10 & 11 \end{array} \\ \begin{array}{c} 00 \\ 01 \\ 10 \\ 11 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

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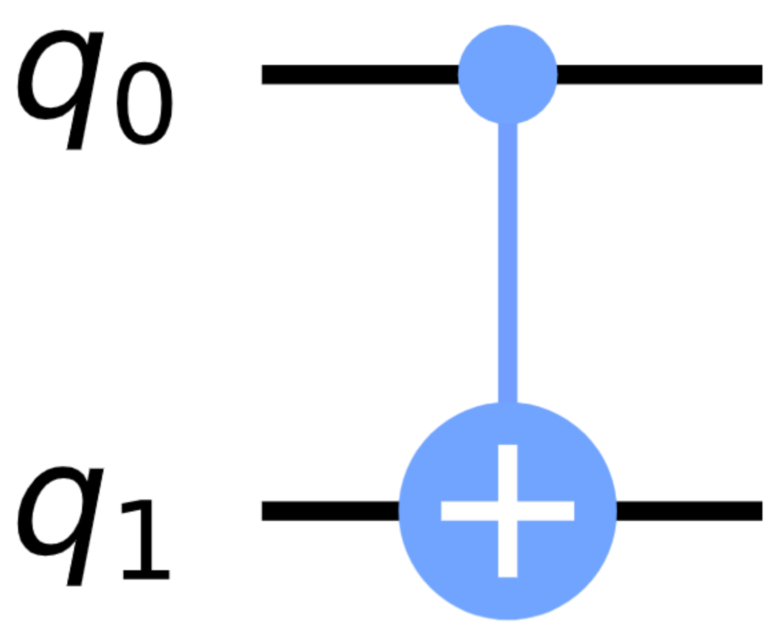
Multi-Qubit Gates

- CNOT gate



$$|a\rangle = \begin{bmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{11} \end{bmatrix}, \quad \text{CNOT}|a\rangle = \begin{bmatrix} a_{00} \\ a_{11} \\ a_{10} \\ a_{01} \end{bmatrix} \begin{matrix} \\ \leftarrow \\ \\ \leftarrow \end{matrix}$$

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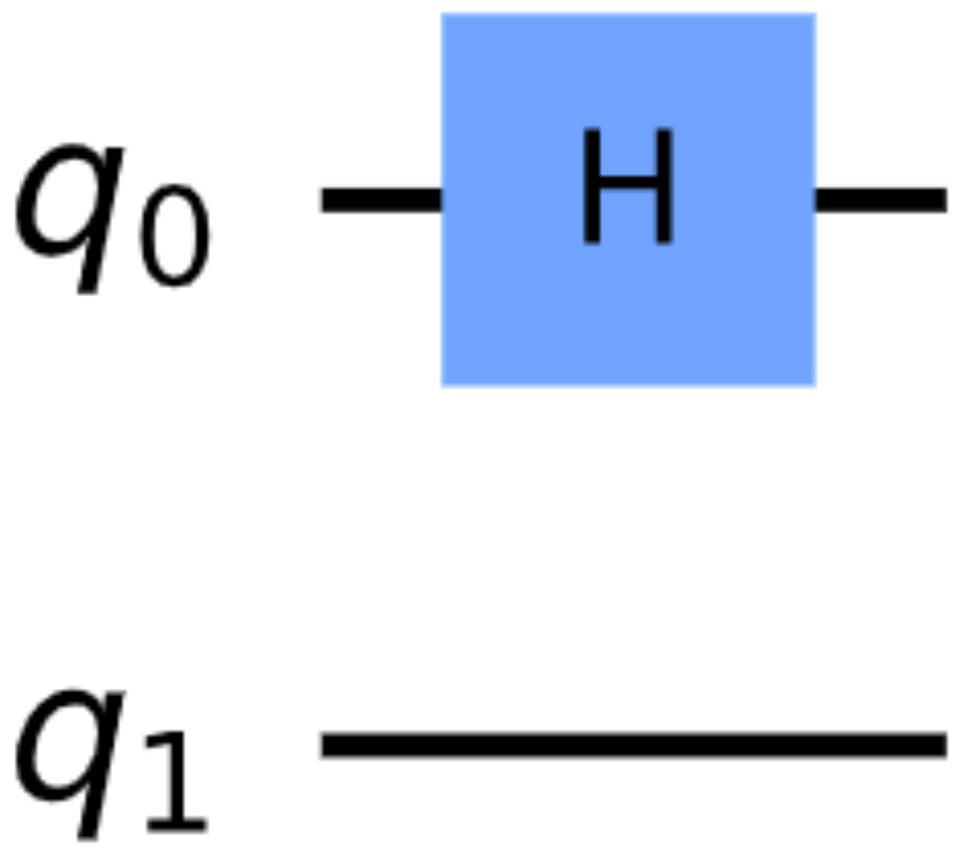
Entanglement

- How CNOT creates **entanglement?**



$$|0+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |01\rangle)$$

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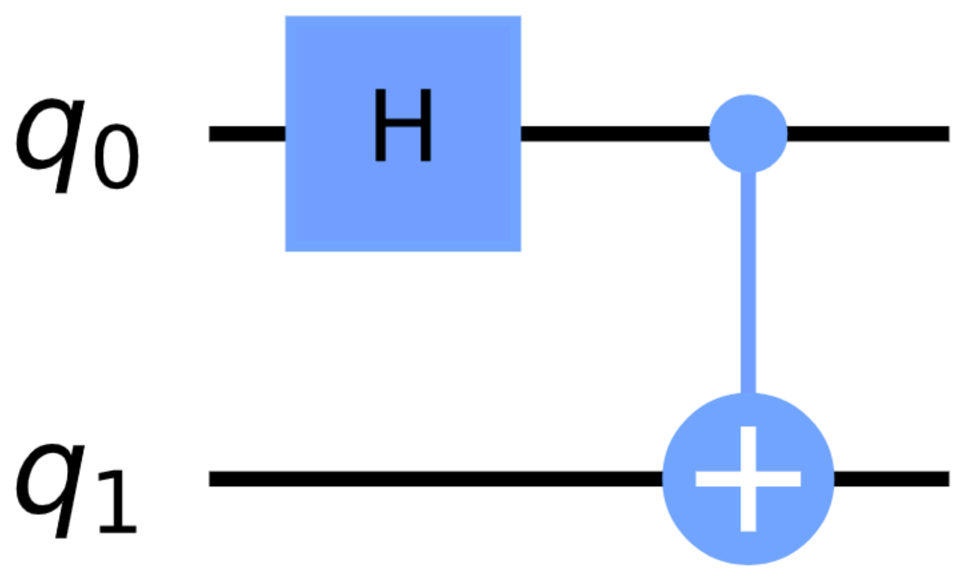
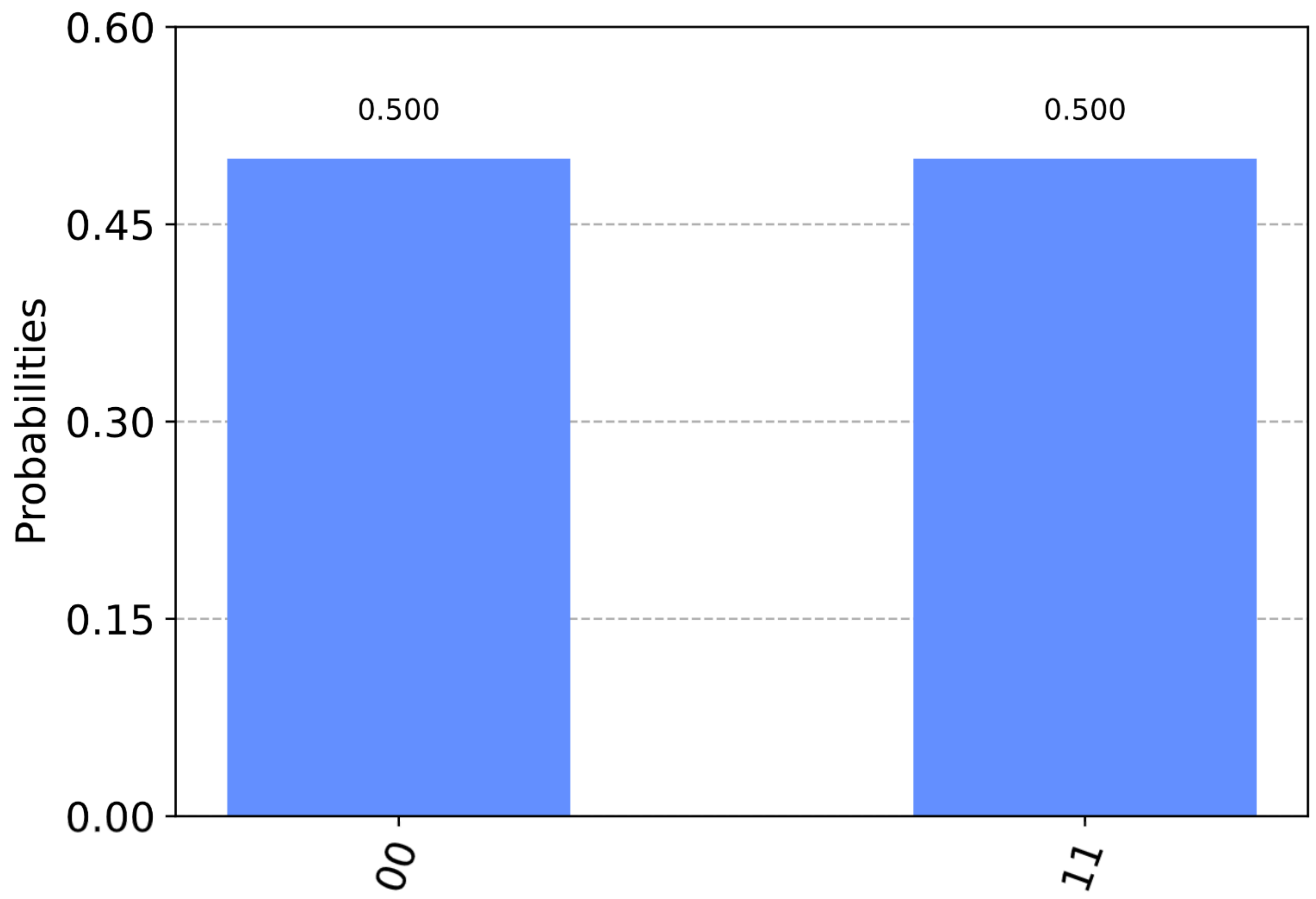
Entanglement

- The classical results **must be the same** of two qubits



$$\mathrm{CNOT}|0+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



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Spooky action at a distance

- “Even if we separated these qubits **light-years away**, measuring one qubit collapses the superposition and appears to have an immediate effect on the other”

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \xrightarrow{\text{measure}} |11\rangle$$

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No-communication theorem

- The measurement result is **random**
- The measurement statistics of one qubit are **not affected** by any operation on the other qubit
- It is not possible to use shared quantum state to **communicate**

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Phase Kickback

- One 1 qubit gate

$$X|-\rangle = -|-\rangle$$

$$\begin{aligned} \mathrm{CNOT}|{-0}\rangle &= |-\rangle \otimes |0\rangle \\ &= |{-0}\rangle \end{aligned}$$

$$\begin{aligned} \mathrm{CNOT}|{-1}\rangle &= X|-\rangle \otimes |1\rangle \\ &= -|-\rangle \otimes |1\rangle \\ &= -|{-1}\rangle \end{aligned}$$

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Phase Kickback

- The CNOT add an additional phase on the **control** qubit

$$
\begin{aligned}
\mathrm{CNOT}|-+\rangle &= \frac{1}{\sqrt{2}}(\mathrm{CNOT}|-0\rangle + \mathrm{CNOT}|-1\rangle) \\
&= \frac{1}{\sqrt{2}}(|-0\rangle + X|-1\rangle) \\
&= \frac{1}{\sqrt{2}}(|-0\rangle - |-1\rangle) \\
&= |-\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \\
&= |--\rangle
\end{aligned}
$$

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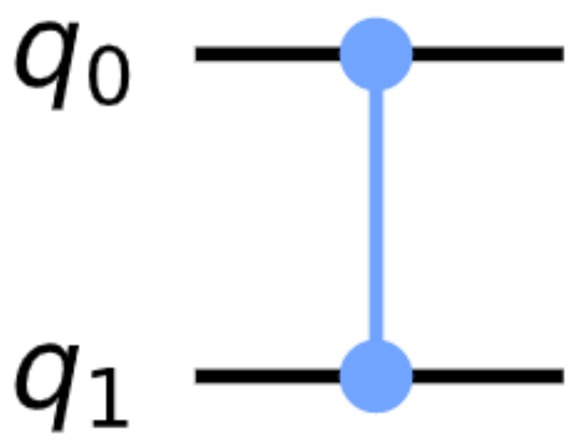
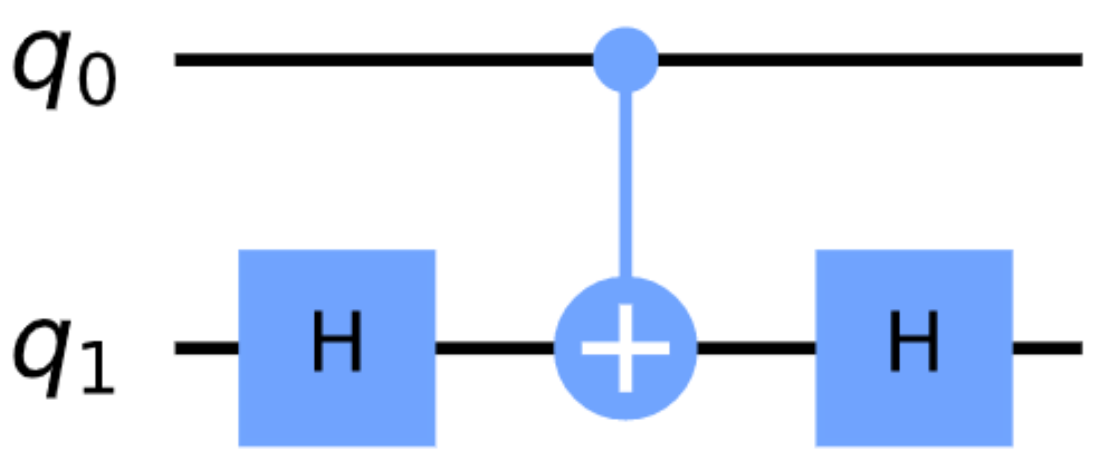
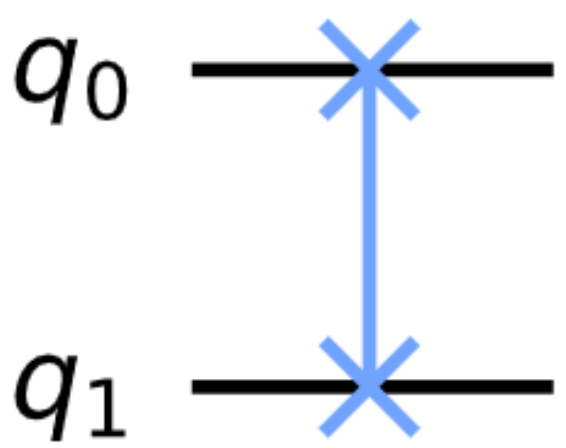
More commonly used multi-qubit gates

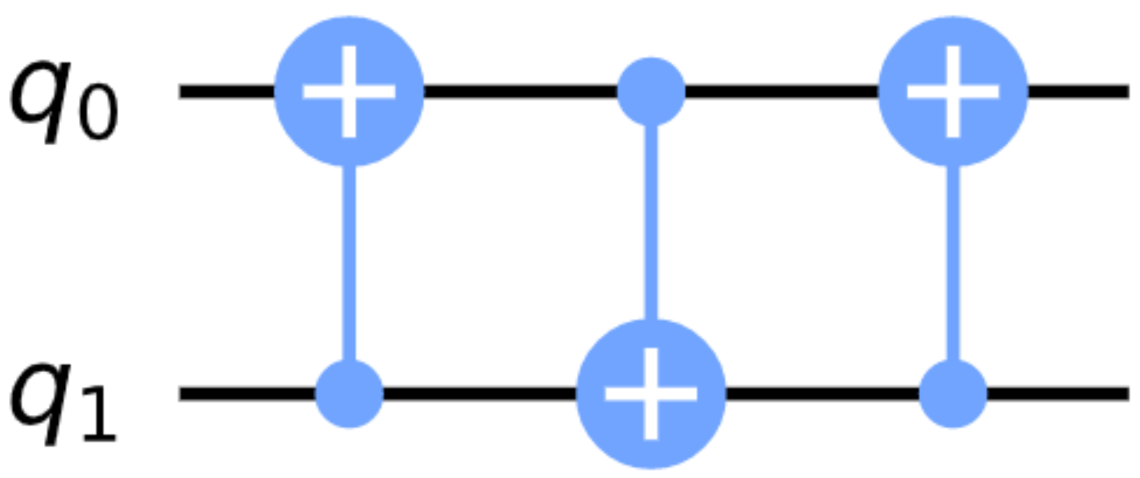
- CZ





- SWAP





- CRX

$$CRX(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ 0 & 0 & -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

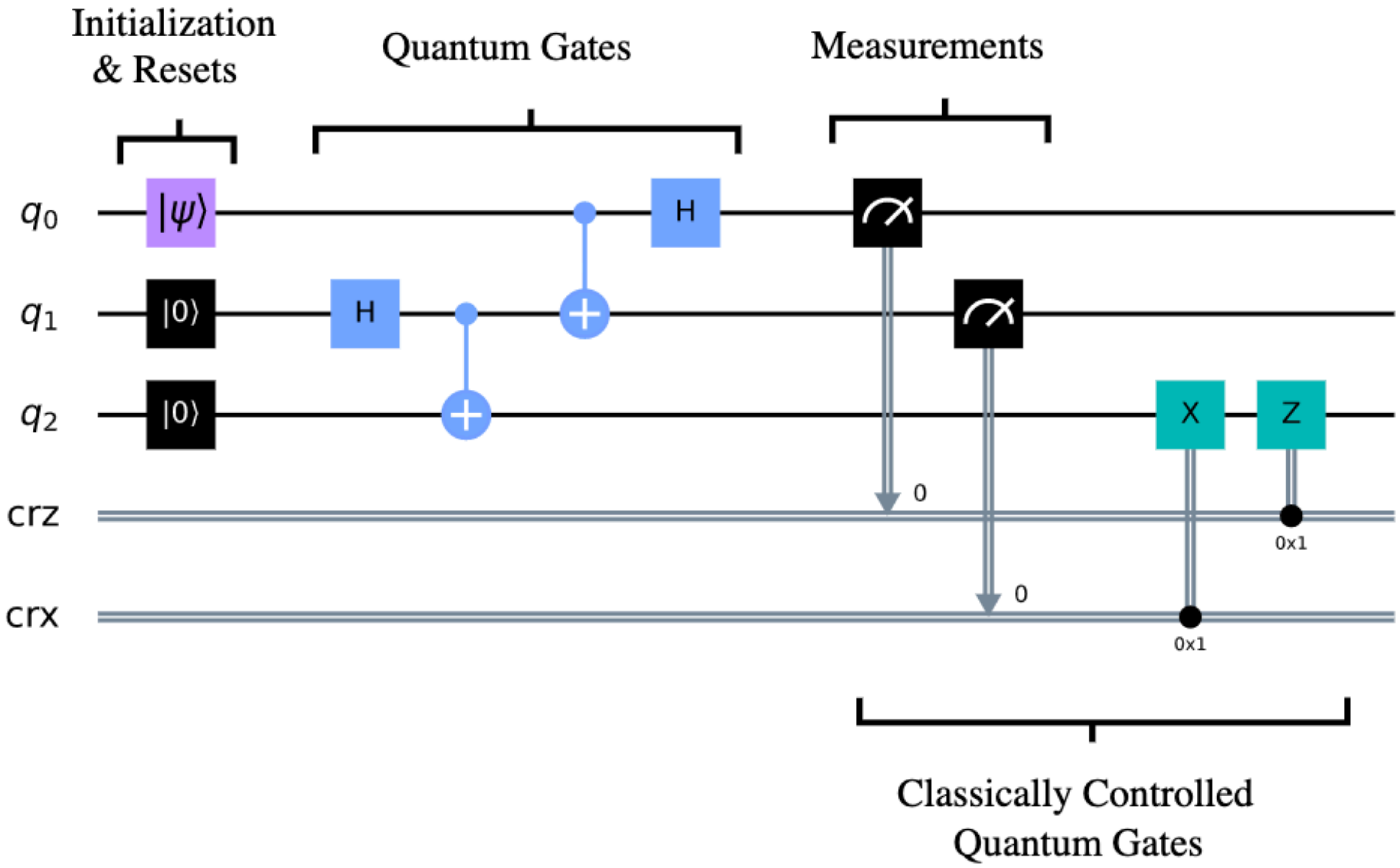
https://qiskit.org/textbook

Section 3

Quantum Circuit

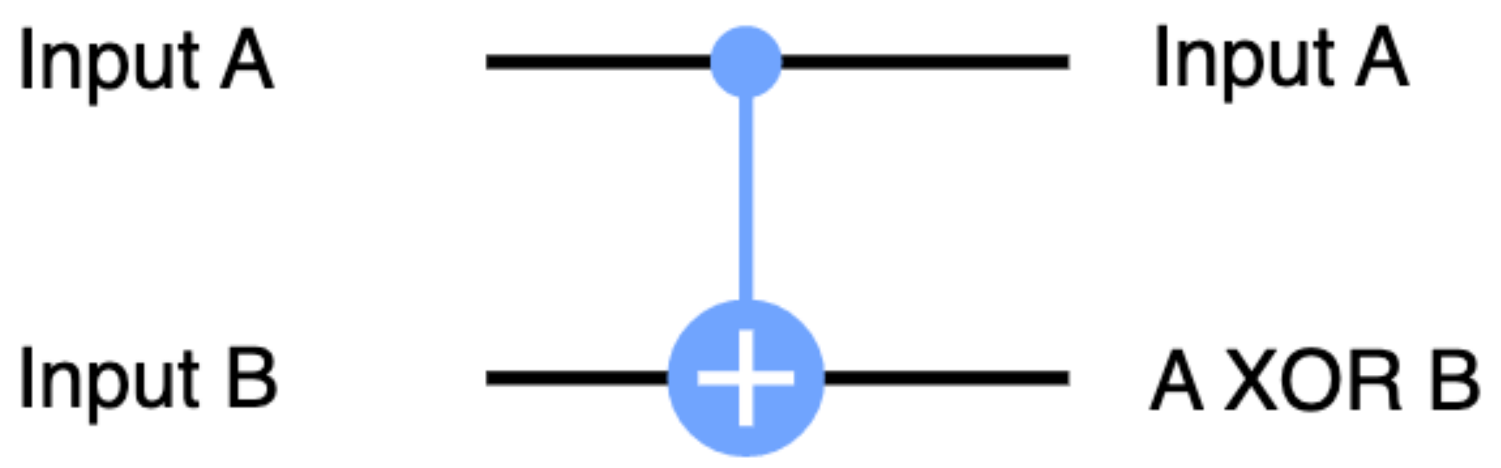
Quantum Circuit

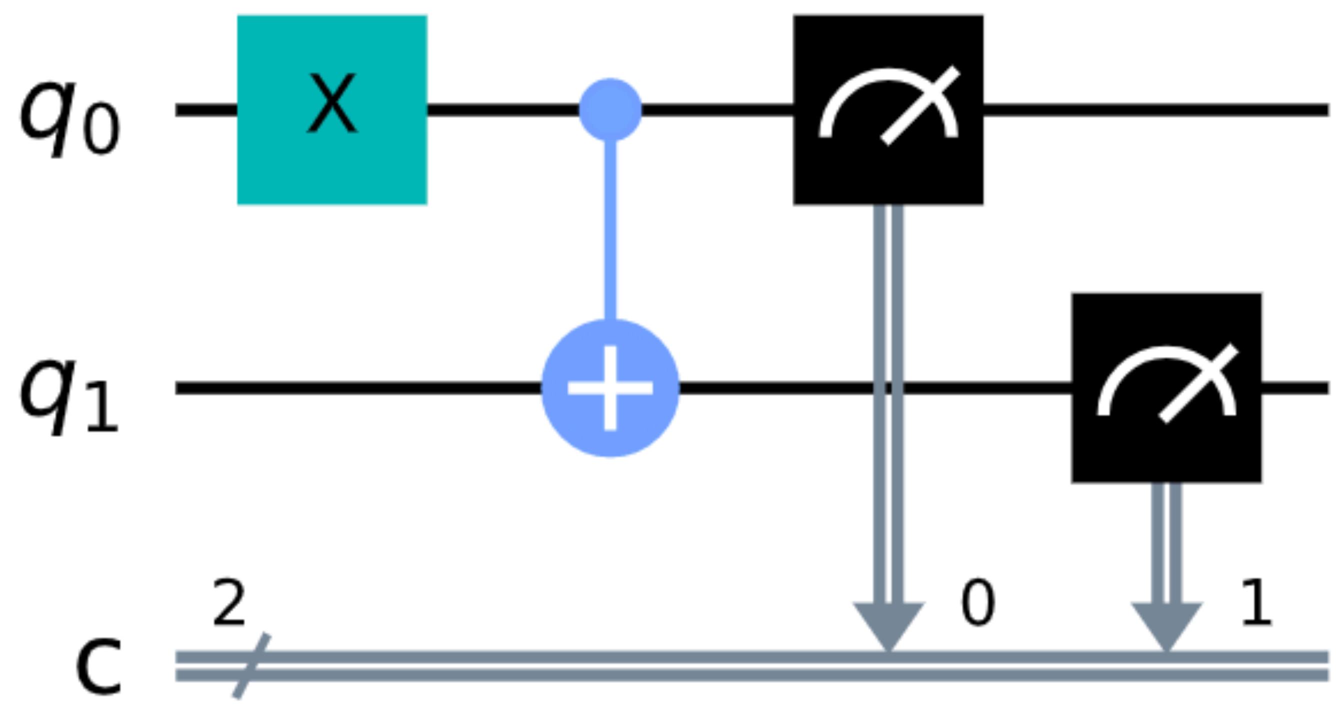
- Different components of a quantum circuit



An Adder Circuit

- CNOT gate again:





Input (q1 q0)	Output (q1 q0)
00	00
01	11
10	10
11	01

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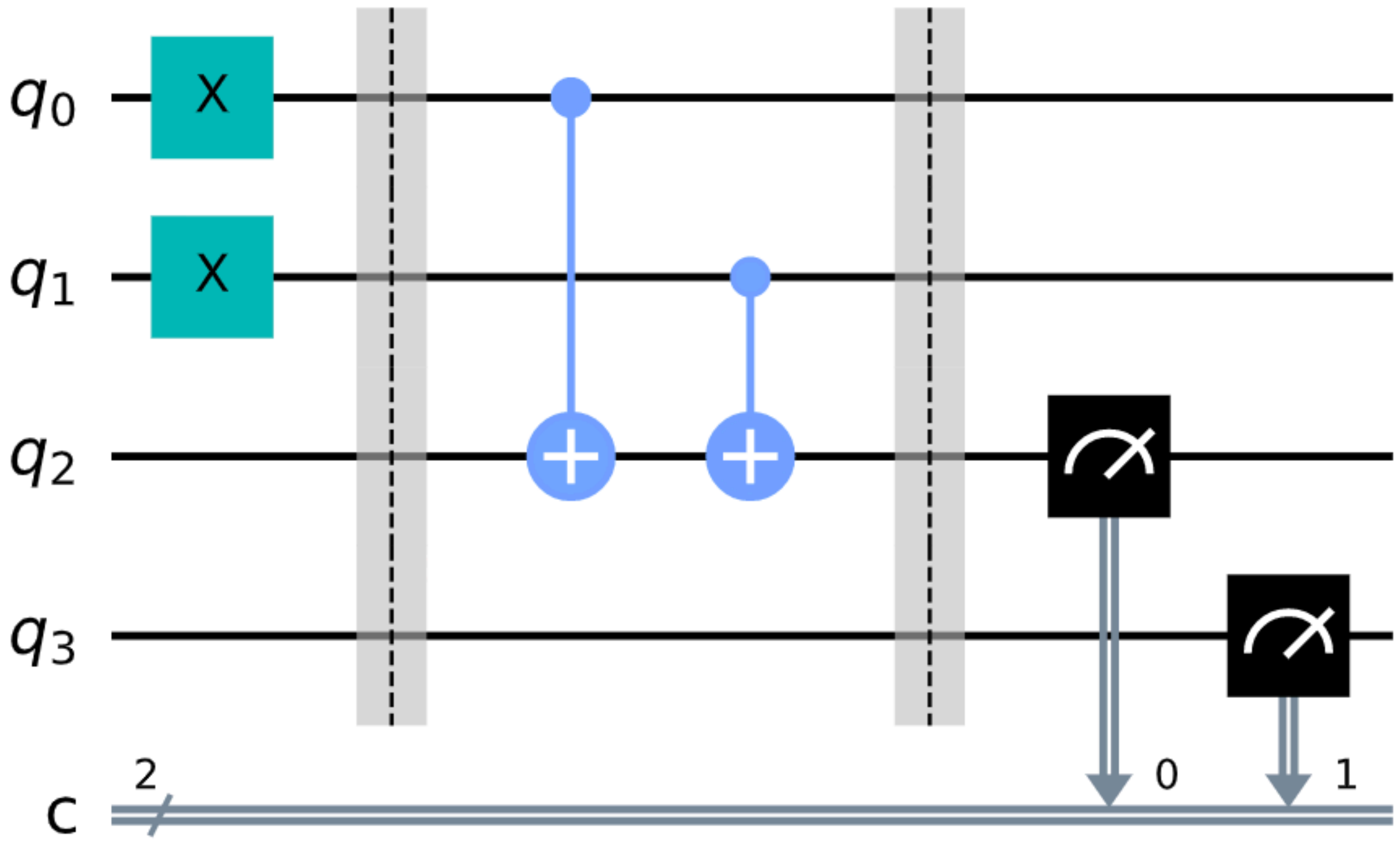
An Adder Circuit

- Half-adder achieved now



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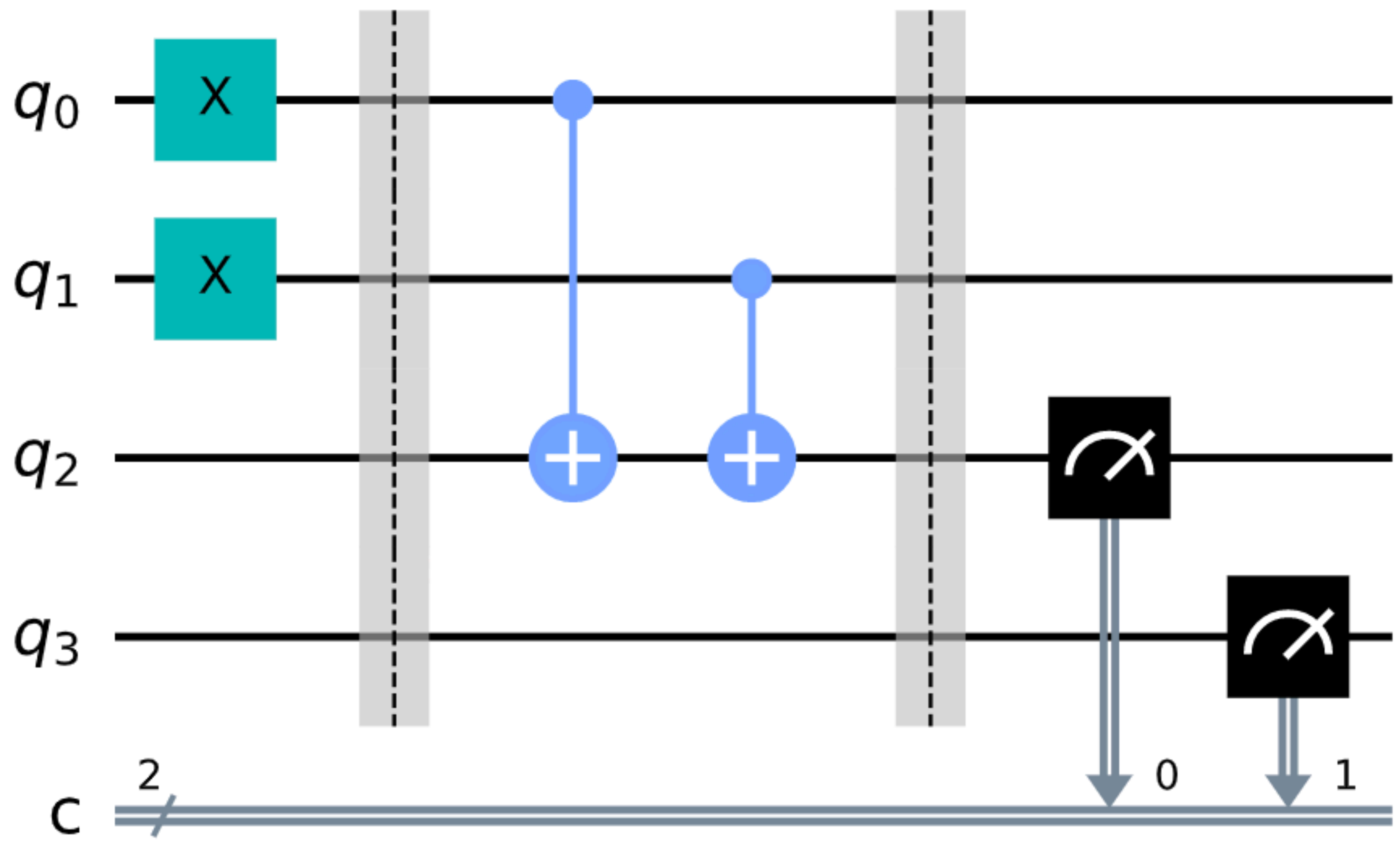
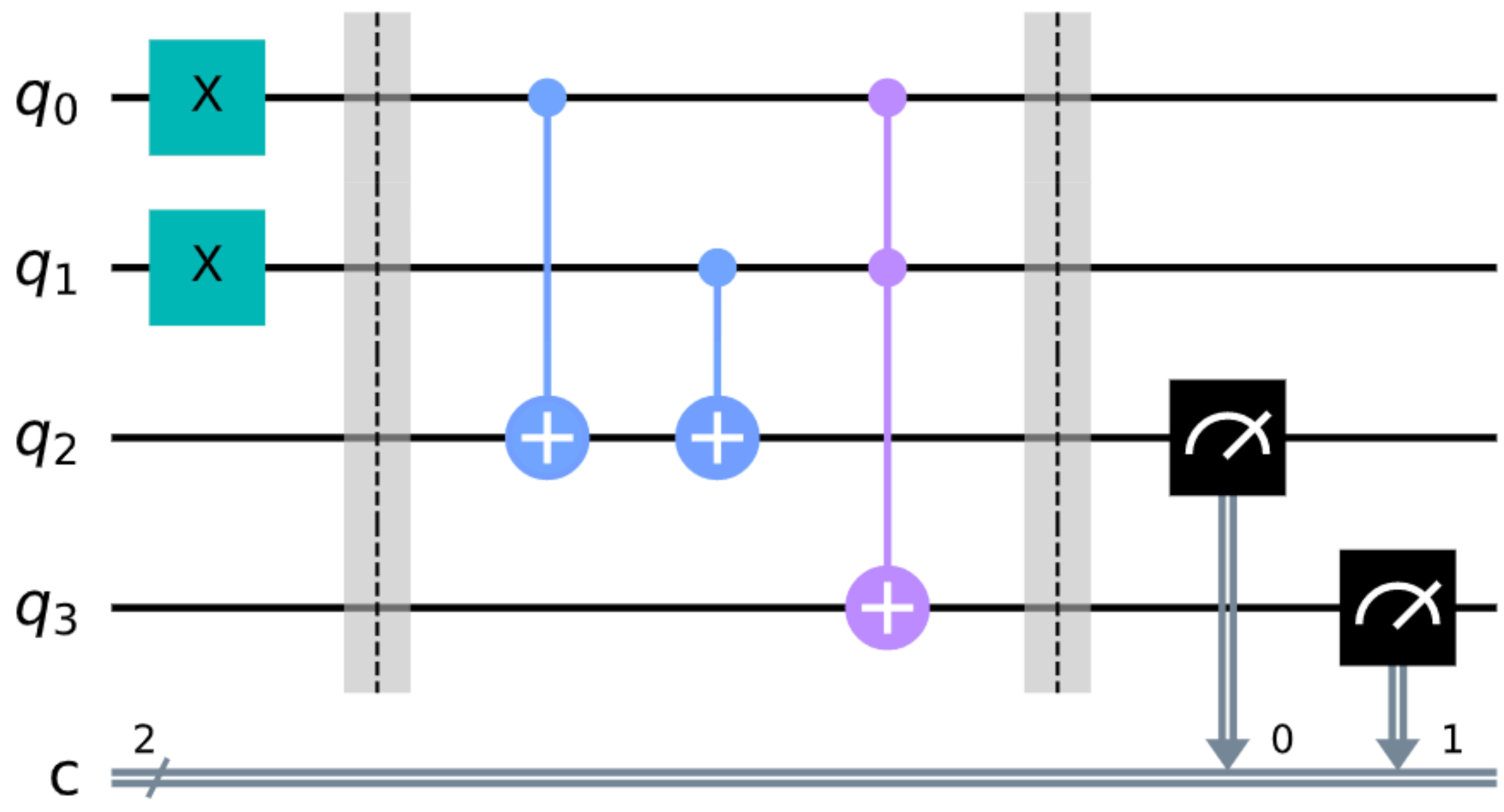
An Adder Circuit

- Half adder -> full adder





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Deutsch Algorithm

- How quantum brings better **parallelism**?
- Imagine we have a black box that computes a function map a single bit to another single bit x -> f(x)
- The computation is complicated, takes **1 day**
- How long does it take to know f(x)?

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Deutsch Algorithm

- Assume we only need to know f(x) is **constant or balanced**
  - constant: f(0) = f(1)
  - balanced: f(0) != f(1)
  - how long does it take to know the whether it is constant or balanced on classical machine?

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Deutsch Algorithm

- Now suppose we have a quantum machine that computes f(x).
- Since quantum compute is unitary and must be invertible, we need a quantum transformation Uf that take two qubits

$$U_f : |x\rangle|y\rangle \rightarrow |x\rangle|y \oplus f(x)\rangle$$

- Can we reduce the time to determine f(x) as constant or balanced?
- Yes!

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Deutsch Algorithm

- Prepare the second qubit in state of $\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$$
\begin{aligned}
U_f : |x\rangle \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \quad &\to \quad |x\rangle \frac{1}{\sqrt{2}}(|f(x)\rangle - |1 \oplus f(x)\rangle) \\
&= \quad |x\rangle (-1)^{f(x)} \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle),
\end{aligned}
$$

- Then suppose we prepare the first qubit in $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

$$
\begin{aligned}
U_f : \quad & \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \to \\
& \frac{1}{\sqrt{2}} \left[(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle\right] \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)
\end{aligned}
$$

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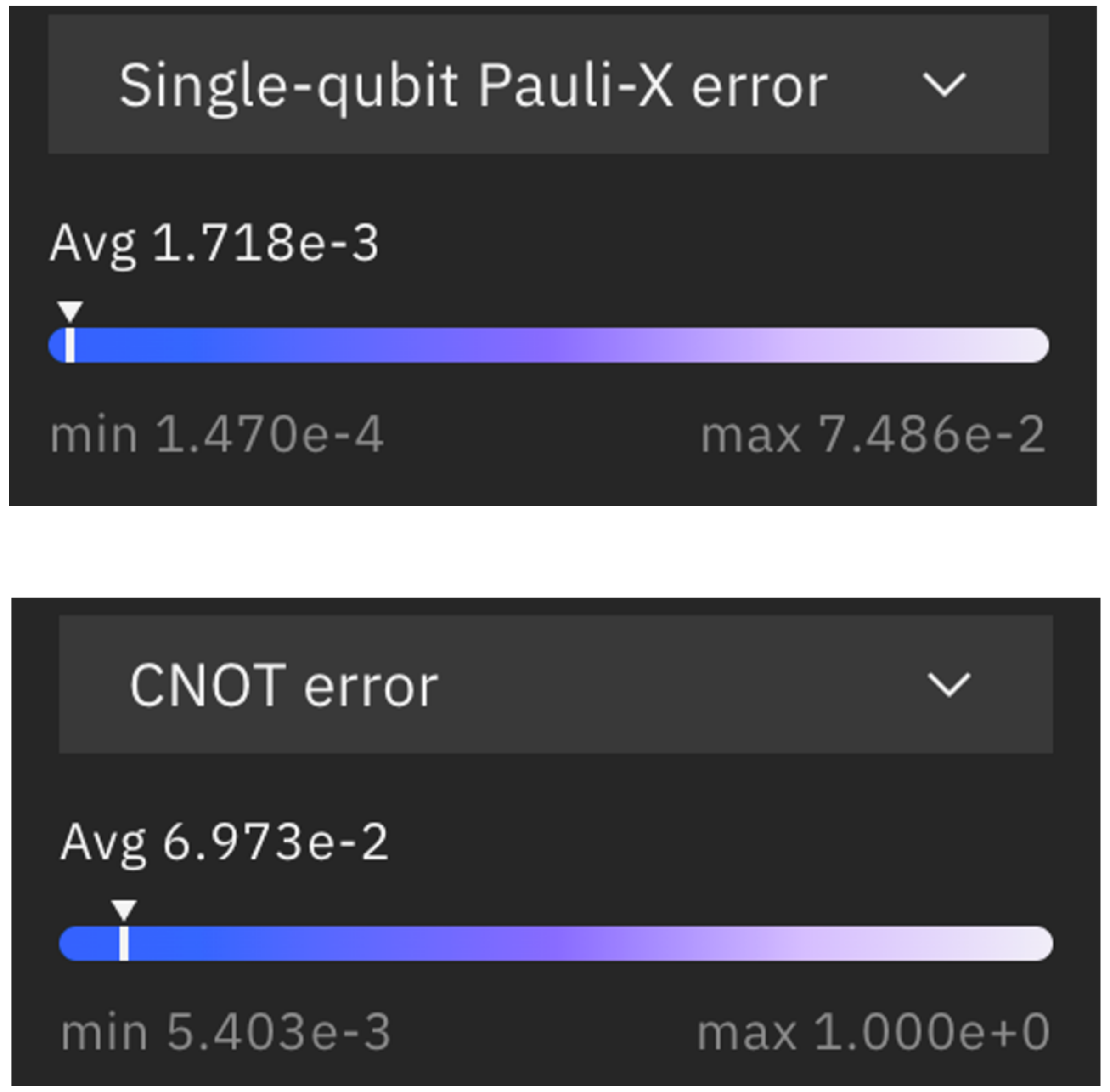
Section 4

NISQ Era

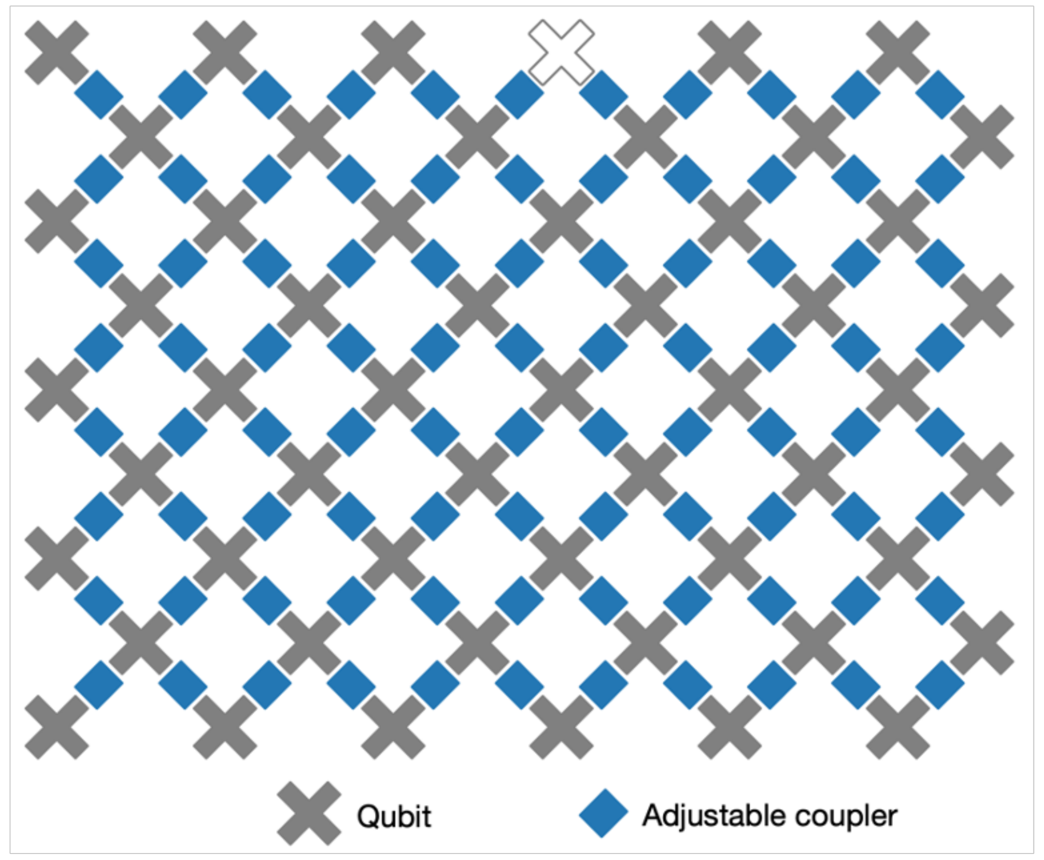
The NISQ Era

- Noisy Intermediate-Scale Quantum (NISQ)
  - **Noisy**: qubits are sensitive to environment; quantum gates are unreliable
  - **Limited number of qubits**: tens to hundreds of qubits
  - Limited connectivity: no all-to-all connections



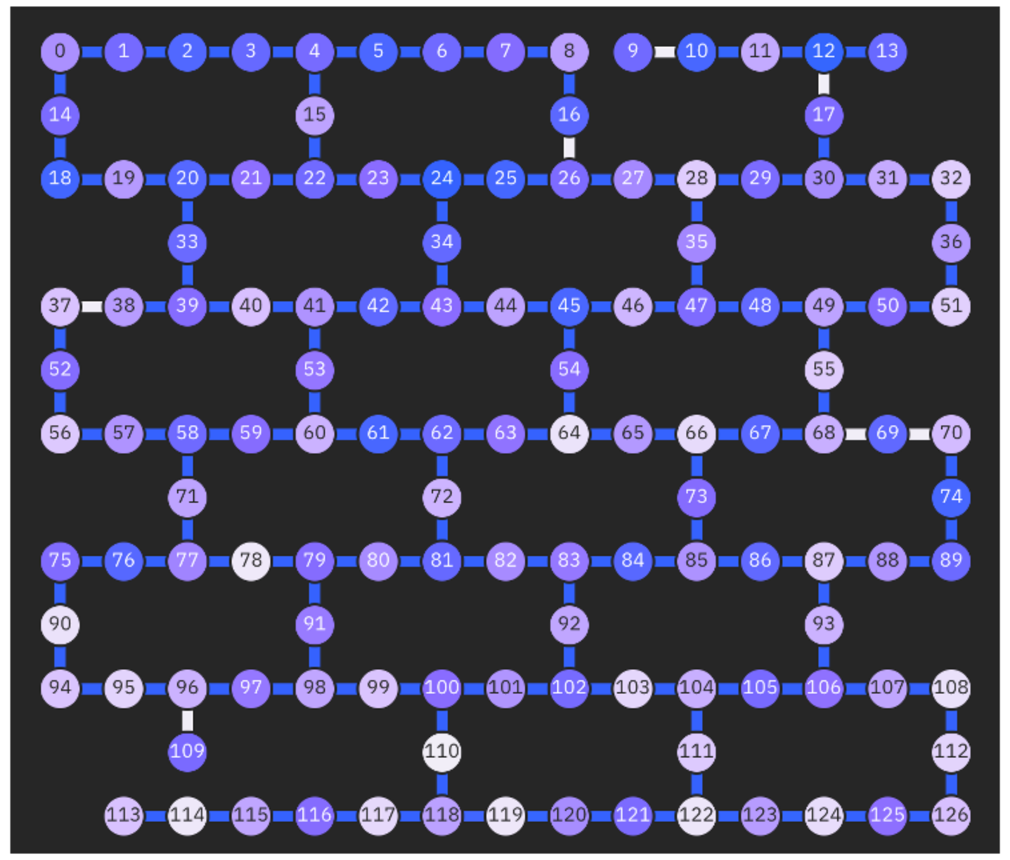
IBMQ Gate Error Rate

https://quantum-computing.ibm.com/



Google Sycamore 53Q

https://www.nature.com/articles/s41586-019-1666-5

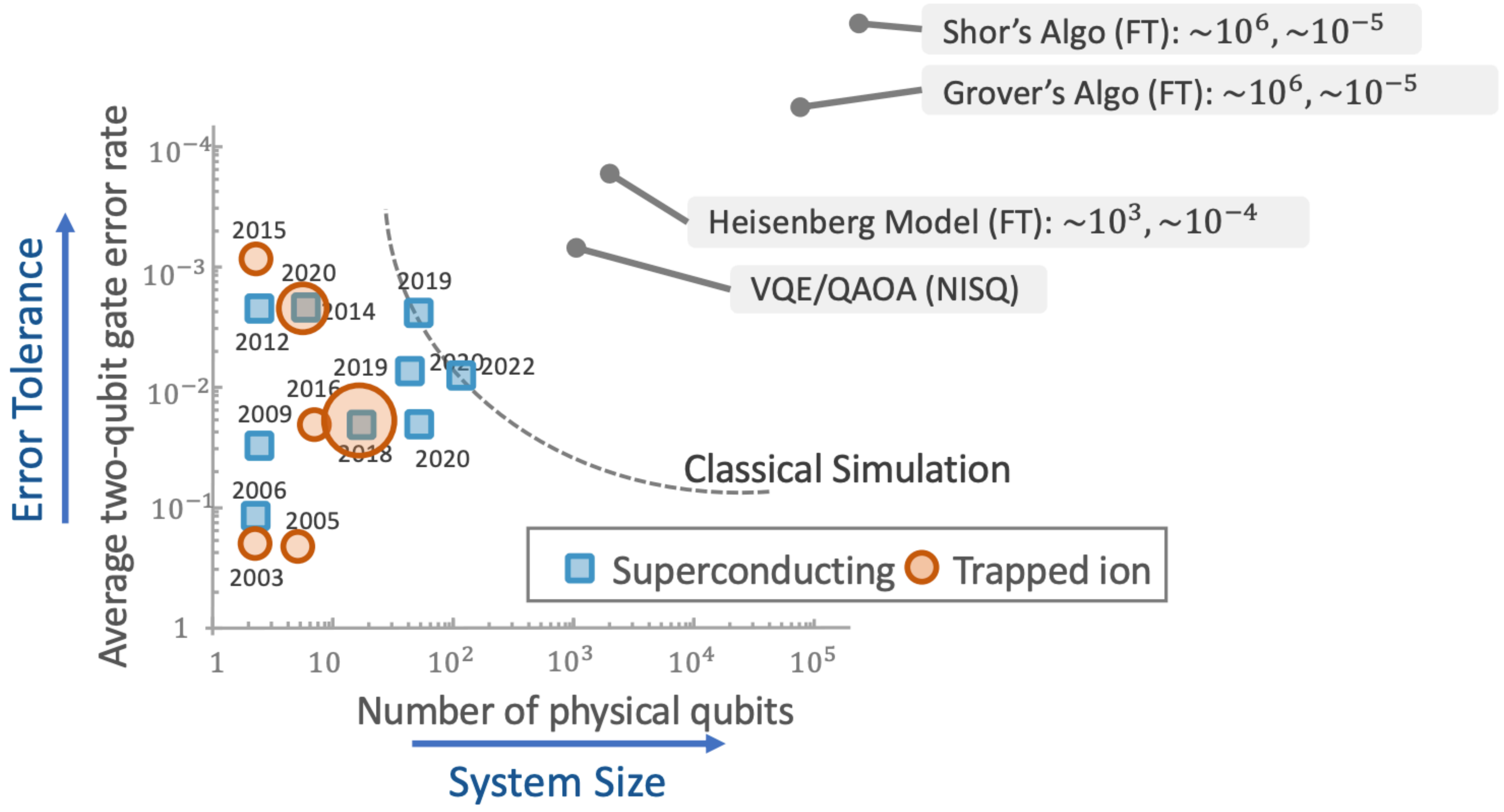


IBM Washington 127Q

https://quantum-computing.ibm.com/

Gap between the Algorithm and Device

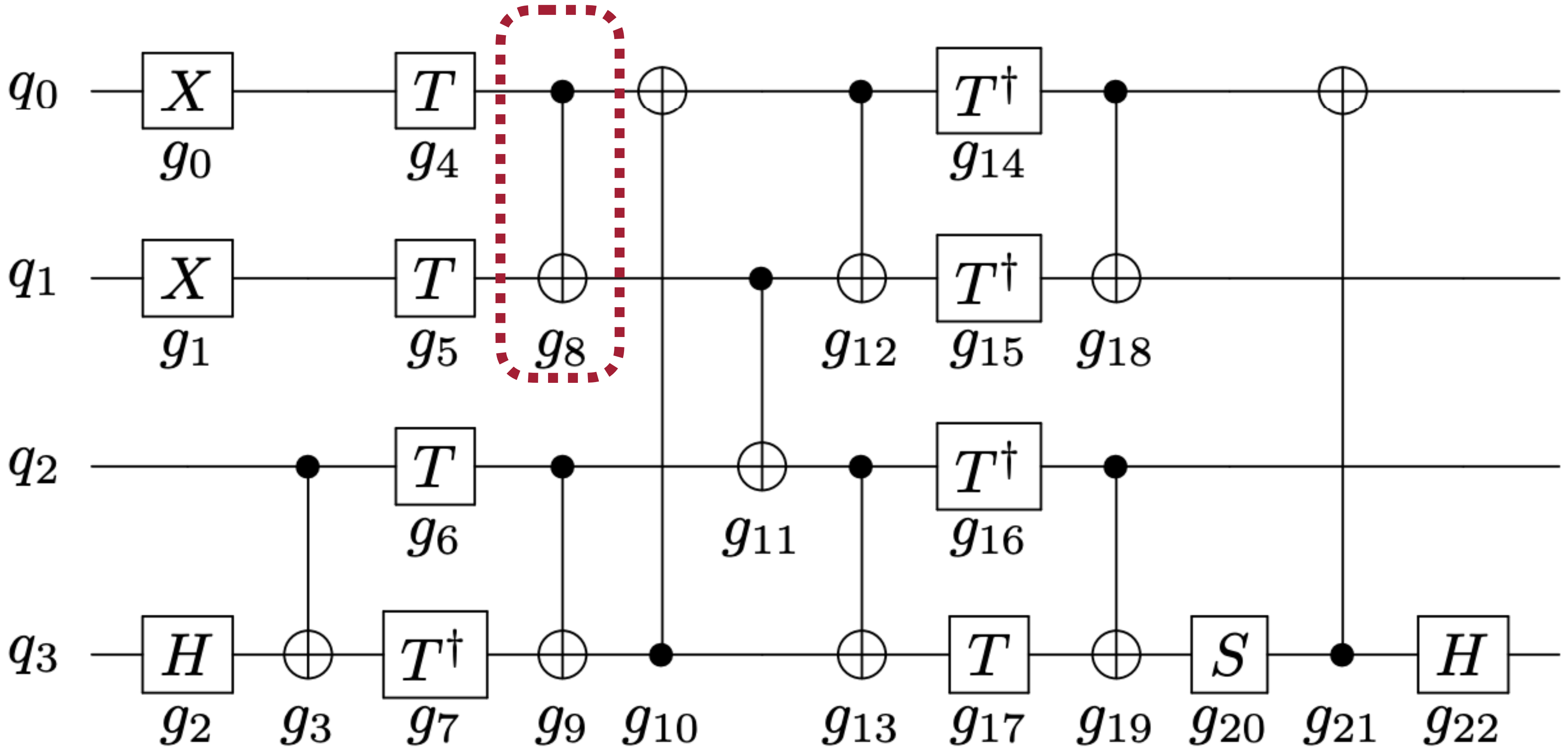
- Gap between algorithm and devices

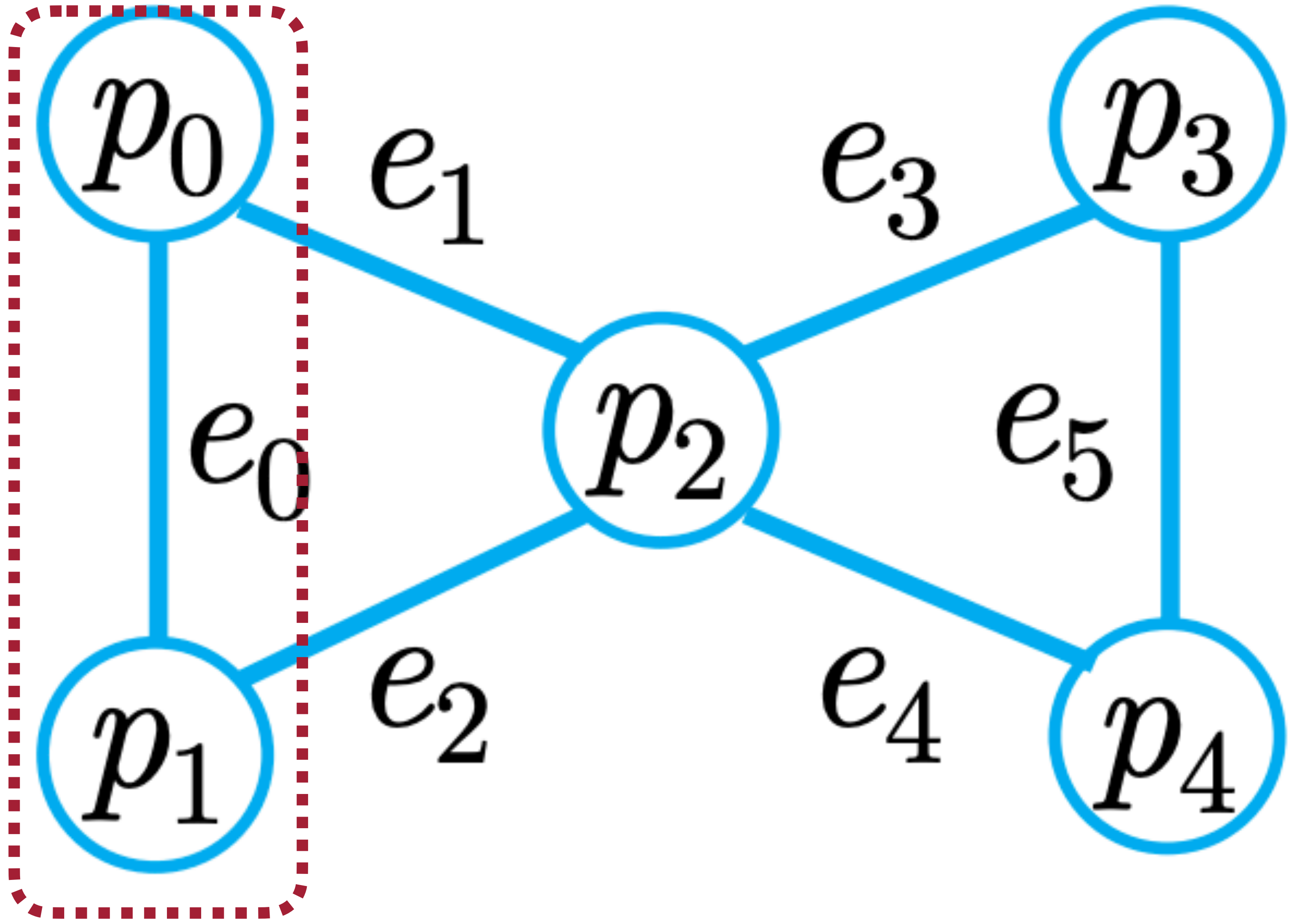


*Size of data point indicates connectivity; larger means denser connectivity.

Ding & Chong

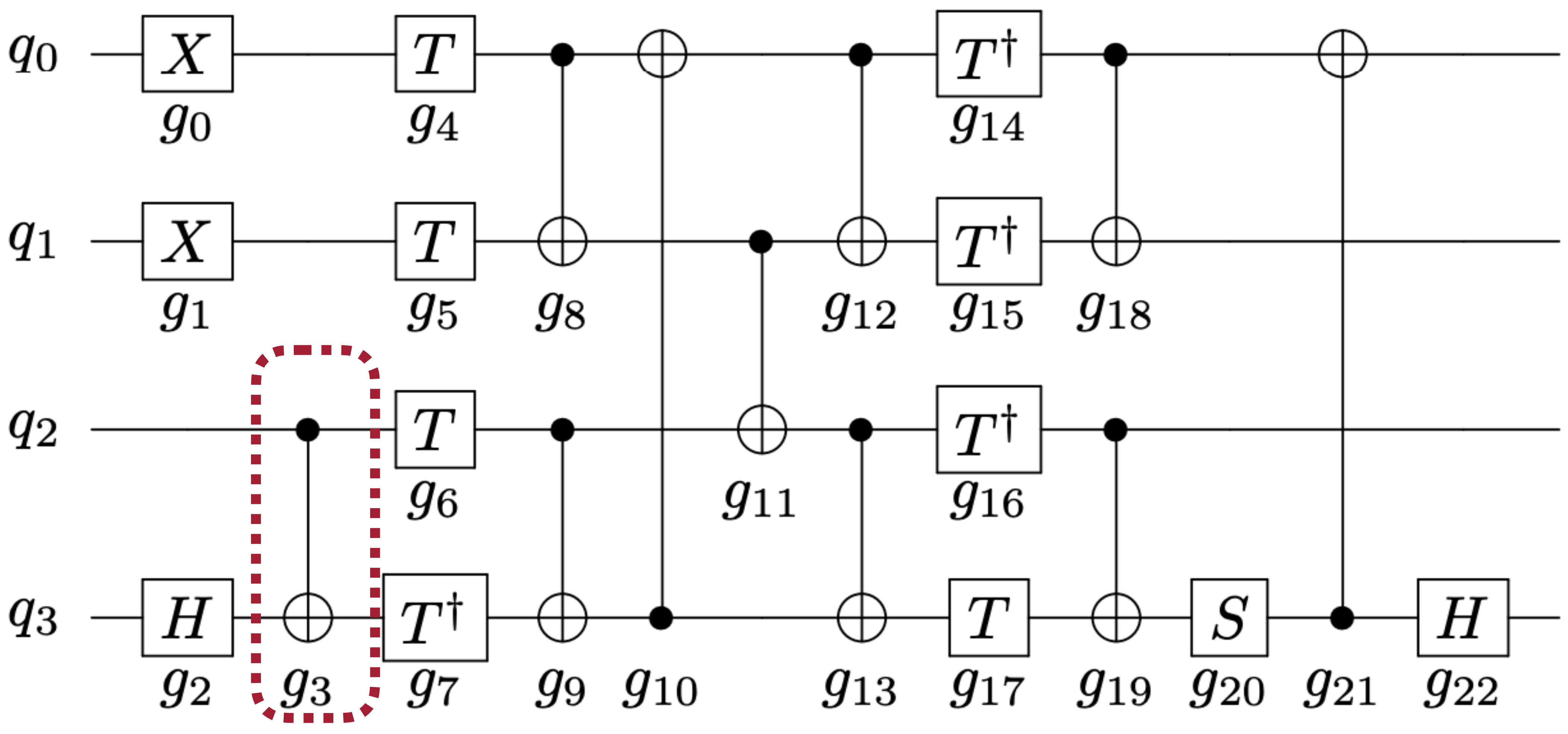
Qubit Mapping

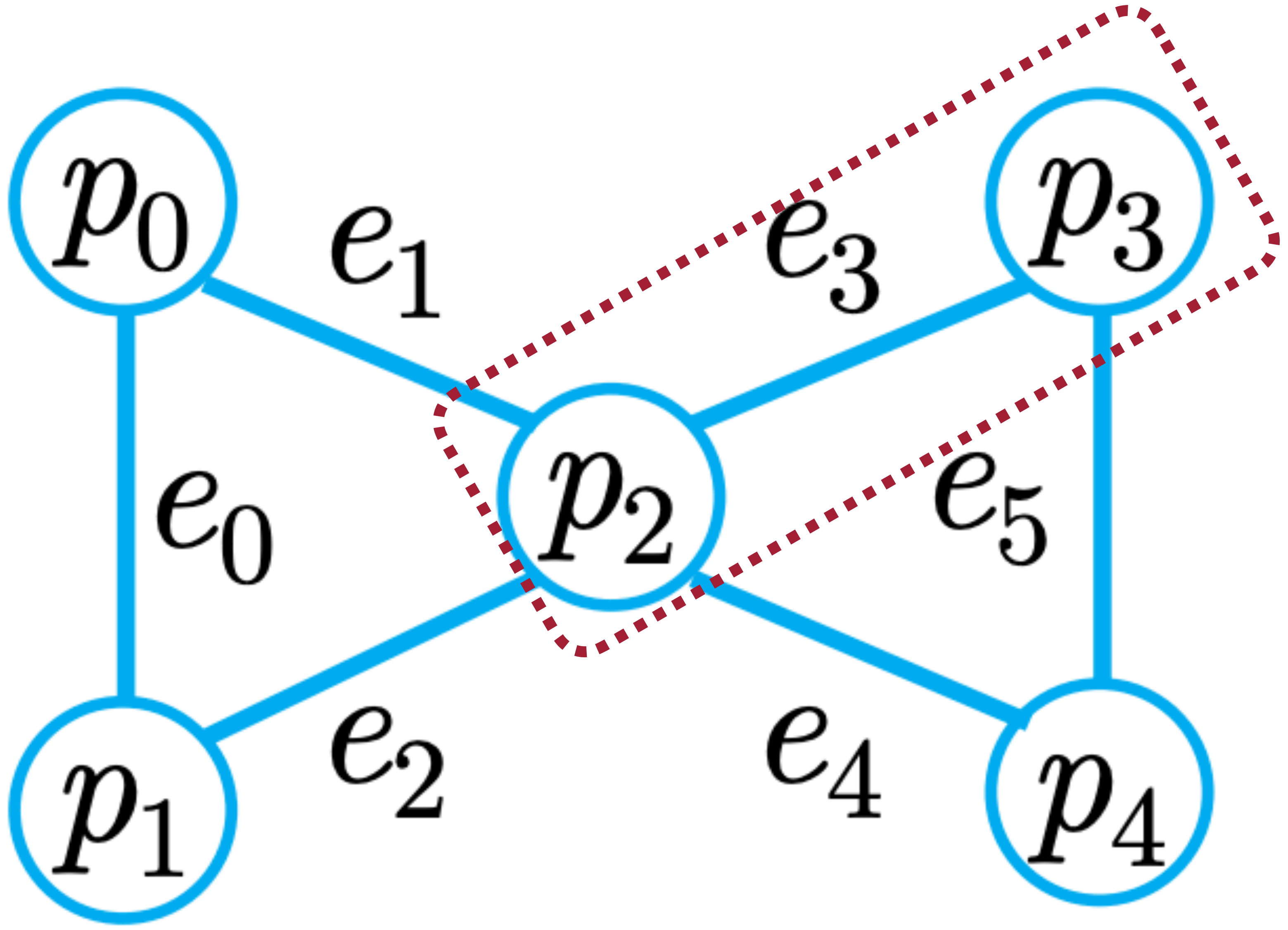




Bochen Tan et al OLSQ

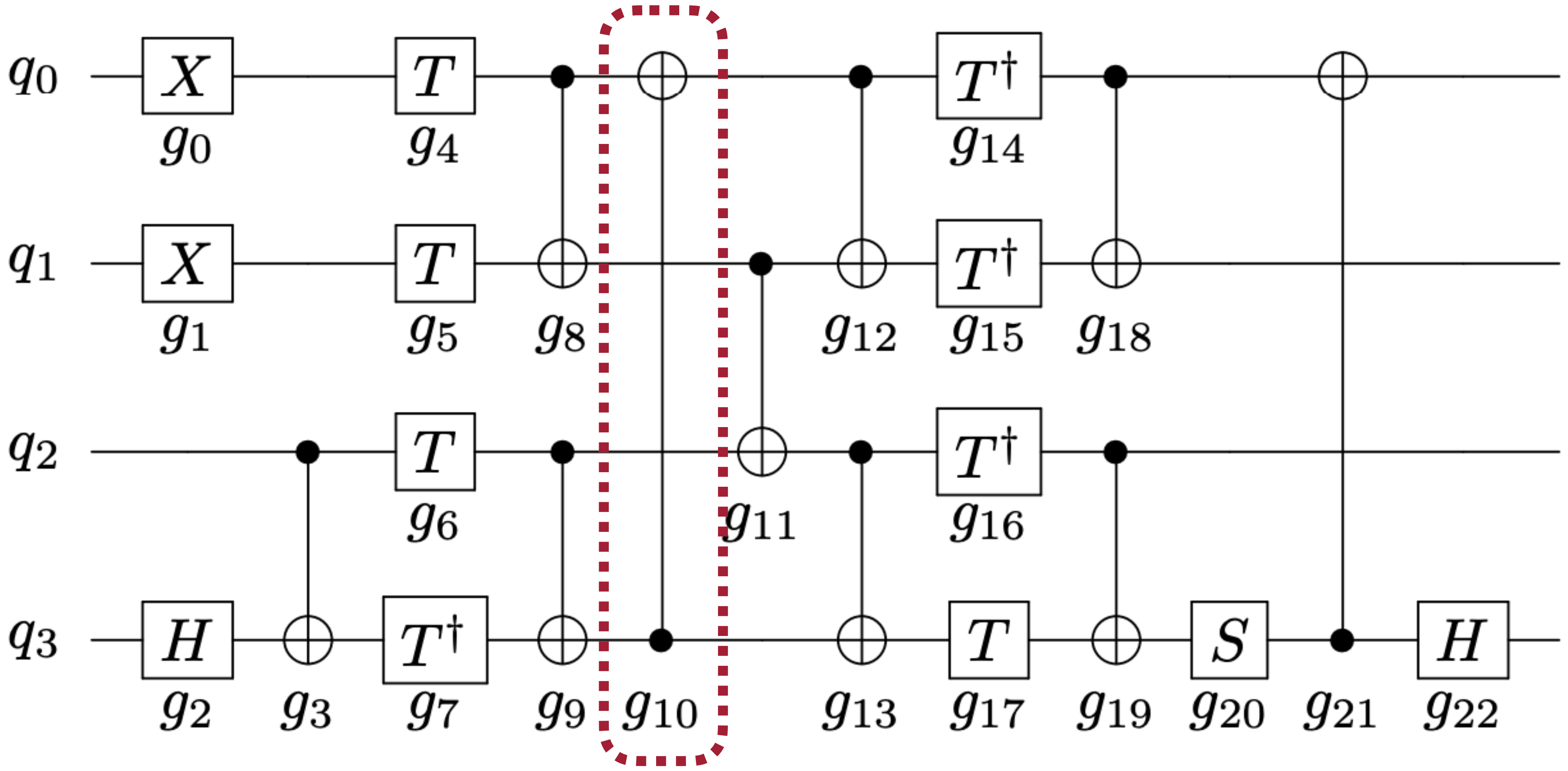
Qubit Mapping

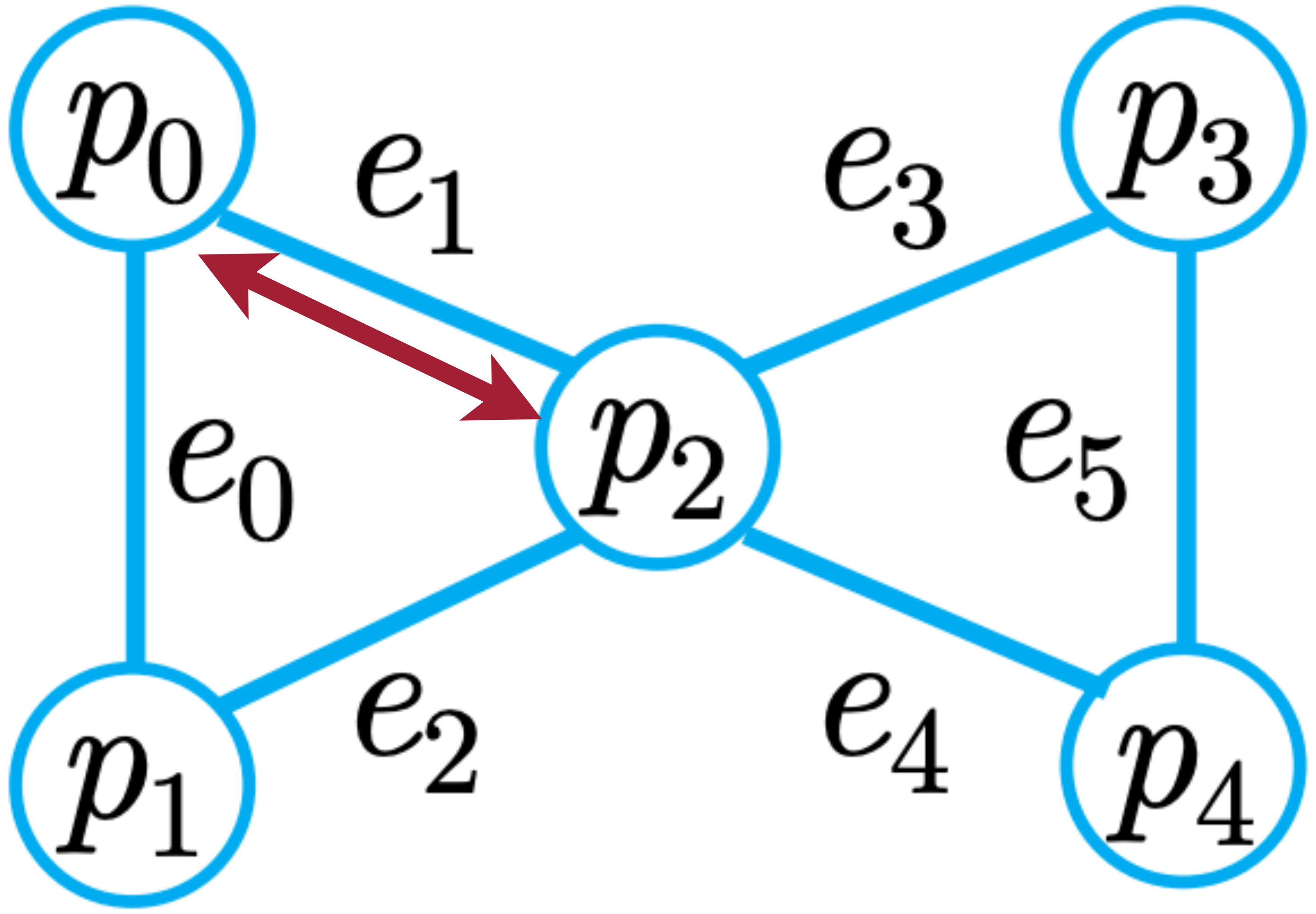




Bochen Tan et al OLSQ

Qubit Mapping

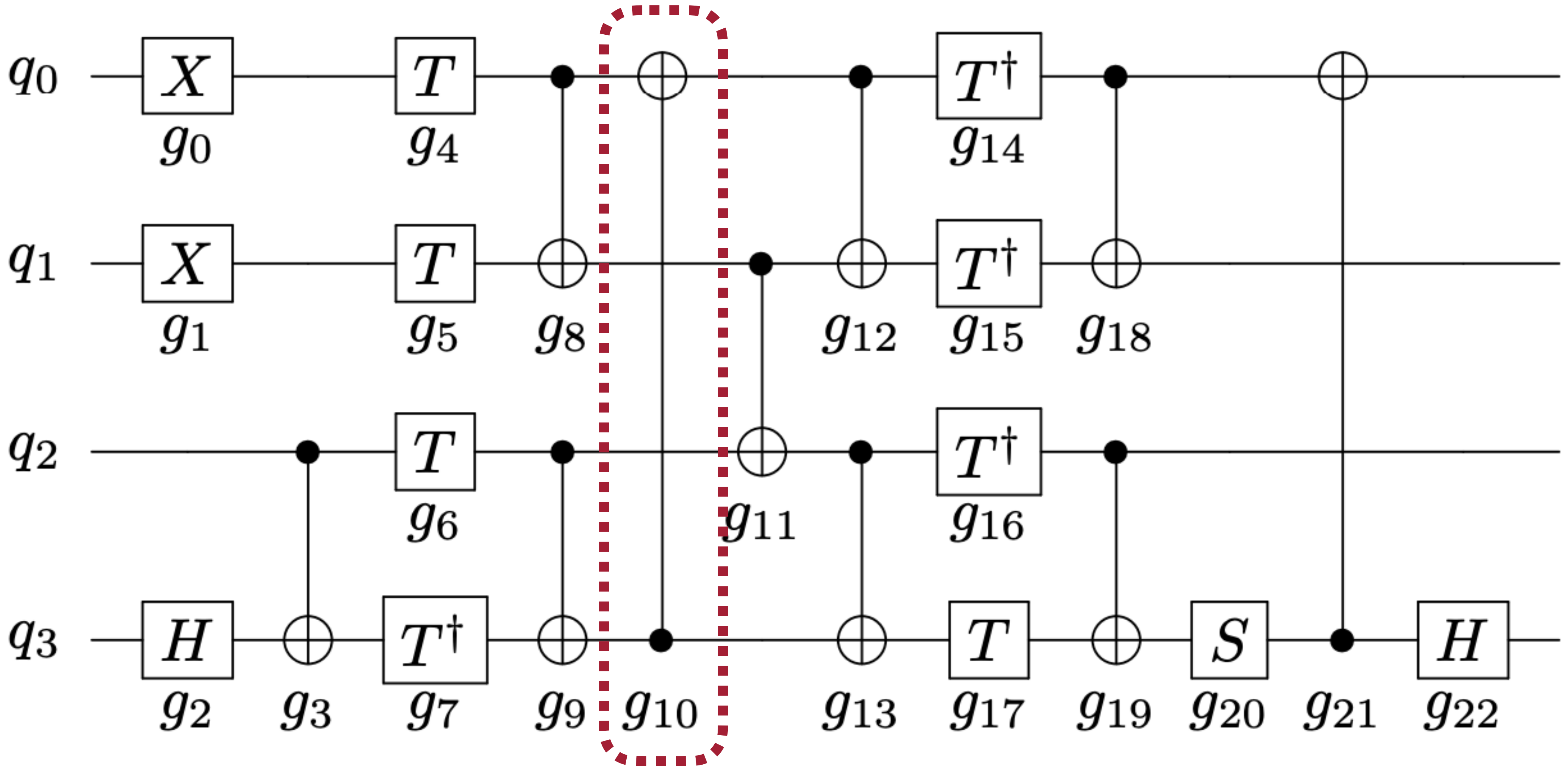


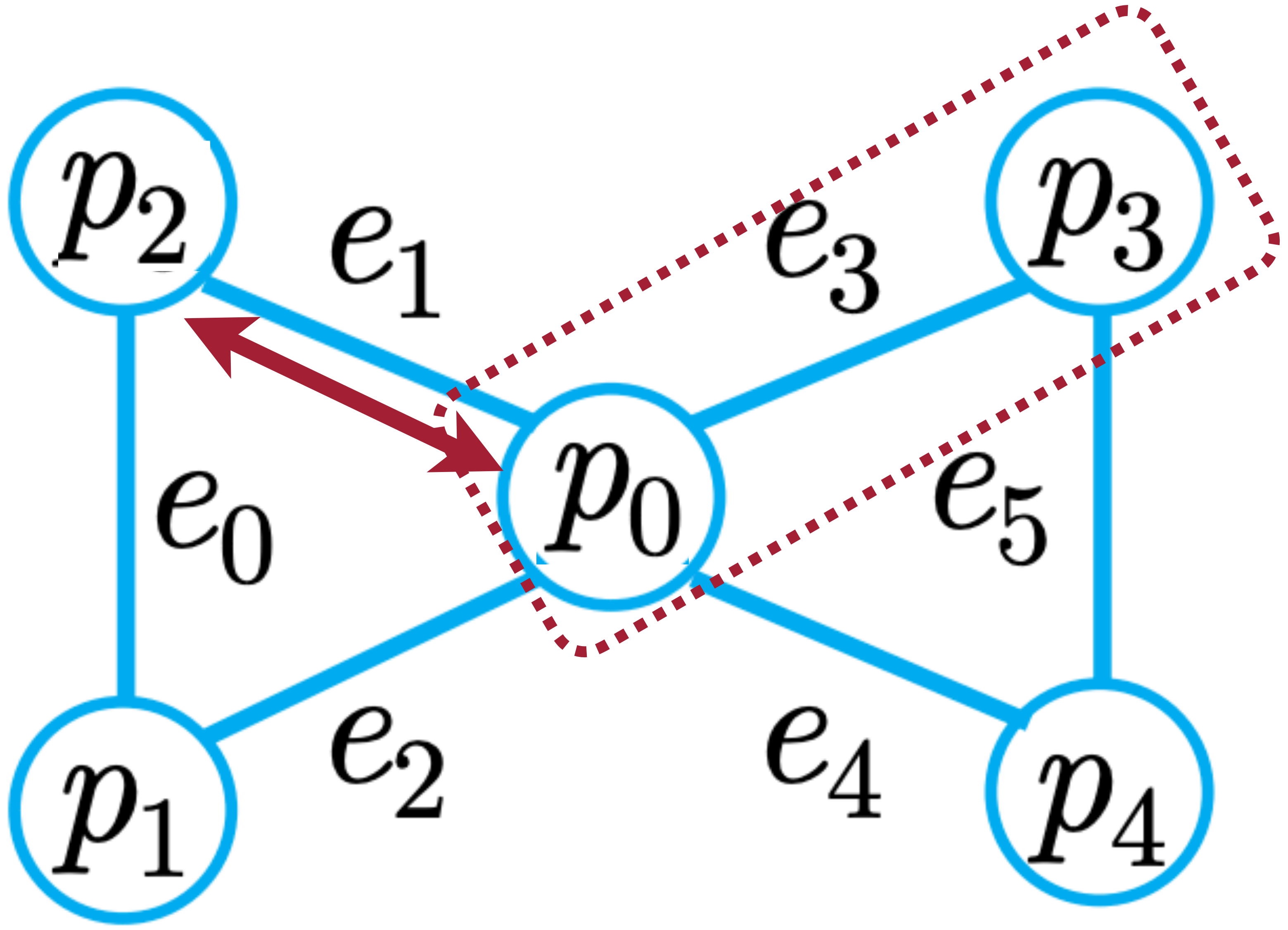


Bochen Tan et al OLSQ

Qubit Mapping

- How to design the best scheduling? When to do the swap?



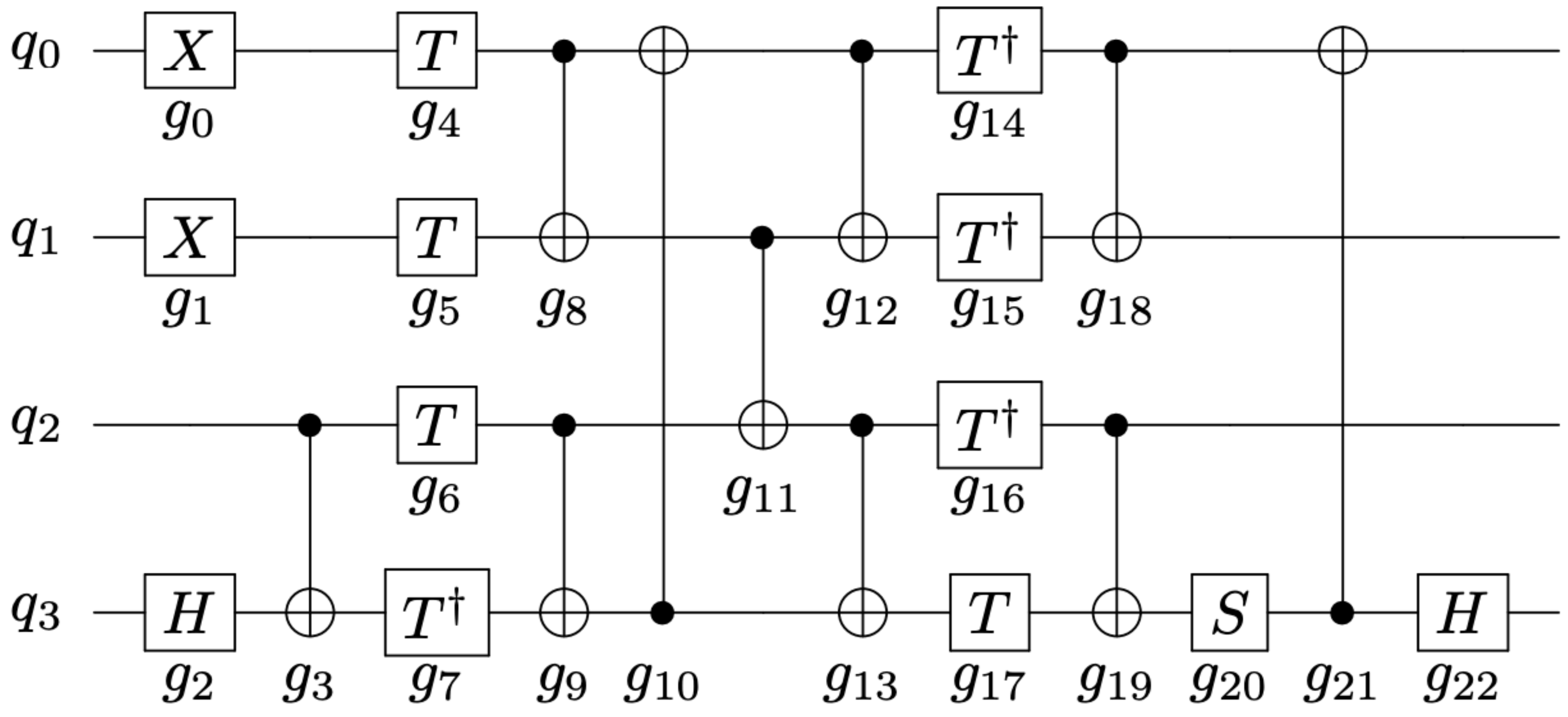


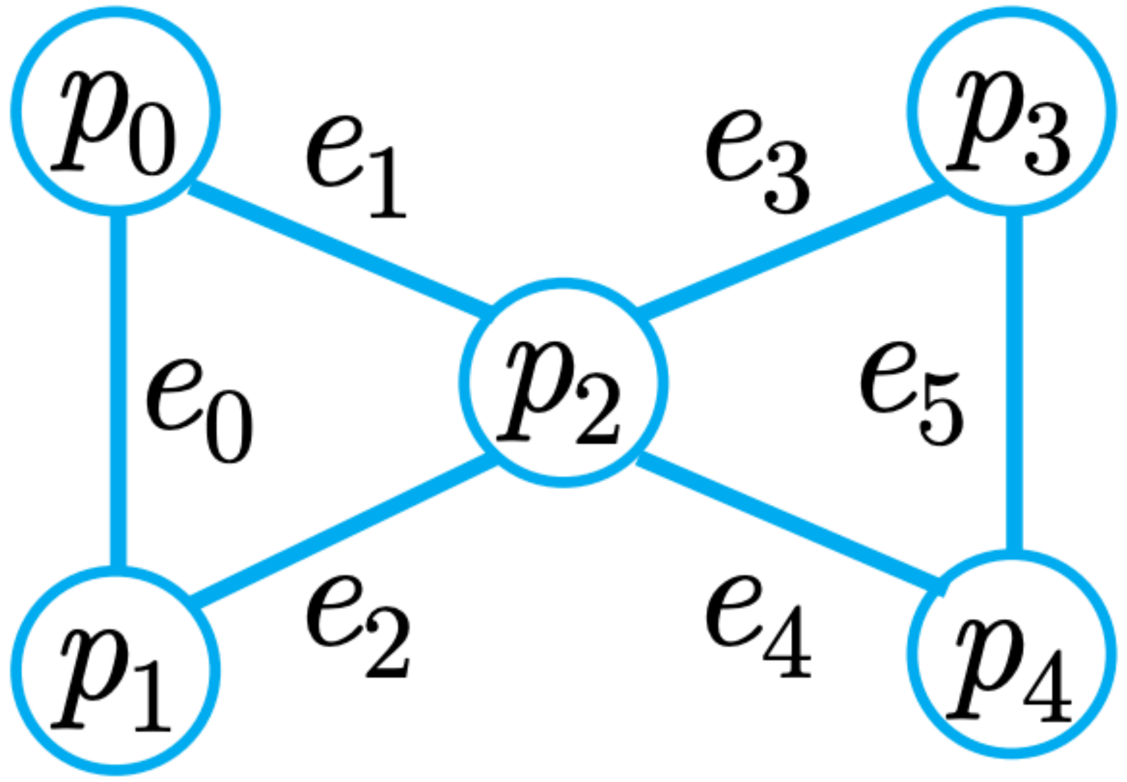
Bochen Tan et al OLSQ

Sabre Qubit Mapping

- How to design the best scheduling? When to do the swap?

$score = [];$
$SWAP\_candidate\_list =$
$Obtain\_SWAPs(F, G);$
for $SWAP$ *in* $SWAP\_candidate\_list$ **do**
  $\pi_{temp} = \pi.update(SWAP);$
  $score[SWAP] =$
  $\mathbf{H}(F, DAG, \pi_{temp}, D, SWAP);$
end
Find the SWAP with minimal score;
$\pi = \pi.update(SWAP);$

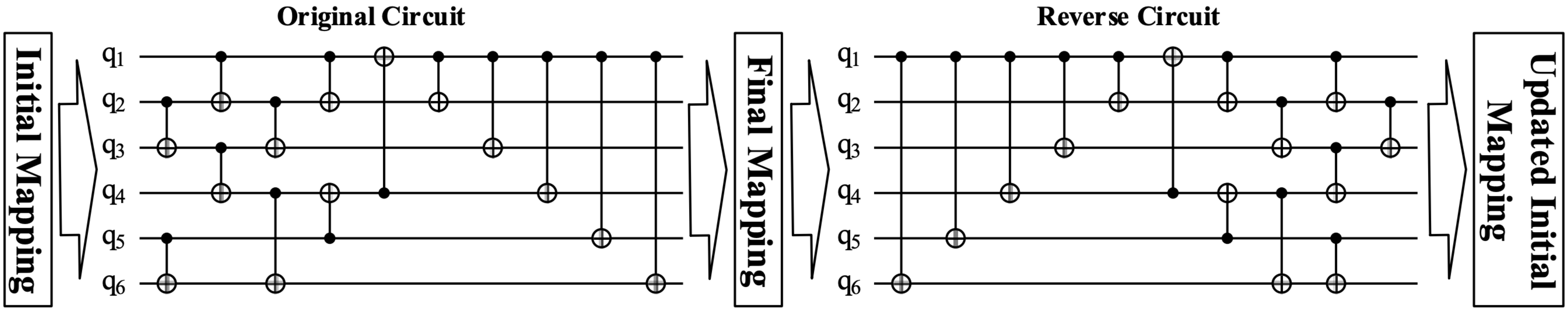




Gushu Li et al Sabre

Sabre Qubit Mapping

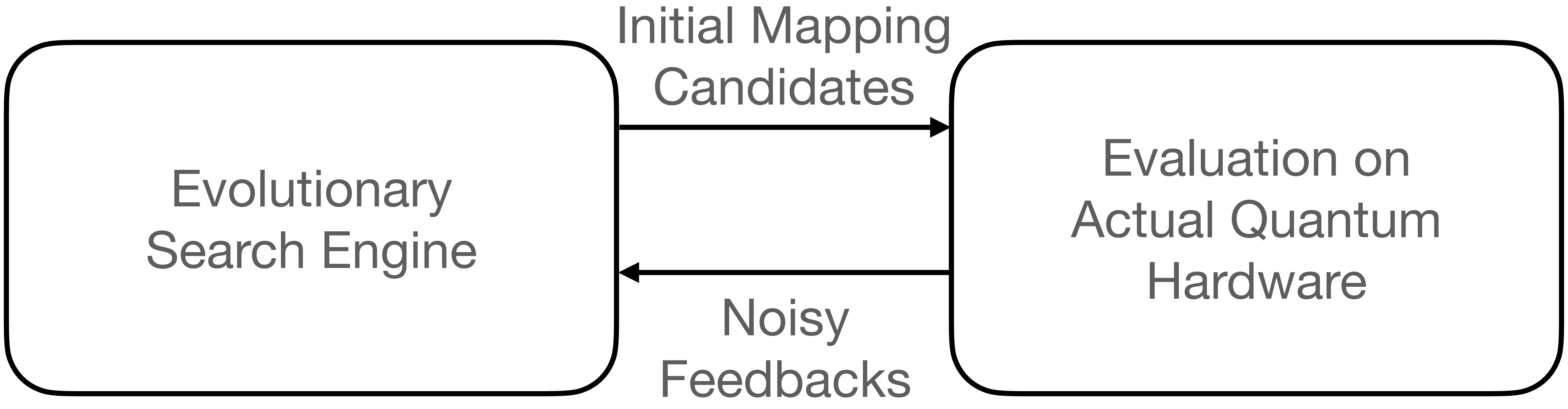
- How to design the initial mapping?



Gushu Li et al Sabre

QuantumNAS Qubit Mapping Search

- Evolutionary Search for initial mapping selection



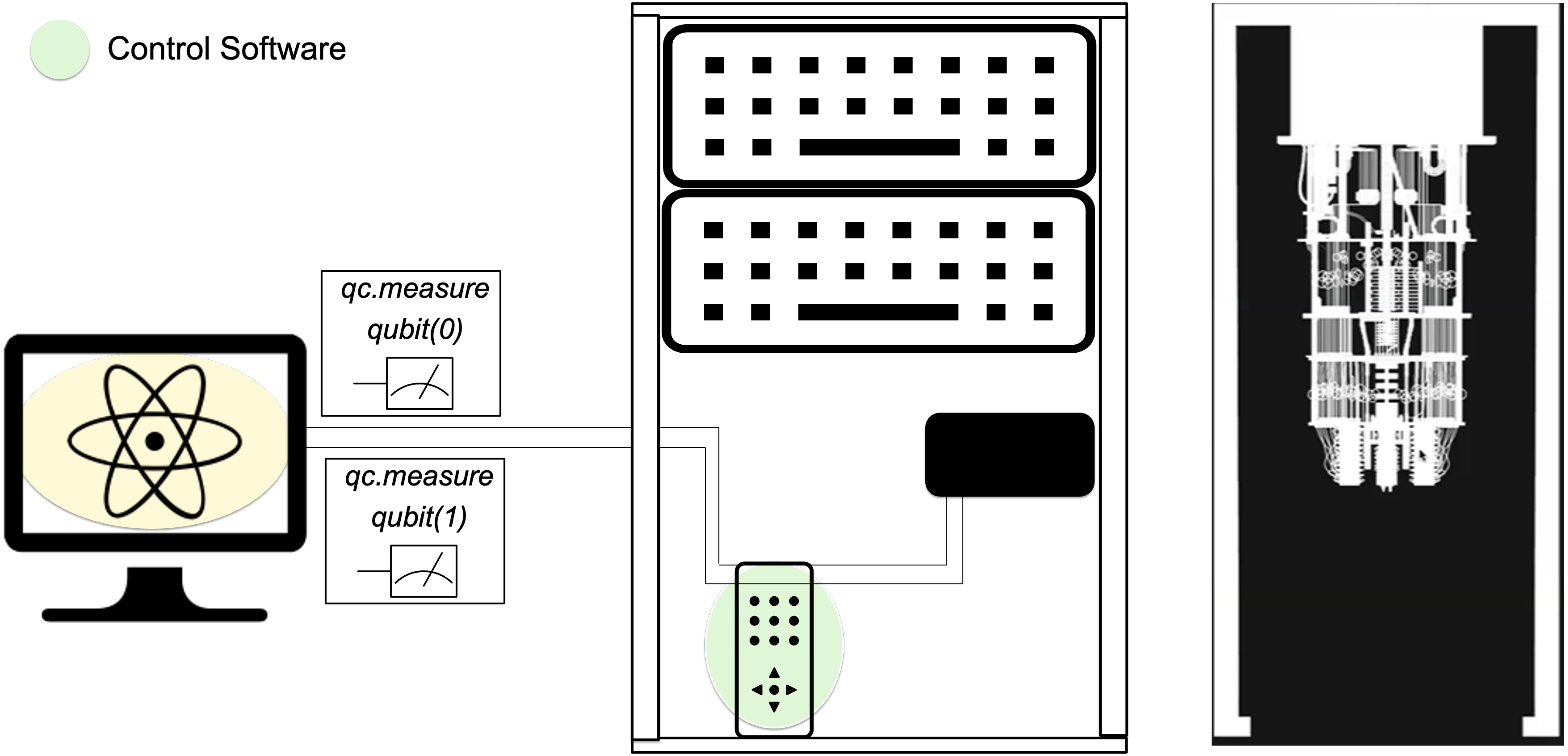
Hanrui Wang et al QuantumNAS

Section 5

Example flow on superconducting quantum computer

Control System

- Application Software
- Control Software



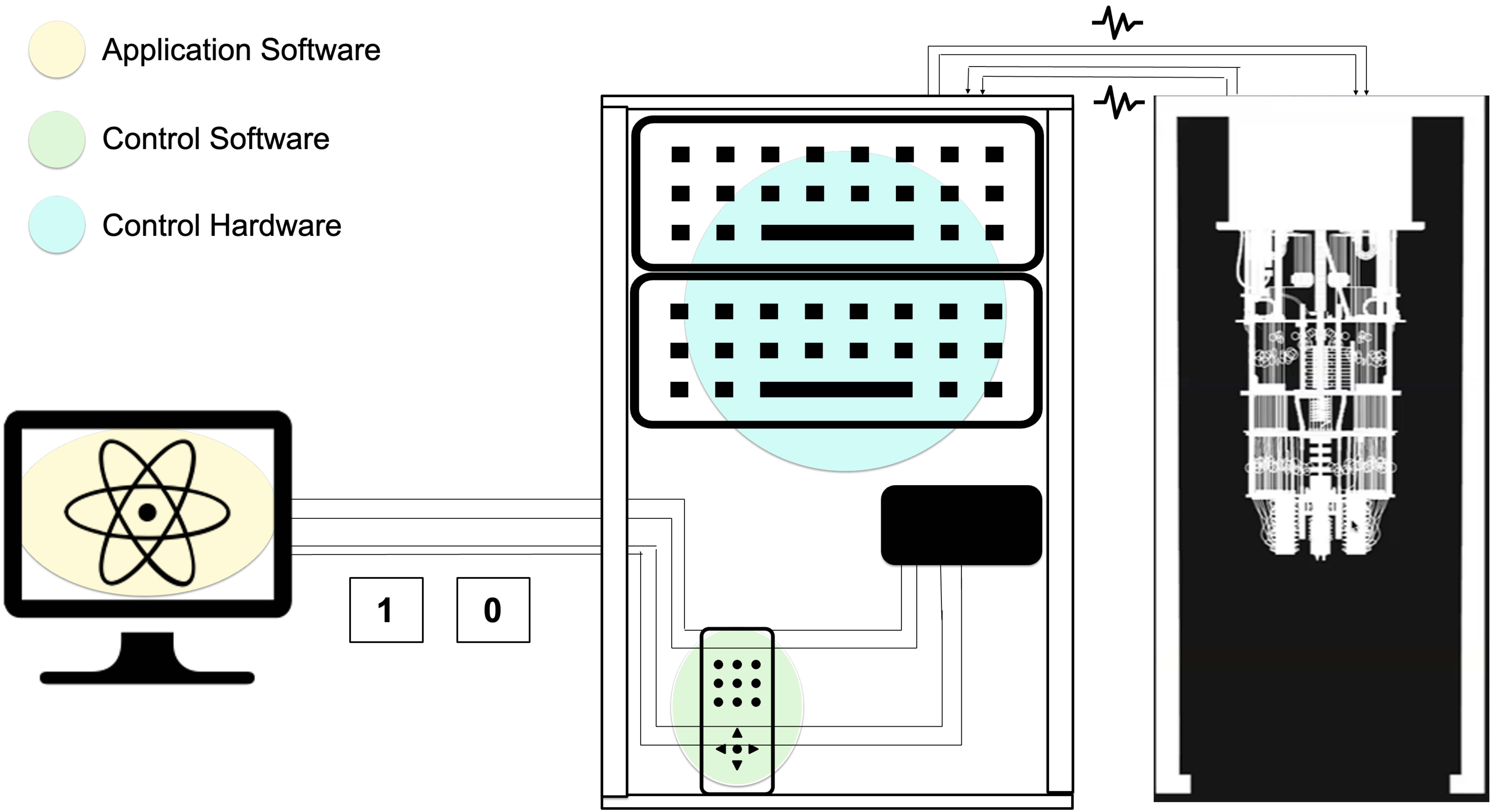
Hanrui Wang et al QuantumNAS

Control System

Application Software

Control Software

Control Hardware



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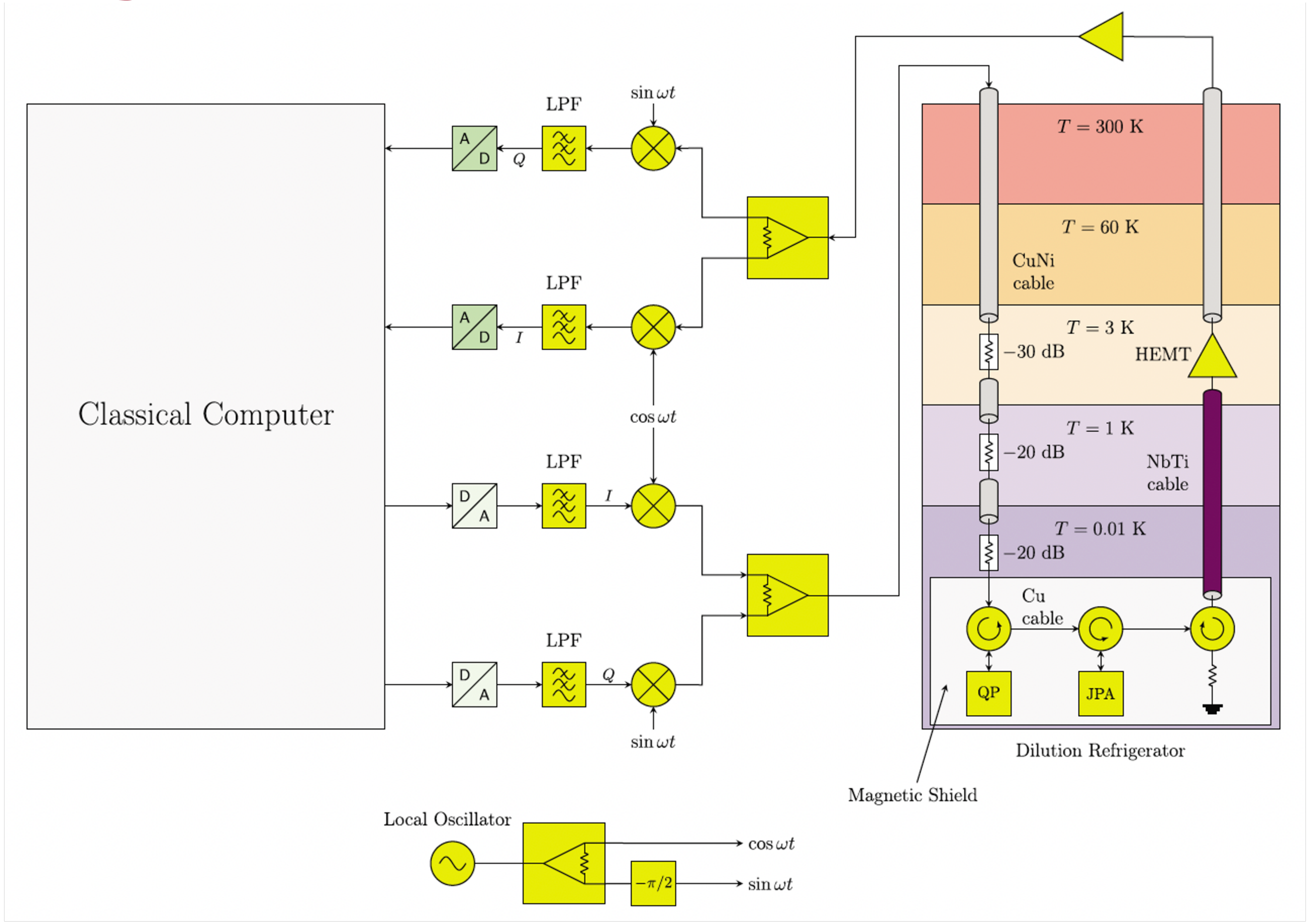
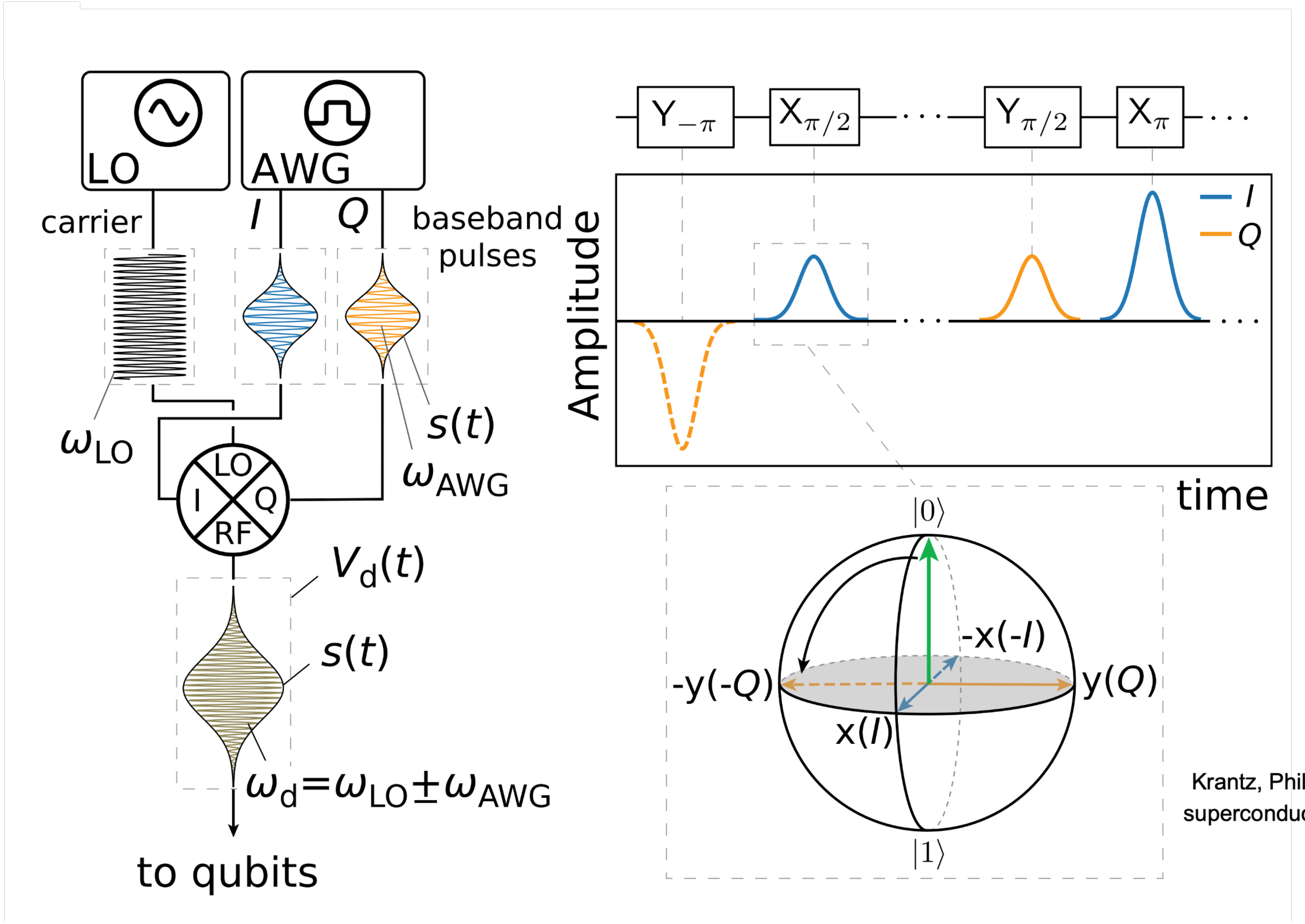
Control System



Stancil, Daniel D., and Gregory T. Byrd. Principles of superconducting quantum computers. John Wiley & Sons, 2022.

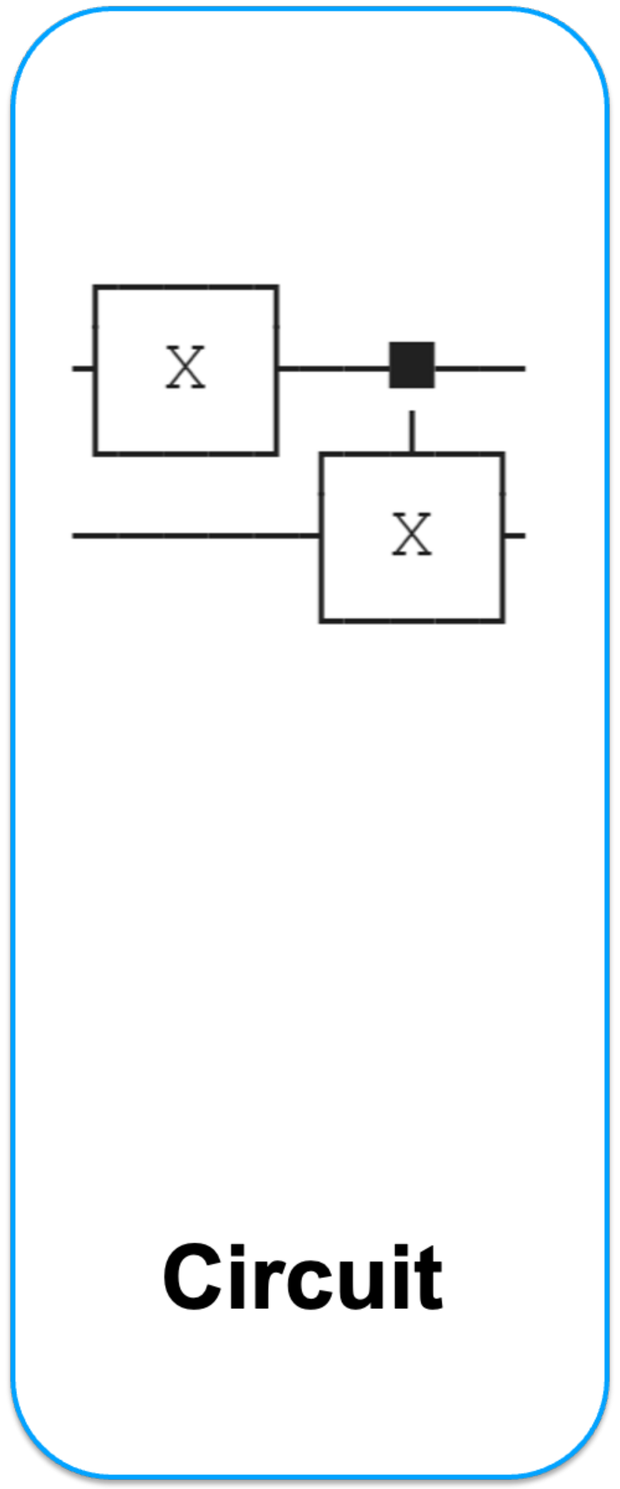
Control Hardware

The microwave is used to control the quantum bits.

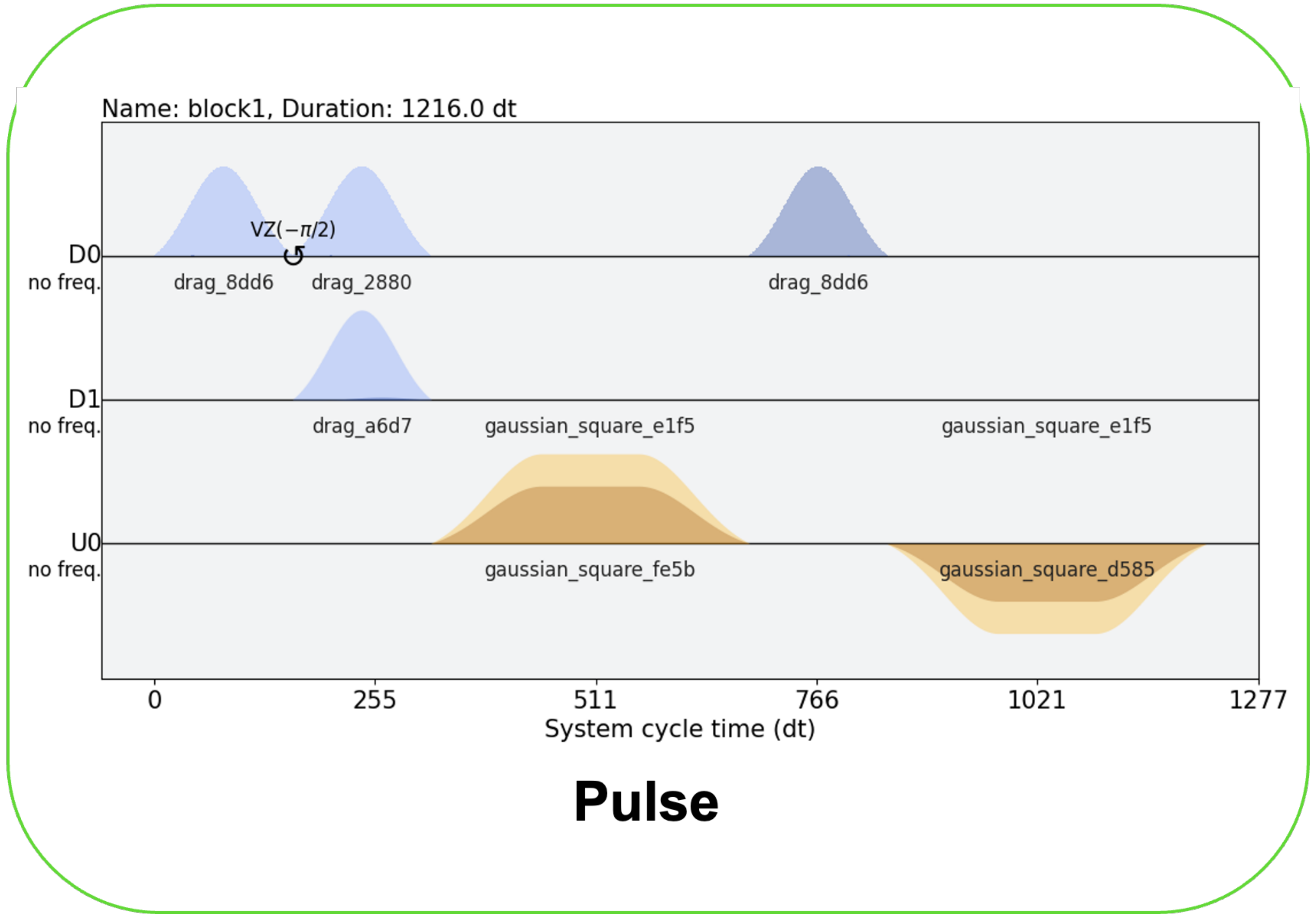


Krantz, Philip, et al. "A quantum engineer's guide to superconducting qubits." Applied physics reviews 6.2 (2019).

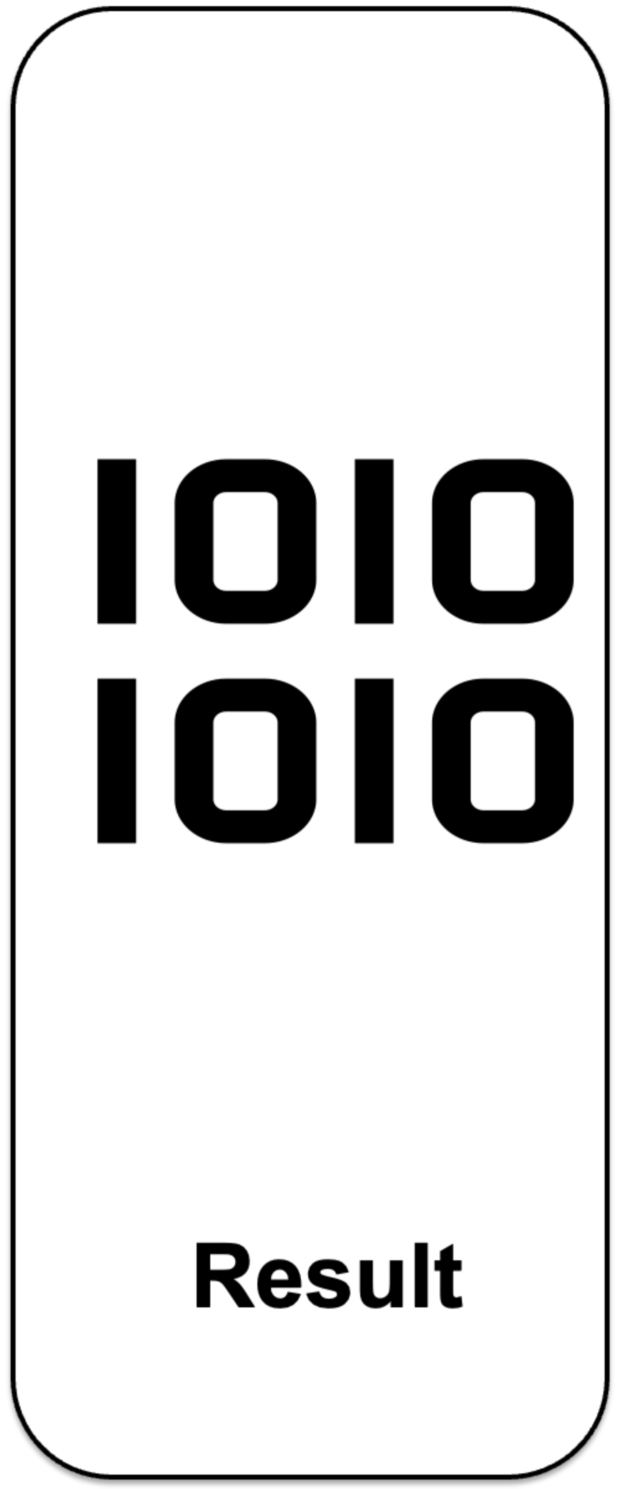
Control Software



Scheduling/ Compilation



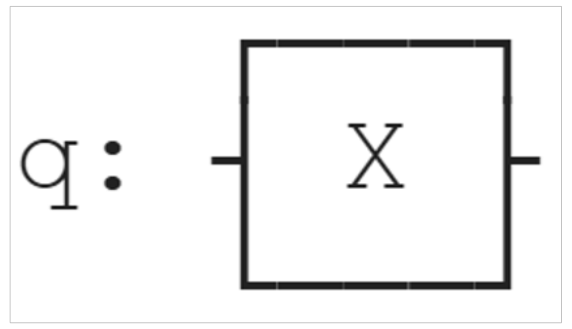
Execution

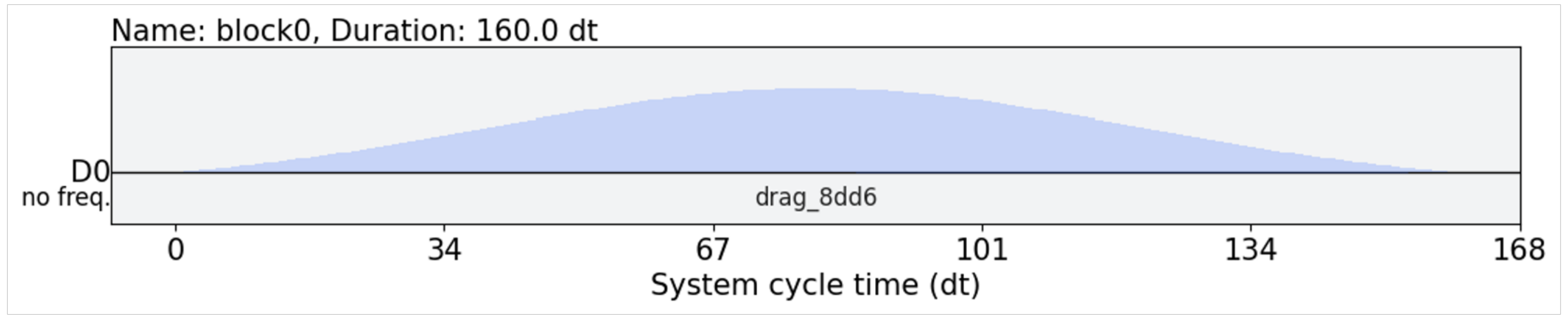


Single Qubit Control

- Mapping from gate to pulses

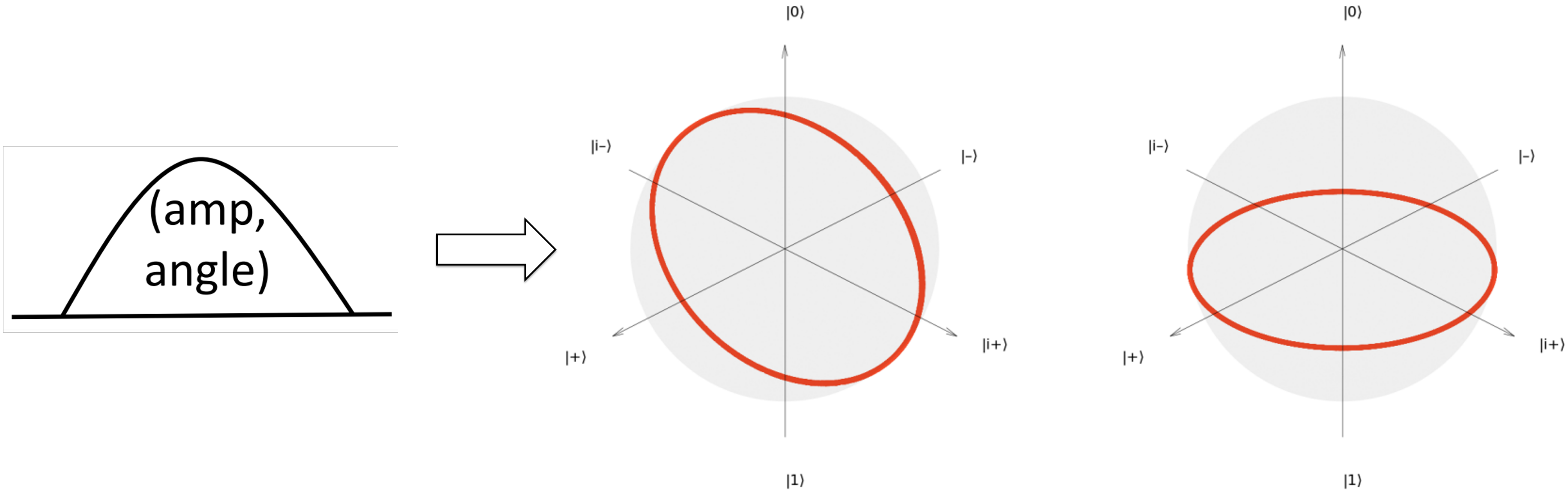
Single qubit gate to pulse, e.g., X gate.





Single Qubit Control

- Microwave pulse rotates the qubit around the bloch sphere



Two Qubit Control

- For IBM superconducting quantum machines, the minimum 2 qubit control basis is cross resonance. The Hamiltonian of cross resonance can be express as following form:

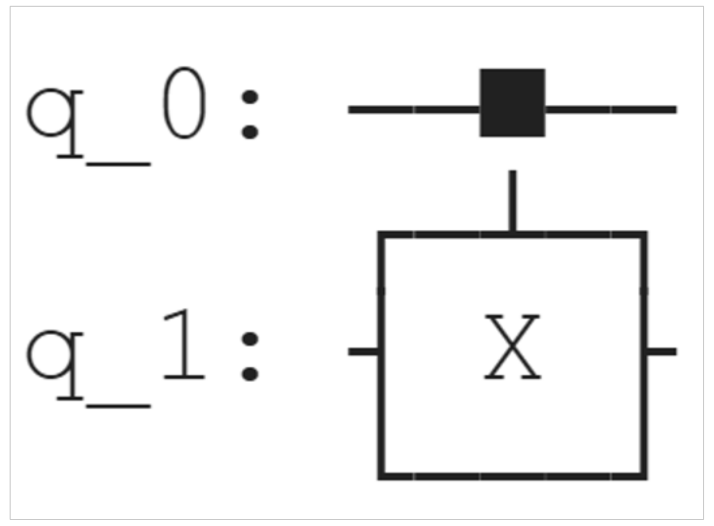
$$\boldsymbol{H} = \frac{\boldsymbol{Z} \otimes \boldsymbol{A}}{\boldsymbol{2}} + \frac{\boldsymbol{I} \otimes \boldsymbol{B}}{\boldsymbol{2}} = \boldsymbol{axZX} + \boldsymbol{ayZY} + \boldsymbol{azZZ} + \boldsymbol{bxIX} + \boldsymbol{byIY} + \boldsymbol{bzIZ}$$

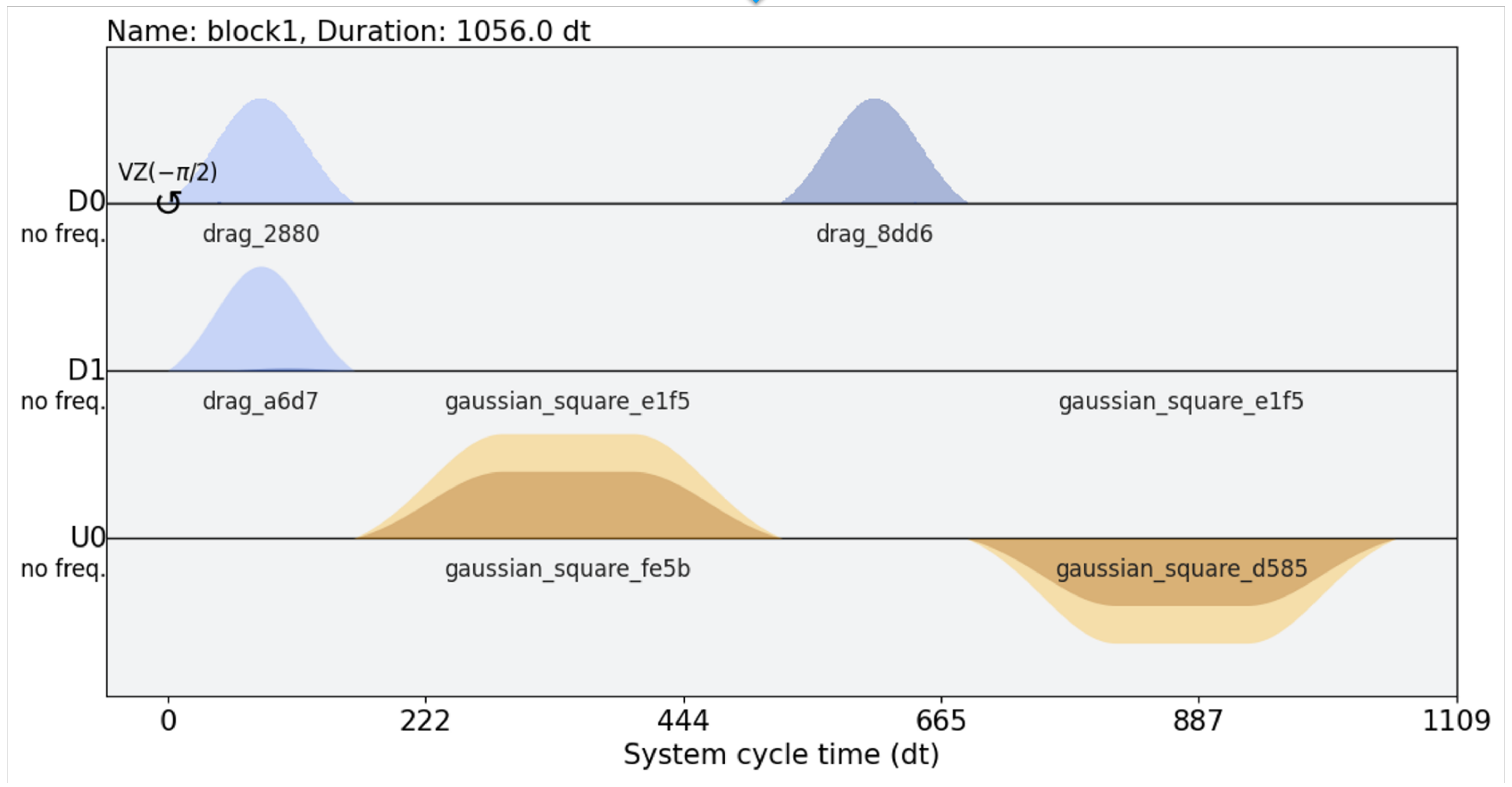
- Since IBM take CX as the 2 qubit basis gate, CX gate only require ZX term, thus, all other terms should be **removed** by calibration.

Two Qubit Control

- Mapping from gate to pulses

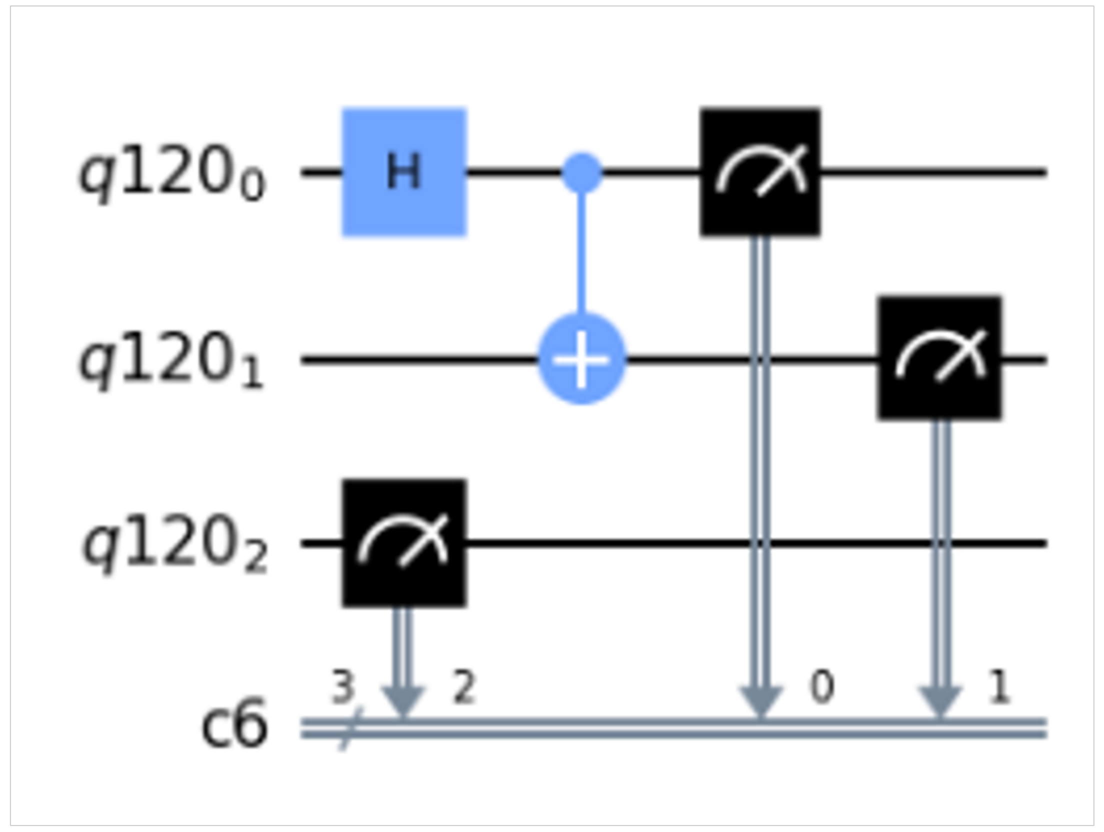
Two qubit gate to pulse, e.g., CX gate.

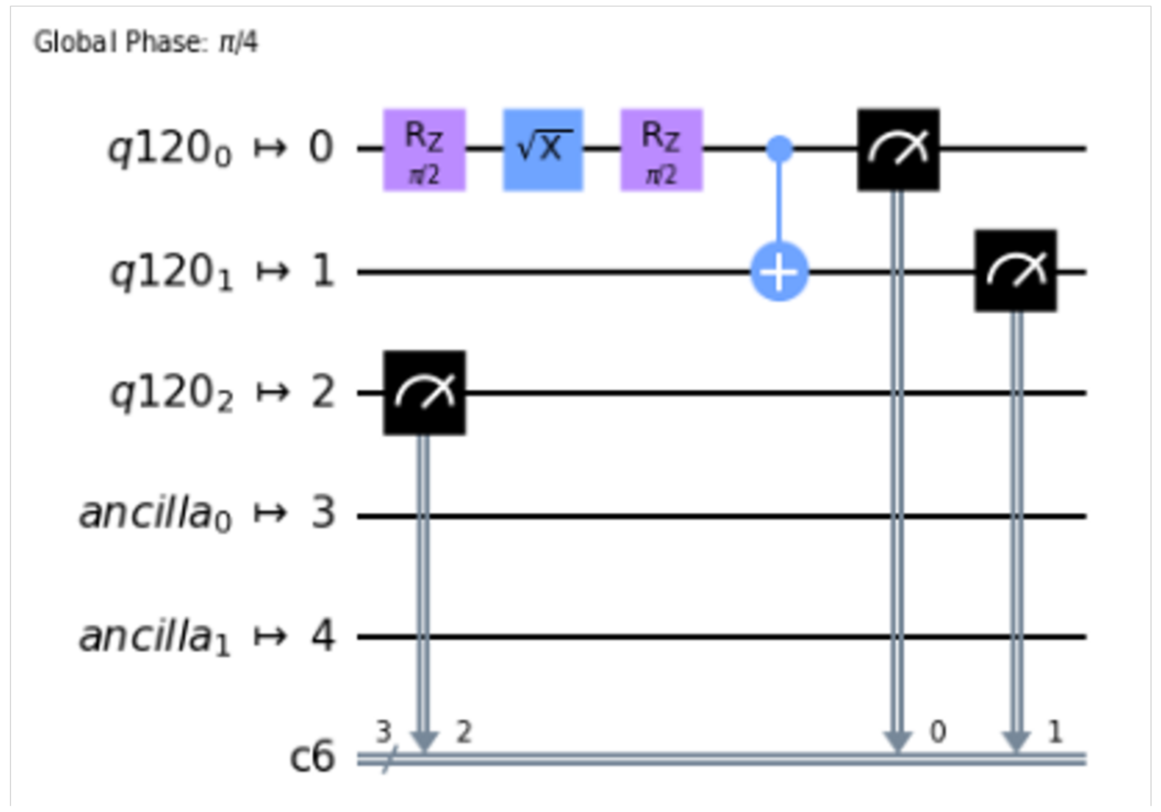


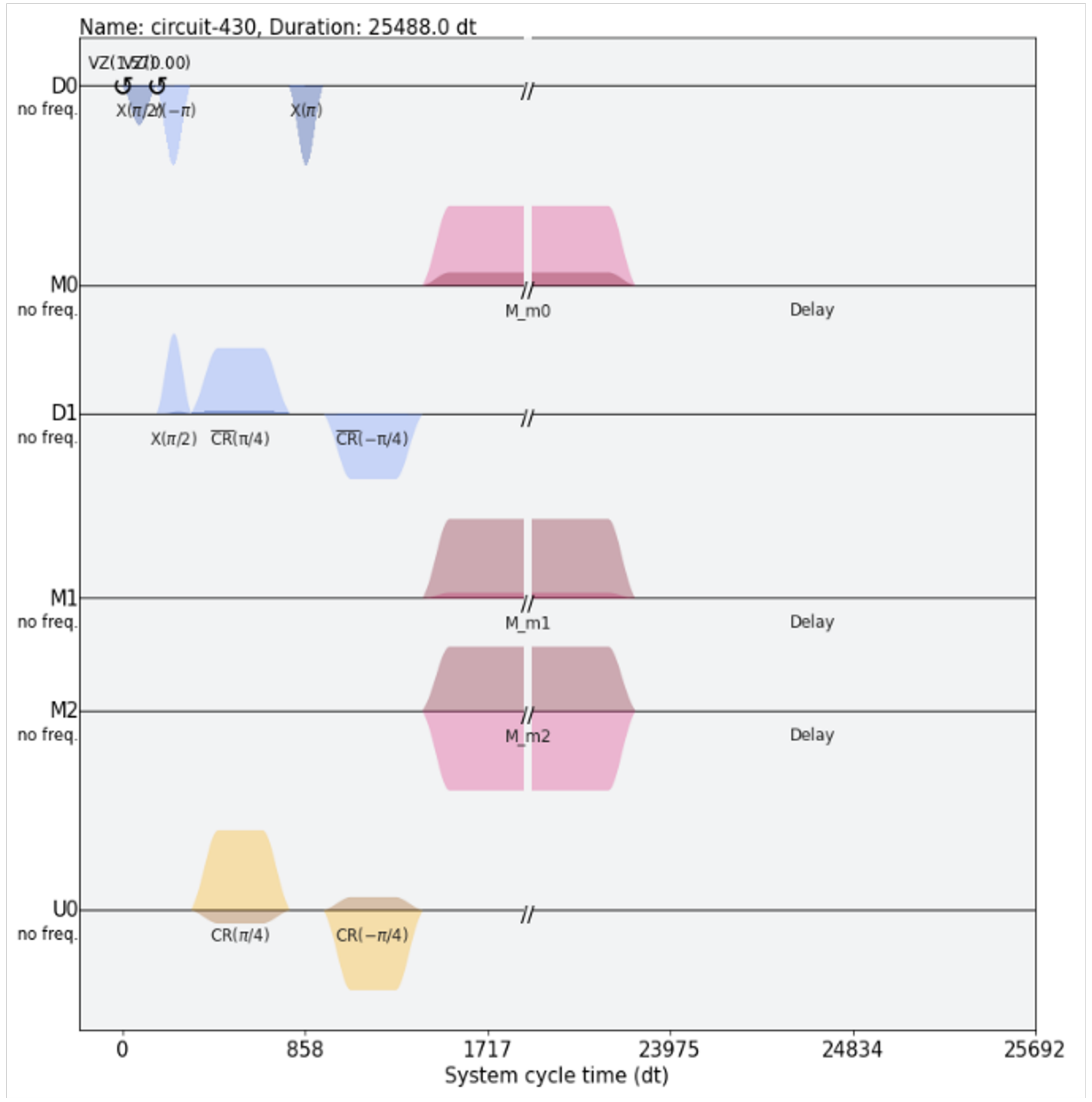


Control Software

- Gate- and Pulse-level are two different abstraction layers.







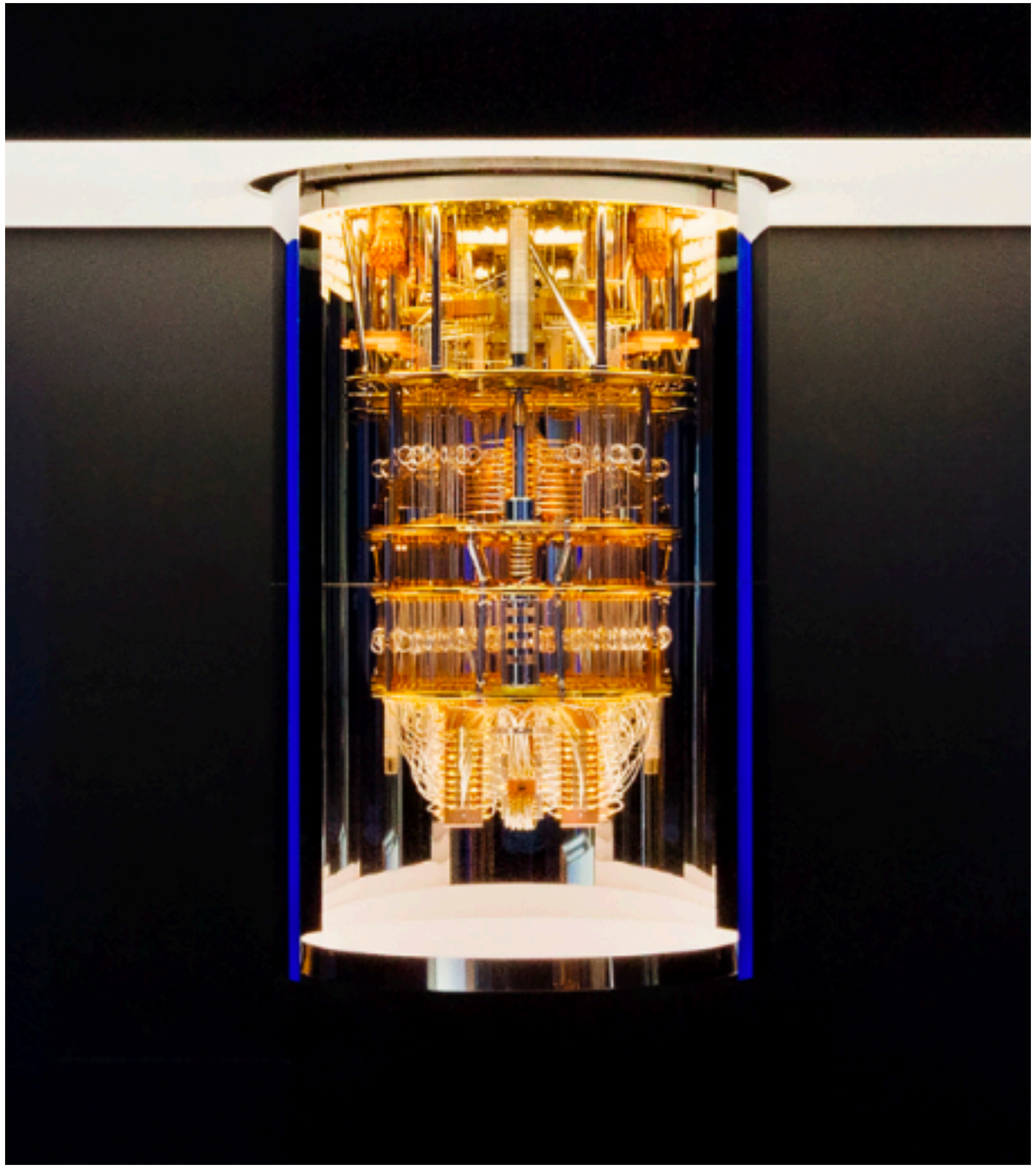
Summary of Today's Lecture

In this lecture, we introduced:

1. Introduce **single** qubit state and gates
2. Introduce **multiple**-qubit state and gates
3. Introduce quantum **circuit**
4. Introduce the **NISQ** Era and compilation problems
5. Introduce the **superconducting workflow**

In next lecture, we will introduce:

Quantum Machine Learning



IBM Quantum

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- Gushu Li, Yufei Ding, and Yuan Xie. 2019. Tackling the Qubit Mapping Problem for NISQ-Era Quantum Devices. In Proceedings of the Twenty-Fourth International Conference on Architectural Support for Programming Languages and Operating Systems (ASPLOS '19). Association for Computing Machinery, New York, NY, USA, 1001–1014. https://doi.org/10.1145/3297858.3304023
- IBM Qiskit Textbook: https://qiskit.org/textbook/preface.html