

# **Strategic Location that accounts for Coverage and Access**

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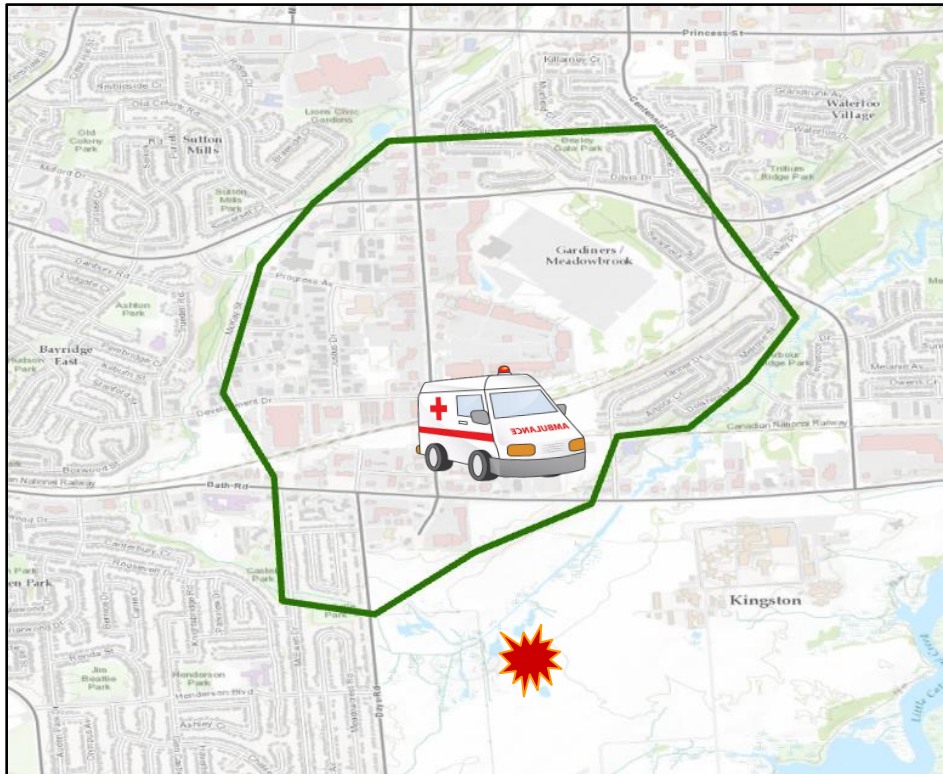
# Strategic Location that accounts for Coverage and Access

## Outline

- ❑ Motivation
- ❑ Key Literature (problem specification)
- ❑ Formal Specification
- ❑ Challenges in Solution
- ❑ Proposed Solution Approach
- ❑ Application Results

# Motivation - Ambulance Response

Effective Emergency Medical Services are dependent on good ambulance staging and/or positioning – the difference between life and death



## Two strategic goals

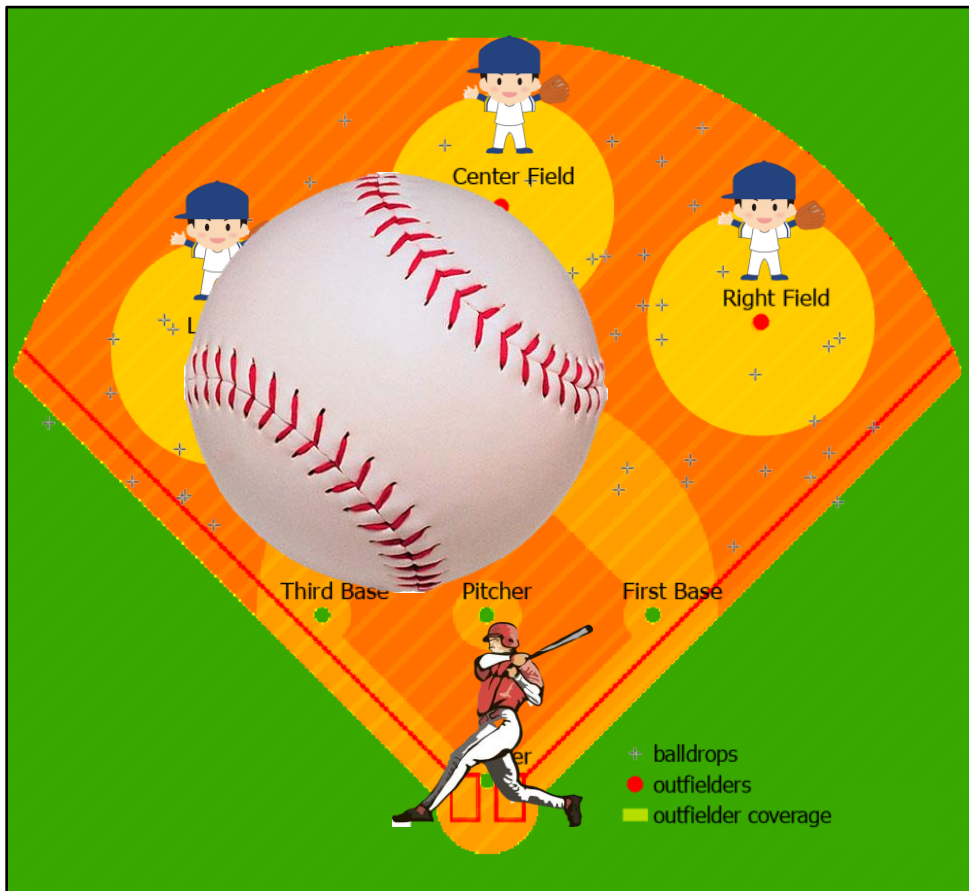
- 1) **Coverage** : respond to a call within 4 minutes
- 2) **Access** : respond to a call as fast as possible

Response beyond desired coverage remains important



# Motivation - Baseball Fielding

Response to hit ball is critical – out or preventing runner from advancing



## Two strategic goals

- 1) **Coverage** : catch a ball before it hits the ground
  - Out
- 2) **Access** : field as quickly as possible
  - Limit runner advancement

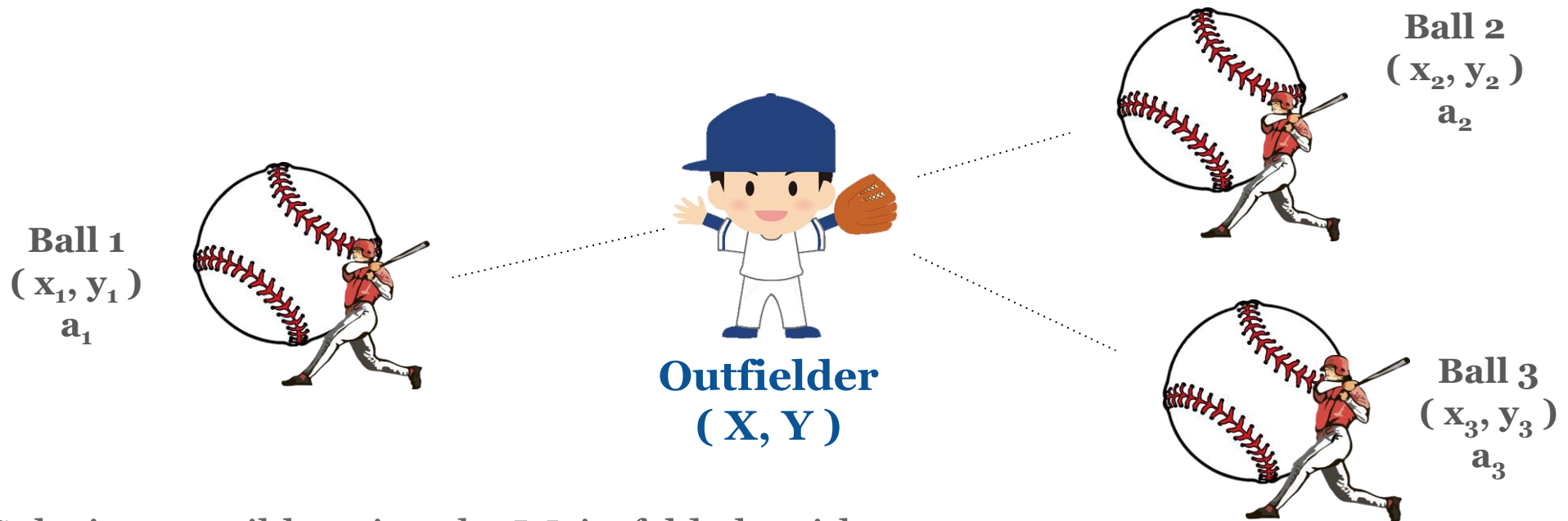
A ball beyond fielder coverage cannot be ignored, making access important.

# Weber Problem

Weber (1909), Simpson (1750), Torricelli (1645)

- ❑ Important spatial analytical problem in human geography
- ❑ Reflects significance of economics, access, social equity, service system design, etc.
- ❑ **Single** facility location problem
- ❑ **Access**: total weighted distance to/from demand sites

**Minimize**  $a_1\sqrt{(X - x_1)^2 + (Y - y_1)^2} + a_2\sqrt{(X - x_2)^2 + (Y - y_2)^2} + a_3\sqrt{(X - x_3)^2 + (Y - y_3)^2}$



- ❑ Solution possible using the Weiszfeld algorithm

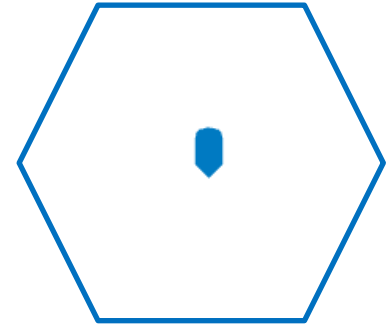
# Coverage

Christaller (1933), Hakimi (1964), ReVelle et al. (1970), Church and ReVelle (1974), Church (1984)

## ❑ Central Place Theory:

### ❑ Range:

the maximum distance consumers are prepared to travel to acquire goods



## ❑ Location Set Covering Problem:

❑ All the demand points should be within the coverage / range.

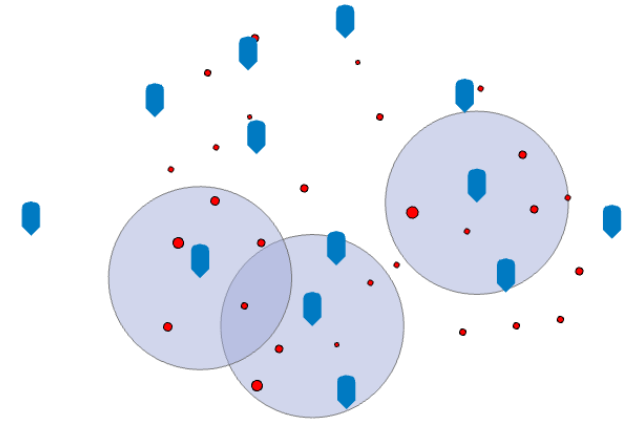
$$\text{Minimize } \sum_{j \in J} x_j \quad \text{Subject to } \sum_{j \in N_i} x_j \geq 1 \quad \forall i$$
$$x_i = (0,1) \quad \forall i$$

## ❑ Maximal Covering Location Problem:

❑ recognized that only **partial** coverage can be optimized.

$$\text{Maximize } \sum_{i \in I} a_i z_i \quad \text{Subject to } \sum_{j \in N_i} x_j + z_i \geq 1 \quad \forall i$$
$$\sum_{j \in J} x_j = P \quad x_j = (0,1) \quad \forall j$$
$$z_i = (0,1) \quad \forall i$$

❑ **Planar** extension: the potential locations can be **anywhere** on the surface



# Bi-objective Problems

Cohon (1978), Solanki (1991)

- ❑ Challenging to solve

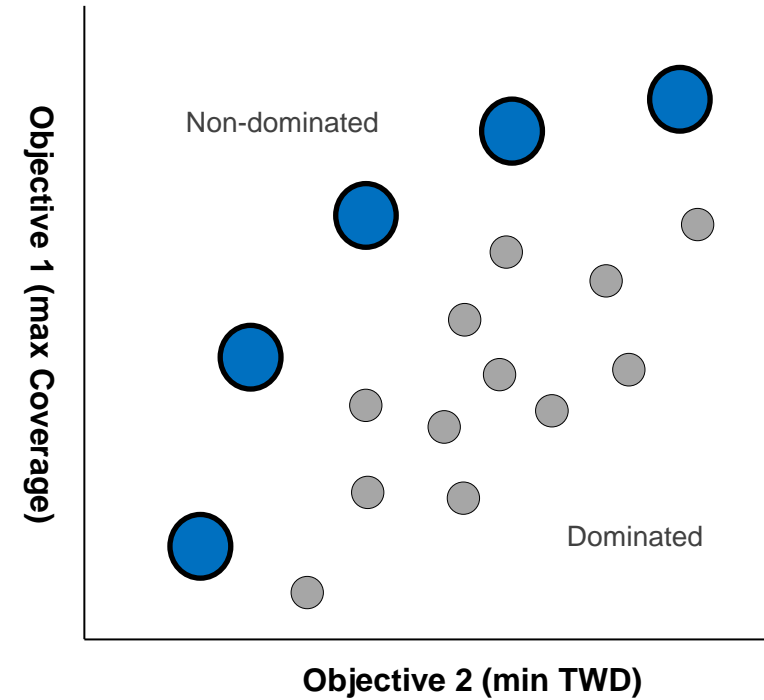
- ❑ **Pareto trade-off:** many optimal solutions
  - ❑ Difficult to find the solutions

- ❑ Dominated solutions:

- ❑ Both objectives are inferior to other solutions

- ❑ Non-dominated Solutions:

- ❑ Cannot improve one objective without compromising another objective



# New Model Formulation

- ❑ The goal is to integrate coverage and access in a spatial optimization problem.
- ❑ Focusing on siting a single facility.

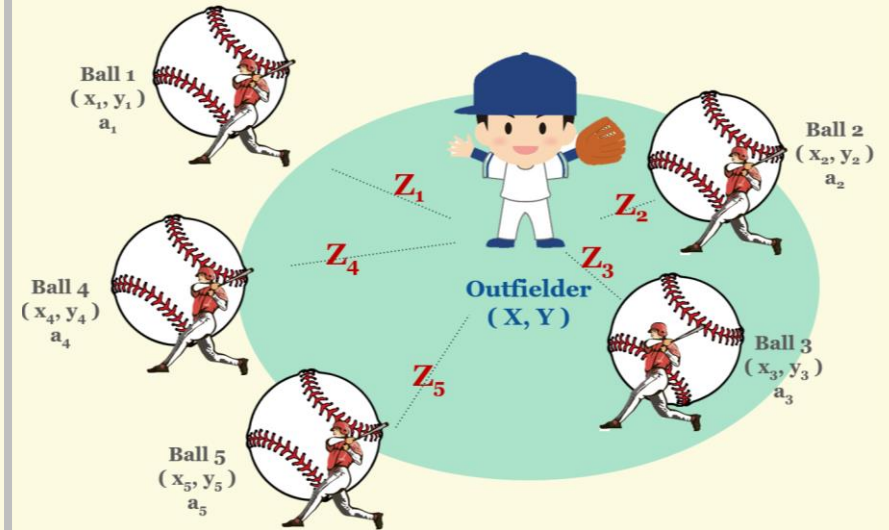
$$\text{Maximize } \sum_i a_i \mathbf{Z}_i$$

$$\text{Minimize } \sum_i a_i \sqrt{(x_i - \mathbf{X})^2 + (y_i - \mathbf{Y})^2}$$

$$\text{Subject to } S(1 - \mathbf{Z}_i) \leq \sqrt{(x_i - \mathbf{X})^2 + (y_i - \mathbf{Y})^2} \quad \forall i$$

$$\mathbf{Z}_i = \begin{cases} 1 & \text{if demand } i \text{ within } s \text{ of facility at } \mathbf{X}, \mathbf{Y} \\ 0 & \text{otherwise} \end{cases}$$

$\mathbf{X}, \mathbf{Y}$  unrestricted in sign





# Solution Challenges

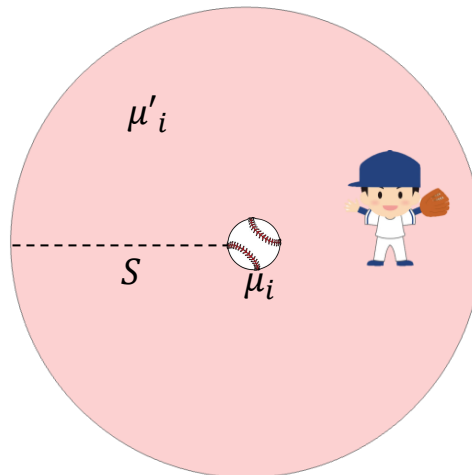
- ❑ There is no existing method for solving this single facility location problem considering coverage and access at the same time.
    - ❑ **New problem formulation**
  - ❑ Linear and non-linear constraints
  - ❑ Multiple optima for bi-objective problems.
    - ❑ **User / analyst / decision maker must select from among them**
- **GIScience** perspective offers potential ...

# Coverage

- ❑ Summarizing continuous space as discrete buffer units.
- ❑ Where in the region could each demand be served?
  - ❑ **GIScience:** regular or irregular **buffer** to represent covering area
- ❑ Mathematical specification of buffer

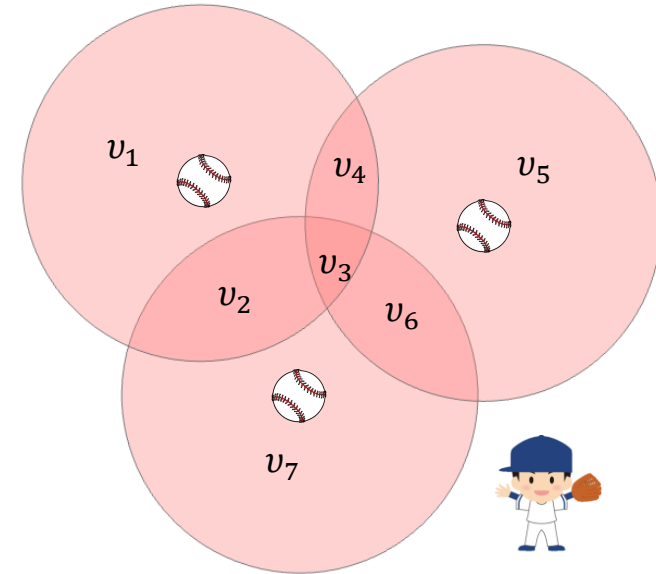
$(x_i, y_i)$ : demand point  $i$

$$\mu_i \leftarrow \{ (x, y) \in \mathbb{R}^2 \mid d_{(x,y)(x_i,y_i)} \leq S \}$$



# Topological (Vector) Overlay

- ❑ An algorithm of computational geometry
- ❑ Spatial operator producing maximally connected subset of faces that do not overlap
  - ❑ Have two or more vector based layers
    - Need/want to combine them as one non-overlapping object layer
  - ❑ Must resolve different and intersecting objects
    - Involves creating new objects
  - ❑ Must reconcile attribute information as well
    - Involves interpolation / estimation / guessing



## ❑ Mathematical specification

\*  $\wp(O)$  is the power set enumerating all combinations of objects to produce overlapping and nonoverlapping components.

$$O = \{ \mu_i \mid \forall (x_i, y_i) \in I \}$$

$$\Psi = \{ v \mid \mu' \in \wp(O) \quad \text{where} \quad \forall v, \hat{v} \subset \mu' \quad v, \hat{v} = \emptyset \quad \wedge \quad \cup_v v = \Phi \}$$

- ❑ **Face** : spatial unit with known coverage capability

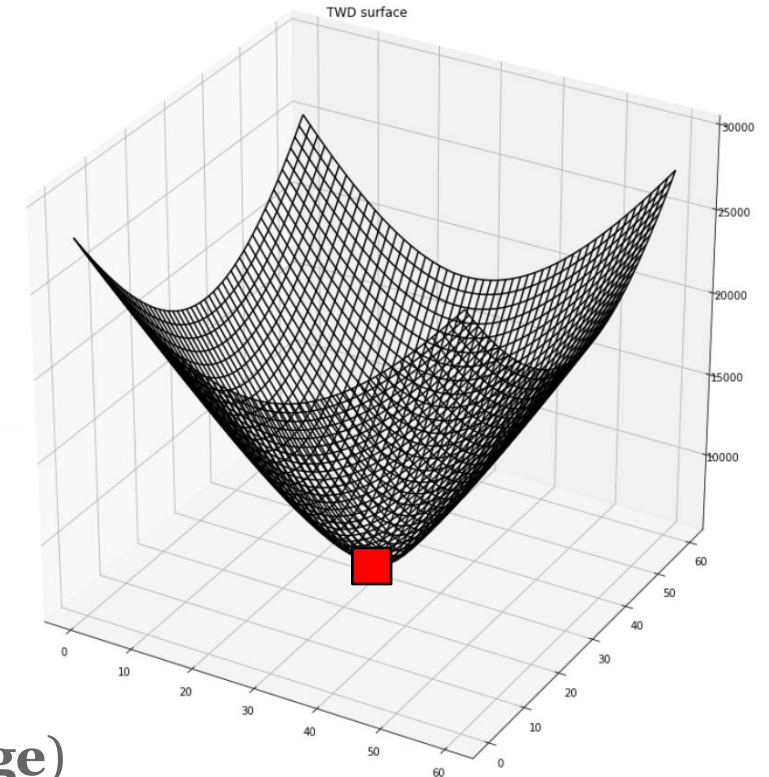
# What about access?

- ❑ So far, we only focused on **coverage**
  - ❑ All the locations in a face has the same coverage value.
  - ❑ But total weighted distance value changes continuously.
- ❑ Minimizing average distance (total weighted distance)
  - ❑ **Regional Weber Point** : Found using **Weiszfeld Algorithm**.  
( $X_R, Y_R$ ) → does not consider coverage

$$(X^{[t+1]}, Y^{[t+1]}) = \left( \frac{\sum_{i \in I} \frac{a_i x_i}{\sqrt{(X^{[t]} - x_i)^2 + (Y^{[t]} - y_i)^2}}, \sum_{i \in I} \frac{a_i y_i}{\sqrt{(X^{[t]} - x_i)^2 + (Y^{[t]} - y_i)^2}} \right)$$

- ❑ **Weber Point for each face,  $v$**   
(minimizes total weighted distance for known level of **coverage**)

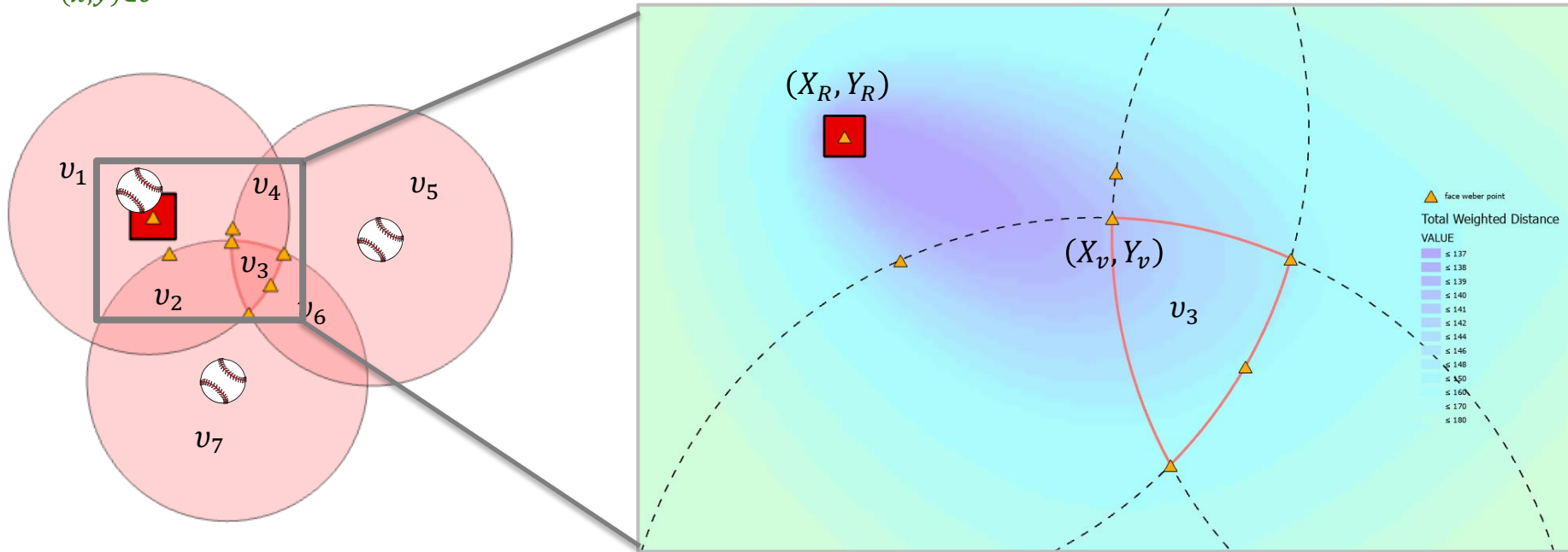
$$(X_v, Y_v) = \min_{(x,y) \in v} \sqrt{(X_R - x)^2 + (Y_R - y)^2}$$



# Optimality

- Can show that  $(X_v, Y_v)$  is optimum in each face  $v$

- $(X_v, Y_v) = \min_{(x,y) \in v} \sqrt{(X_R - x)^2 + (Y_R - y)^2}$



- As a result, a search process that evaluates each face will result in the location within each face that provides the best access and coverage
  - Non-dominate solutions can be found from among these finite number of options

# Developed Solution Algorithm

❑ **Algorithm:** finite steps to find the solutions with guaranteed optimality

❑ Major Components

- ❑ Regional Weber Point
- ❑ Buffer
- ❑ Vector Overlay
- ❑ The Nearest Point
- ❑ Non-dominated Solutions

**Input:**  $I, S$

**Output:** *Non-dominated*

**Initialize:**  $O = \{ \}$

$(X_R, Y_R) \leftarrow Weiszfeld(I)$

**for each**  $i \in I$  **do**

$O \leftarrow \{(x, y) \in R^2 \mid d_{(x,y)(x_i, y_i)} \leq S\}$

**end-for**

$\Psi(O) = \{v \mid \mu' \in \wp(O) \text{ where } \forall v, \widehat{v} \subset \mu' \quad v, \widehat{v} = \emptyset \wedge \cup_v v = \Phi \}$

**for each**  $v \in \Psi$  **do**

$(X_v, Y_v) \leftarrow \min_{(x,y) \in v} \sqrt{(X_R - x)^2 + (Y_R - y)^2}$

$N_v = \{i \mid d_{v(x_i, y_i)} \leq S \quad \forall i\}$

$cov_v = \sum_{i \in N_v} a_i$

$twd_v = \sum_{i \in I} a_i \sqrt{(X_v - x_i)^2 + (Y_v - y_i)^2}$

**end-for**

**for each**  $n \in I$  **do**

**for each**  $m \in I$  **do**

**if**  $cov_m \leq cov_n \wedge twd_m \geq twd_n$  **then**  
 $Dominated \leftarrow m$

**end-if**

**end-for**

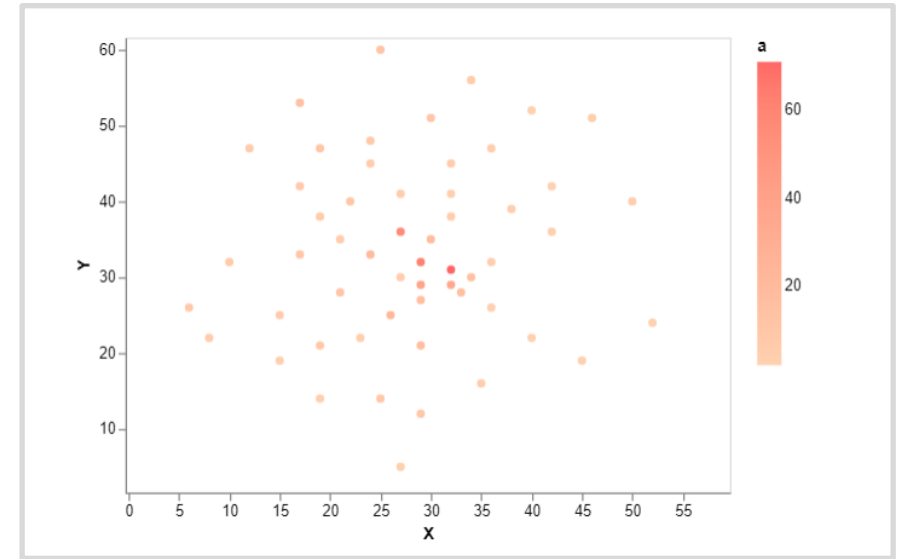
**end-for**

$Non - dominated \leftarrow \{n \mid \forall n \text{ where } n \notin Dominated\}$

# Applications

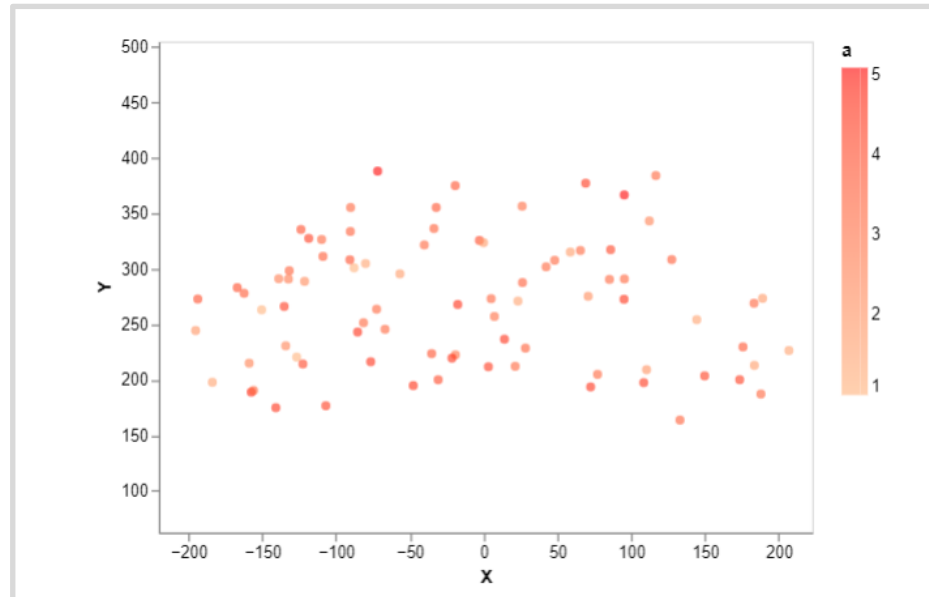
## ❑ Swain Data

- ❑ Swain (1971) (see also Church and Baez 2020)
- ❑ 55 demand points
- ❑  $S=10$

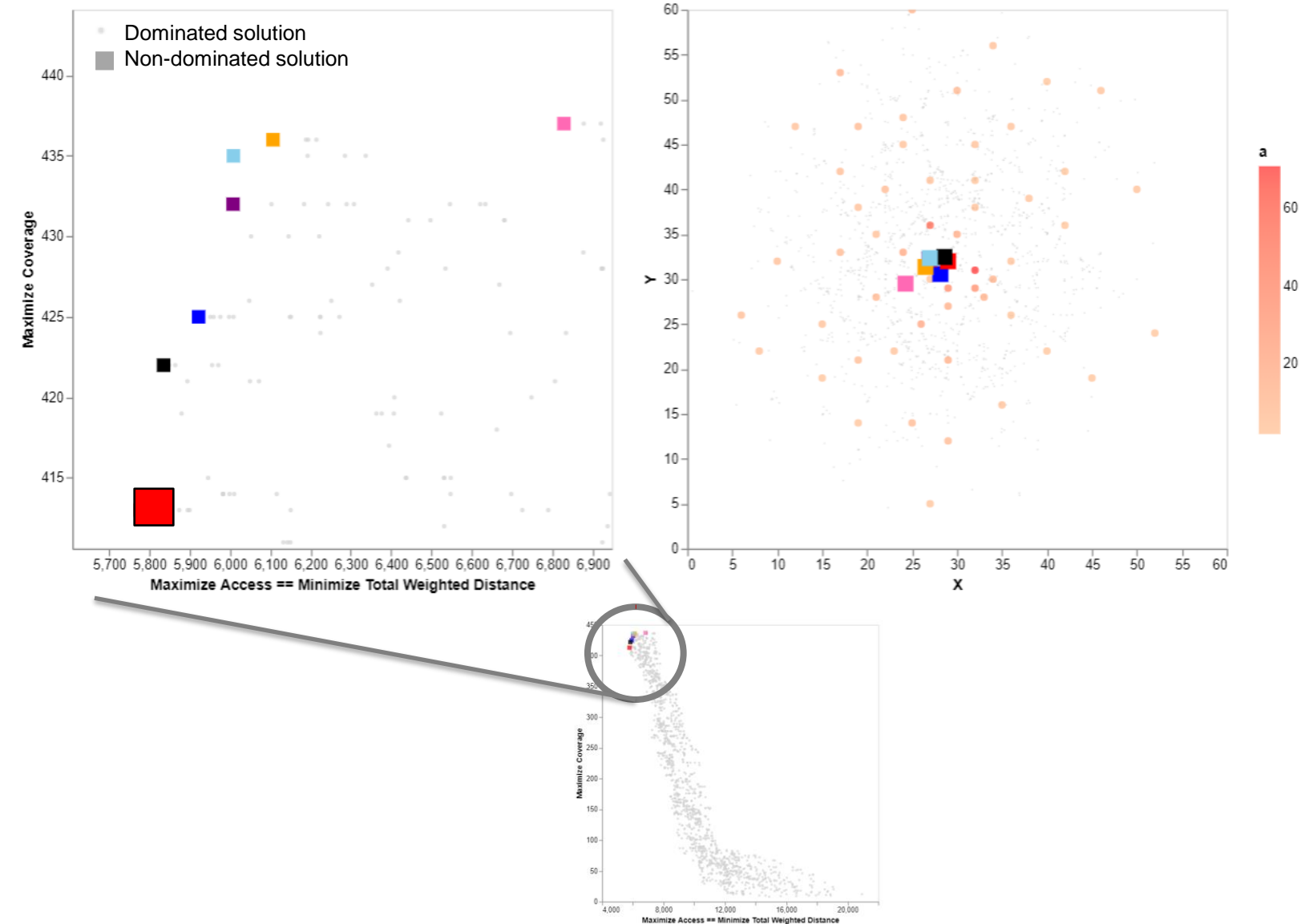


## ❑ UCSB baseball

- ❑ 85 batted balls
- ❑  $S=90$  ft



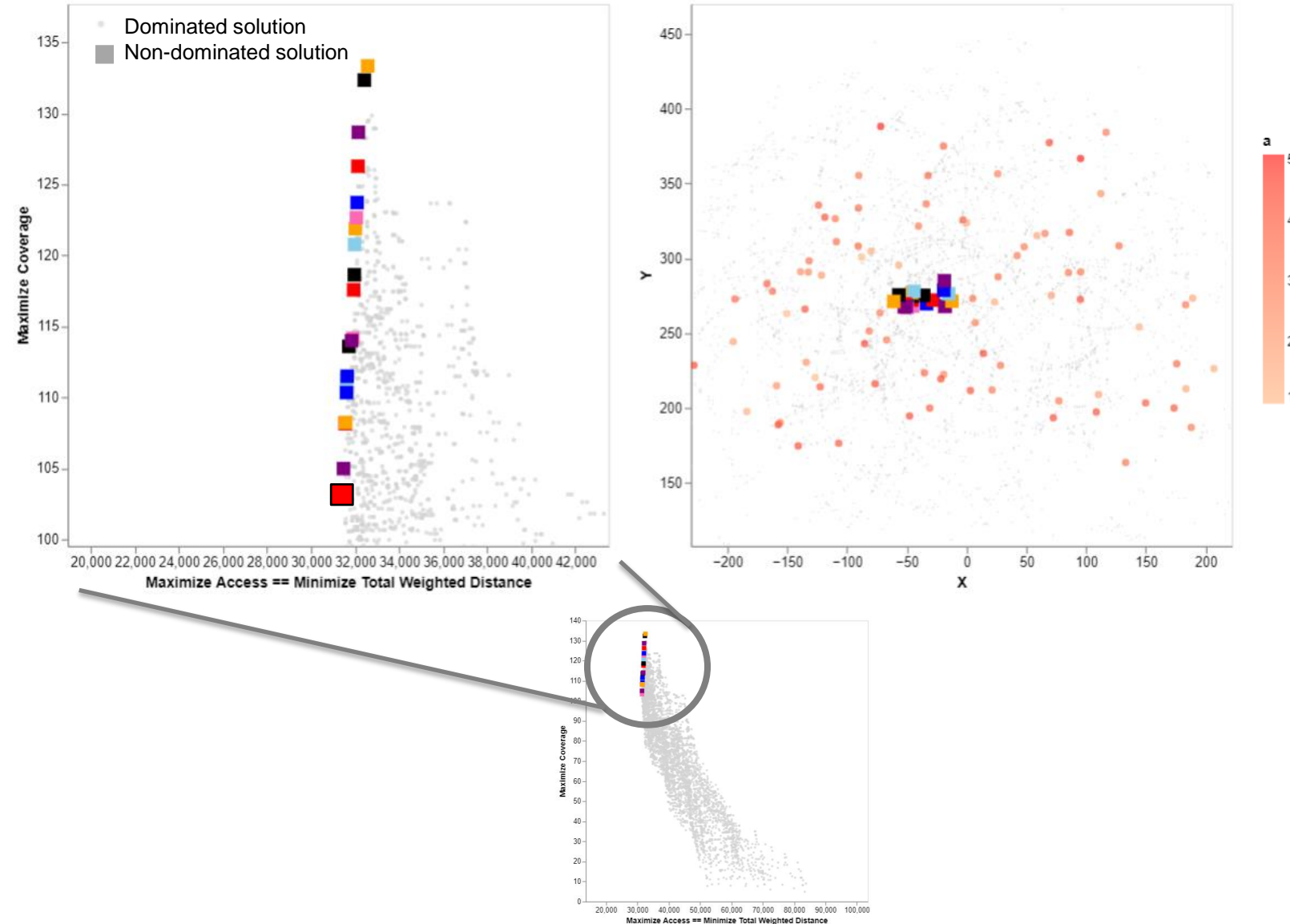
# Swain Results



Regional Weber Point
(29.0, 32.0) with objective of 5777.838798
Buffer and Vector Overlay
55 points and buffer for S=10 → 1557 faces
Face Weber Points
1557 faces with coverage and access values
Non-dominated Solutions
7 Non-dominated solutions
Total Solution Time
<b>15.541 seconds</b>



# UCSB Baseball Results



Regional Weber Point
(-18.62, 267.72 ) with objective of 31478.21
Buffer and Vector Overlay
85 points ... buffer for S=90 ft → 4666 faces
Face Weber Points
4666 faces with coverage and access values
Non-dominated Solutions
20 Non-dominated solutions
Total Solution Time
<b>45.39 seconds</b>

# Conclusion

- ❑ Strategic location in a **planar** context, accounting for **coverage** and **access** at the same time is challenging. But it's an important problem that is applicable to various facility siting contexts.
- ❑ GIS and computational geometry are invaluable for devising a solution algorithm
  - ❑ Exploits spatial knowledge
  - ❑ Enables the problem to be solved optimally