Program Design and Algorithms
Lecture 1: Introduction

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Outline

- About Me
- Course Details
- A.M. Turing Award Winners for Algorithms
- What Is This Course About
- What Are Algorithms
- What Does It Mean to Analyze An Algorithm
- Comparing Time Complexity

Instructor: Yongxin Tong

- Beihang University (2015.4 - Current)
  - “Zhuoyue Program” Associate Professor
  - State Key Lab. of Software Development Environment
  - Research Interests: **Big Data** and **Crowd Intelligence**

- HKUST (2010.8 – 2015.3)
  - Research Assistant Professor (2014.2 – 2015.3)
    - CSE Department, focused on data mining and crowdsourcing
  - Ph.D. Student and Candidate (2010.8 – 2014.1)
    - CSE Department, focused on uncertain data mining

Contact and TAs

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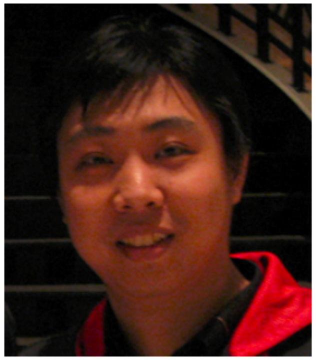
    http://www.cse.ust.hk/~yxtong/

Contact and TAs

www.cse.ust.hk/~yxtong/



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[**Short Bio**] [**Research**] [**Publications**] [**Awards**] [**Experiences**] [**Professional Services**] [**Misc.**]

Short Biography

Yongxin Tong is an Associate Professor in the State Key Laboratory of Software Development Environment (SKLSDE) of the School of Computer Science and Engineering at Beihang University (BUAA). He received a Ph.D. degree in Computing Science and Engineering from the Department of Computer Science and Engineering, The Hong Kong University of Science and Technology (HKUST), under Prof. Lei Chen's supervision. He also received a Master degree in Software Engineering at Beihang University and a Double Bachelor degree in Economics from China Centre for Economic Research (CCER) at Peking University.

Research Interests

- Crowdsourcing
- Spatio-temporal Data Processing and Analysis
- Uncertain Data Mining and Management
- Social Network Analysis

Our Recent Tutorials

- New **Yongxin Tong**, Lei Chen, Cyrus Shahabi. "Spatial Crowdsourcing: Challenges, Techniques, and Applications", in *Proceedings of the 43rd International Conference on Very Large Databases* (**VLDB 2017**), Munich, Germany, August 28 - September 1, 2017. [Tutorial Slides]

Selected Publications [My DBLP Entry] [Full Publication List]

- New **Yongxin Tong**, Libin Wang, Zimu Zhou, Bolin Ding, Lei Chen, Jieping Ye, Ke Xu. "Flexible Dynamic Task Assignment in Real-Time Spatial Data", in *Proceedings of the 43rd International Conference on Very Large Databases* (**VLDB 2017**), Munich, Germany, August 28 - September 1, 2017. [Slides] [Poster]
- New **Yongxin Tong**, Yuqiang Chen, Zimu Zhou, Lei Chen, Jie Wang, Qiang Yang, Jieping Ye. "The Simpler The Better: A Unified Approach to Predicting Original Taxi Demands on Large-Scale Online Platforms", in *Proceedings of the 23rd ACM SIGKDD Conference on Knowledge Discovery and Data Mining* (**SIGKDD 2017**), Halifax, Nova Scotia, Canada, August 13 - 17, 2017. [Slides] [Poster] [Short Promotional Video]
- New Jieying She, **Yongxin Tong**, Lei Chen, Tianshu Song. "Feedback-Aware Social Event-Participant Arrangement", in *Proceedings of the 36th ACM SIGMOD International Conference on Management of Data* (**SIGMOD 2017**), Chicago, IL, USA, May 14-19, 2017. [Slides] [Poster]

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    http://www.cse.ust.hk/~yxtong/

- TAs
  - Qian Tao (Ph.D. Student)
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Faculty Members in SKLSDE


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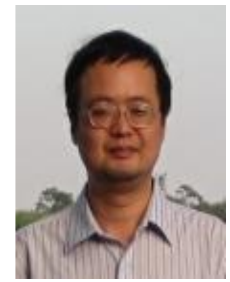
马殿富教授


吕卫锋教授


尹宝林教授


蔡维德教授


马世龙教授


张玉平教授


许可教授


张辉教授


郎波教授


杨钦教授


吴文峻教授


朱皞罡教授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李吉刚博士


罗杰博士


杜博文博士


王德庆博士

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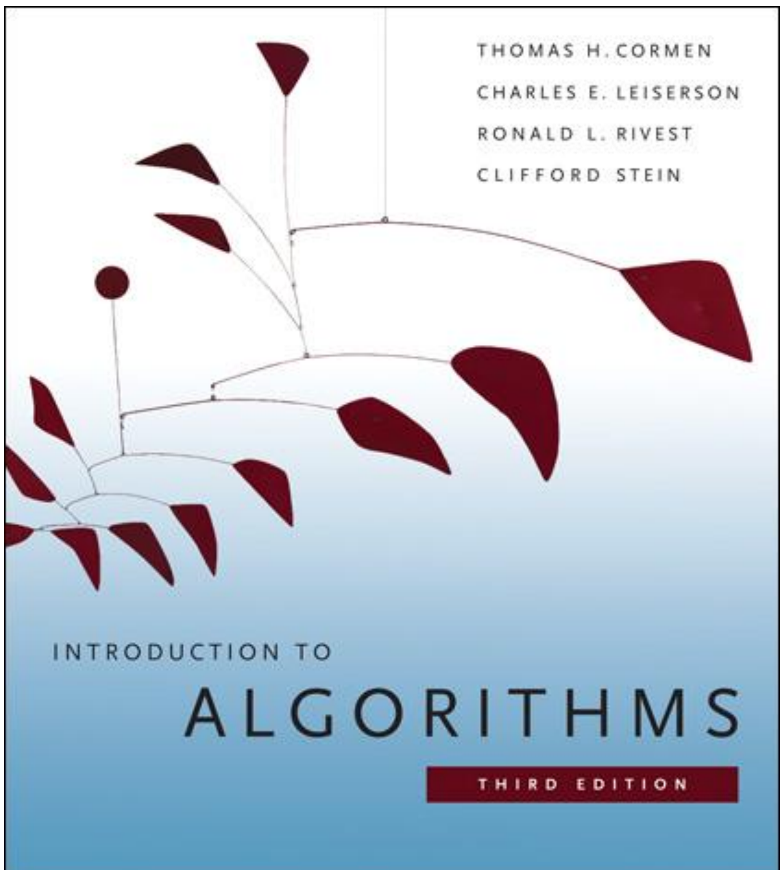
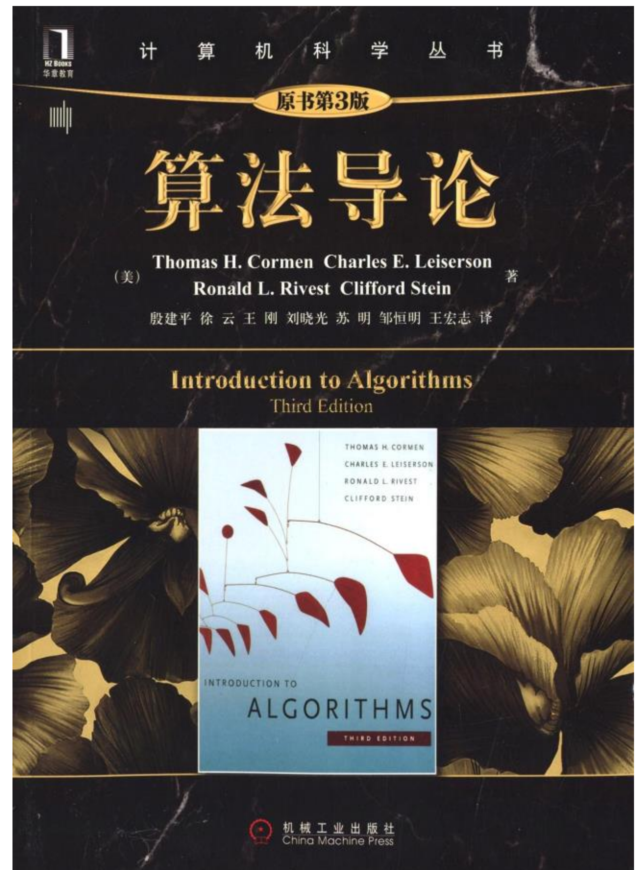
Textbook

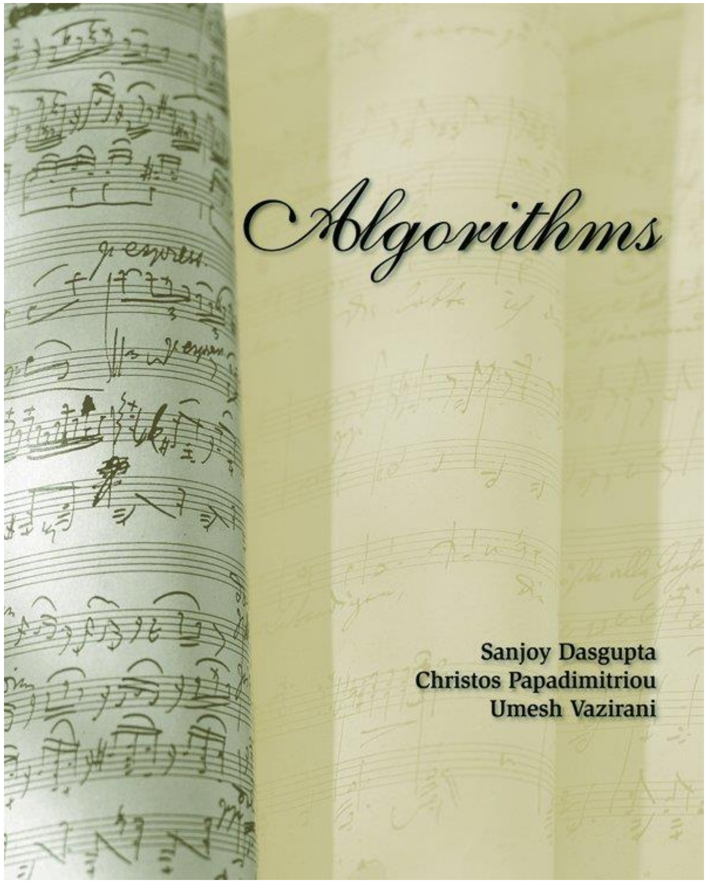
- Textbook: *Introduction to Algorithms* (3rd ed.)
  - by Cormen, Leiserson, Rivest and Stein (CLRS)
  - Prepublication version available online

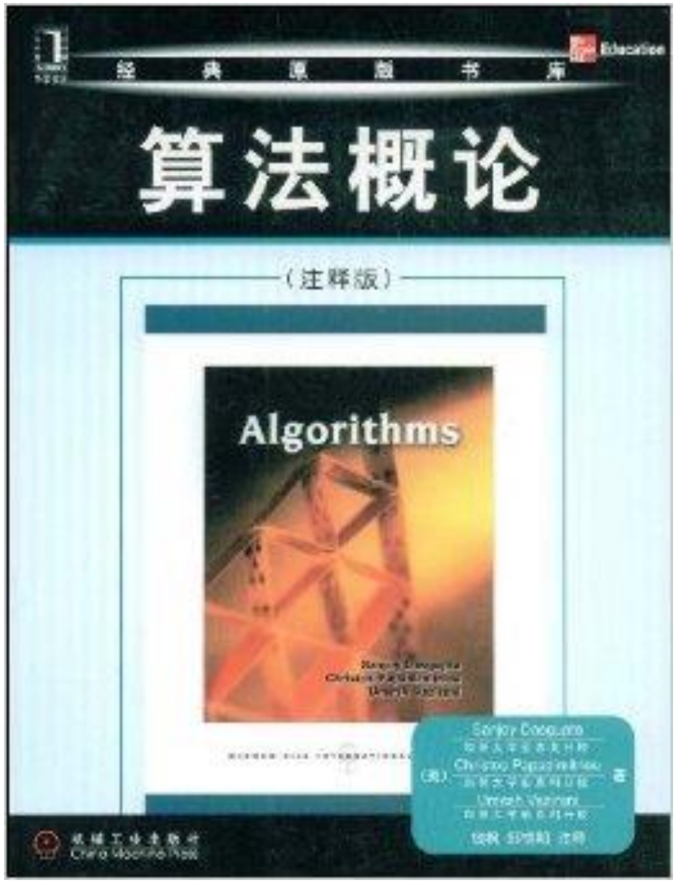




References (1)

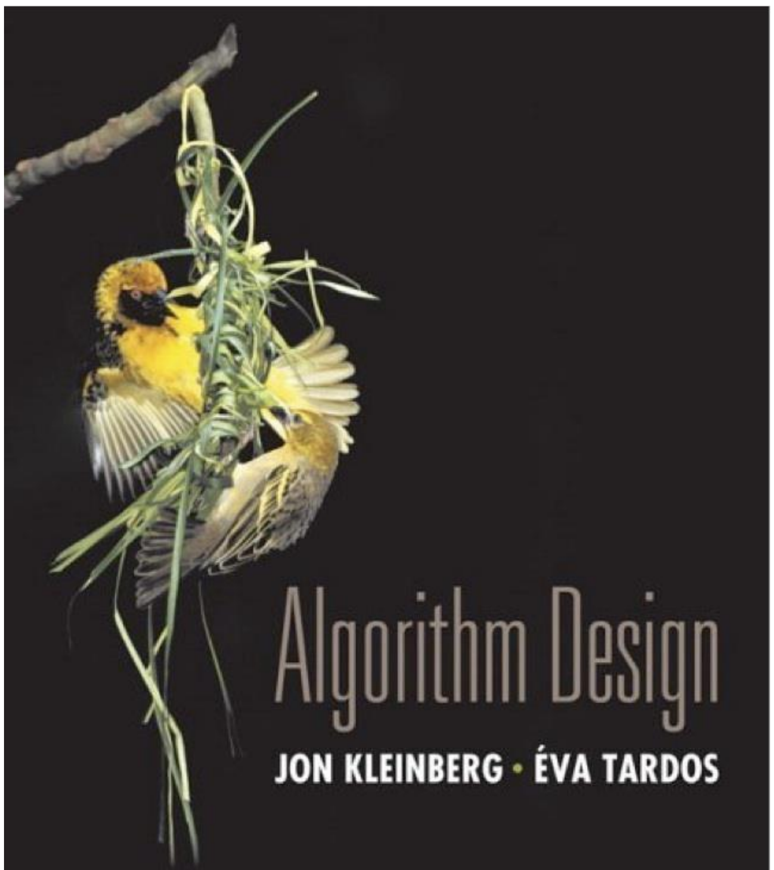
- Reference: *Algorithms*
  - by Dasgupta, Papadimitriou, and Vazirani (DPV)
  - Prepublication version available online

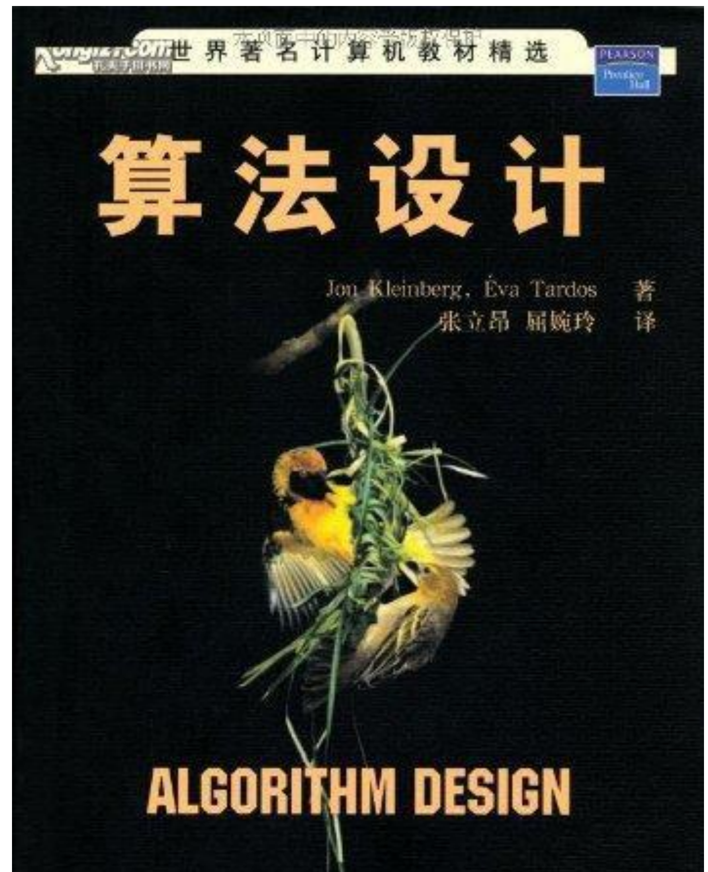




References (2)

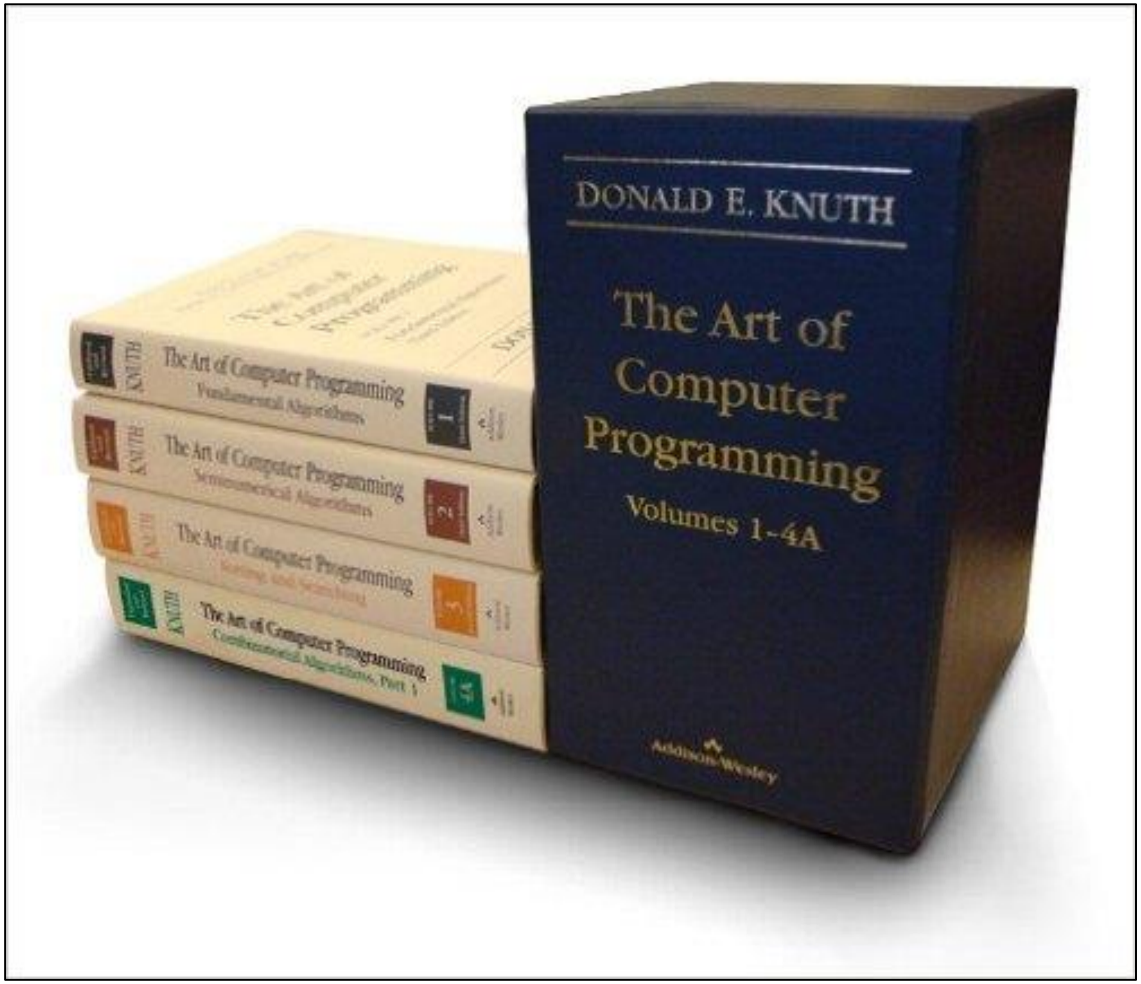
- Reference: *Algorithm Design*
  - by Kleinberg and Tardos (KT)





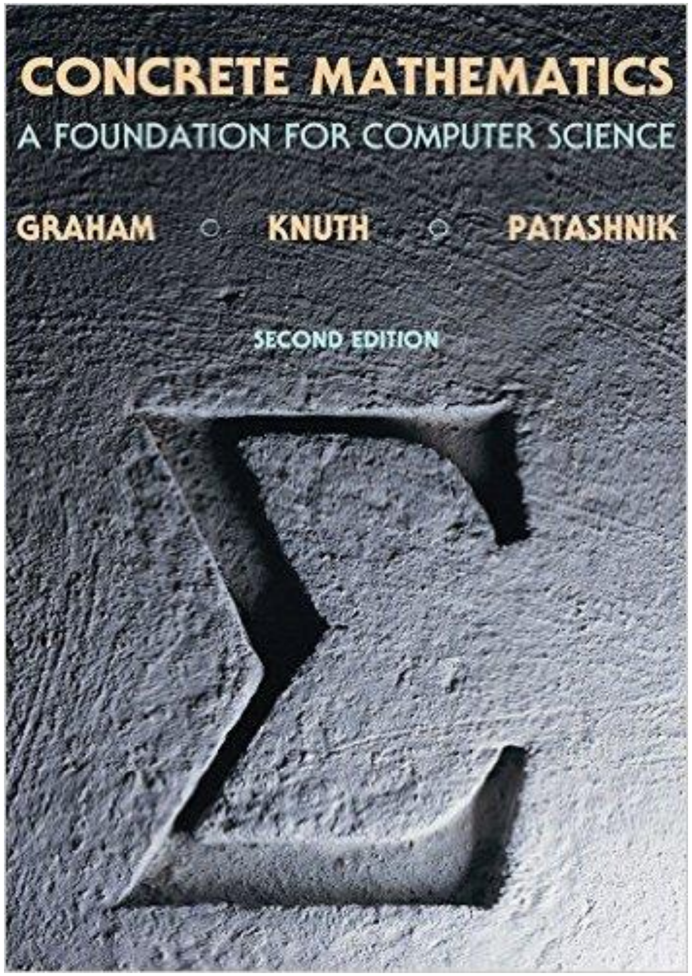
References (3)

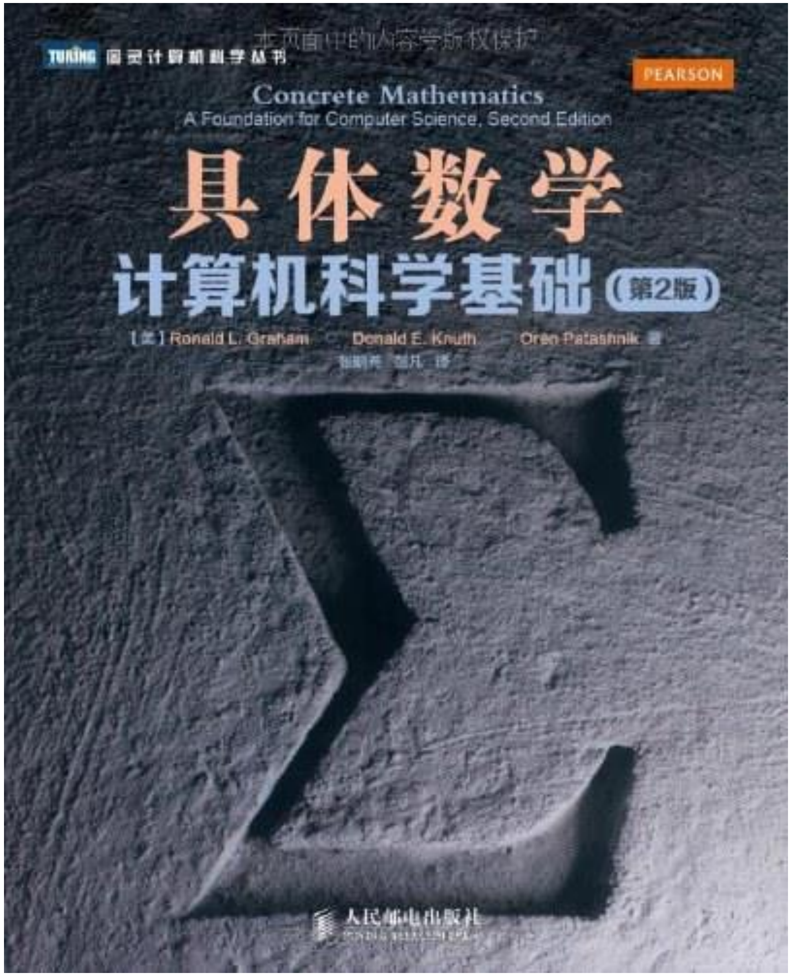
- Reference: *The Art of Computer Programming*
  - by Donald E. Knuth



References (4)

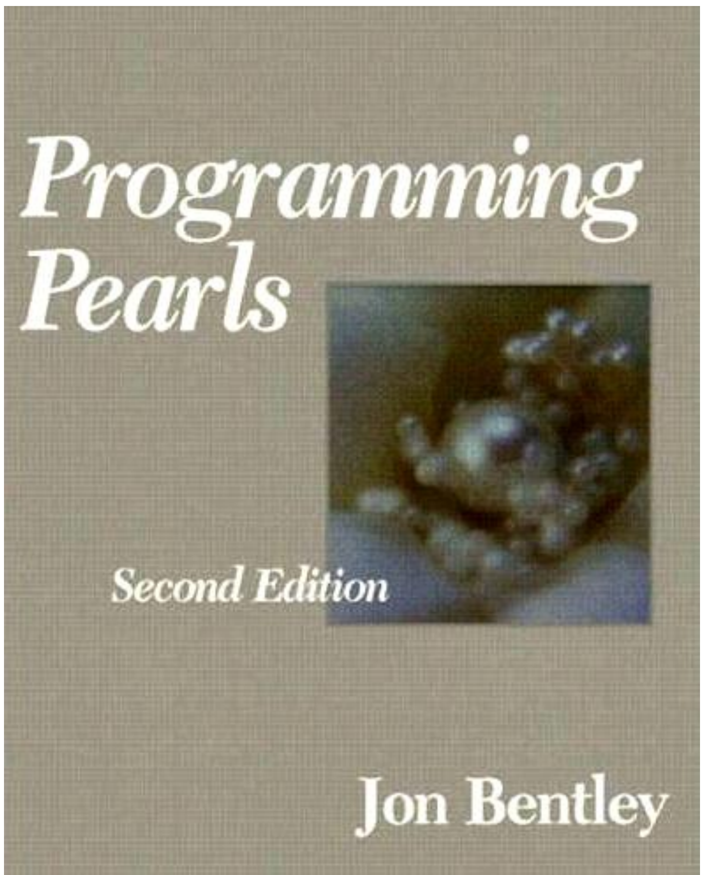
- Reference: *Concrete Mathematics* (2nd ed.)
  - by Graham, Knuth, Patashnik (GKP)

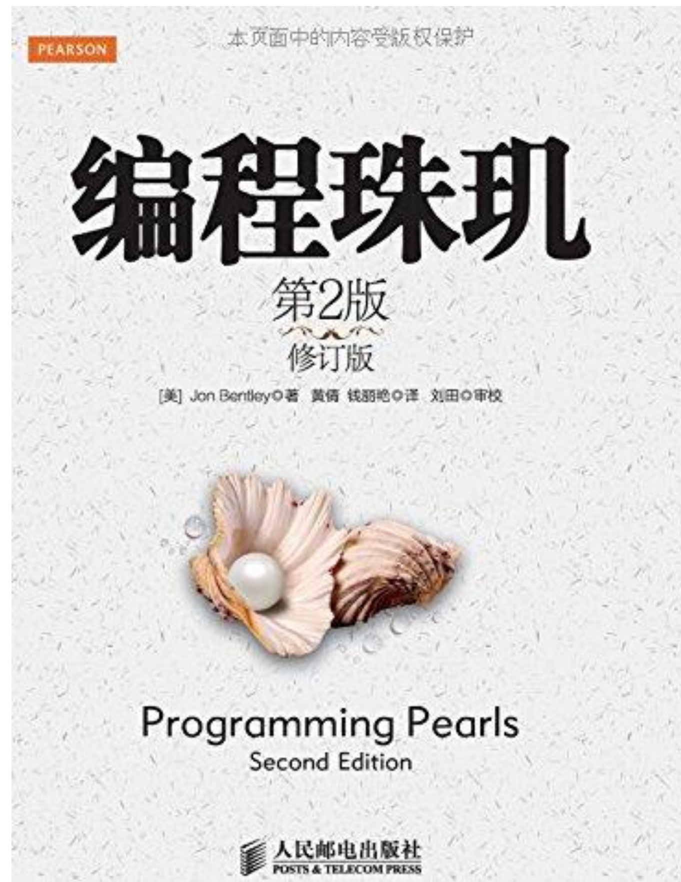




References (5)

- Reference: *Programming Pearls (2nd ed.)*
  - by Jon Bentley





Prerequisites

- We assume you know:
  - Linked Lists, Stacks, Queues
  - Binary Search Trees
    - Traversals
    - Searching (but not analysis)
- What have you learnt previously?
  - Graph algorithms
    - Breadth-first search (BFS)
    - Depth-first search (DFS)
    - Topological sort (TS)
    - Minimum Spanning Trees (MST)
    - Dijkstra's shortest path algorithm (SP)

Tentative Syllabus

- Basics
  - Asymptotic Notations and Recurrences
- Divide and Conquer Algorithms
  - MCS Problem, PM Problem, and Quicksort
- Graph Algorithms
  - BFS, DFS, SP, MST, Max Flow and Matching
- Greedy Algorithms
  - Huffman Coding and Fractional Knapsack
- Dynamic Programming Algorithms
  - 0-1 Knapsack, Rod-Cutting, CMM, LCS, and APSP
- Dealing with Hard Problems
  - Problem Classes (P, NP, NPC) and Approximation Alg.

ACM-ICPC

- Full Name
  - ACM International Collegiate Programming Contest
- Contest Rules (Team Competitions)
  - Each team consist of three university students.
  - Students who have previously competed in two World Finals or five regional competitions are ineligible to compete again.
- History of ACM-ICPC in China
  - Four champions (2002/2005/2010:SJTU & 2011: ZJU).
  - Beihang Univeristy: Rank 14th in 2016 (Top-3 in China).
- Invited Talk of ACM-ICPC Participant
  - *I will invite at least one ACM-ICPC participant to share her/his experiences of learning algorithms in our course.*

Lectures and Tutorials

- Lectures
  - Slides will be available on our course WeChat group.
- Tutorials (补充练习)
  - There will be 9 tutorials in this semester.
  - The tutorials will provide more examples to illustrate the material you learnt in class.
  - The first tutorial will be released on next week.

Grading Scheme

- (30%) Four Assignments
  - Each requires designing algorithms and analyzing correctness/run time.
  - Each will take 10-14 days. The first one will be released in the next week.
  - After each submission due, we will post the solution and **WON’T** accept any assignment.
- (10%) Project
  - Each project is completed by a group.
  - The number of students in a group is at most 8.
  - Each group needs to submit a final report and codes.
  - The topics of the project will be released at the end of Oct.
- (60%) Final Exam
  - It covers entire semester’s material.

Classroom Etiquette

- **No roll-call in our class !**
- Turn off cell phone ringers.
  - No phone conversations in room.
- Latecomers should enter quietly.
- No LOUD talking among selves during lectures.

WeChat Group

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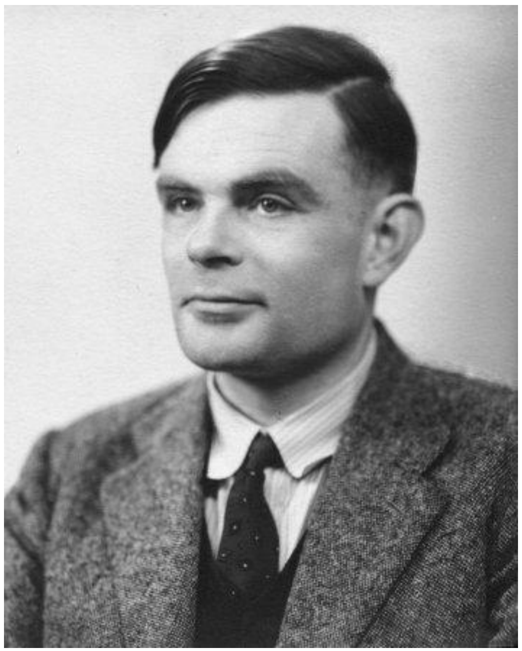
A.M. Turing Award

From 2007 to 2013, the award was accompanied by a prize of US $250,000 by Intel and Google. Since 2014, the award has been accompanied by a prize of US $1 million by Google.



Alan M. Turing

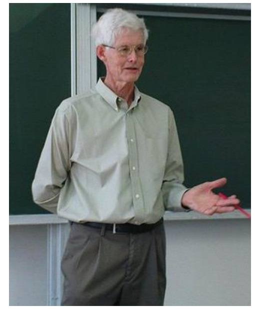
Nobel Prize of Computing

Since 1966，there have been 65 recipients of A.M. Turing Award!
This year is the 50th anniversary of A.M. Turing Award!

A.M. Turing Award Winners for Algorithms

Donald E. Knuth
1974, USA

Robert W. Floyd
1978, USA

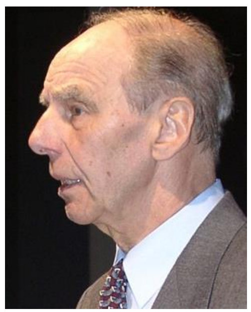


Stephen A. Cook
1982, USA

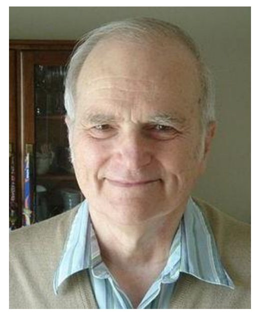
Richard M. Karp
1985, USA

John Hopcroft
1986, USA

Robert Tarjan
1986, USA



Juris Hartmanis
1993, Latvia



Richard E. Stearns
1993, USA

Manuel Blum
1995, Venezuela

Andrew Yao
2000, China

Leslie G. Valiant
2010, Hungarian

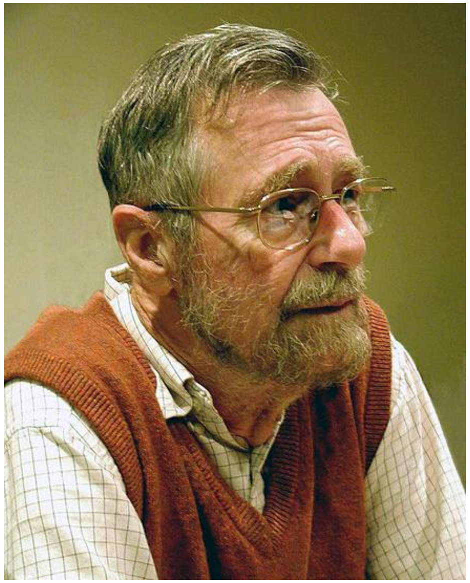
Silvio Micali
2012, Italy

Shafi Goldwasser
2012, USA

Martin Hellman
2015, USA

Whitfield Diffie
2015, USA

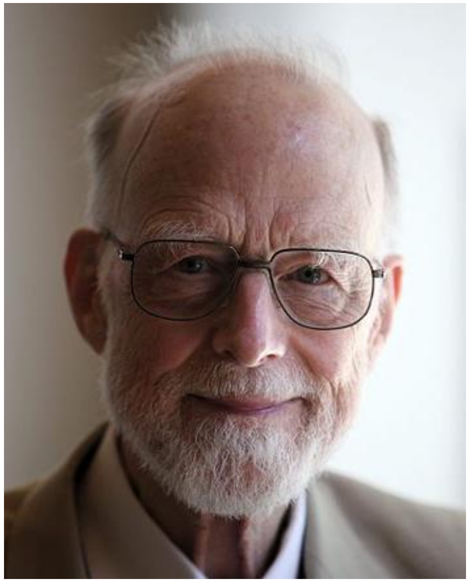
Other Related A.M. Turing Award Winners



Edsger W. Dijkstra

The Recipient in 1972, Netherlands

Contributions: ALGOL Father Dijkstra Algorithm



Tony Hoare

The Recipient in 1980, UK

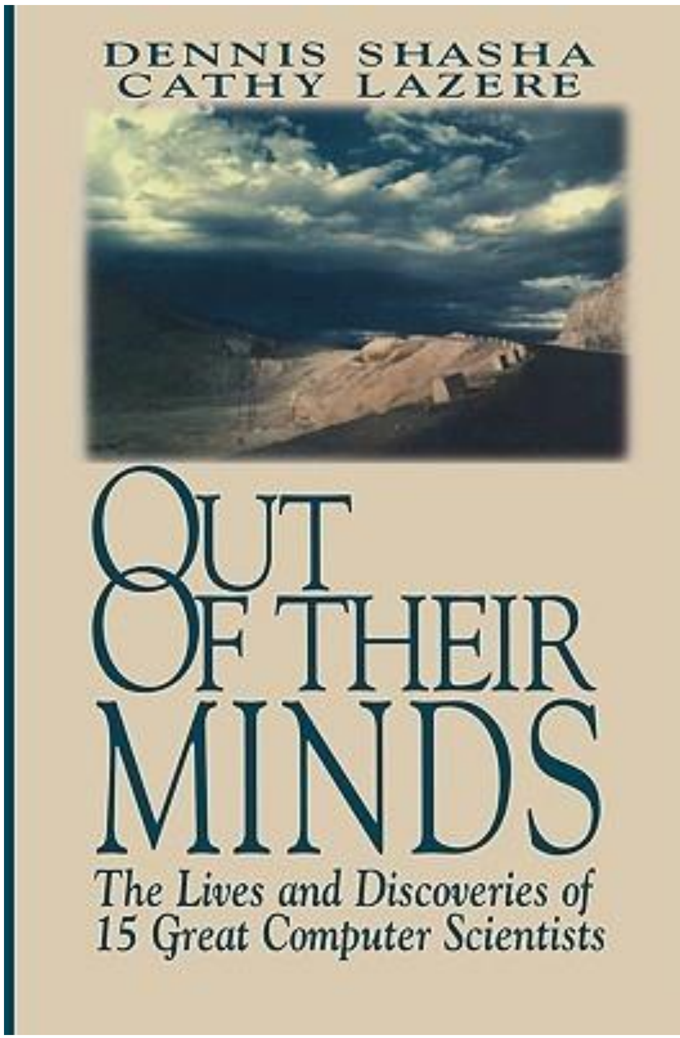
Contributions: Hoare logic, QuickSort

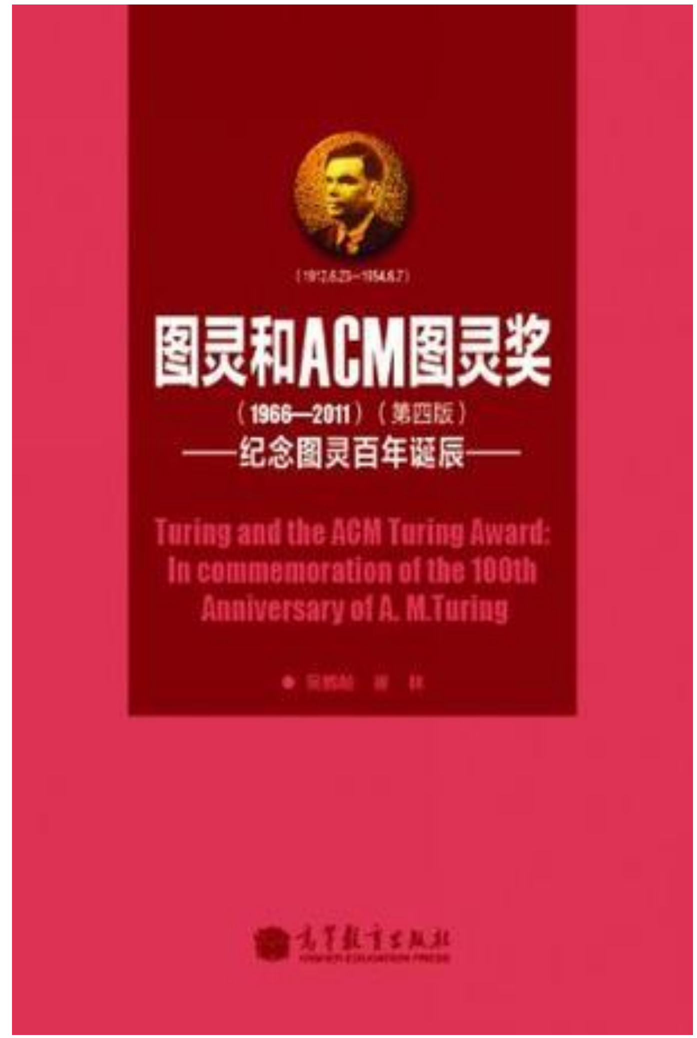
Tim Berners-Lee

The Recipient in 2017, UK

Contributions: World Wide Web, The first web browser

Books of A.M. Turing Award Winners





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What is this course about?

Example (Chain Matrix Multiplication)

$$A = C = \begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}.$$

$$B = D = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}.$$

Want: $ABCD = ?$

- Method 1: $(AB)(CD)$
- Method 2: $A((BC)D)$

Method 1 is much more efficient than Method 2.
(Expand the expression on board)

What is this course about?

- There is usually more than one algorithm for solving a problem.
- Some algorithms are more efficient than others.
- We want the most efficient algorithm.

What is this course about?

- If we have a number of alternative algorithms for solving a problem, how do we know which is the most efficient?
- To do so, we need to analyze each of them to determine its efficiency.
- Of course, we must also make sure the algorithm is correct.

What is this course about?

- In this course, we will discuss fundamental techniques for:
  - Designing efficient algorithms,
  - Proving the correctness of algorithms,
  - Analyzing the running times of algorithms

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- Note:
  - Analysis and design go hand-in-hand: *By analyzing the running times of algorithms, we will know how to design fast algorithms*

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Computational Problem

Definition

A computational problem is a specification of the desired input-output relationship

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Example (Computational Problem)

Sorting

- Input: Sequence of n numbers $\langle a_1, \cdots, a_n \rangle$
- Output: Permutation (reordering)

$$\langle a'_1, a'_2, \cdots, a'_n \rangle$$

such that $a'_1 \leq a'_2 \leq \cdots \leq a'_n$

Instance

Definition

A problem instance is any valid input to the problem.

Instance

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A problem instance is any valid input to the problem.

Example (Instance of the Sorting Problem)

$$\langle 8, 3, 6, 7, 1, 2, 9 \rangle$$

Algorithm

Definition

An algorithm is a well defined computational procedure that transforms inputs into outputs, achieving the desired input-output relationship

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An algorithm is a well defined computational procedure that transforms inputs into outputs, achieving the desired input-output relationship

Definition

A correct algorithm halts with the correct output for every input instance. We can then say that the algorithm solves the problem

Example: Insertion Sort

Pseudocode:

Input: $A[1 \ldots n]$ is an array of numbers
for $j \leftarrow 2$ **to** n **do**
    key $\leftarrow A[j]$;
    $i \leftarrow j - 1$;
    while $i \geq 1$ *and* $A[i] > key$ **do**
        $A[i+1] \leftarrow A[i]$;
        $i \leftarrow i - 1$;
    end
    $A[i+1] \leftarrow$ key;
end

	key	

Sorted   Unsorted

Where in the sorted part to put "key"?

How Does It Work?

- An incremental approach: To sort a given array of length n,
  at the *i*th step it sorts the array of the first *i* items by making use of the sorted array of the first *i* - 1 items

Example

Sort $A = \langle 6, 3, 2, 4, 5 \rangle$ with insertion sort

Step 1: $\langle 6, 3, 2, 4, 5 \rangle$

Step 2: $\langle 3, 6, 2, 4, 5 \rangle$

Step 3: $\langle 2, 3, 6, 4, 5 \rangle$

Step 4: $\langle 2, 3, 4, 6, 5 \rangle$

Step 5: $\langle 2, 3, 4, 5, 6 \rangle$

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Analyzing Algorithms

- Predict resource utilization
  - Memory (space complexity)
  - Running time (time complexity) -- focus of this course

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Analyzing Algorithms

- Predict resource utilization
  - Memory (space complexity)
  - Running time (time complexity) -- focus of this course
    - depends on the speed of the computer
    - depends on the implementation details
    - depends on the input, especially on the size of the input
- In light of the above factors, how can we compare different algorithms in terms of their running times?
- We want to find a way of measuring running times that is mathematically elegant and machine-independent.

Machine-independent running time

- We will measure the running time as the number of primitive operations (e.g., addition, multiplication, comparisons) used by the algorithm

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Machine-independent running time

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- Input size ***n***: rigorous definition given later
  - Sorting: number of items to be sorted

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- Input size ***n***: rigorous definition given later
  - Sorting: number of items to be sorted
  - Graphs: number of vertices and edges

Three Kinds of Analysis: I

Best Case: An instance for a given size n that results in the fastest possible running time.

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Example (Insertion sort)

$$A[1] \leq A[2] \leq A[3] \leq \cdots \leq A[n]$$

Three Kinds of Analysis: I

Best Case: An instance for a given size n that results in the fastest possible running time.

Example (Insertion sort)

$$A[1] \leq A[2] \leq A[3] \leq \cdots \leq A[n]$$

The number of comparisons needed is equal to

$$\underbrace{1 + 1 + 1 + \cdots + 1}_{n-1} = n - 1 = \Theta(n)$$

	key	
Sorted		Unsorted

"key" is compared to only the element right before it.

Three Kinds of Analysis: II

Worst Case: An instance for a given size n that results in the slowest possible running time.

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Example (Insertion sort)

$$A[1] \geq A[2] \geq A[3] \geq \cdots \geq A[n]$$

Three Kinds of Analysis: II

Worst Case: An instance for a given size n that results in the slowest possible running time.

Example (Insertion sort)

$$A[1] \geq A[2] \geq A[3] \geq \cdots \geq A[n]$$

The number of comparisons needed is equal to

$$1 + 2 + \cdots + (n-1) = \frac{n(n-1)}{2} = \Theta(n^2)$$

	key	

Sorted Unsorted

"key" is compared to everything element before it.

Three Kinds of Analysis: III

Average Case: Running time averaged over all possible instances for the given size, assuming some probability distribution on the instances.

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Example (Insertion sort)

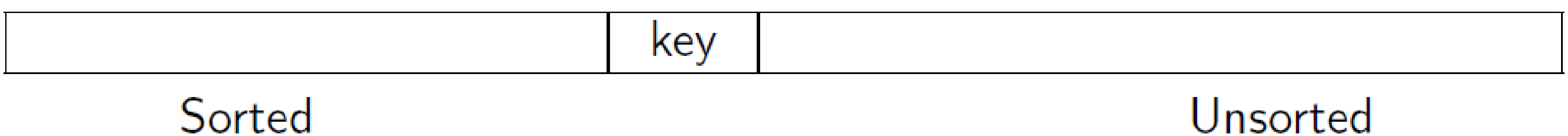
$\Theta(n^2)$, assuming that each of the $n!$ instances is equally likely (uniform distribution).

Three Kinds of Analysis: III

Average Case: Running time averaged over all possible instances for the given size, assuming some probability distribution on the instances.

Example (Insertion sort)

$\Theta(n^2)$, assuming that each of the $n!$ instances is equally likely (uniform distribution).



On average, "key" is compared to half of the elements before it.

Three Kinds of Analysis

- Best case: Clearly useless

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- Worst case: Commonly used, will also be used in this course
  - Gives a running time guarantee no matter what the input is
  - Fair comparison among different algorithms

Three Kinds of Analysis

- Best case: Clearly useless
- Worst case: Commonly used, will also be used in this course
  - Gives a running time guarantee no matter what the input is
  - Fair comparison among different algorithms
- Average case: Used sometimes
  - Need to assume some distribution: real-world inputs are seldom uniformly random!
  - Analysis is complicated

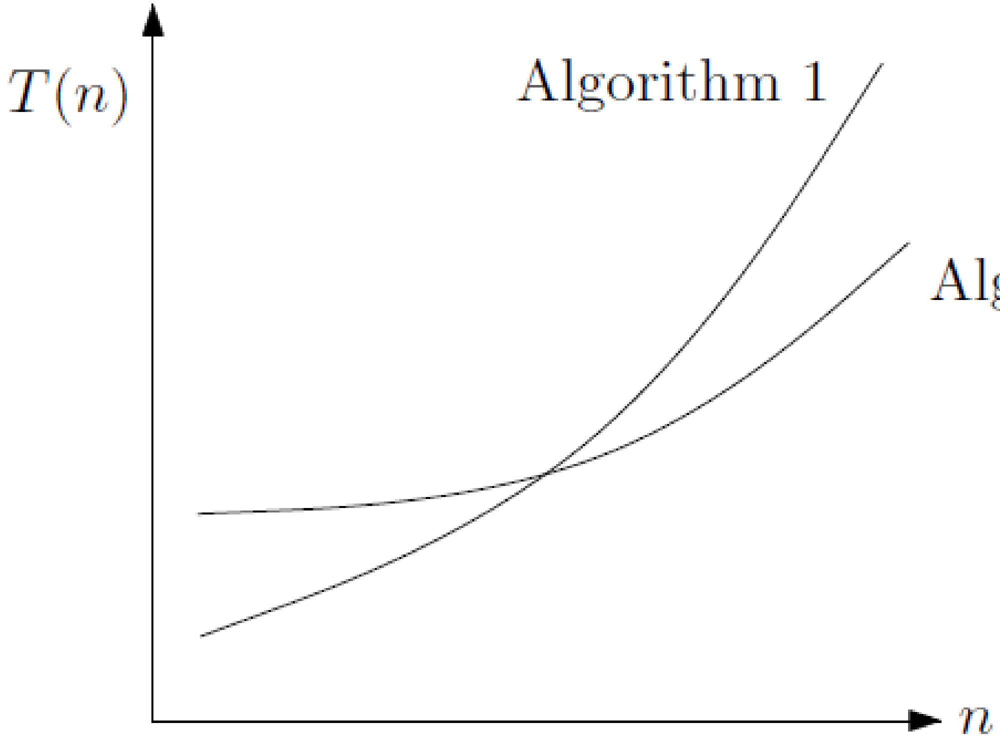
Three Kinds of Analysis

- Best case: Clearly useless
- Worst case: Commonly used, will also be used in this course
  - Gives a running time guarantee no matter what the input is
  - Fair comparison among different algorithms
- Average case: Used sometimes
  - Need to assume some distribution: real-world inputs are seldom uniformly random!
  - Analysis is complicated
  - Will not be used in this course

Outline

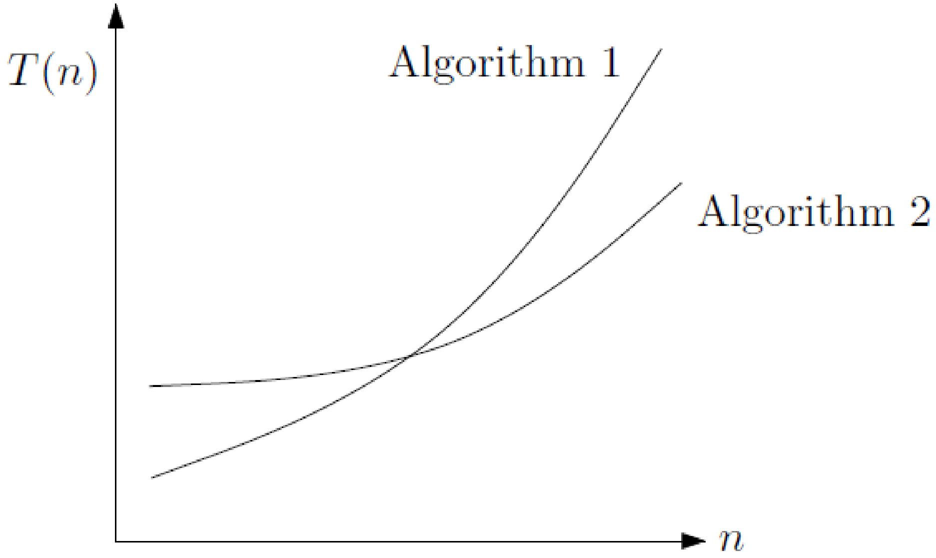
- About Me
- Course Details
- A.M. Turing Award Winners for Algorithms
- What Is This Course About
- What Are Algorithms
- What Does It Mean to Analyze An Algorithm
- Comparing Time Complexity

Comparing Time Complexity



- Which algorithm is superior for large n?
  - T(n) for Algorithm 1 is $3n^3 + 6n^2 - 4n + 17$
  - T(n) for Algorithm 2 is $7n^2 - 8n + 20$
- Clearly, Algorithm 2 is superior.

Asymptotic Analysis



- T(n) for Algorithm 1 is $3n^3 + 6n^2 - 4n + 17 = \Theta(n^3)$

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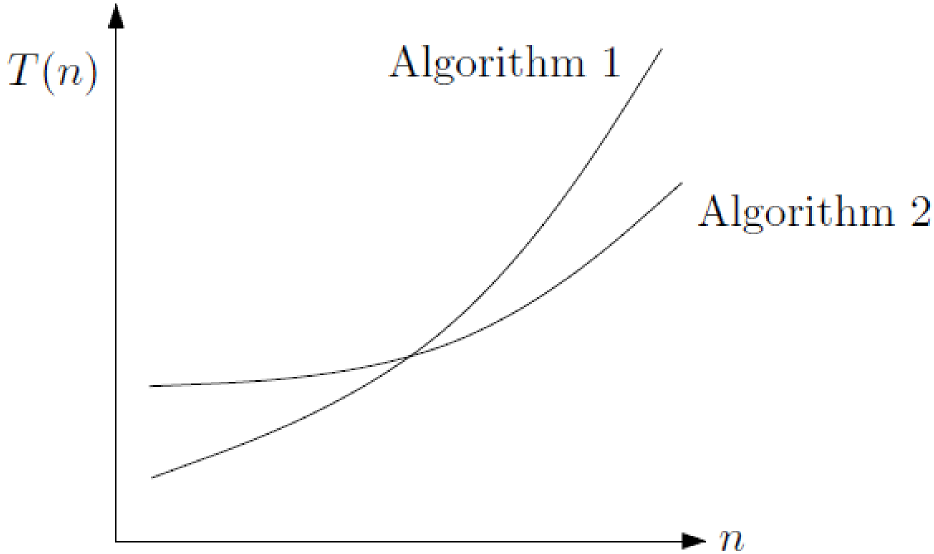

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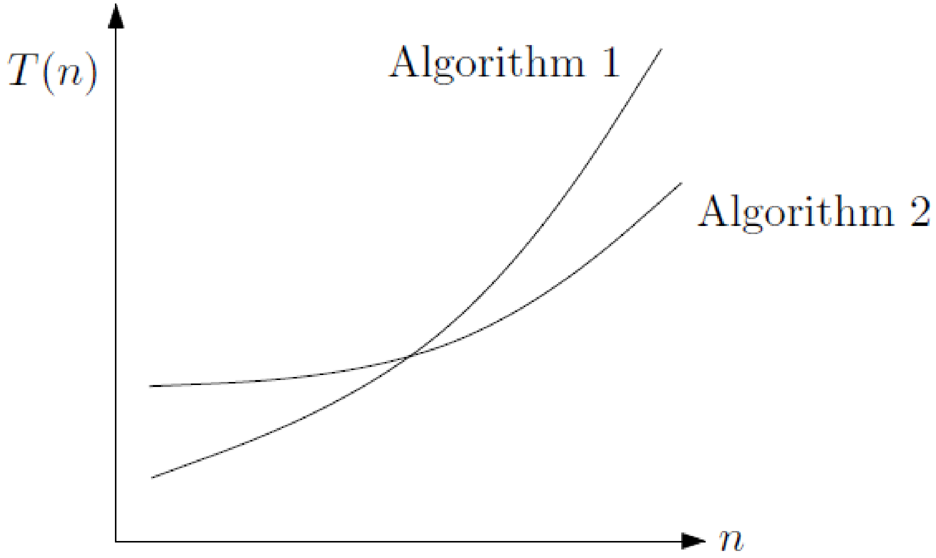

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Θ-notation

- Drop low-order terms; ingore leading constants

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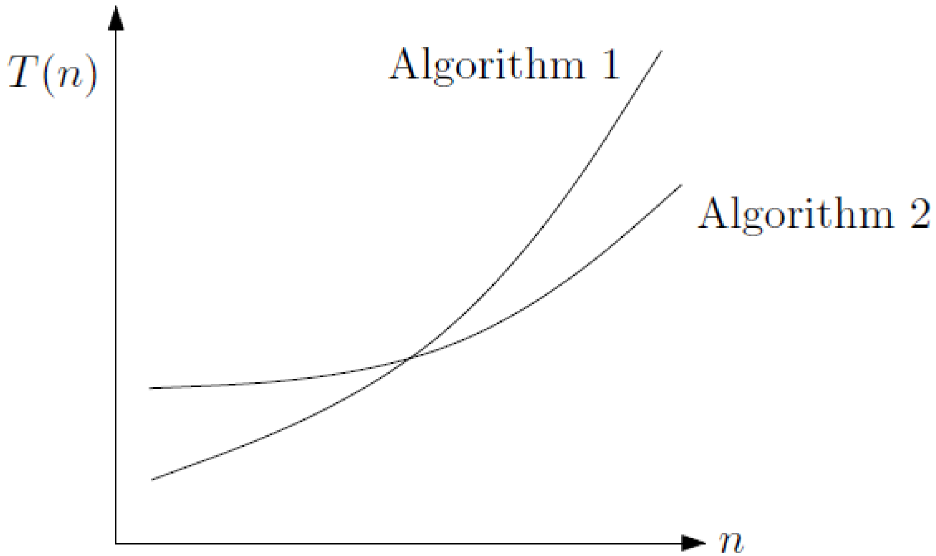

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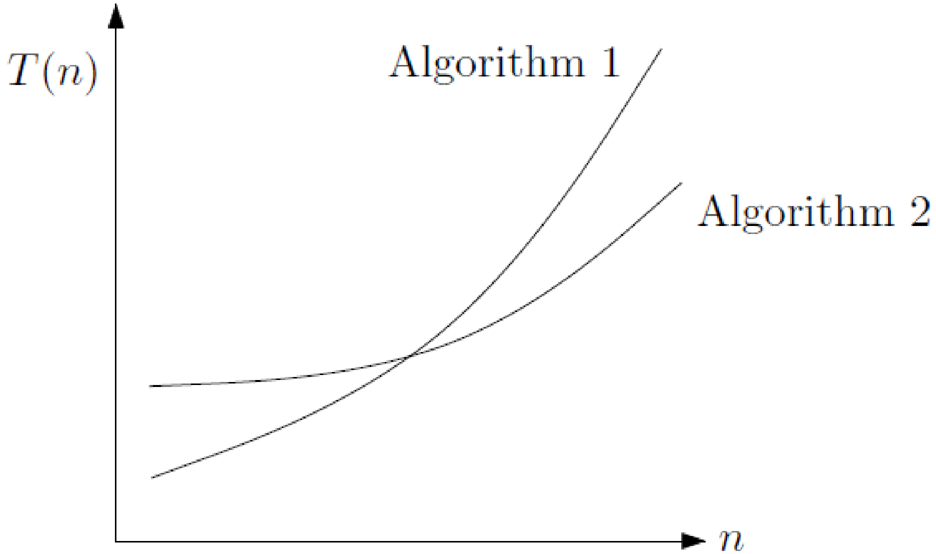

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Θ-notation

- Drop low-order terms; ignore leading constants
- Look at growth of T(n) as $n \rightarrow \infty$
- When n is large enough, a $\Theta(n^2)$ algorithm always beats a $\Theta(n^3)$ algorithm

Merge Sort

Mergesort(A, left, right)

if *left* $<$ *right* **then**
    center $\leftarrow \lfloor(\text{left} + \text{right})/2\rfloor$;
    Mergesort(A, left, center);
    Mergesort(A, center+1, right);
    "Merge" the two sorted arrays;
end

- To sort the entire array $A[1 \ldots n]$, we make the initial call Mergesort(A, 1, n).

Merge Sort

Mergesort(A, left, right)

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if left < right then
    center ← ⌊(left + right)/2⌋;
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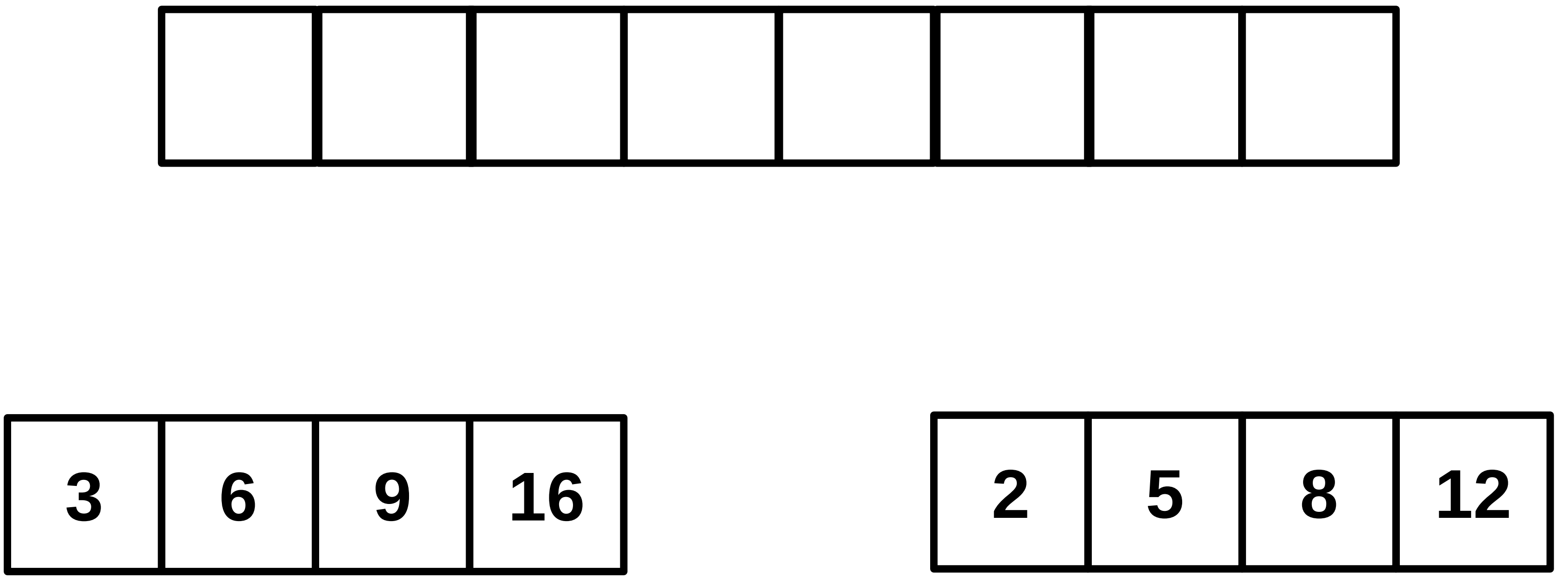
```

- To sort the entire array $A[1 \dots n]$, we make the initial call Mergesort(A, 1, n).
- Key subroutine: "Merge"

Merge two sorted arrays



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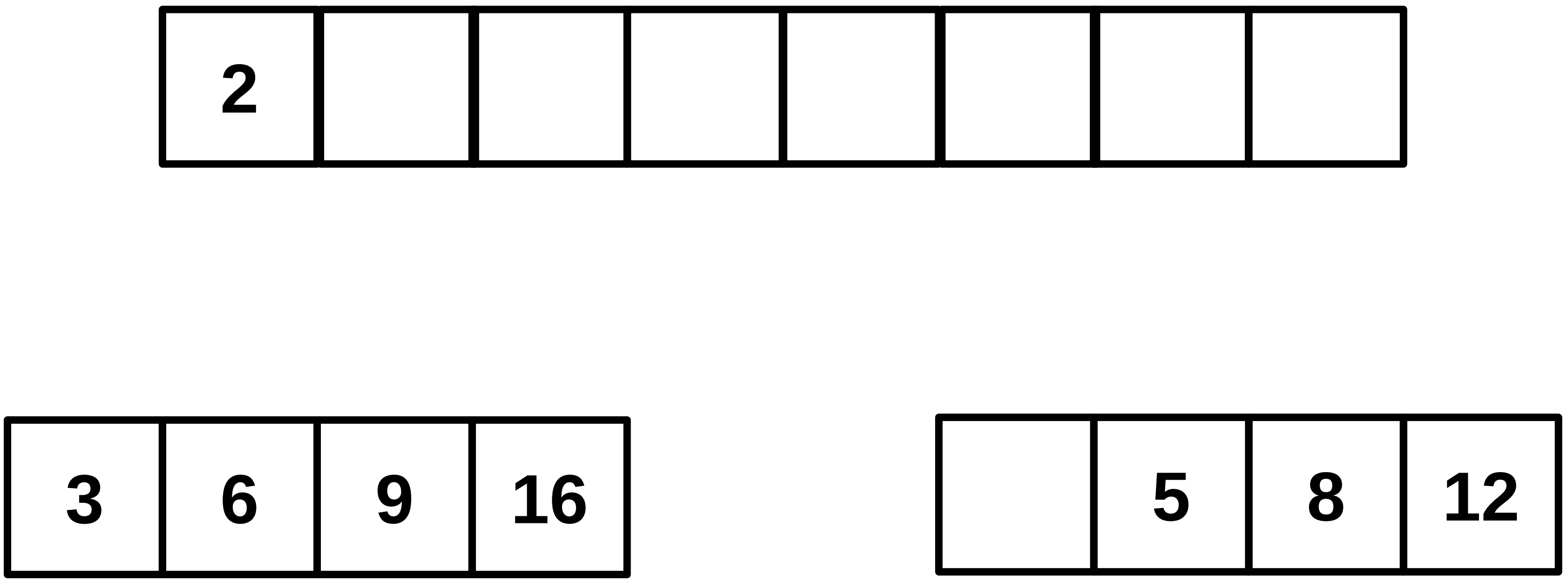

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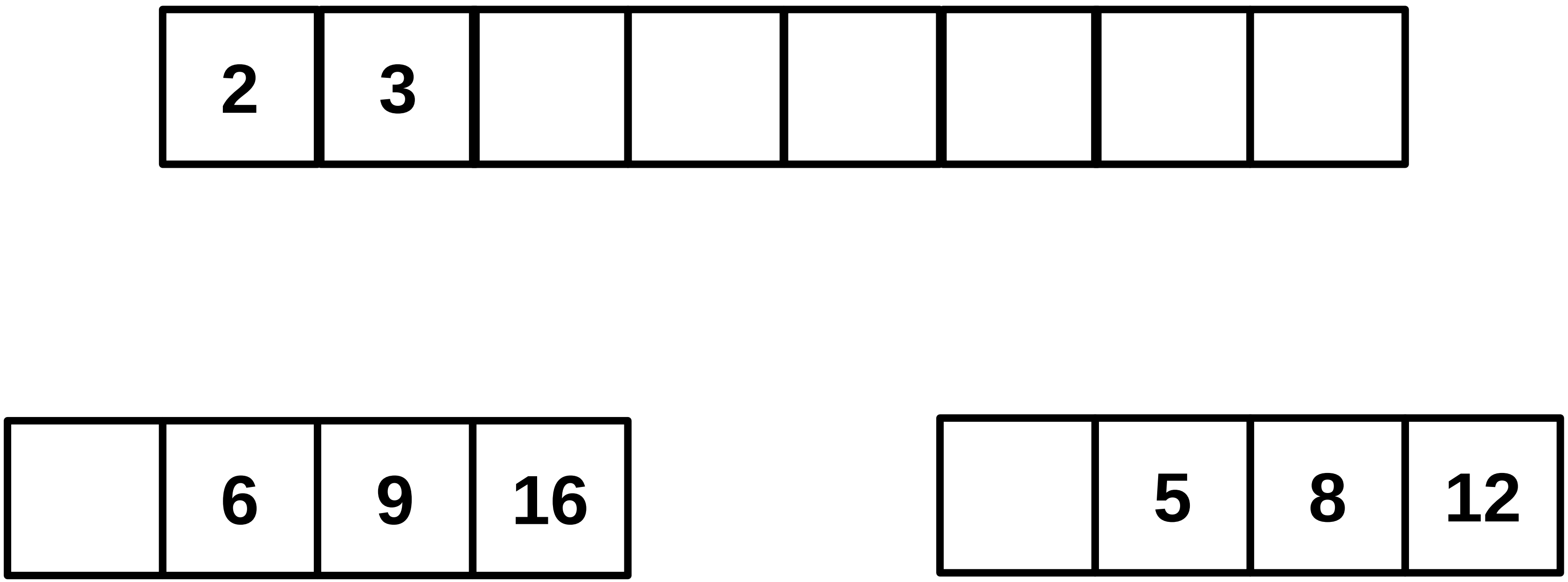

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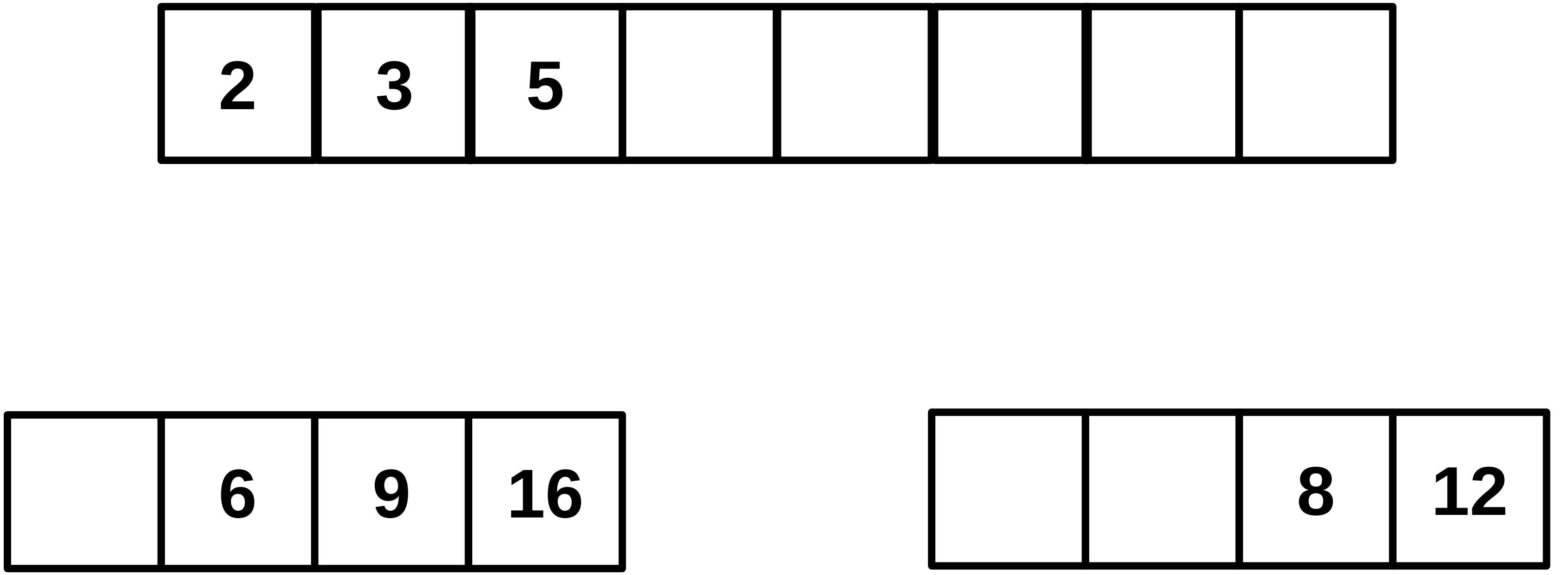

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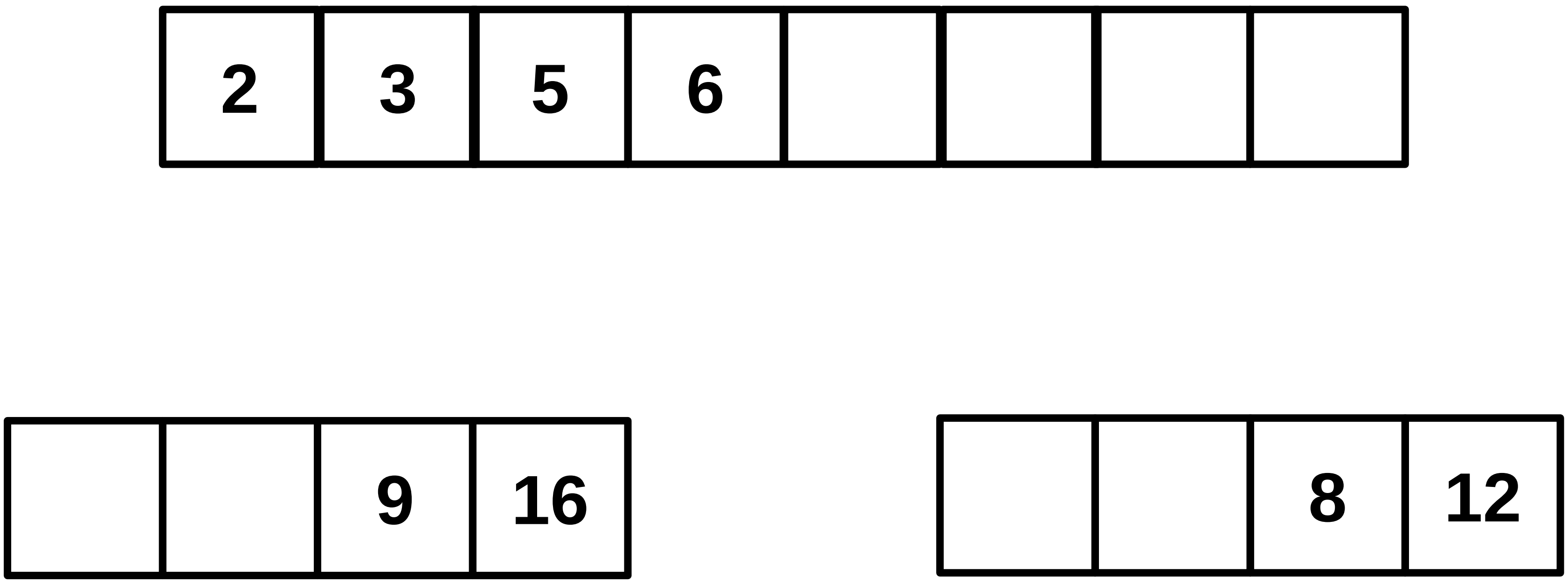

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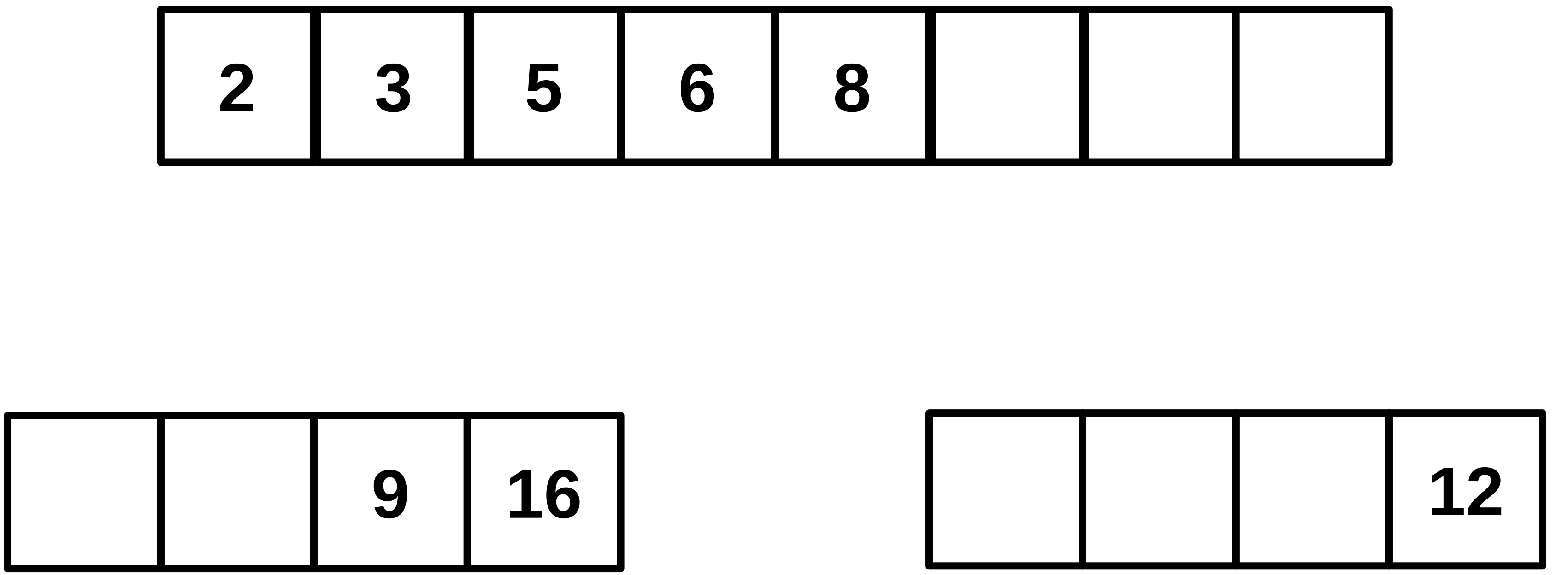
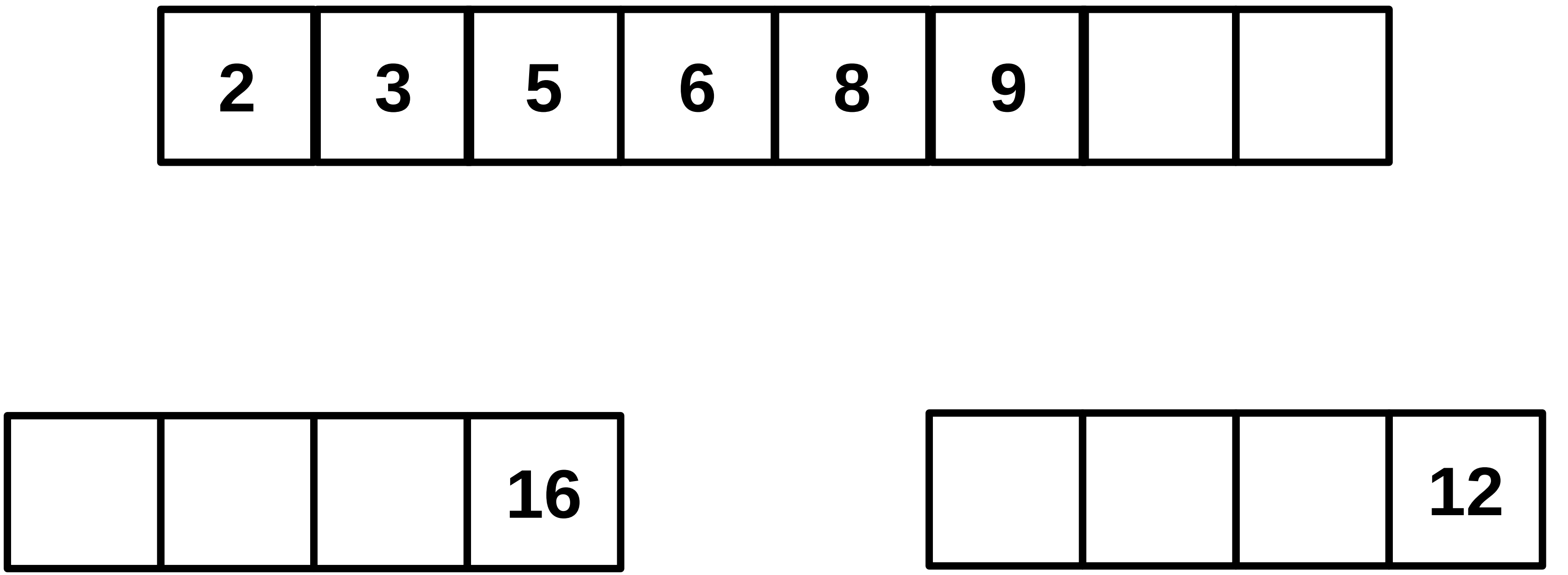


Merge two sorted arrays

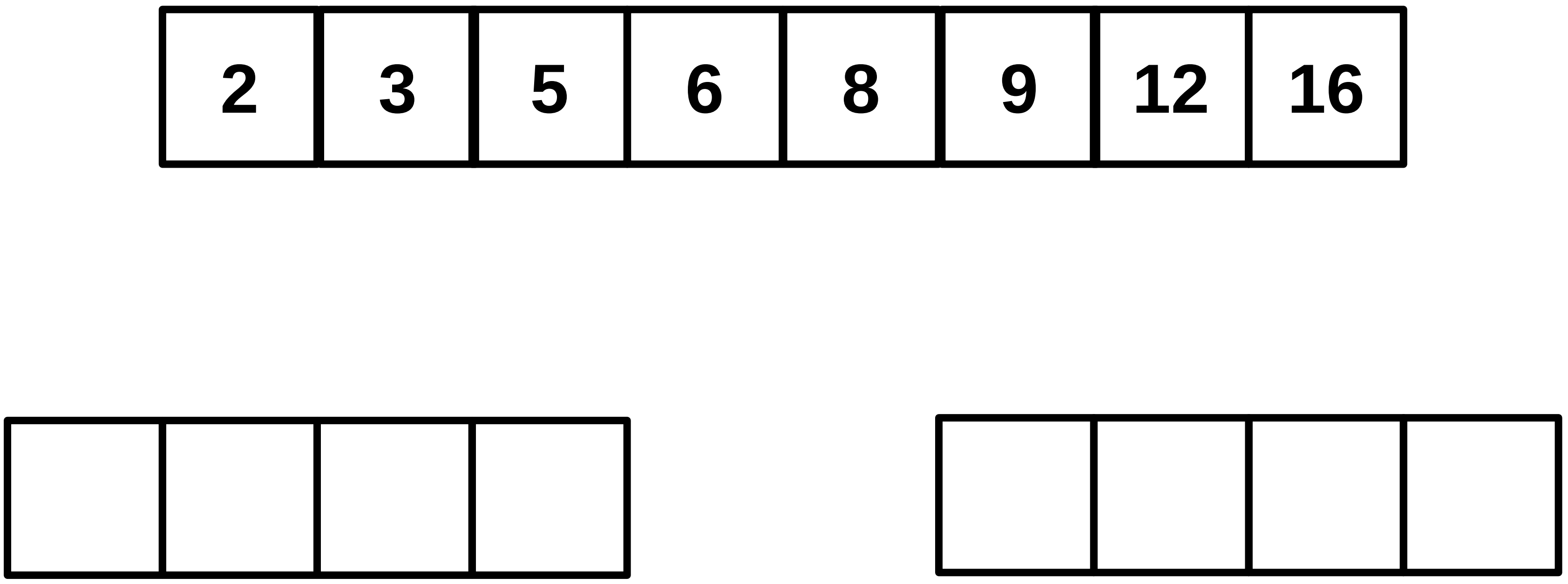


Merge two sorted arrays

2	**3**	**5**	**6**	**8**	**9**	**12**	

			16

Merge two sorted arrays



Three Kinds of Analysis

- T(n): time needed to run Mergesort(A,1,n)
- Assume *n* is a power of 2 for simplicity

Mergesort(A, left, right)

---

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end
```

$$T(n) = \begin{cases} 2T(n/2) + \Theta(n), & \text{if } n > 1, \\ \Theta(1), & \text{if } n = 1. \end{cases}$$

Three Kinds of Analysis

Solve

$$T(n) = \begin{cases} 2T(n/2) + n, & \text{if } n > 1, \\ 1, & \text{if } n = 1. \end{cases}$$

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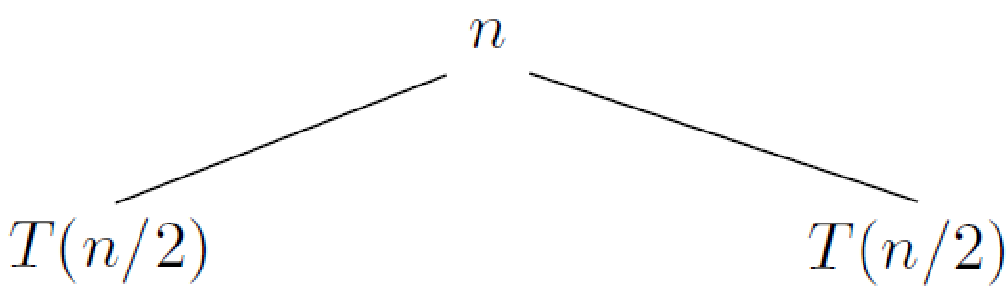

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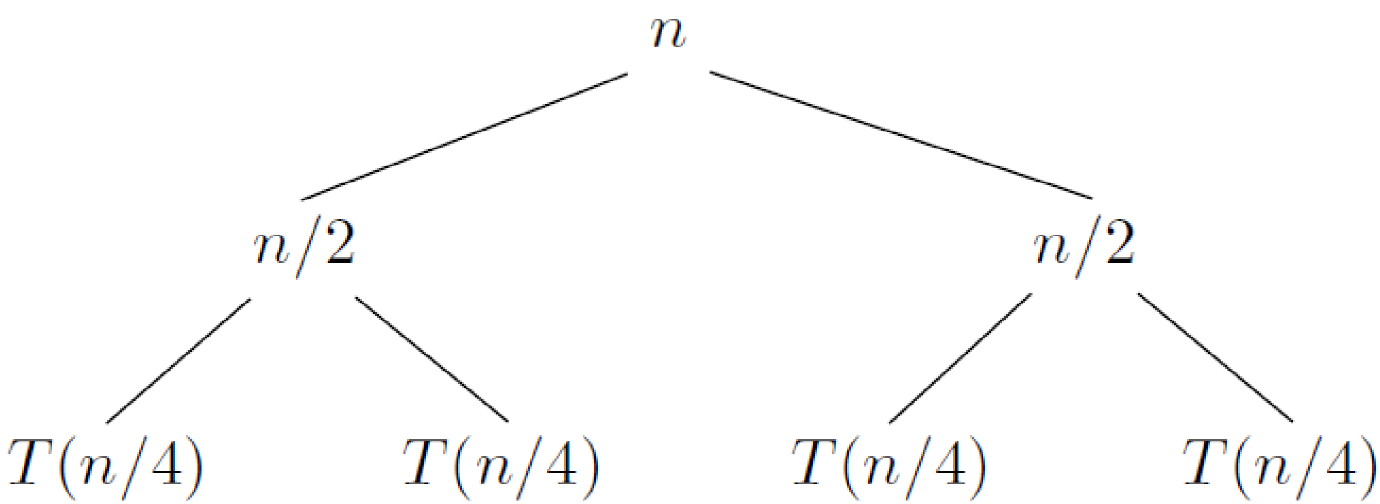

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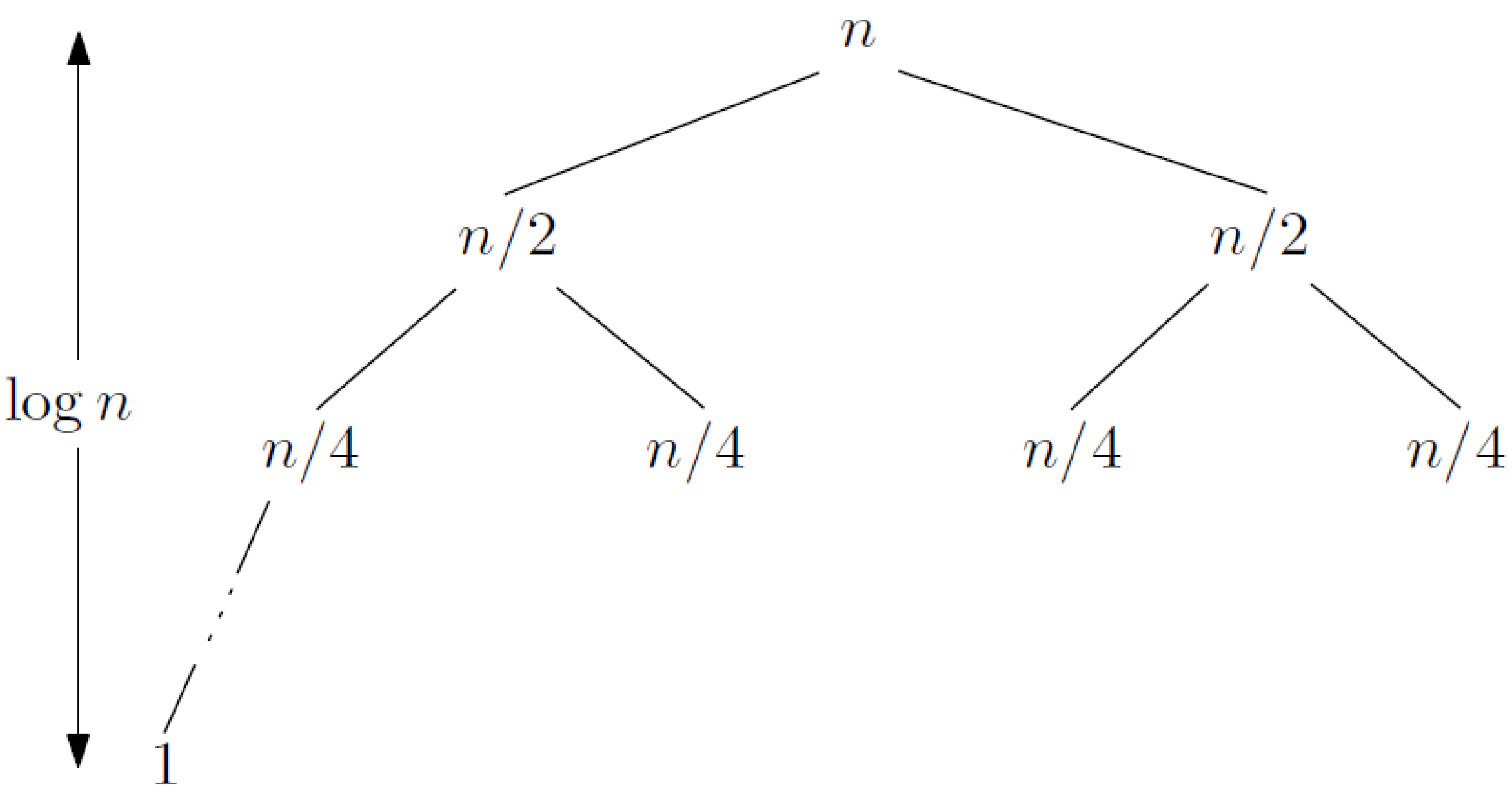

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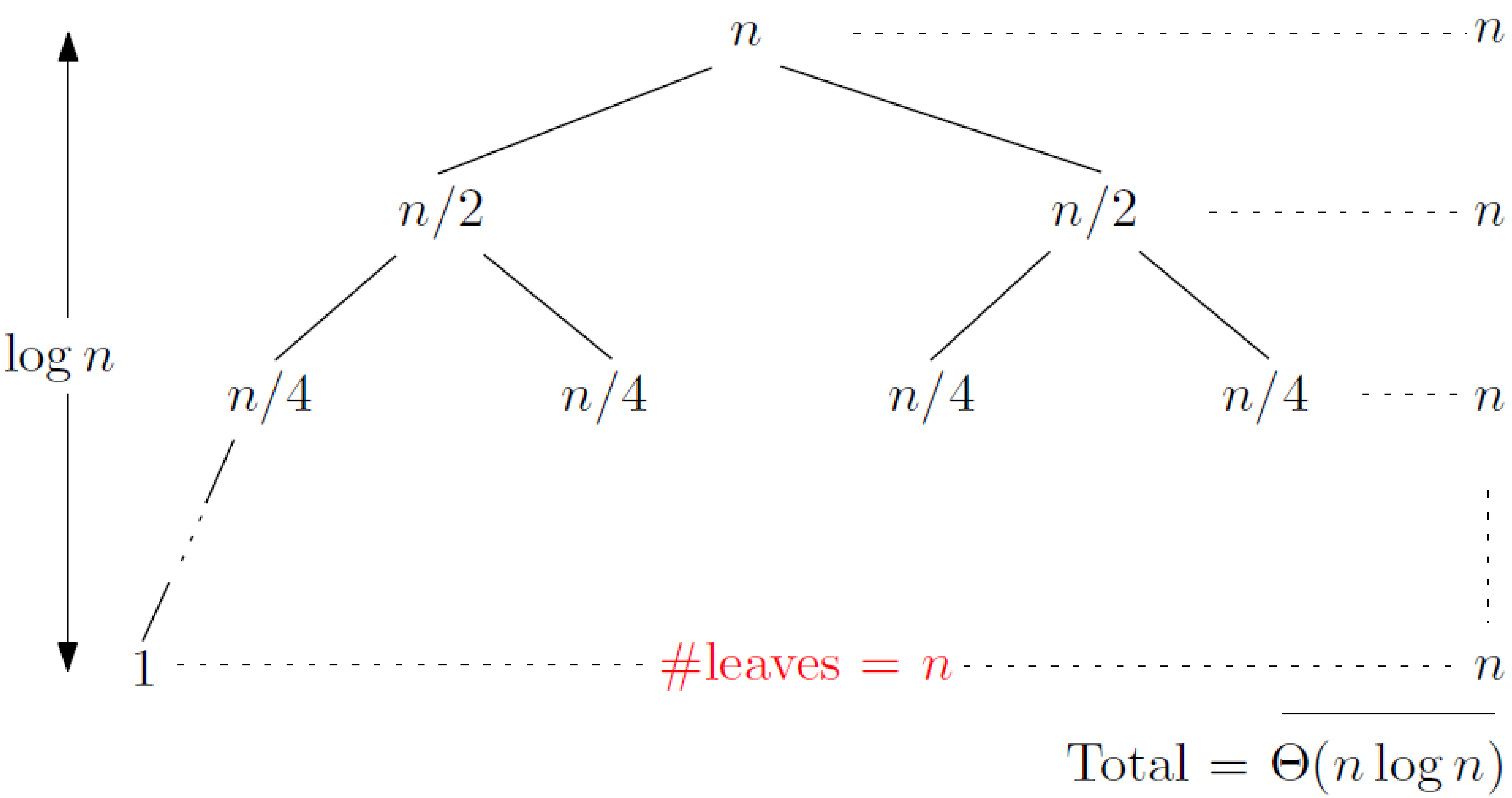


dank u
ju faleminderit
Tack
谢谢
Tak
mulţumesc
Asante
kiitos
Salamat!
Gracias
Terima kasih
Aliquam
Merci
Dankie
Obrigado
ありがとう
köszönöm
grazie
Aliquam
Go raibh maith agat
děkuji
Thank you
gam