Strategic Location that accounts for Coverage and Access

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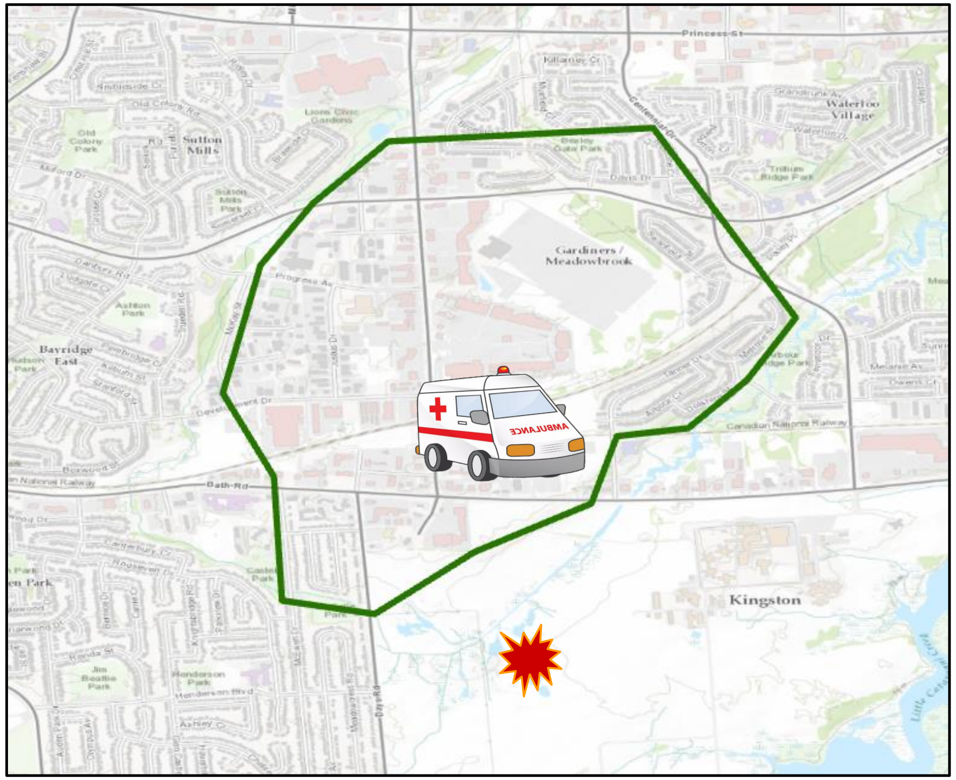
Outline

- ❑ Motivation
- ❑ Key Literature (problem specification)
- ❑ Formal Specification
- ❑ Challenges in Solution
- ❑ Proposed Solution Approach
- ❑ Application Results

Motivation - Ambulance Response

Effective Emergency Medical Services are dependent on good ambulance staging and/or positioning – the difference between life and death



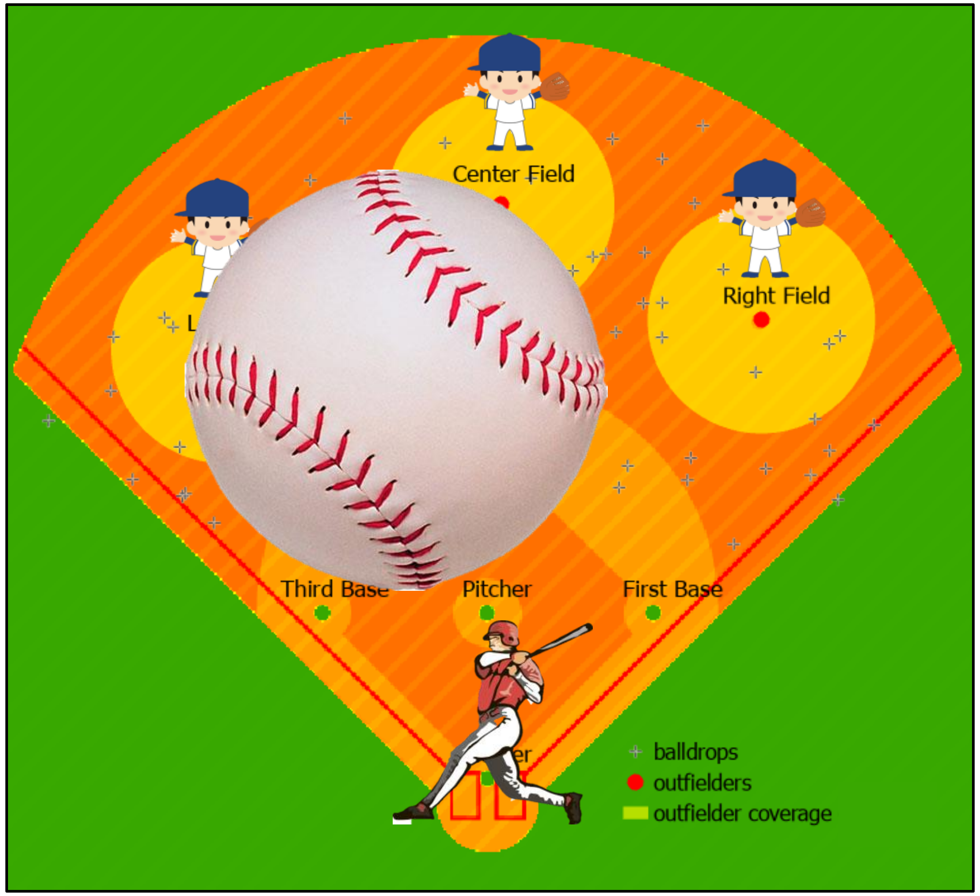
Two strategic goals

1) **Coverage : respond to a call within 4 minutes**
2) **Access : respond to a call as fast as possible**

Response beyond desired coverage remains important

Motivation - Baseball Fielding

Response to hit ball is critical – out or preventing runner from advancing



Two strategic goals

1) **Coverage** : catch a ball before it hits the ground
   - Out
2) **Access** : field as quickly as possible
   - Limit runner advancement

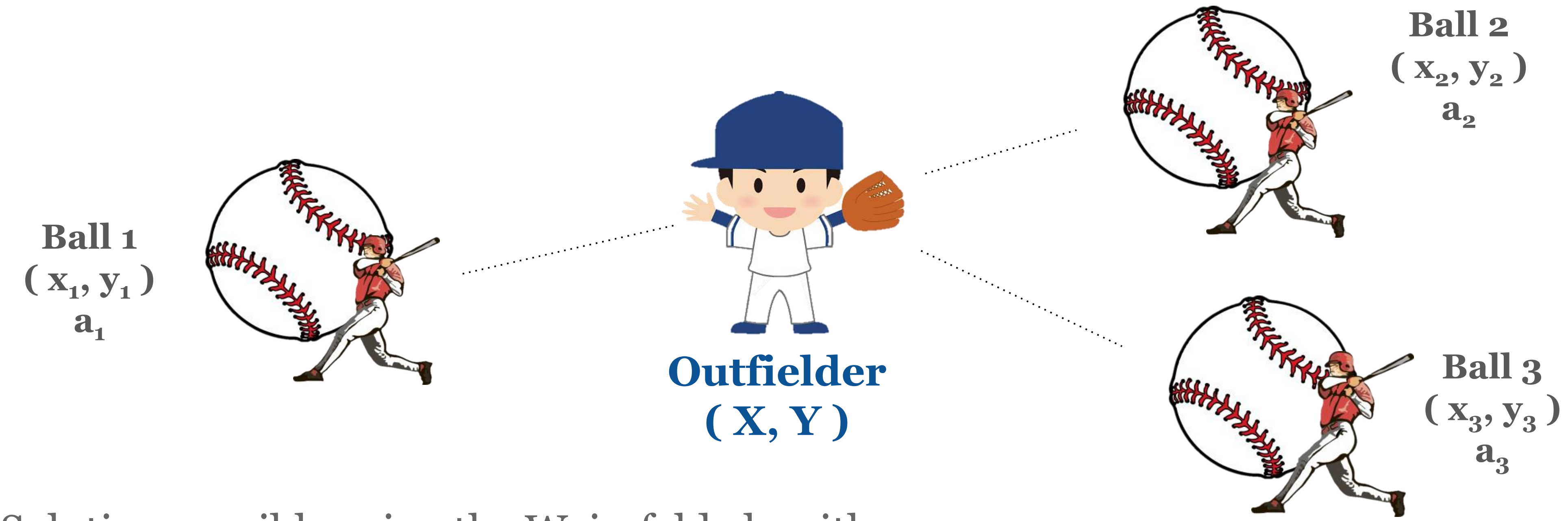
A ball beyond fielder coverage cannot be ignored, making access important.

Weber Problem

Weber (1909), Simpson (1750), Torricelli (1645)

- Important spatial analytical problem in human geography
- Reflects significance of economics, access, social equity, service system design, etc.
- **Single** facility location problem
- **Access**: total weighted distance to/from demand sites

Minimize $a_1\sqrt{(X-x_1)^2+(Y-y_1)^2}+a_2\sqrt{(X-x_2)^2+(Y-y_2)^2}+a_3\sqrt{(X-x_3)^2+(Y-y_3)^2}$

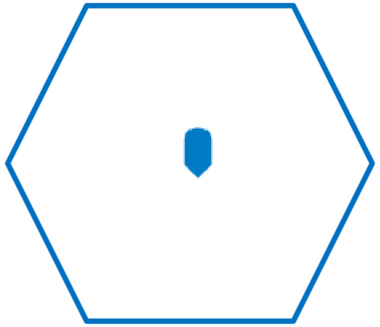


- Solution possible using the Weiszfeld algorithm

Coverage

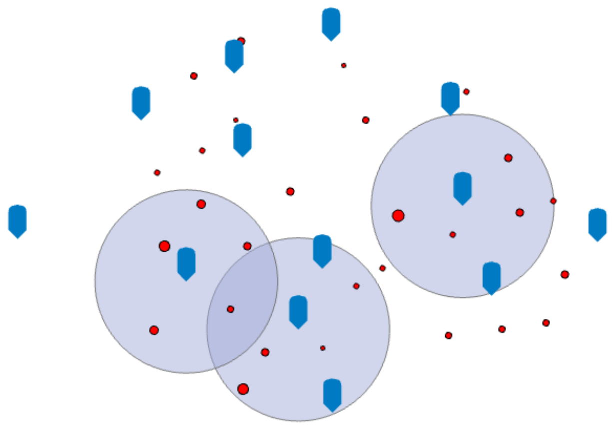
Christaller (1933), Hakimi (1964), ReVelle et al. (1970), Church and ReVelle (1974), Church (1984)

- **Central Place Theory:**
  - **Range:**
    the maximum distance consumers are prepared to travel to acquire goods



- **Location Set Covering Problem:**
  - **All** the demand points should be within the coverage / range.

$$Minimize \sum_{j \in J} X_j \qquad Subject\ to \sum_{j \in N_i} X_j \geq 1 \quad \forall i$$
$$X_i = (0,1) \quad \forall i$$



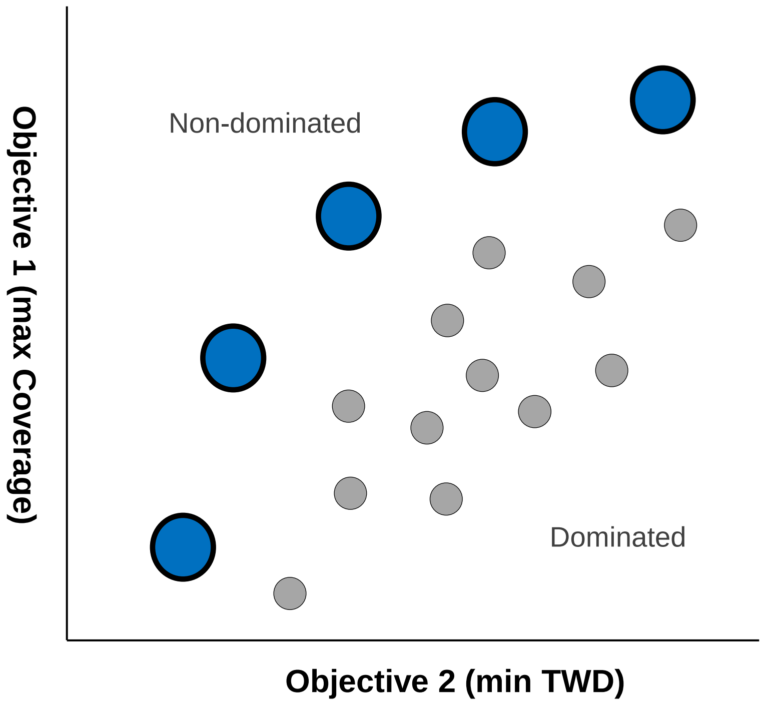
- **Maximal Covering Location Problem:**
  - recognized that only **partial** coverage can be optimized.

$$Maximize \sum_{i \in I} a_i Z_i \qquad Subject\ to \sum_{j \in N_i} X_j + Z_i \geq 1 \quad \forall i$$
$$\sum_{j \in J} X_j = P \qquad X_j = (0,1) \quad \forall j$$
$$Z_i = (0,1) \quad \forall i$$

- **Planar** extension: the potential locations can be **anywhere** on the surface

Bi-objective Problems Cohon (1978), Solanki (1991)

- ❑ **Challenging to solve**
  - ❑ **Pareto trade-off**: many optimal solutions
  - ❑ Difficult to find the solutions



- ❑ **Dominated solutions:**
  - ❑ Both objectives are inferior to other solutions
- ❑ **Non-dominated Solutions:**
  - ❑ Cannot improve one objective without compromising another objective

New Model Formulation

- ❑ **The goal is to integrate coverage and access in a spatial optimization problem.**
- ❑ **Focusing on siting a single facility.**

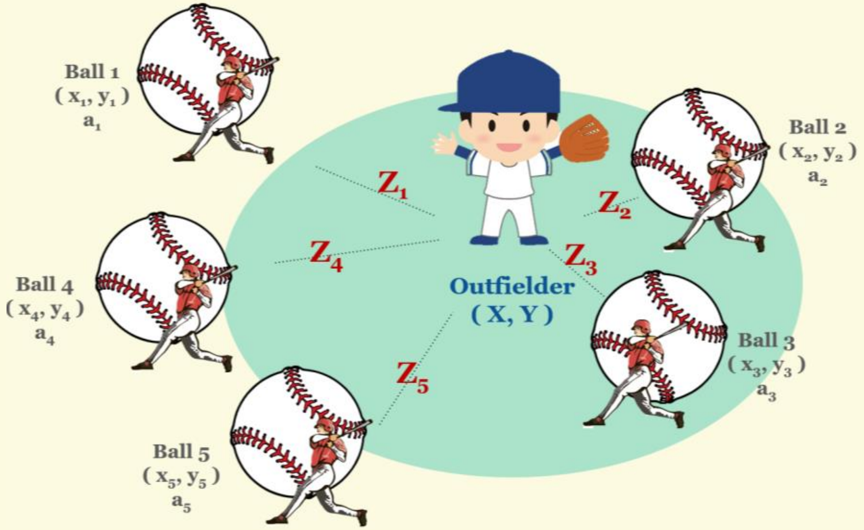
$$\textbf{Maximize} \quad \sum_i a_i \boldsymbol{Z_i}$$

$$\textbf{Minimize} \quad \sum_i a_i \sqrt{(x_i - \boldsymbol{X})^2 + (y_i - \boldsymbol{Y})^2}$$

$$Subject\ to \qquad S(1 - \boldsymbol{Z_i}) \le \sqrt{(x_i - \boldsymbol{X})^2 + (y_i - \boldsymbol{Y})^2} \qquad \forall i$$

$$\boldsymbol{Z_i} = \begin{cases} 1 \text{ if demand i within s of facility at X,Y} \\ 0 \text{ otherwise} \end{cases}$$

$$\boldsymbol{X}, \boldsymbol{Y}\ unrestricted\ in\ sign$$



Solution Challenges

- There is no existing method for solving this single facility location problem considering coverage and access at the same time.
  - **New problem formulation**
- Linear and non-linear constraints
- Multiple optima for bi-objective problems.
  - **User / analyst / decision maker must select from among them**

➢ **GIScience** perspective offers potential …

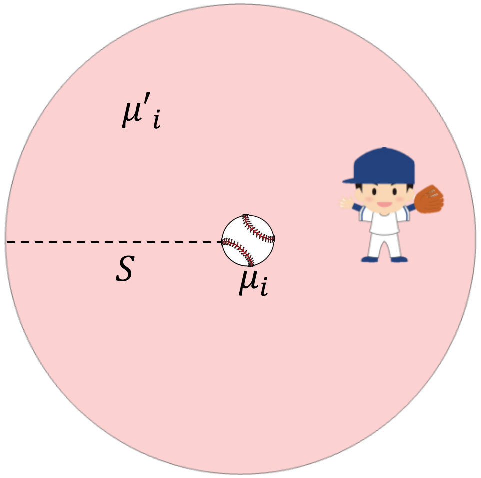
Coverage

- **Summarizing continuous space as discrete buffer units.**
- Where in the region could each demand be served?
  - **GIScience:** regular or irregular **buffer** to represent covering area

- Mathematical specification of buffer

(x_i , y_i): demand point i

$$\mu_i \leftarrow \{ (x, y) \subset \mathbb{R}^2 \mid d_{(x,y)(x_i ,y_i)} \leq S\}$$



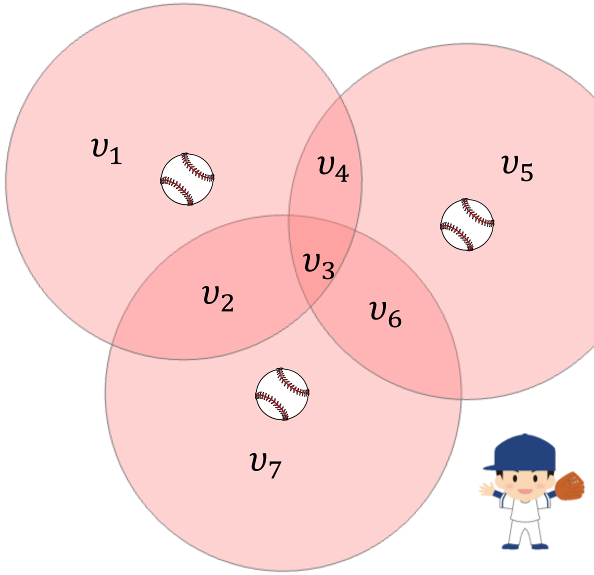
Topological (Vector) Overlay

- An algorithm of computational geometry
- Spatial operator producing maximally connected subset of faces that do not overlap
  - Have two or more vector based layers

    Need/want to combine them as one non-overlapping object layer
  - Must resolve different and intersecting objects

    Involves creating new objects
  - Must reconcile attribute information as well

    Involves interpolation / estimation / guessing



- Mathematical specification

  * $\wp\,(O)$ is the power set enumerating all combinations of objects to produce overlapping and nonoverlapping components.

$$O = \{\, \mu_i \mid \forall (x_i\,, y_i) \in I \,\}$$

$$\Psi = \{\, \nu \mid \mu' \in \wp\,(O) \quad where \;\; \forall\, \upsilon, \hat{\upsilon} \subset \mu' \quad \upsilon, \hat{\upsilon} = \emptyset \quad \wedge \quad \cup_\nu\, \nu = \Phi \,\}$$

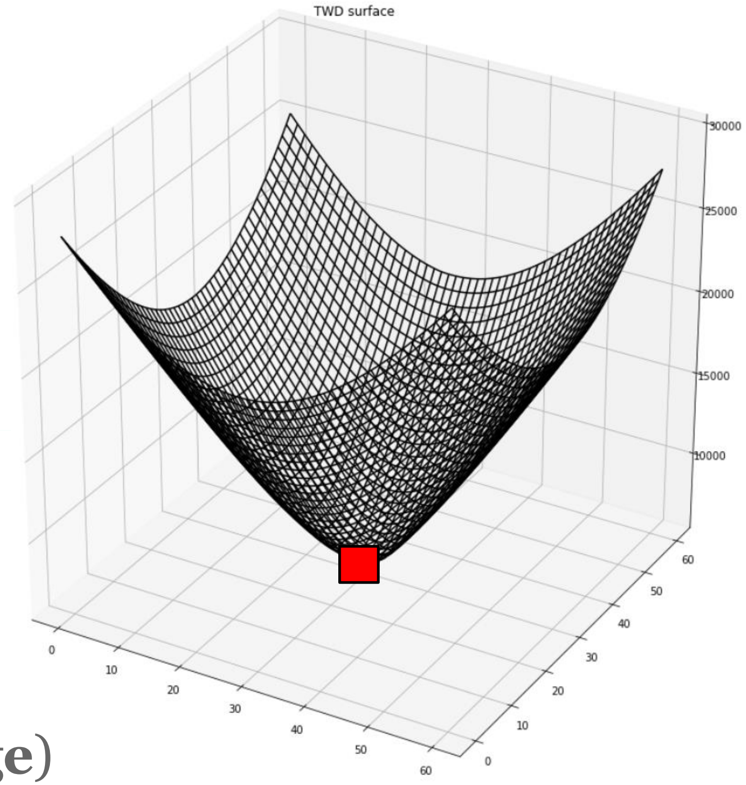
- **Face** : spatial unit with known coverage capability

What about access?

- So far, we only focused on **coverage**
  - All the locations in a face has the same coverage value.
  - But total weighted distance value changes continuously.

- Minimizing average distance (total weighted distance)
  - **Regional Weber Point** : Found using **Weiszfeld Algorithm**.
    (X_R, Y_R) $\rightarrow$ does not consider coverage

$$\left(X^{[t+1]}, Y^{[t+1]}\right) = \left(\frac{\sum_{i \in I} \frac{a_i x_i}{\sqrt{\left(X^{[t]} - x_i\right)^2 + \left(Y^{[t]} - y_i\right)^2}}}{\sum_{i \in I} \frac{a_i}{\sqrt{\left(X^{[t]} - x_i\right)^2 + \left(Y^{[t]} - y_i\right)^2}}}, \frac{\sum_{i \in I} \frac{a_i y_i}{\sqrt{\left(X^{[t]} - x_i\right)^2 + \left(Y^{[t]} - y_i\right)^2}}}{\sum_{i \in I} \frac{a_i}{\sqrt{\left(X^{[t]} - x_i\right)^2 + \left(Y^{[t]} - y_i\right)^2}}} \right)$$

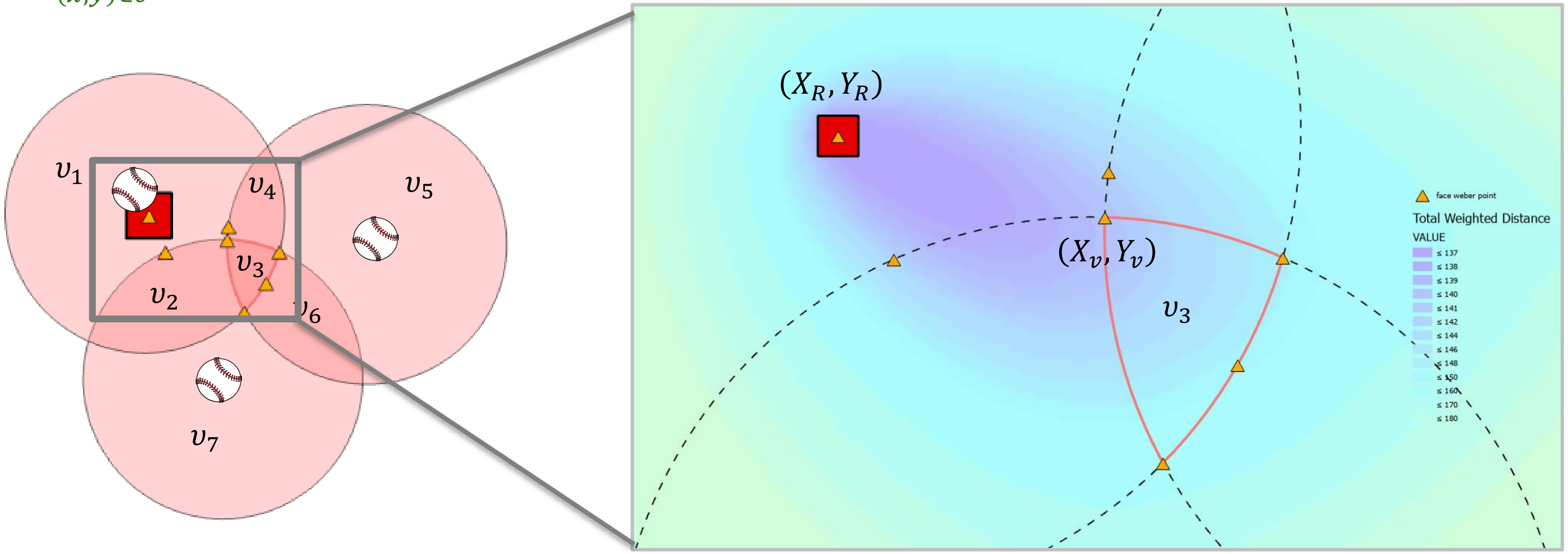


- **Weber Point for each face, *v***
  (minimizes total weighted distance for known level of **coverage**)

$$(X_v, Y_v) = \min_{(x,y) \in v} \sqrt{(X_R - x)^2 + (Y_R - y)^2}$$

Optimality

- Can show that (X_v, Y_v) is optimum in each face v
  - $(X_v, Y_v) = \min_{(x,y)\in v} \sqrt{(X_R - x)^2 + (Y_R - y)^2}$



- As a result, a search process that evaluates each face will result in the location within each face that provides the best access and coverage
  - Non-dominate solutions can be found from among these finite number of options

Developed Solution Algorithm

- **Algorithm**: finite steps to find the solutions with guaranteed optimality

- Major Components
  - Regional Weber Point
  - Buffer
  - Vector Overlay
  - The Nearest Point
  - Non-dominated Solutions

Input: $I,\ S$
Output: $Non-dominated$
Initialize: $O = \{\ \}$

$(X_R,\ Y_R) \leftarrow Weiszfeld(I)$

for each $i \in I$ **do**
$\quad O \leftarrow \{(x,y) \subset R^2 \mid d_{(x,y)(x_i,y_i)} \le S\}$
end-for

$\Psi(O) = \{\nu \mid \mu' \in \wp(O)\ where\ \forall\ \upsilon, \widehat{\upsilon} \subset \mu' \quad \upsilon, \widehat{\upsilon} = \emptyset\ \wedge \cup_\nu \nu = \Phi\}$

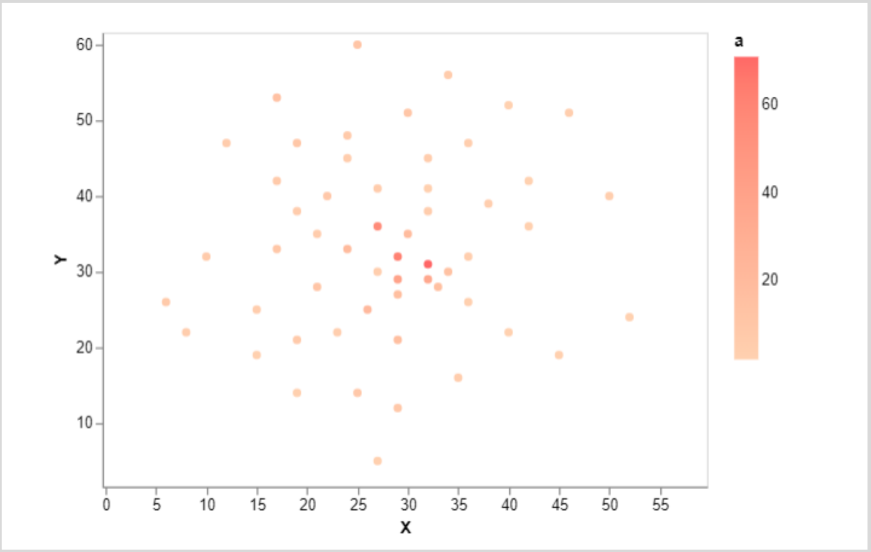
for each $\nu \in \Psi$ **do**
$\quad (X_\nu,\ Y_\nu) \leftarrow min_{(x,y)\in\nu} \sqrt{(X_R - x)^2 + (Y_R - y)^2}$
$\quad N_\nu = \{i \mid d_{\nu(x_i,y_i)} \le S\ \forall i\}$
$\quad cov_\nu = \sum_{i \in N_\nu} a_i$
$\quad twd_\nu = \sum_{i \in I} a_i \sqrt{(X_\nu - x_i)^2 + (Y_\nu - y_i)^2}$
end-for

for each $n \in I$ **do**
$\quad$**for each** $m \in I$ **do**
$\quad\quad$**if** $cov_m \le cov_n \ \wedge\ twd_m \ge twd_n$ **then**
$\quad\quad\quad Dominated \leftarrow m$
$\quad\quad$**end-if**
$\quad$**end-for**
end-for

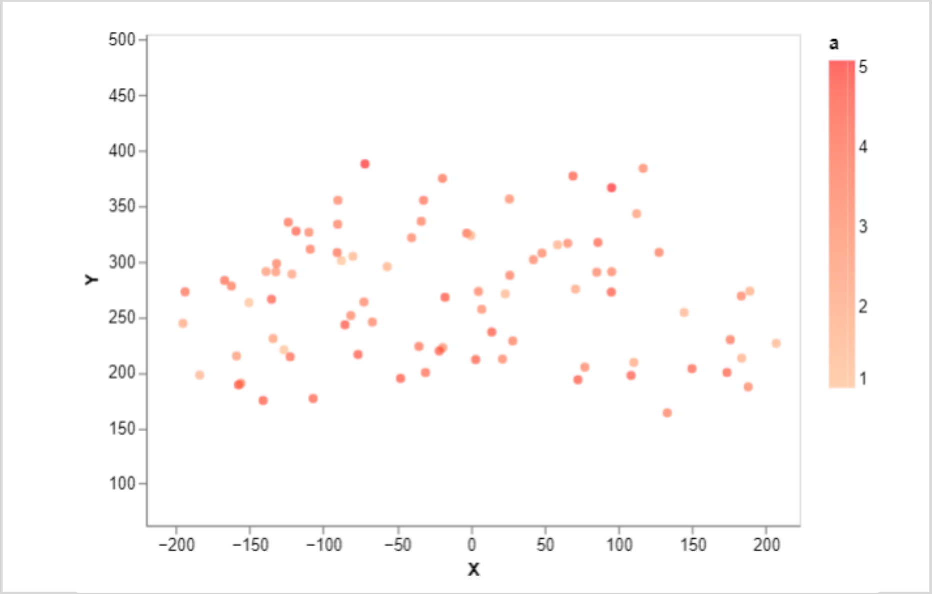
$Non-dominated \leftarrow \{n \mid \forall n\ where\ n \notin Dominated\}$

Applications

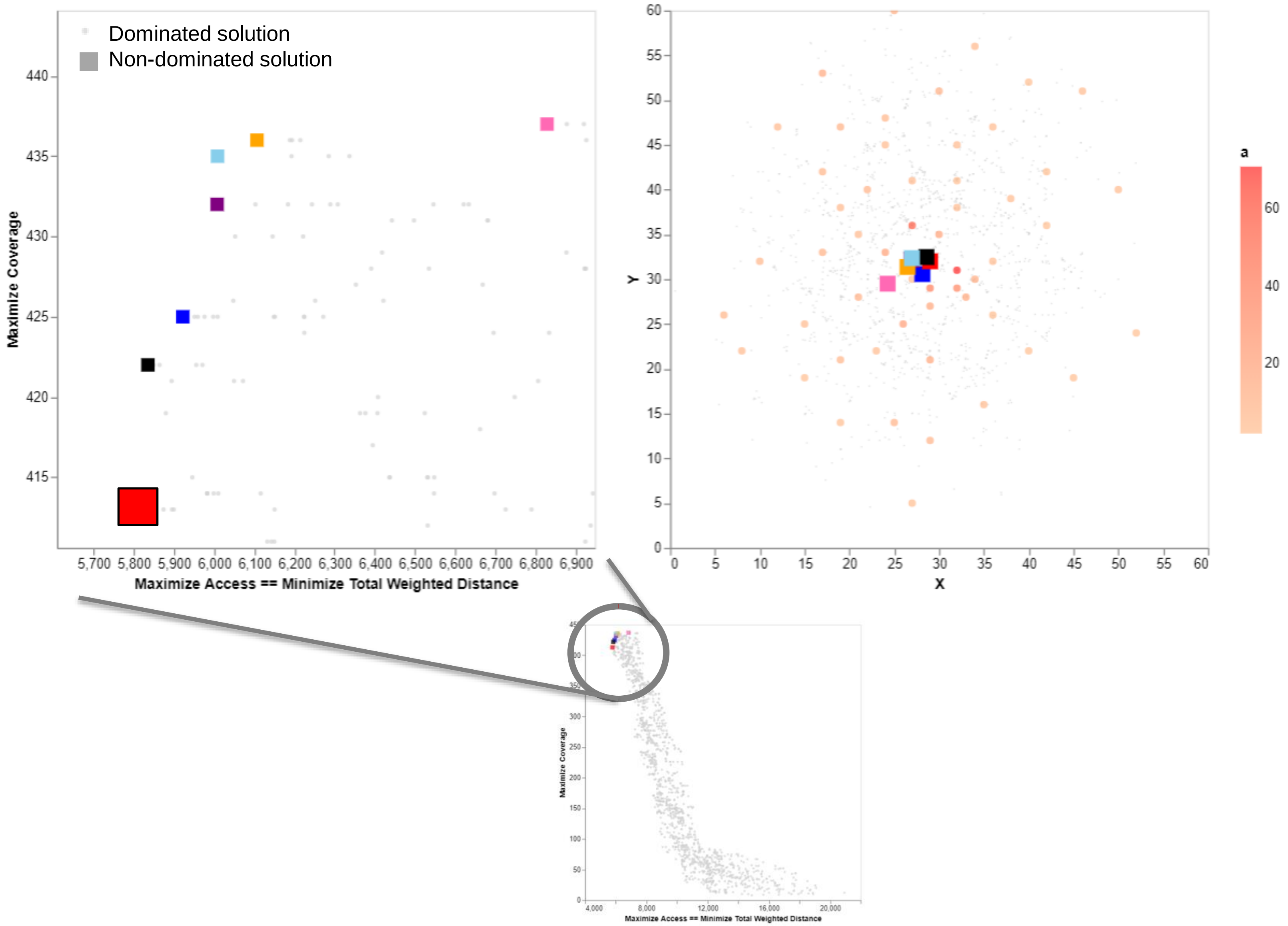
- Swain Data
  - Swain (1971) (see also Church and Baez 2020)
  - 55 demand points
  - S=10



- UCSB baseball
  - 85 batted balls
  - S=90 ft

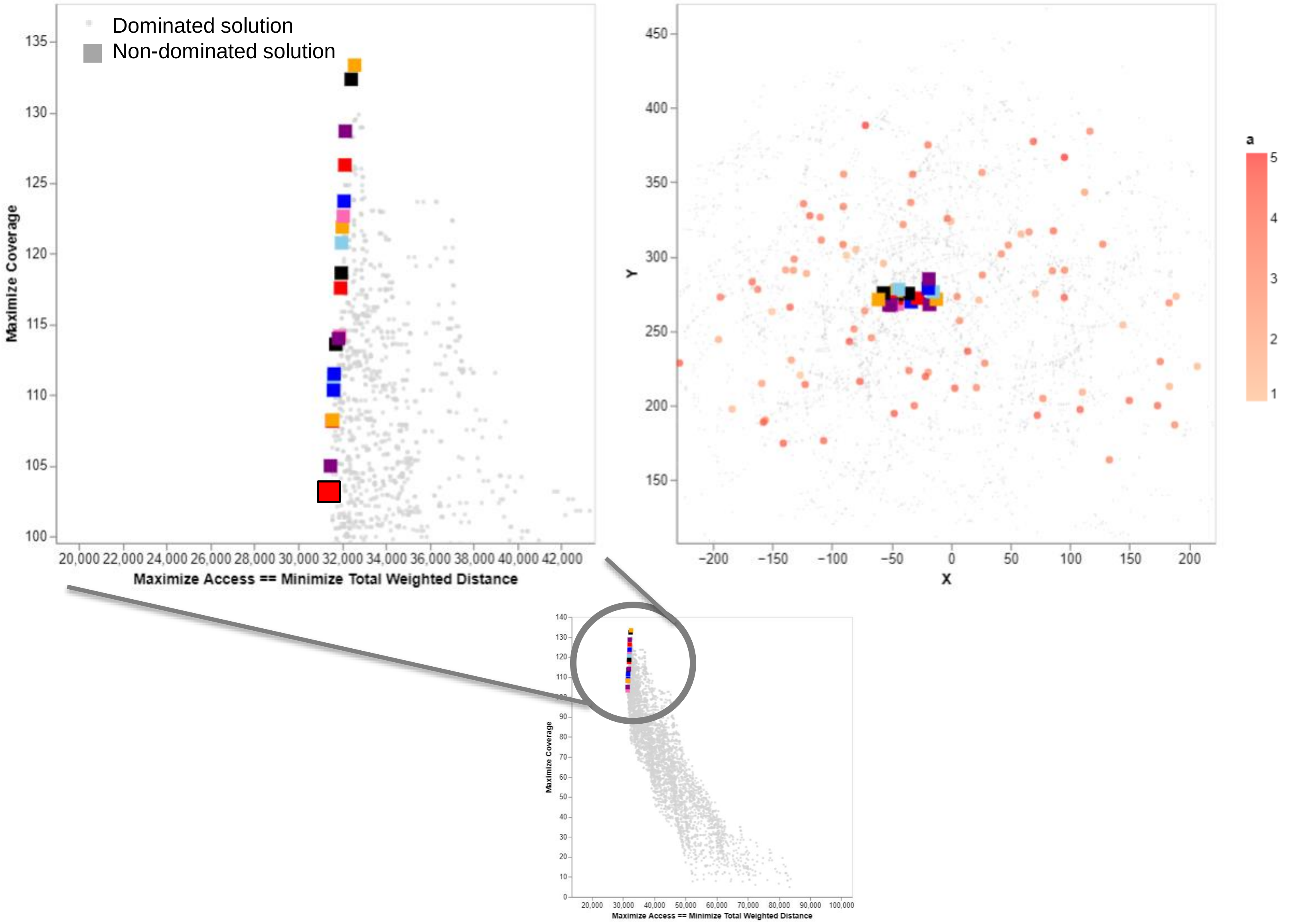


Swain Results



Regional Weber Point
(29.0, 32.0) with objective of 5777.838798
Buffer and Vector Overlay
55 points and buffer for S=10 → 1557 faces
Face Weber Points
1557 faces with coverage and access values
Non-dominated Solutions
7 Non-dominated solutions
Total Solution Time
15.541 seconds

UCSB Baseball Results



Regional Weber Point
(-18.62, 267.72) with objective of 31478.21
Buffer and Vector Overlay
85 points … buffer for S=90 ft → 4666 faces
Face Weber Points
4666 faces with coverage and access values
Non-dominated Solutions
20 Non-dominated solutions
Total Solution Time
45.39 seconds

Conclusion

- Strategic location in a **planar** context,
  accounting for **coverage** and **access** at the same time is challenging.
  But it's an important problem
  that is applicable to various facility siting contexts.

- GIS and computational geometry are invaluable for devising a solution algorithm
  - Exploits spatial knowledge
  - Enables the problem to be solved optimally