Modeling Evaporative Cooling For A Cup of Hot Water

Jiyu Xie

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Prof. Nicolas Xuanlai Fang

Abstract

Evaporation and cooling are processes with tremendous industrial and scientific applications, ranging from meteorology to food processing. These problem would be of special interest to food and drink companies, such as Starbucks, who would like to figure out the best cup design to improve the customer experiences with the lowest cost; or inspired us design new type of self-filling bottle for the people in the arid area. If wildly being used, the device could contribute to the happiness of millions of hot beverage consumers and help billions of people who lack of drinking water. The present work investigated the thermodynamic properties of moist air around a cup of hot water, which was placed in each of the cups in the same amounts and left on a counter to cool. Temperature, humidity data were taken over the course of twenty minutes while they were cooling, with data points taken every ten seconds. Experimental data and simulation model suggests that the temperature is highest in the shadow zone of the cup, and therefore, the relative humidity becomes very low.

1. Introduction

For more than a century, scientists have been interested in the properties of the moist air around hot water, since the age of steam. Evaporation and cooling is important for a wide range of scientific and industrial processes, which is an important component in the climate system, and plays a key role in food processing, especially would be of great interests to drink company like Starbucks, or to design a self-water-filling device. However, a major problem with this kind of application is investigating the thermodynamic properties of moist air around a cup/bottle of hot water. Factors such as the turbulent flow of air, transport of water vapor and heat transfer by convection and conduction, which is founded to be influencing the evaporation and cooling process, have been explored in a large number of published studies[1,2].

Knowing the properties of a moist air around a cup of hot water is of significant importance to meteorology, as well as food and drink industry, and to the larger customer base of drinkers of coffee and other hot beverages. A better design and material use of beaker/coffee cup would keep them hot for a longer time, based on these approaches, much ergonomic and aesthetically pleasing design can be made; companies such as Starbucks could make more benefits with a better customer experience; and people can thus find the best way to cool the coffee/tee/milk with lowest costs, when the drink is too hot to drink right away.

This paper investigates the factors that determine the evaporation and cooling around a cup of hot water, describes the design and implementation of the experimental set-ups for the measurement of relative humidity (RH) and temperature value, and it also offers a COMSOL® simulation model for understanding the evaporative cooling process. Based on the data I got from this investigation, I could conceive, design, prototype, evaluate and improve the performance of other cups. Temperature, relative humidity (RH) value were measured at three different places near the cup..

2. Background

2.1 Principles of Evaporation

Evaporation occurs when some liquid vaporizes into a gaseous phase that is not saturated with the liquid.

The value of the saturation pressure p_{sat} is strongly temperature dependent, at which the thermal equilibrium with the liquid or solid state is reached. According to the following approximation from Principles of environmental physics by J. L. Monteith and M. H. Unsworth (1990), we have:

$$(1)\quad p_{sat}(T) = 610.7 Pa \cdot 10^{7.5\frac{T-273.15K}{T-35.85K}}$$

The saturation concentration for ideal gas at which the relative humidity is 100% can be evaluated as:

$$(2)\; c_{sat} = \frac{p_{sat}(T)}{RT}$$

where R is the ideal gas constant.

The thermodynamic properties of moist air depend on the fraction of water vapor. A mixture formula is used to describe the properties with the proportional amount of dry air and water vapor. Assuming air behaves as an ideal gas, the density reads:

$$(3)\; \rho_m = \frac{\rho}{RT}(M_a X_a + M_v X_v)$$

2.2 Heat Transfer

Latent heat would be released from the water surface during evaporation, which cools down the water in addition to convective and conductive cooling by the surrounding. This additional heat flux depends on the amount of evaporated water. The latent heat source then is:

$$(4)\; q_{evap} = L_v g_{evap}$$

The latent heat of vaporization L_{vap} is given in J/kg. At the water surface, the evaporation occurs, to evaluate the amount of water which is evaporated from the beaker into the air, the evaporative flux g_{evap} at the surface is:

$$(5)\quad g_{evap} = K(c_{sat} - c_v)M_v$$

with the evaporation rate K, the molar mass of water vapor M_v, the vapor concentration c_v and the saturation concentration c_{sat} which can be calculated from the correlation p_{sat}

$$(6)\; c_{sat} = \frac{p_{sat}}{R_g T}$$

2.3 Newton's law of cooling

When the hot water cools, heat is transferred from the liquid to the cup by conduction and then to the air by convection. The following equation is Newton's Law of cooling where Q is the thermal energy in joules, h is the heat transfer coefficient, A is the surface area, T is temperature and t is time.

$$(7)\; \frac{dQ}{dt} = h \cdot A(T_{environment} - T_0)$$

3. Experimental Design

At the water surface, heat is released by evaporation. The heat of vaporization for water is approximately (it’s actually temperature dependent, but using a constant value here is a reasonable approximation) with the molar mass of water . The amount of heat released depends on how much vapor escapes from the water surface into the air. This relates the heat source to the

Transport of Diluted Species interface via the total flux in normal direction to the surface, which can be understood as the net flux of water vapor into air.

At the water surface, the relative humidity will always be 100%. Hence, the saturation concentration is reached. It defines the concentration of water vapor at the water surface according to Equation (2), with the saturation pressure determined by the Heat Transfer in Fluids interface. All in all, it is a strongly coupled phenomena that is implemented in no time.

In this report, a slight air draft around the beaker is assumed to accelerate the cooling by transporting heat and removing water vapor from the surface. At the water-air interface, vapor escapes from the liquid into the air, causing additional cooling by evaporation. 20 minutes of cooling was investigated, the initial coffee temperature was 80 ℃ and air at 27 ℃ with a relative humidity of 69% causing the cooling. Below is the resulting temperature and relative humidity distribution, both after 20 minutes.

3.1 Preparing Cooling Samples

20 minutes of cooling was investigated, the initial coffee temperature was 80 ℃ and air at 27 ℃ with a relative humidity of 69% causing the cooling. The inner diameter of the beaker is 5cm.

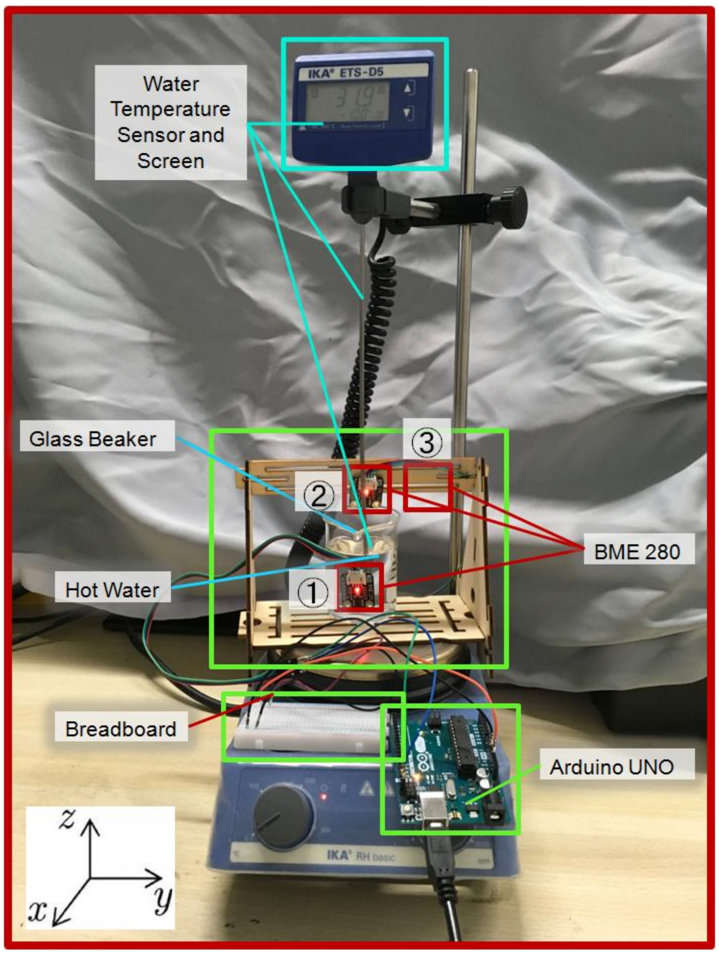

Figure 1: Experimental set-up for the test of the cooling at an initially water temperature of 80℃.

Hot water at 80℃ in a glass beaker were placed on the build platform so that the sensors can be easily settled. Three BME280 humidity and temperature sensors were arranged as Figure 2 shown. Additionally,a IKA ETS-D5 RH basic® real time water temperature sensor was also used to test the cooling of hot water.

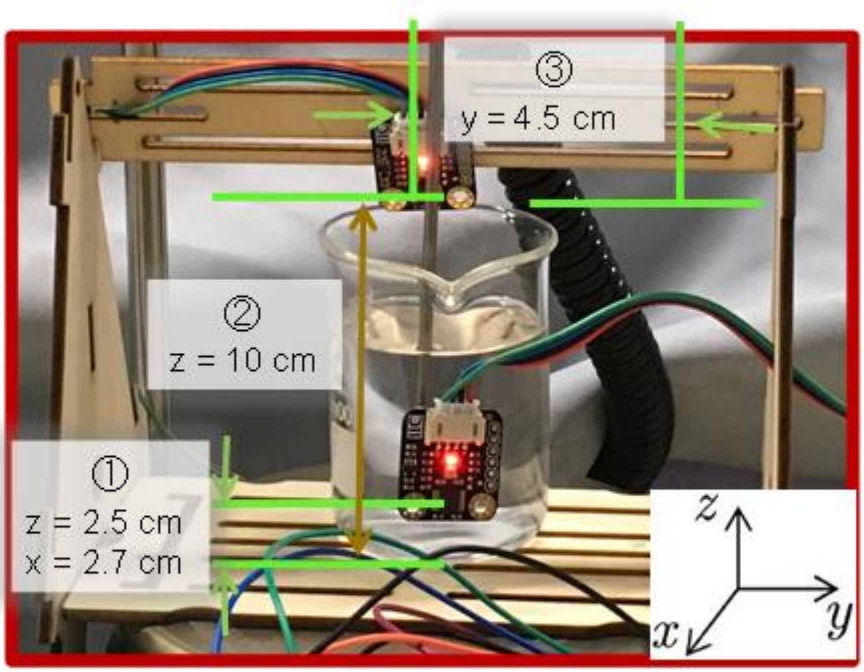

Figure 2: The arrangement of the beaker and BME 280 sensors.w

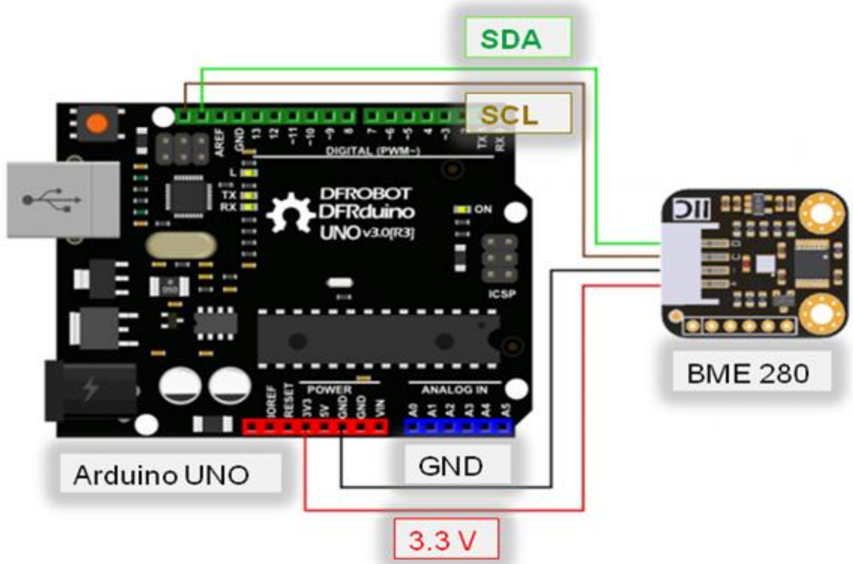

Figure 3: Arduino electrical circuit design.

4. Results and Discussion

20 minutes of cooling was investigated, the initial water temperature was 80℃ and air at

27℃ with a relative humidity of 69% causing the cooling. The inner diameter of the beaker is 5cm. The wate surface area is measured to be approximately π*5cm*5cm = 25πcm^2 = 78.54 cm^2. and after 20 minitues of cooling, the change of the mass for the hot water at 80℃ is about 1.49g. (from 75.24g to 73.75g)

The sensors in ①, ② and ③ position, measures of the temperature and relative humidity value. The relative humidity value in ② and ③ position deceased in a vibrating tendency, while the temperature at these two positions is almost the same through 20 minutes of cooling (39.54 ℃ in average at ②, 40.15 ℃ in average at ③). However, an interesting phenomenon occurs at position ①. The temperature at ① reaches a peak value of 59.97 ℃ at 150s, which is very high comparing to the room temperature of 27.4℃, then, after 150s, the temperature constantly increases. The relative humidity value at ① is decreasing at 0s-170s, at 170s, it reached a valley value of 16.17%, and after 170s, it rising in a approximately linear relationships, like the tendency of temperature goes. The relative humidity value is very low comparing to the room relative humidity of 69 %.

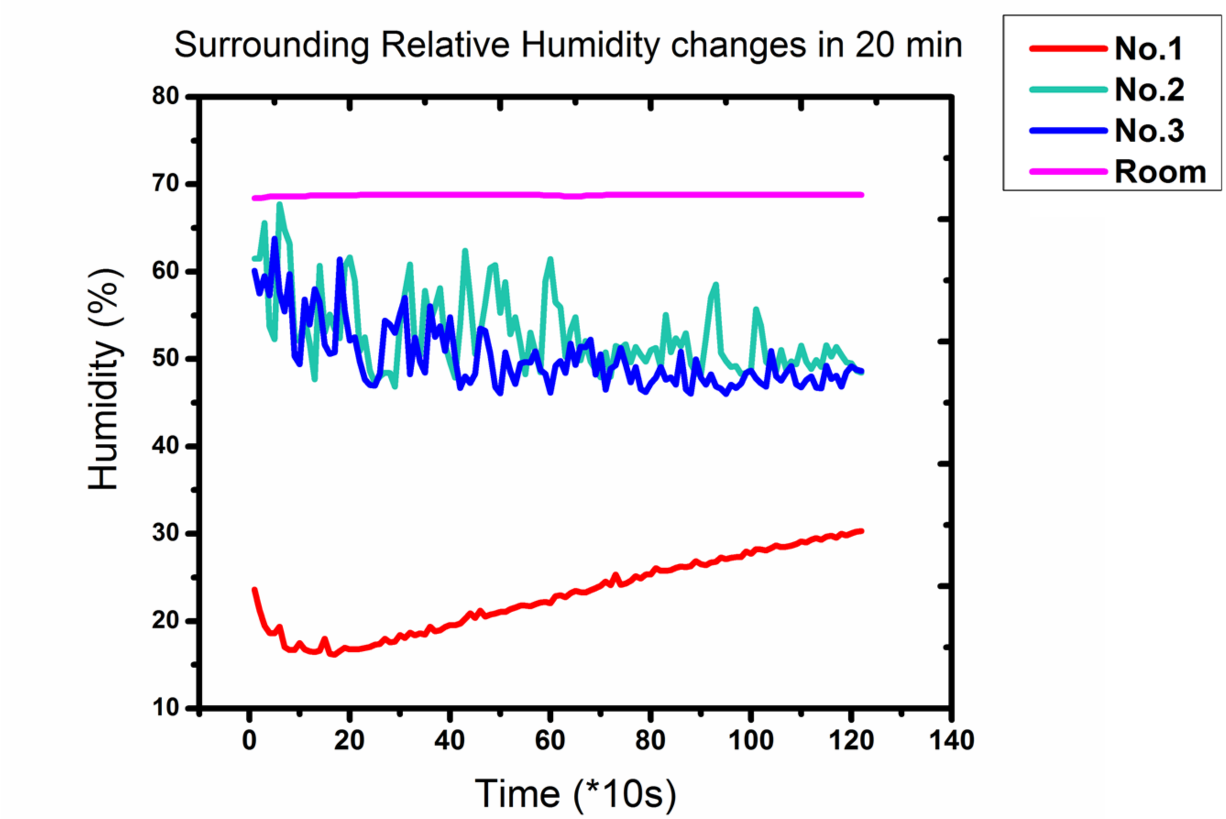

Figure 4: The average Room's relative humidity (RH) value is 69 %, which can be considered as a constant number . The RH value of ③ is lower than ②'s and Room's RH in average, and the RH value of ① is decreasing at 0s-170s, after 170s, it rising in a approximately linear relationships.

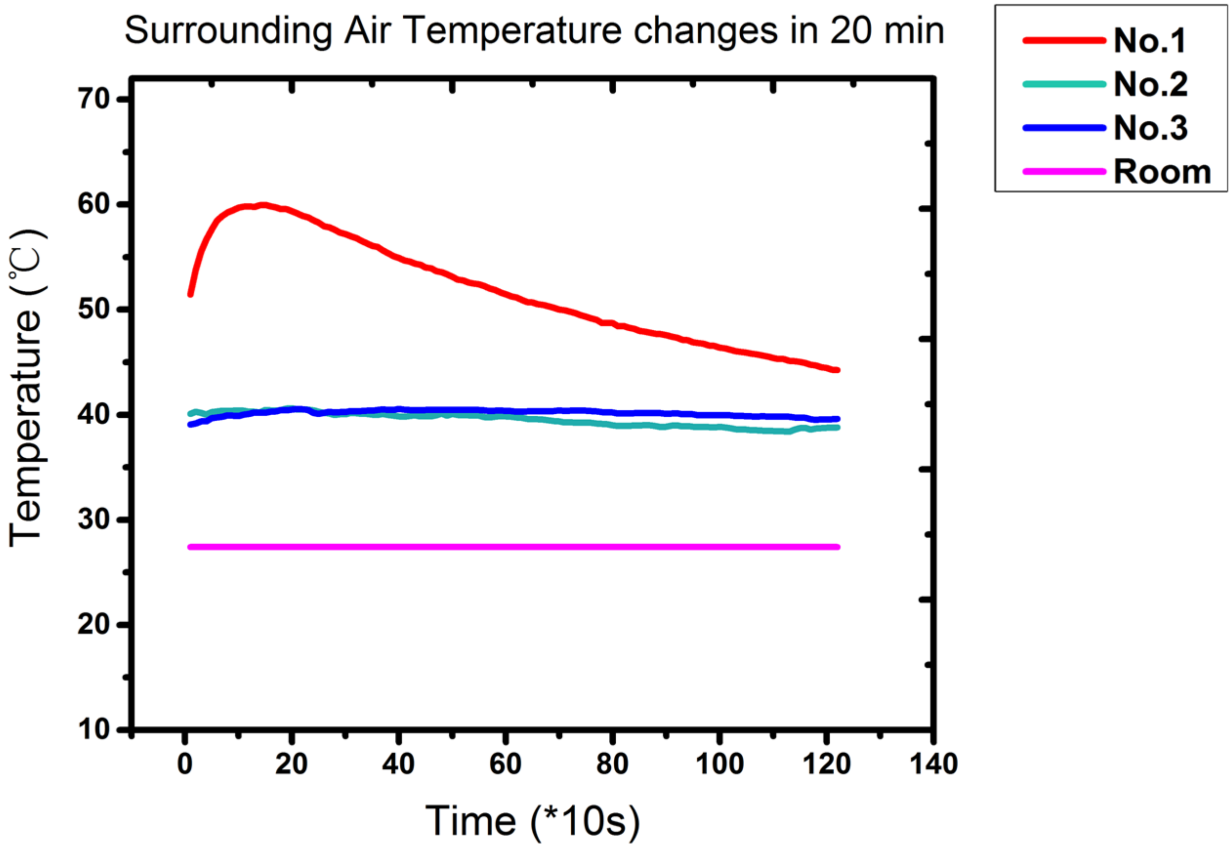

Figure 5: The Room's temperature value is 27.4 ℃, which can be considered as a constant number. The temperature of ① reaches a peak value of 59.97 ℃ at 150s. The temperature of ② starts to be lower than ③ since 570s (9.5 min), rather than justified with margins, while the average temperature of ② is 39.54 ℃, and ③ is 40.15 ℃.

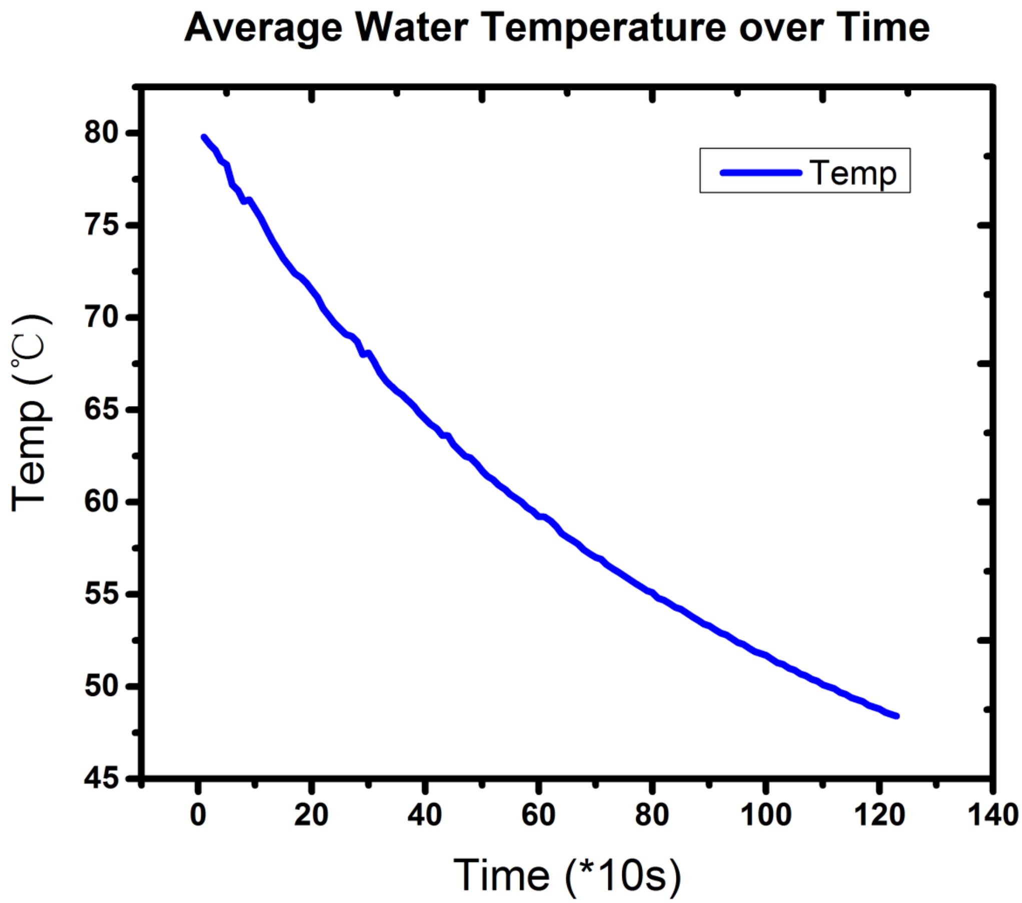

Figure 6: Experiment data on the Average Water Temperature over 20 minutes.

To explain this phonomenon, a COMSOL® simulation model were built governing by the relationship (equation 1-7) mentioned above, where the governing equation is:

And the simulation results shows below:

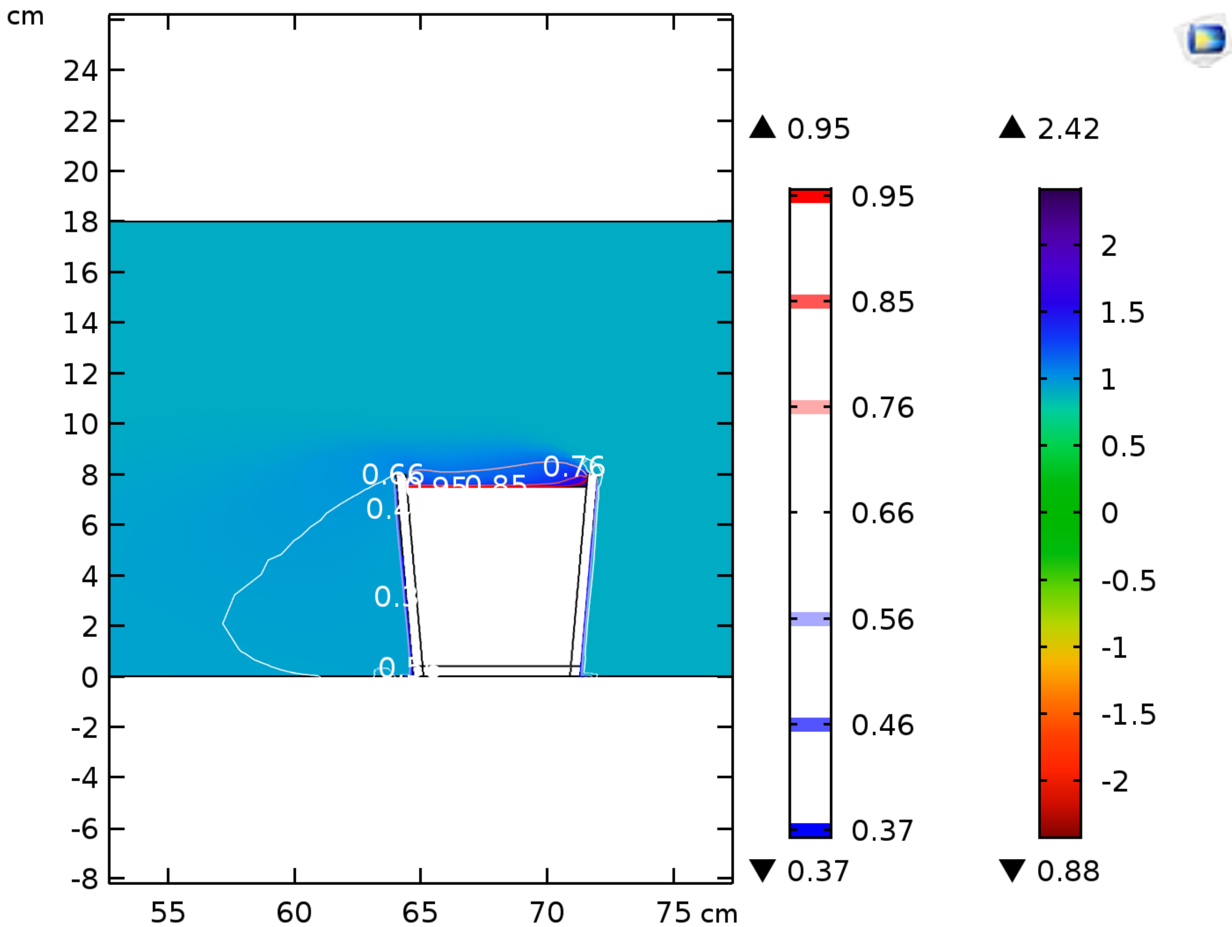

Figure 7: This figure shows the concentration and relative humidity at the symmetry plane. Close to the water surface, the relative humidity is about 100% as expected. Behind the beaker the relative humidity can become even smaller than 20%. Due to the high temperature, air can absorb a higher amount of water.

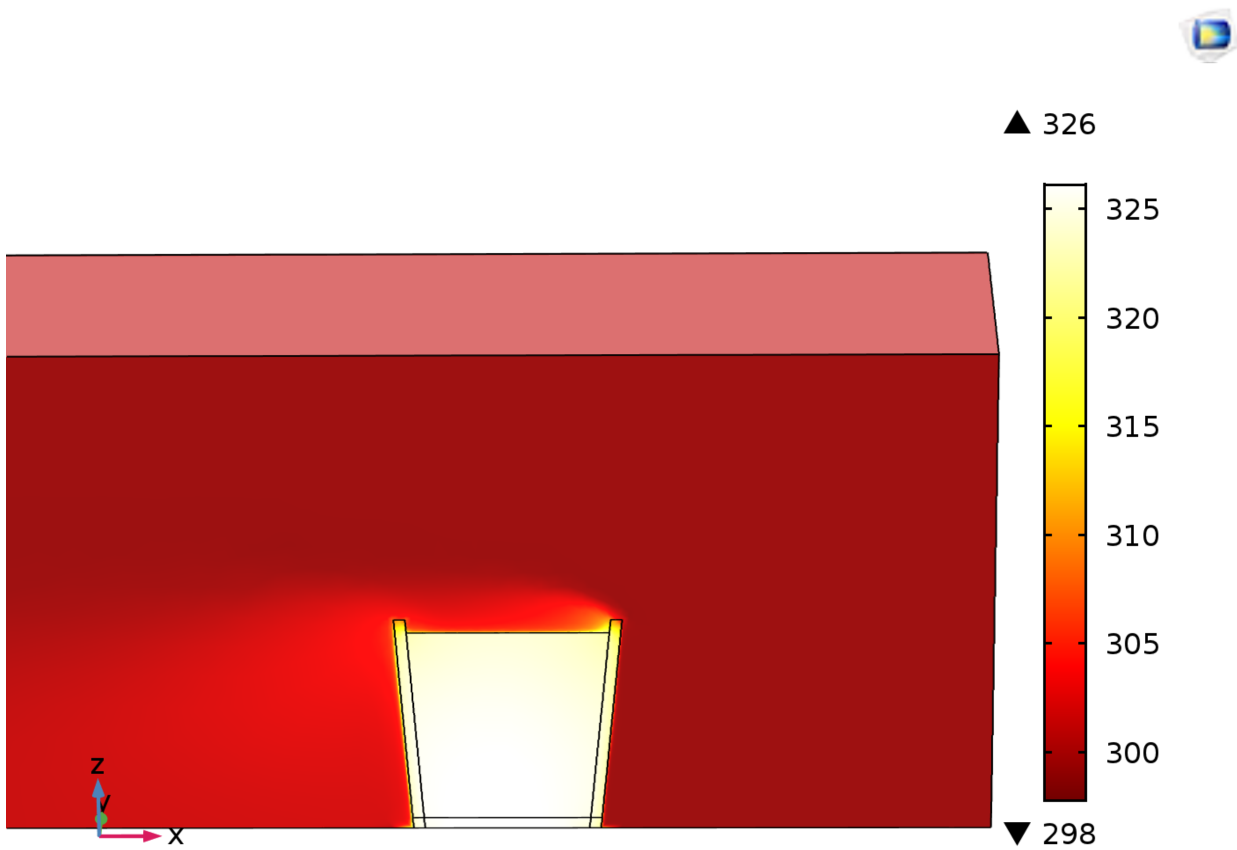

Figure 8: This figure shows the temperature simulation distribution at the symmetry plane. Close to the water surface, the temperature is about 308 Kelvins (34.85 ℃). Behind the beaker the temperature can become even higher than 304 Kelvins (30.85 ℃).

5. Conclusions

An interesting feature was the relative humidity becomes very low in the shadow zone of the cup while the temperature becomes high at the same place. Several theories could potentially explain this phenomenon. One hypothesis was that this effect was deflection caused by the turbulent flow of air, the transport of water vapor, and the heat transfer by convection. But more details about the cause of this deceleration remains unclear; it could be a result of the air or water properties or have some other unknown source.

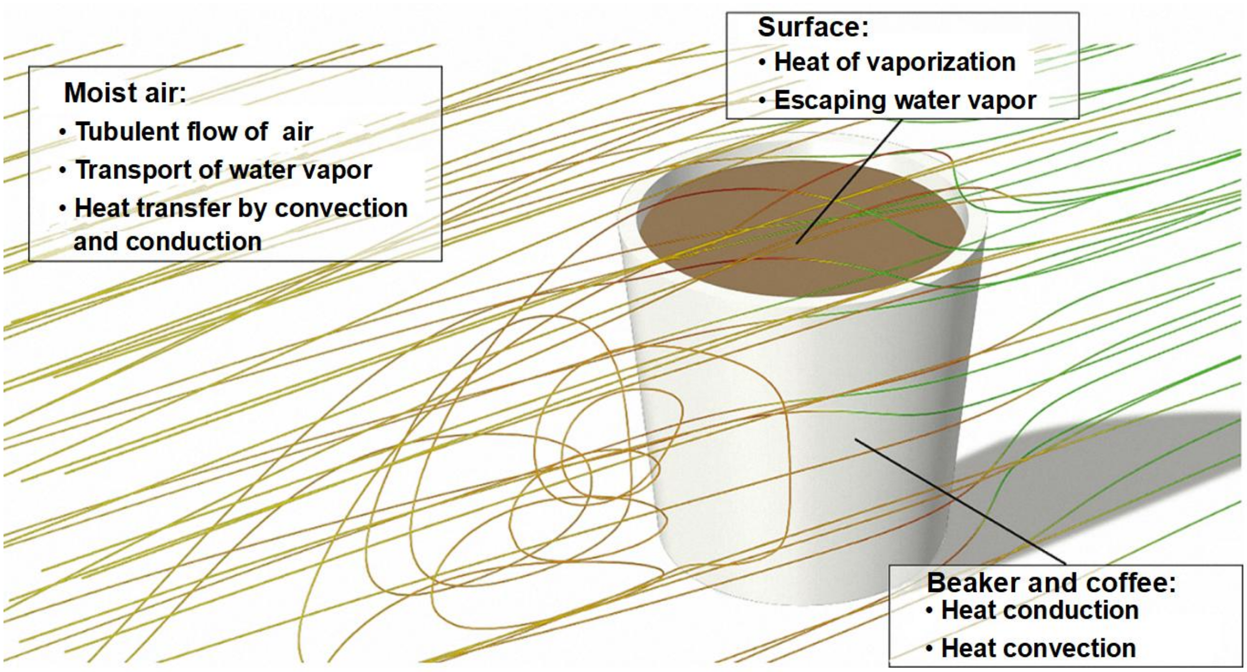

Figure 10: Sketch of the participating effects in a coffee cup[1].

Due to the time limitation, there still lack of enough data to determine the evaporation rate **K** and the heat transfer coefficient **h** value. So, for further experimentation regarding the determination of the evaporation rate **K** and the heat transfer coefficient **h** in equation (6) and (7), and determing the mathematical relationship of the temperature and relative humidity value in the shadow zone of the cup. By studying the evaporative cooling procedure for a cup of hot water and the moist air's properties, we can have better cup designs such as a better Starbucks coffee cup with a better taste, or a self-filling water bottle that can be used in some remote and arid area.

Acknowledgments

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