

Adv Algo Practicals

Question 1 : Write a program to sort the elements of an array using Randomized Quick Sort (the program should report the number of comparisons)

Solution :

- **Time Complexity:**
 - Best/Average Case: $O(n \log n)$ – Due to balanced partitioning with random pivot selection.
 - Worst Case: $O(n^2)$ – Occurs when the partitioning is highly unbalanced.
- **Space Complexity:** $O(\log n)$ – Recursion depth for the average case, as Quick Sort is in-place.

Output :

```
Initial array: 15 3 45 10 29 8 19 42
Partitioning with pivot: 10 at position: 7
Pivot finalized at position: 2
Partitioning with pivot: 3 at position: 1
Pivot finalized at position: 0
Partitioning with pivot: 29 at position: 7
Pivot finalized at position: 5
Partitioning with pivot: 15 at position: 4
Pivot finalized at position: 3
Partitioning with pivot: 42 at position: 7
Pivot finalized at position: 6
```

```
Sorted array: 3 8 10 15 19 29 42 45
Total comparisons made: 14
```

Question 2 : Write a program to find the i th smallest element of an array using Randomized Select.

Solution :

Time Complexity:

Average Case: $O(n)$ – The randomized pivot selection ensures that on average, the array is divided evenly.

Worst Case: $O(n^2)$ – Occurs when the pivot selection repeatedly results in highly unbalanced partitions.

Space Complexity: $O(1)$ – The algorithm operates in-place with no additional data structures, but recursive calls are replaced by an iterative loop to maintain $O(1)$ space.

Output :

```
Initial array: 15 3 45 10 29 8 19 42
Enter the value of i (to find the ith smallest element): 3
Pivot chosen: 15
Pivot chosen: 3
Pivot chosen: 8
Pivot chosen: 10
The 3th smallest element is: 10
```

Question 3 : Write a program to determine the minimum spanning tree of a graph using Kruskal's algorithm

Solution :

Overall Time Complexity: $O(E * \log(E) + E * \alpha(V))$ which simplifies to $O(E * \log(E))$ in most cases since $\alpha(V)$ is nearly constant.

Overall Space Complexity: $O(V + E)$.

Output :

```
Edges in the Minimum Spanning Tree:  
2 -- 3 == 4  
0 -- 3 == 5  
0 -- 1 == 10
```

Question 4 : Write a program to implement the Bellman-Ford algorithm to find the shortest paths from a given source node to all other nodes in a graph

Solution :

Time Complexity Analysis:

Overall Time Complexity: $O(V * E)$.

Space Complexity Analysis:

Overall Space Complexity: $O(V + E)$.

Output :

Vertex	Distance from Source
0	0
1	1
2	4
3	3
4	3

Question 5 : Write a program to implement a B-Tree

Solution :

Time Complexity Analysis:

1. Insertion: $O(t * \log_t(N))$ where t is the degree of the tree, and N is the number of nodes. This complexity arises from splitting the nodes and traversing the tree.
2. Search: $O(\log_t(N))$ where t is the degree of the tree, and N is the number of nodes.
3. Traversal: $O(N)$ where N is the total number of keys in the tree.

Space Complexity Analysis:

1. The space complexity is $O(N)$ for storing keys and child pointers, where N is the number of keys.

Output :

```
Traversal of the constructed B-Tree is:
```

```
5 6 7 10 12 17 20 30
```

```
Searching for element: 6
```

```
Present
```

```
Searching for element: 15
```

```
Not Present
```

Question 6 : Write a program to implement the Tree Data structure, which supports the following operations:

- a. Insert
- b. Search

Solution :

Time Complexity Analysis:

- 1. Insertion: $O(h)$, where h is the height of the tree. In the worst case (unbalanced tree), $h = O(n)$.
- 2. Search: $O(h)$, where h is the height of the tree. In the worst case, $h = O(n)$.
- 3. Deletion: $O(h)$, where h is the height of the tree. In the worst case, $h = O(n)$.
- 4. Traversal (in-order): $O(n)$, where n is the number of nodes in the tree.

Space Complexity Analysis:

- 1. Space complexity is $O(n)$ for storing all nodes in the tree.

Output :

```
Initial tree (in-order traversal):  
20 30 40 50 60 70 80
```

```
Searching for elements:  
Element 40 found in the tree.  
Element 90 not found in the tree.
```

```
Deleting nodes:  
Deleted key: 20  
30 40 50 60 70 80  
Deleted key: 30  
40 50 60 70 80  
Deleted key: 50  
40 60 70 80
```

Question 7 : Write a program to search a pattern in a given text using the KMP algorithm

Solution :

Time Complexity Analysis:

1. Preprocessing the Pattern (LPS Array): $O(m)$, where m is the length of the pattern.
2. Searching the Pattern in Text: $O(n)$, where n is the length of the text.

Overall Time Complexity: $O(n + m)$.

Space Complexity Analysis:

1. Space Complexity for LPS Array: $O(m)$.
2. Overall Space Complexity: $O(m)$.

Output :

```
Text: ababcabababd
Pattern: ababd
Pattern found at index 10
```

Question 8 : Write a program to implement a Suffix tree

Solution :

Time Complexity Analysis:

1. Building the Suffix Tree: $O(n)$,
2. **Space Complexity:** $O(n * |\Sigma|)$, where $|\Sigma|$ is the alphabet size. The space complexity is determined by the number of nodes and edges in the suffix tree.

Output :

```
Suffix tree built successfully.
```

```
Suffix Tree Structure:
```

```
+-- $
+-- a
    +-- bxac$
    +-- c$
+-- bxac$
+-- c$
+-- xa
    +-- bxac$
    +-- c$
```