Simple Book Example

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thesis

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Chapter 1

Simulations of a plenoptic 1.0 system

In this chapter we present some results obtained simulating raw data images using the simulation platform described in chapter 2. We also describe how we used the digitally generated light fields data to design, develop and test the data processing algorithms, such as the rendering, the synthetic refocus and the depth estimation algorithms. Light fields are generated from the raw images obtained propagating light in the system using the methods described in chapter 2. All the light field processing algorithms use the third parametrization of the light field, described in 1.4.2, that is the four dimensional array of the intensity recorded on the sensor whose indexes are the position x and y, and the the direction θ_x and θ_y.

1.1 Description of the system

The plenoptic 1.0 camera described in section 1.4 has been modelled with a simple 2f system composed by a main lens, a micro lens array and a sensor as shown in figure 3.1. The micro lens array is placed at a distance $z = 2f$ form

the main lens, where f is the focal length of the main lens, and the sensor is placed at a distance $f_m u$ from the micro array plane. $z_3 = f_\mu$ is also the focal length of the lens lets. In this configuration the lens let array plane is conjugated with the object plane, while the sensor plane is conjugated with the main lens plane. In order to satisfy the f-number matching condition described in section 1.4, once fixed the propagation distances z_1, z_2, z_3, the resolution N of the input field, the wavelength of the light used and the micro array parameters, the software automatically sets the aperture of the main lens.

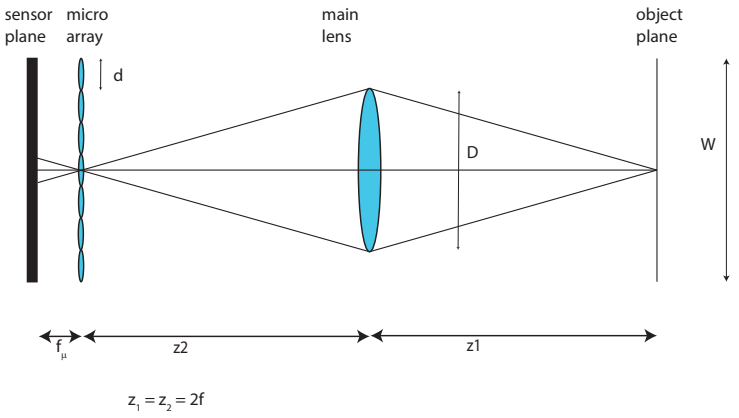

Figure 1.1: The parameters that characterize the micro lens array are: the pitch p, defined as the distance between the centres of two neighbours lens lets, the diameter d, the number of lens lets per raw N and the size of the micro array W.

The micro array parameters are illustrated in figure 3.2. and are the pitch between two close lens lets, the diameter of the single lens let, the number of lens let in a row and the size of the array. In all our simulations the pitch of the lens lets is equal to the size of its diameter and the lens lets are arranged in a square matrix.

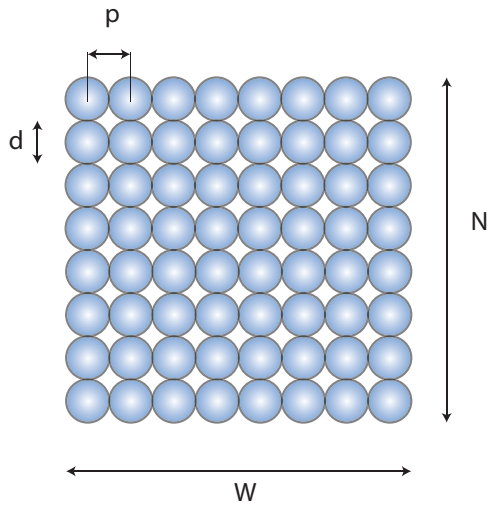

Figure 1.2: The parameters that characterize mi micro lens array are: the pitch p, defined as the distance between the centres of two neighbours lens lets, the diameter d, the number of lens lets per raw N and the size of the micro array W.

1.1.1 Diffraction effects on the System

Diffraction form the lens let circular aperture generates cross talk between neighbours sub images since the light form a single lens let falls in the sub image of the neighbour lens let. This facts generates a blur in the rendered image an a loss of spatial resolution. To avoid this loss of resolution the diameter of each lens let should be larger than the size of its Airy disk diffraction pattern. As seen in equation 2.99 for a lens with a circular aperture, the diameter of the Airy disk at it's focal plane is $d = 1.22\lambda z/D$. For the system described the lens let a diameter D and z is equal to its focal length f_μ. Therefore the lens let diameter should be at least twice the Airy disk diameter. We have:

$$D > 2.44\frac{\lambda z}{D} \tag{1.1}$$

Therefore the condition of D is:

$$D > \sqrt{2.44\lambda f_{\mu}} \tag{1.2}$$

as shown in figure 3.3

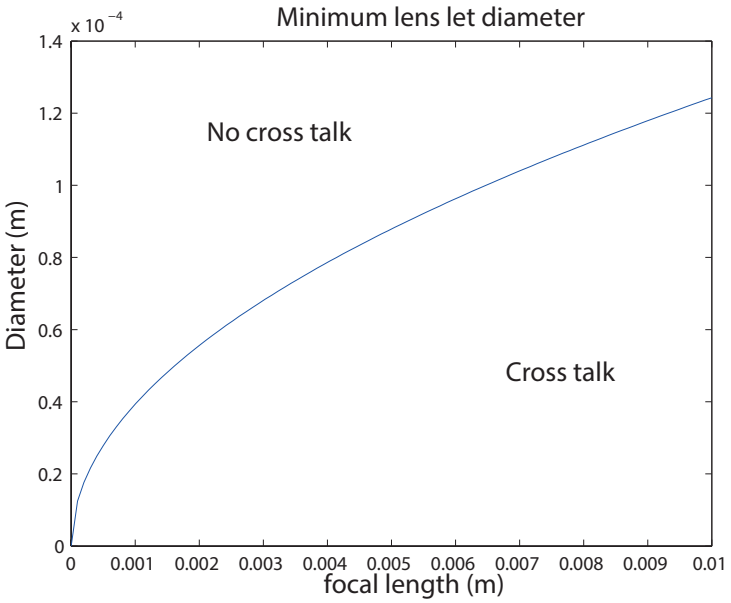

Figure 1.3: Values of acceptable lens let diameter to avoid diffraction induced cross talk.

To conclude this section an example of cross talk raw image is shown. In figure 3.4.

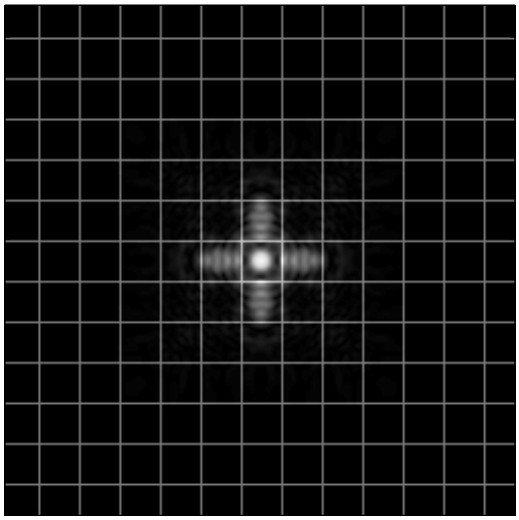

Figure 1.4: Raw image of a point source.The grid represents the edges of the sub images. If the point source is in focus, it should be represented by only one lens let. Because of diffraction some light goes also on the neighbours sub images.

The effects of cross talk are more clear if we analyse the phase space. Figure 3.5 shows the phase space of point source in presence of cross talk, and without cross talk.

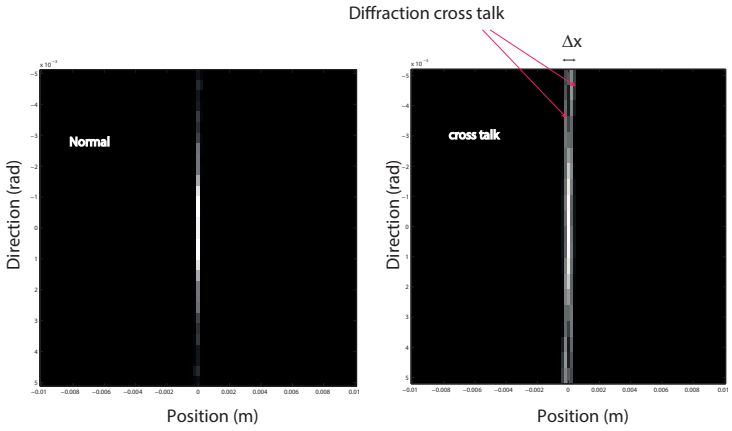

Figure 1.5: Phase space of a single point light field: on the left when the condition 3.2 is respected; on the right when diffraction induces cross talk arise between neighbours lens lets. cross talk introduces a blur in the rendered image whose width is Δx.

1.2 Advanced Rendering Techniques

As described in chapter 1.5, one pixel in the rendered image is obtained as a mean of all the pixels of a sub image. This methods generates an image that is equivalent to the image obtained with a conventional camera, without any additional information contained in the light field. In addition to that each lens let sub image generates only one pixel of the final image and the final resolution depends on the number of lens lets in the micro array that is smaller than the resolution that can be obtained by a conventional camera. More advanced forms of enable interesting post processing features. The first two post processing features we present are the change of aperture and the change of point of view. These features are achieved integrating the each sub image along a sub set of directional coordinates, that correspond to a smaller number of pixels.

1.2.1 Changing Aperture

The possibility to change the aperture of the main lens in post processing is a vary interesting feature. The quality of the image captured by a conventional camera depends, with a lot of simplifications, by the combination of shutter speed and aperture chosen in order to get the optimum total exposure [23]. In our simulations the shutter speed is not an issue since all the objects imaged where static with constant and uniform illumination. Therefore what determine the exposure is the aperture of the main lens or the f number. The f number is proportional to the depth of field of the imaging system that can be defined as the range of depths that appears sharp in the resulting photograph

[2]. If the f number increases and the aperture gets smaller, the depth of field increases. Therefore a plenoptic camera can control the depth of field in post processing. If we look at the raw plenoptic image, each sub image is formed by a number of samples of the directions of the rays. If in equation 1.6 we reduce the range of integration along the directional coordinates, this is equivalent in reducing the aperture of the main lens, since less rays will be considered to form the intensity of each pixel. The result is a rendered image that looks like it has been captured with a narrower aperture, therefore with less light. From a computational point if we want to reduce the aperture of a k pixels we have:

$$I(x,y) = \frac{1}{N'^2} \sum_{i=k/2}^{N-\frac{k}{2}} \sum_{j=k/2}^{N-\frac{k}{2}} L(x,y,i,j) \tag{1.3}$$

where $N' = N - k$. This operation in the phase space is shown in figure:

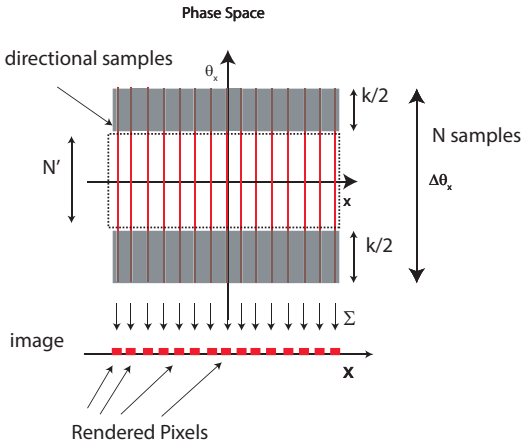

Figure 1.6: Rendering seen from the (x, θ_x) slice of the phase space. In grey are shown the directional pixels not considered for the integration.

An example of implementation of the change of the aperture can be seen in figure 3.7. The two images has been rendered from the raw data described

in section 1.4.2. Two examples of imaged rendered changing the aperture in post processing can be seen in figure 3.7.

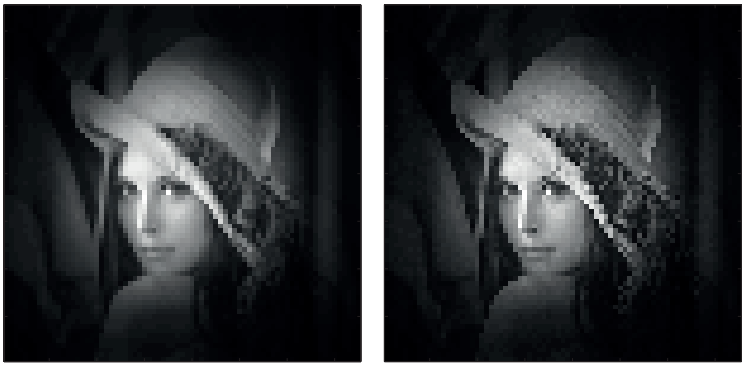

Figure 1.7: The two images are rendered from the same raw data. The image on the left has been rendered integrating along all the sub image, while the image on the right has been rendered taking only the central pixel form each sub image. Since the object is a 2 dimensional image, the effect on the depth of field is not present.

The object, as explained in section 1.5, is a two dimensional picture, the extended depth of filed is not noticeable. What can be quantified is the amount of light present in the two images. With full aperture we have a total intensity of NUMBER and a mean intensity of NUMBER, while with the smallest aperture the total intensity and the mean intensity are respectively:NUMBER AND NUMBER. As expected the total amount of light decrease if we close the aperture, even in the post processing case. Regarding the extended depth of field achievable reducing the aperture in post processing to a single ppixel, applied the home made algorithm to raw data downloaded from Stanford microscopy group.

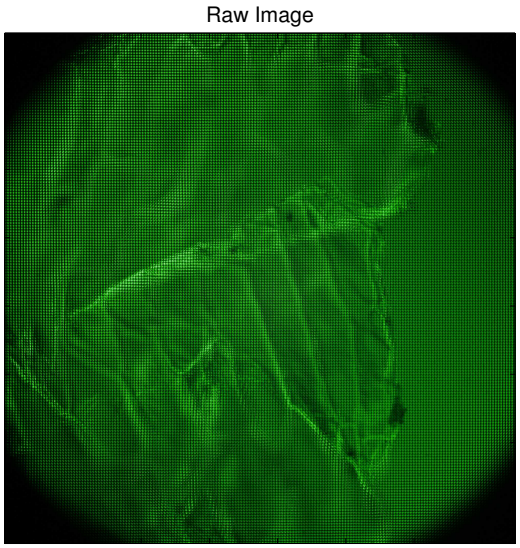

Figure 1.8: Raw plenoptic 1.0 image obatined by Marc Levoy. The microscope was a Zeiss Axiovert with a 20x/0.5NA (dry) objective. The microlens array was 24mm x 36mm, with square 125-micron x 125-micron f/20 microlenses, held in front of the Axiovert's side camera port using an optical bench mount. The camera was a Canon 5D full-frame digital SLR. The specimen was the thin silky skin separating two layers of an onion, immersed in oil to improve transparency [4].

1.2.2 Changing point of view

Selecting the directional components to integrate is possible to change to point of view of the rendered image, in the limit of the directions contained in the f-number of the main lens. Viewpoint can be chosen both in x and y direction. It is possible to choose only one specific viewpoint (θ_x, θ_y), that rendering the image using only one single pixel per sub image, as shown in figure, or it is possible to select a point on view in θ_x or θ_y and integrate over

all the other direction. The rendering equation then becomes:

$$\begin{aligned} I(x,y) &= \frac{1}{N}\sum_{i=0}^{N} L(x,y,i,\theta_y) \\ I(x,y) &= \frac{1}{N}\sum_{j=0}^{N} L(x,y,\theta_x,j) \end{aligned} \tag{1.4}$$

1.3 Depth estimation

In this section we will explain how it is possible to extract depth information from light fields data. In the specific case of plenoptic 1.0 raw data, depth can be decoded looking at the phase space. In section 1.4.1 is explained that a point source on the focal plane of the main lens looks like a straight vertical line in the phase space. The physical meaning of this is that for a single position, one lens let, all the directional coordinates are sampled. If the point source is out of focus the situation is different. If the source is closer to the main lens respect the focal plane, it will be imaged on a plane that is behind the micro array. This situation is shown in figure 3.9 in the middle. Both directional and positional coordinates are sampled by more than one lens let. Each sub image will sample a set of directional coordinates correspondent to the part of the main lens that is being imaged. In figure 3.10 it is shown what happens in the phase space. On the lens let plane the spot size will have a width of Δx proportional to the distance, therefore the directions will be sampled by the lens lets included in this interval. Each lens let receive light coming form a particular area of the main lens, its correspondent sub image only records the pixels linked with those coordinates as can be seen comparing figure 3.9 with figure 3.10. The resultant phase space representation is still a straight line, but with positive

slope proportional to the distance of the point source 3.10. If the source is further away from the plane where the main lens is focused, the main lens image is formed before the lens lets plane. The point is still sampled by more than one lens let included in the spot size Δx. The difference with the previous case is that now in the phase space the point source is represented by a straight line with a negative slope. The physical meaning of the slope of the phase space line can be understood looking at figure 3.11. When the main lens image is formed in front of the micro lens plane, if the increasing directions on the θ_x are mapped on decreasing positions in the x plane therefore the line on the phase space representing the point has a negative slope. If the image is formed behind the micro lens plane the image is this inversion is absent and the line in the phase space has a positive slope [2]. This characteristic of the phase space permits to discriminate between points in the scene that are in front or behind the main lens focal plane, giving a first rough depth estimation.

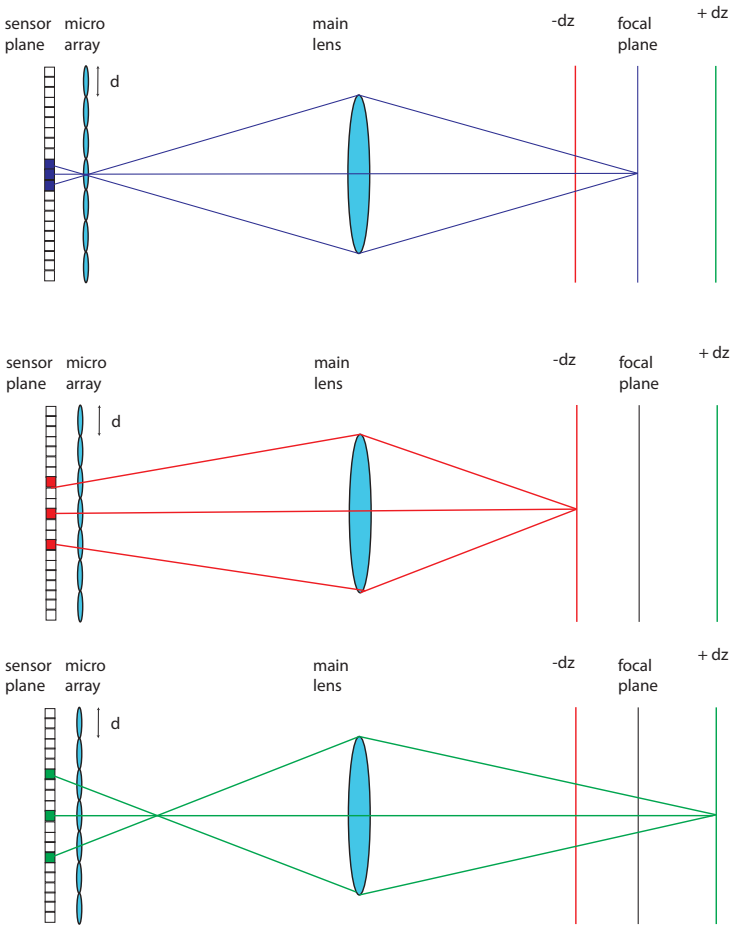

Figure 1.9: Sampling of the light field of a point source in focus, on the top, closer to the main lens, centre, and further away, bottom. When the source is out of focus, both position and directions are sampled by more than one lens let since the main lens image is not formed any more on the lens let plane.

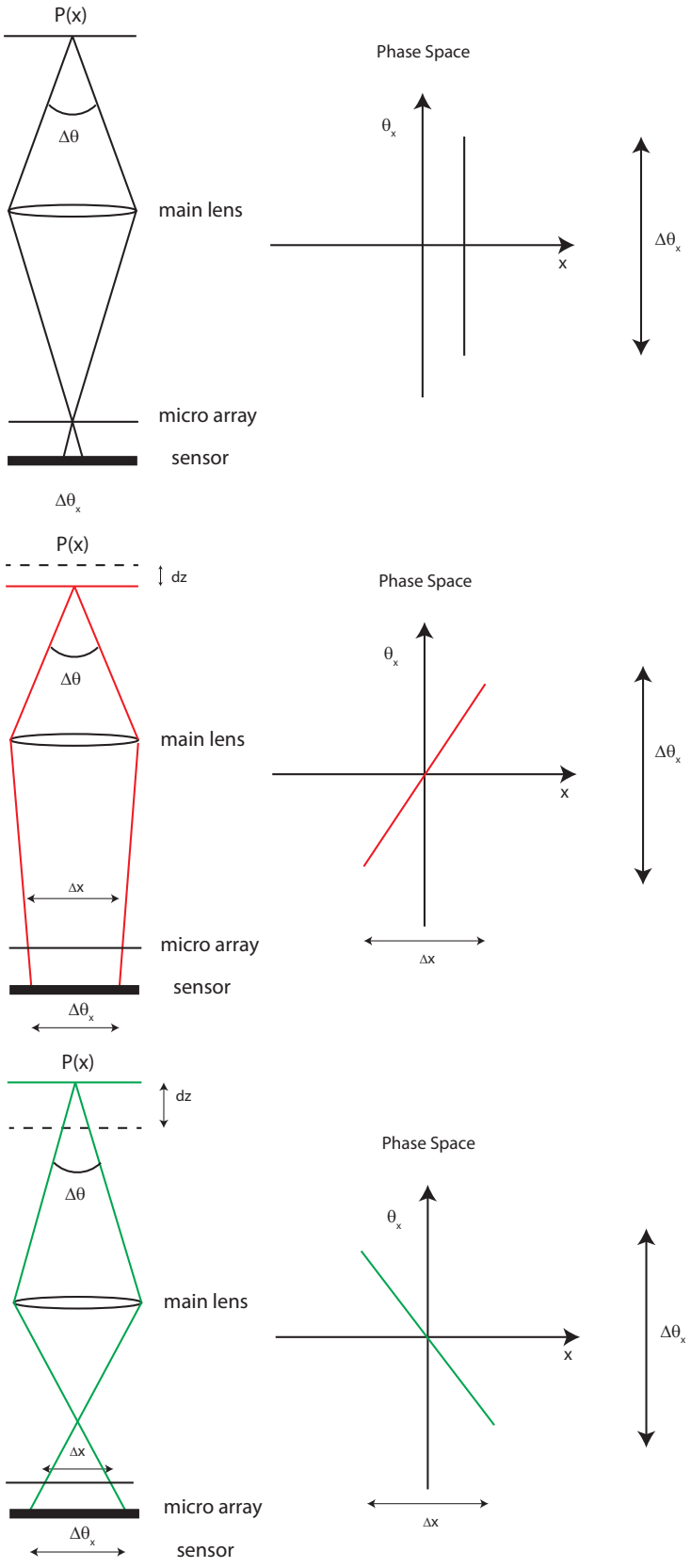

Figure 1.10: Information on the depth of a point source imaged by a plenoptic 1.0 system. As explained in section 1.4.1 If the point source is in focus, on the top, it's position is sampled by one lens let as well as the whole set of directions .

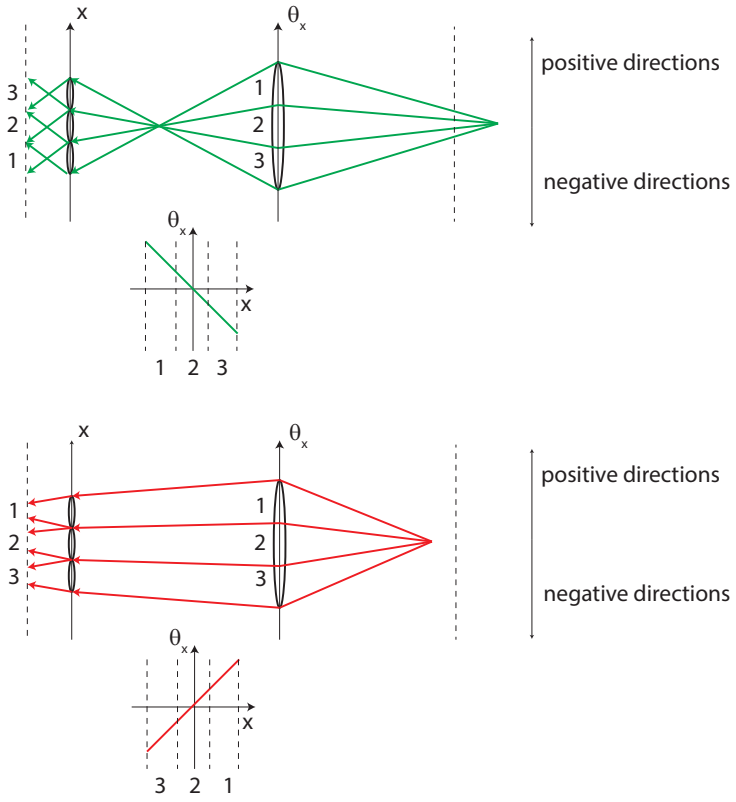

Figure 1.11: Physical meaning of the slope in the phase space. If the point source is further then the camera focal plane, the phase space line has a negative slope. If the point source is closer then the focal plane, the slope of the phase space line is positive.

A simulation of plenoptic 1.0 camera has been made to verify this fact. The system was composed by a main lens with a focal length of *120 mm*, ina *2f* configuration creating an image on the micro array plane. The Micro lens array

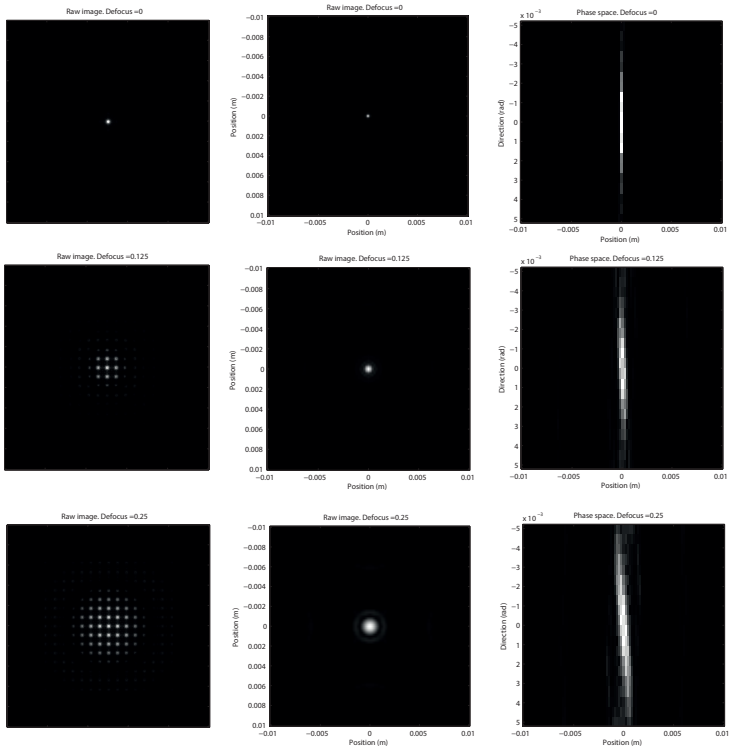

Figure 1.12: Numerical simulation of a point source imaged by a plenoptic 1.0 imaging system. The point source was placed at three different distances from the main lens. On the top we have the focused image. in the centre we have a defocus of 0.125 m far away from the main lens and on the bottom the de focus is 0.25. For these three cases we represent the raw data image, on the left, the rendered image on the center and the phase space line on the left.

1.4 Synthetic refocus

To conclude this chapter we examine one of the most fascinating post processing features enabled by collecting the light field is possibility to refocus an image after it has been captured. This features is called synthetic refocusing and is based on the fact that the recorded light field can be used to compute images as if they were taken by a synthetic camera positioned and focused differently form the actual camera. In this section we will explain the

method to manipulate light field to refocus in post processing an image. We will also show how this method, based on a synthetic camera model obtained with ray tracing, can be implemented in a wave optics approach, and we will discuss the limitations and the issues arising from a wave approach respect the simple and ideal ray optics approach. This method is based on the fact that it possible to define a synthetic camera composed by an aperture and a sensor plane, for which the defocused object plane results in focus. Referring to figure 3.13, the main lens produce an out of focus image on the micro lens plane. The directional set of coordinates Δu is mapped by along the range of spatial coordinates Δx. The light field can be re parametrized in terms of a synthetic light field generated by a synthetic aperture, represented by the dashed lens, that focus the object on a synthetic focal plane, represented by the dashed line. This second light field is parametrized with the coordinates u' v' and x' y'.

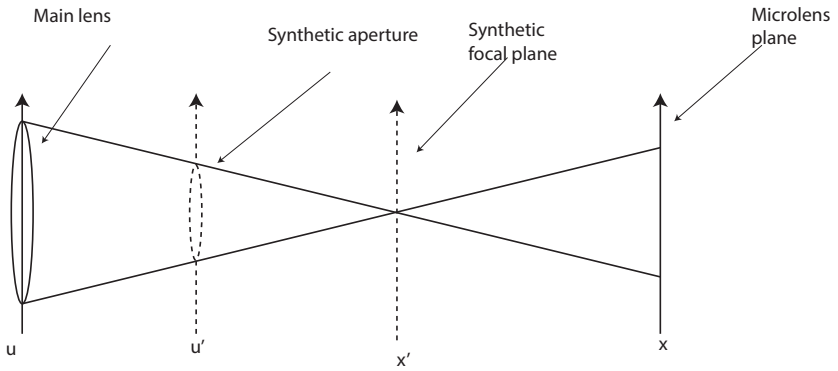

Figure 1.13: The synthetic camera refocusing method is based on the fact that is always possible to define a virtual aperture that is focused on the lens let plane.

Following the work of Ng *et al.* [10] and in analogy with what explained in section 1.5, we define the intensity obtained rendering a synthetic light

field $L'(x', y', u', v')$ parametrized by the synthetic planes *(x',y')* and *(u',v')* as:

$$I(x, y) = \iint L'(x', y', u', v')A(u', v')dudv \tag{1.5}$$

The goal is to express this intensity as a function of the captured light field $L(x, y, u, v)$ finding the relationship that links the set of coordinates *x,y,u,v* with the coordinates *x',y',u',v'*.

1.4.1 Synthetic refocus algorithm

Changing the focal plane is equal to change the distance between the main lens and the sensor. Therefore correctly refocus an image there is no need to shift the main lens plane or changing the aperture, and we can do the following simplifications:

- We assume that to refocus an object, only the synthetic sensor plane *x',y'* will be moved. Therefore the main lens plane *u',v'* is at the same position of the plane *u,v*, and no transformation is performed on the directional coordinates, so we can write *(u,v)=(u',v')*.
- the aperture of the main lens will not change, $A(x', y') = 1$.

In figure 3.14 is shown a two dimensional diagram explaining the re parametrization under the hypotheses explained above. The diagram only shows the coordinates x and u, and the y and v dimensions share an identical relationship. F is the distance of the main lens from the lens let array plane, and F' is the distance from the synthetic focal plane. We define the parameter α as the ratio between these two distances:

$$\alpha = \frac{F}{F'} \tag{1.6}$$

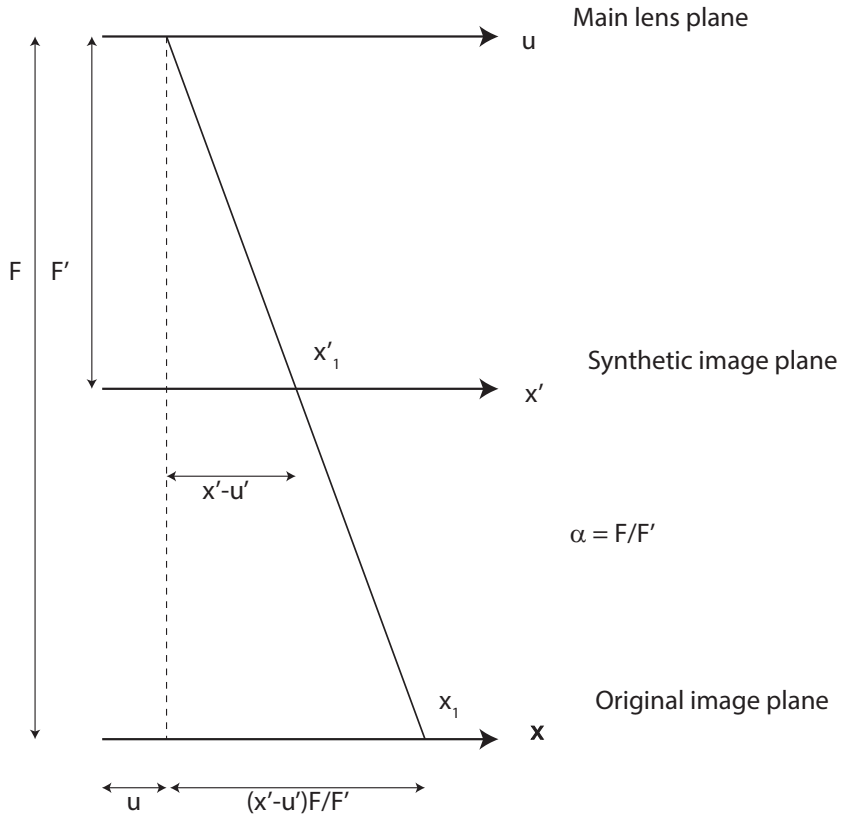

Figure 1.14: Changing the focal plane of the camera is equual to re parametrize the light field according to the new coordinates. The re parametrization is basically a shift proportional to the ratio between the refocused plane and the original camera plane α .

The factor α can be:

- $\alpha = 1$ when the synthetic focal plane is at the same position of the main lens focal plane

- $\alpha > 1$ when the distance between the main lens and the synthetic image plane is smaller than the distance between the main lens and its original focal plane. This occur when we are refocusing on a plane that it further away respect the main lens focal plane.

- $\alpha < 1$ when the distance between the main lens and the synthetic image plane is bigger than the distance between the main lens and its original focal plane. This occur when we are refocusing on a plane that it closer to the main lens.

A ray of light that crosses the main lens plane at the coordinate u and that then intercepts the synthetic image plane at the point x_1', can be represented using with the coordinates of the original image plane x_1. Because of similar triangles, the value of the x coordinate on the image plane can be expressed as a function of the coordinates u, x'. We have:

$$x_1 = u + (x_1' - u)\frac{F}{F'} = u + (x_1' - u)\alpha \tag{1.7}$$

Therefore extending to the four dimensional case, the points belonging to the synthetic image plane can be expressed as function of the directional coordinates at the main lens plane and the spatial coordinates at the original image plane as:

$$\begin{aligned} x' &= \frac{x}{\alpha} + u\left(1 - \frac{1}{\alpha}\right) \\ y' &= \frac{y}{\alpha} + v\left(1 - \frac{1}{\alpha}\right) \end{aligned} \tag{1.8}$$

With this change of coordinates the light field at the synthetic focal plane x' can be written substituting equations 3.8 into equation 3.5.

$$I(x,y) = \iint L\left(\frac{x}{\alpha} + u\left(1 - \frac{1}{\alpha}\right), \frac{y}{\alpha} + v\left(1 - \frac{1}{\alpha}\right), u, v\right) A(x', y')dudv \tag{1.9}$$

Equation 3.9 represent the rendered image obtained integrating along the directional coordinates the light field re parametrized as if it were captured by a camera whose focal plane is the same as the synthetic focal plane.

Focusing at different depths corresponds to changing the separation between the lens and the film plane, resulting in a shearing of a factor α the four dimensional the light field recorded at the micro array plane. The shearing of the light field can be seen as the composition of two transformations:

- a scaling due to the factor α that depends by the distance of the synthetic focal plane from the main lens.
- a translation term $u(1 - 1/\alpha)$ that increases with the directional coordinates and with the magnitude of the factor alpha.

The synthetic light field is therefore obtained shifting and rescaling by a factor α the light field captured by the camera and the effects of synthetic refocus can be described by a linear operator acting on the four dimensional light field

1.4.2 Shearing Operator

The synthetic refocus equation 3.9 has been obtained with ray optics considerations only. However, it can be applied to light fields obtained light fields obtained with wave optics simulations. The only difference will be that particular attention should be given in the design of the imaging system in order to avoid cross talk effects induced by diffraction, as explained in section 3.1.1. The effects of this operator in the phase space is a change of the slope of the lines describing the points of the image in the phase space. Since the total light field is conserved, the area in the phase space remains the same, hence the projection of the light field on the phase space results sheared respect the original one.

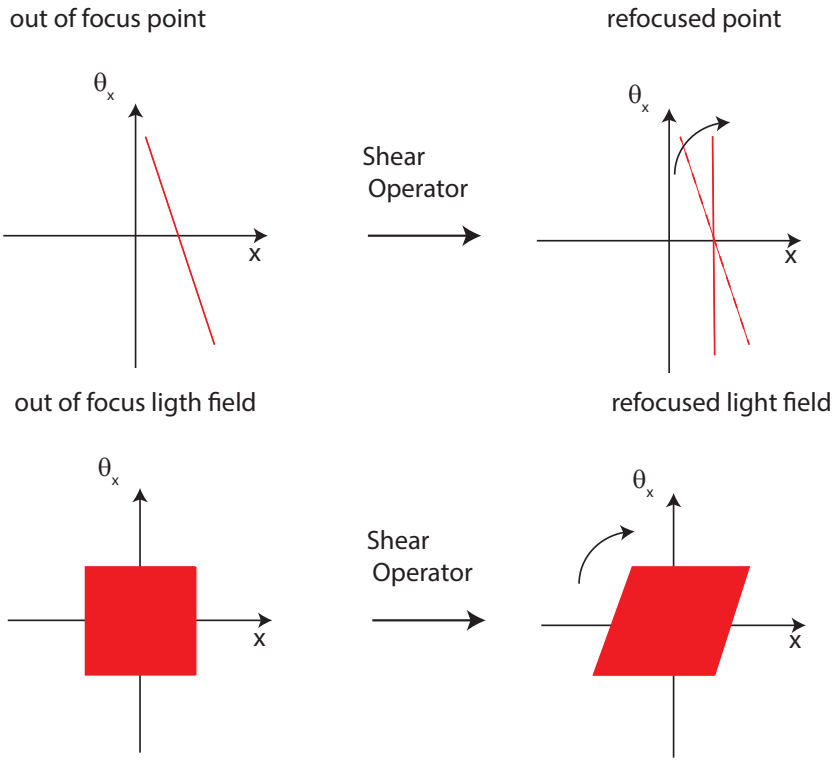

Figure 1.15: Changing the osition of the focal plane is equal to shear the light field in the phase space. The total area remains the same since the total light field is conserved.

From a computation point of view we implemented the synthetic refocus operator in MATLAB. The operator is composed by three stages. The first stage it creates a new set of spatial coordinates shifting and rescaling the the original set of coordinates as shown in equation 3.8. Then the output synthetic light field is created interpolating the input light field using as a base the new set of coordinates. The flow chart can be seen in figure 3.16

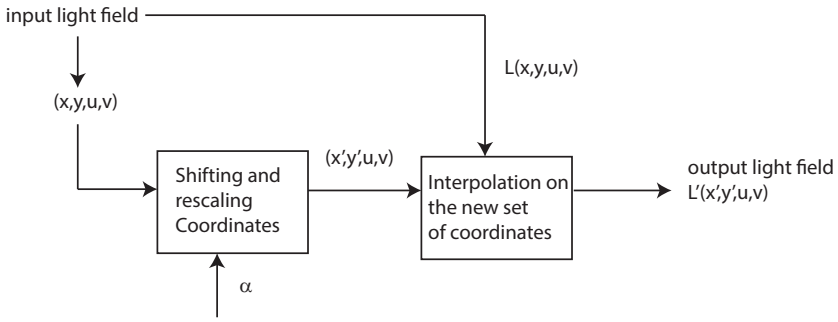

Figure 1.16: Flow chart of the operator shearing.

1.5 Results of the Simulations

To test and explore the potentials of the synthetic refocusing algorithm we generated a set of light field using the wave opticas simulation toolbox described in section **??**. The System simulated is tha one described in section 2.6.1 and shown in figure 3.17 and the optical parameters can be found in the following table.

Main lens focal length f	120mm
Lens aperture D	3.6 mm
Micro lens focal length f_μ	10 mm
Micro lens diameter d	150 μm
Micro array pitch p	150 μm
Field of view W	20 mm
f-number	33.6
sensor resolution	3030 by 3030 pixel

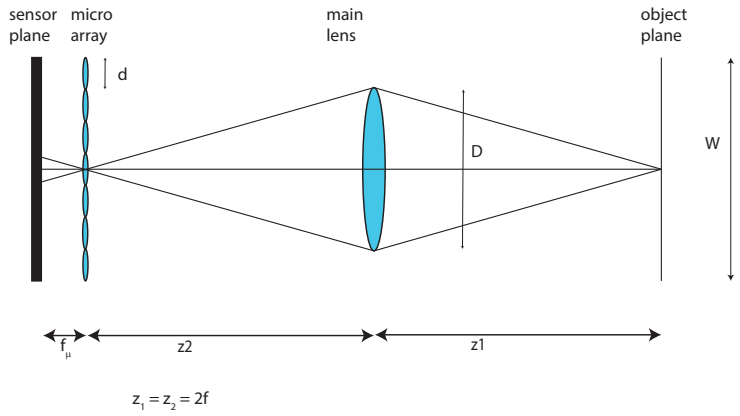

Figure 1.17: Set up simulated.

With this set up the final rendered image resolution is of 101 by 101 pixel. Each pixel of the final image corresponds to a lens let, therefore the spatial resolutio iun the final image is equa to the diameter of the single lens let. Each sub image has 30 by 30 lens let, meaning that the full set of directional coordinates is sampled by $N_{sub} = 30$ pixels. The angular resolution is therefore given by:

$$\delta\theta = \frac{\Delta\theta}{N_{sub}} = \frac{NA}{N_{sub}} \tag{1.10}$$

and is equal to $\delta\theta = 2.5x10^{-4}rad$. The first object to be imaged was a volume V of dimension 20 mm x 20 mm x 100 mm containing three point sources. Each point source has been simulated by a circle of 10 μm diameter and are positioned as shown in figure 3.18

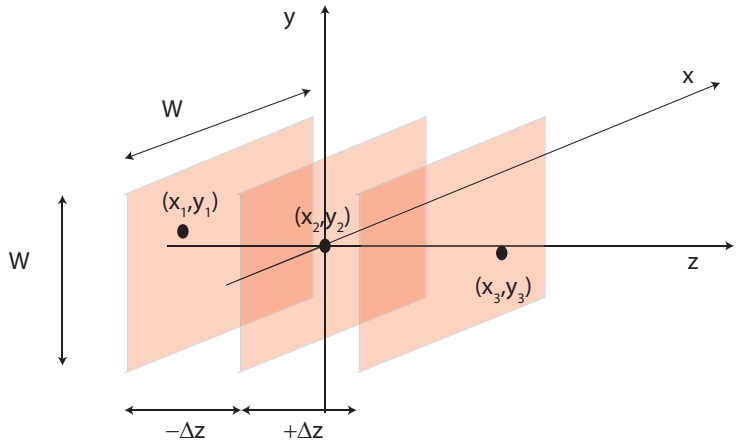

Figure 1.18: Position of the point sources imaged inside the volume V simulated.

Each plane has been imaged singularly setting the distance z_1 equal to the sum of the distance of the main lens focal plane and the defocus δz. The Coordinates of the points where:

- A: $x_1 = -3mm, y_1 = -3mm, z_1 = -50mm$
- B: $x_2 = 0mm, y_2 = 0mm, z_2 = 0mm$
- C: $x_3 = +3mm, y_3 = +3mm, z_3 = +50mm$

After that the light of the three planes has been propagated into the system, the final raw image has been obtained summing the three separate raw images normalized by the total amount of energy contained. This can be made possible since the Fresnel simulation toolbox is time independent. In figure are shown the total raw image and the rendered image integrated without any refocusing. 3.18

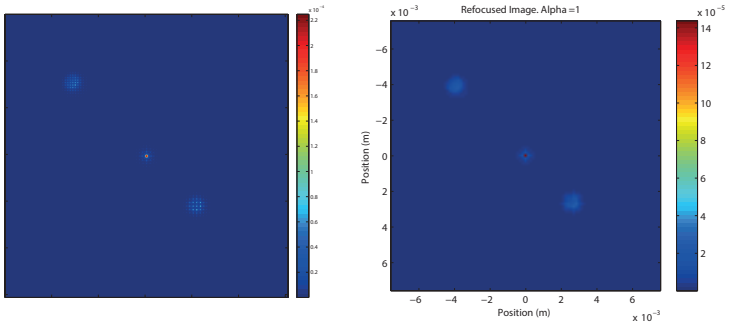

Figure 1.19: Raw image on the left and rendered image on the right. Intensity is shown in false colour in order to appreciate variations in energy distribution.

For a defocus of 50 mm the factor alpha can be obtained using reversing the lens equation. For the 2f system simulated and referring to figure 3.14 for notations, we have

$$\frac{1}{f} = \frac{1}{F'} + \frac{1}{z + \Delta z)} \tag{1.11}$$

Since $\alpha = F/F'$, and in a 2f system F=2f=z, we have:

$$\frac{1}{f} = \frac{\alpha}{2f} + \frac{1}{2f + \Delta z)} \tag{1.12}$$

Resolving for α we have:

$$\alpha = \frac{2f + s\Delta z}{2f + \Delta z} \tag{1.13}$$

In figure are shown the values of α obtained by equation3.13 for defocus values varying between plus and minus 10 cm.

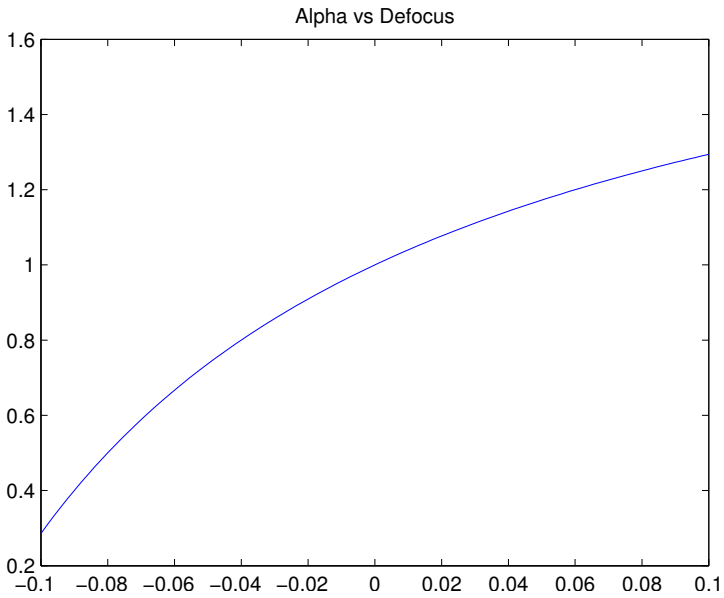

Figure 1.20: Raw image on the left and rendered image on the right. Intensity is shown in false colour in order to appreciate variations in energy distribution.

We can see from figure 3.20 that it is not linear with defocus, and therefore the axial resolution in not linear. To conclude this section we show the refocused imaged obtained shearing the light field extracted by thw raw image in figure 3.19. According to equation 3.13, the point A is refocused for a value of α equal to 0.7 and point B for $\alpha = 1.17$. Results are shown in figure 3.21

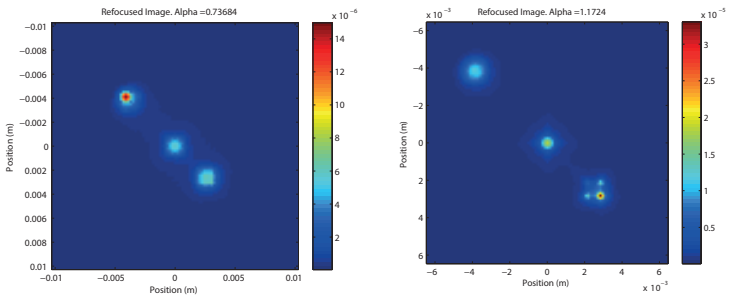

Figure 1.21: Refocused images...

1.6 depth estimation through synthetic refocus

The synthetic refocus algorithm provides a method to estimate the depth of a point source in a volume. If we have a look at the Fourier transform of a rendered image of a point source before and after refocus, we note that when the point source is in focus, its spectrum is broader than the blurred one. This fact is true for normal images as well, when the presence of more sharp details in the in focus image, causes its spectrum to have a larger amount of high spatial frequencies. Inj figure is shown the difference between the spectrum of the out of focus image(blue) and the in focus image(red) for a defocus of 5 cm.

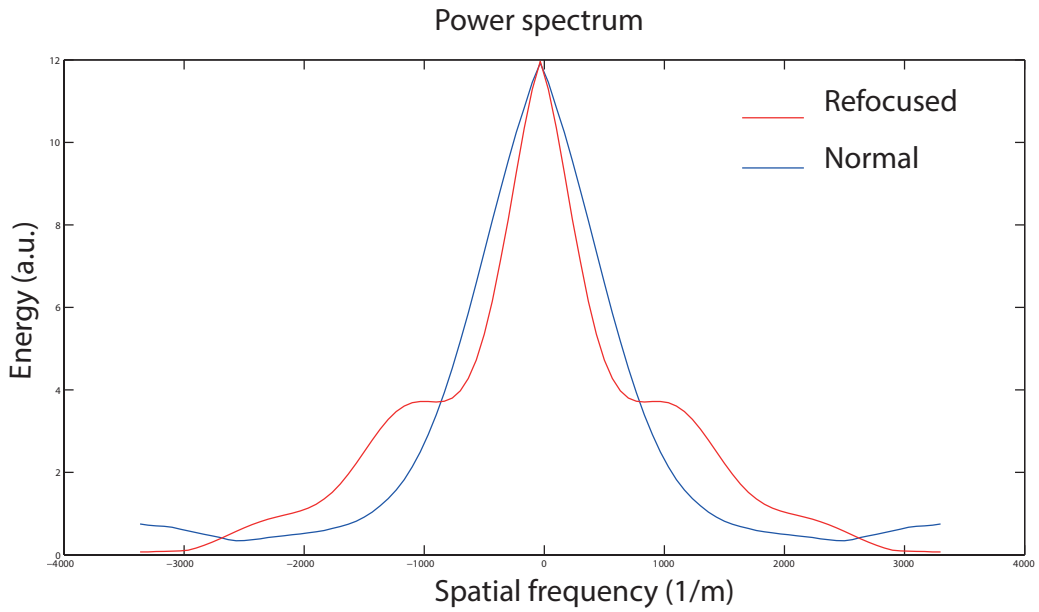

Figure 1.22: Refocused images...

The action of shearing the light field produces a redistribution of the power spectrum of the signal towards high frequency. This effect causes the

sharpness of the image and therefore the refocus. This effect can be used to estimate the depth of object imaged just evaluating its level of blur. The method consist in creating a focal stack shearing the light field at different values of the coefficient alpha, that correspond to different focal planes, and rendering the images. After the focal stack is created the power spectrum of each image is calculated using the fast Fourier transform algorithm. It is then removed the low frequency terms and it calculated the total energy contained in the high frequency term. Plotting the values of energy as a function of the different alpha coefficients, we obtain a curve with presents a maximum in the proximity of the most in focus image. This maximum corresponds to the value of alpha that best estimates the actual value of alpha. In figure 3.23 are shown the energy as a function of the refus parameter alpha for a point source placed at different depths. The vertical red line indicate the position of the maximum. The values of the estimated alpha and the real one are shown in the following table:

Estimated α	Real *alpha*	Error	Defocus
0.85	0.738	0.1132	-0.05 m
0.925	0.8837	0.0413	-0.025 m
1.080	1.0943	-0.0143	+0.025 m
1.105	1.1724	0.0674	+0.05 m

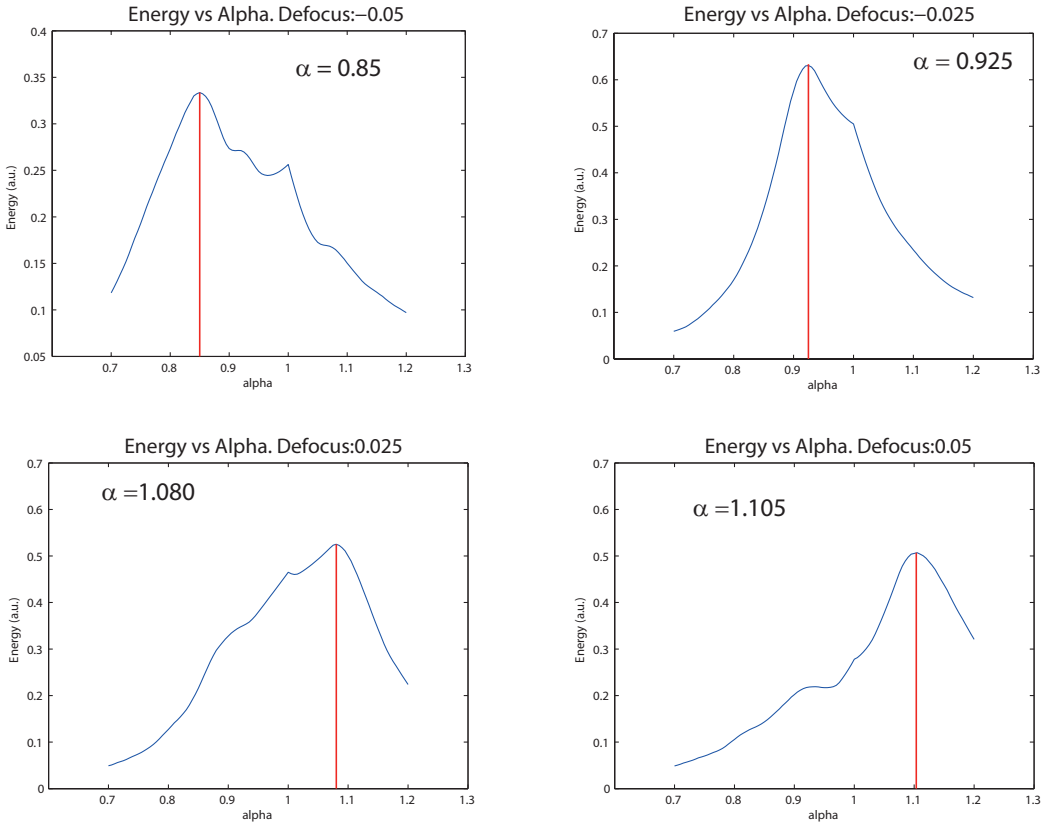

Figure 1.23: Refocused images...

In figure are shown the distribution of the estimated values of alpha in comparison with the actual values of alpha from equation 3.13. It is clear that when the object is close to the main lens, defocus is negativ, the error in evaluating alpha is bigger, while the more accurate values are in the range of depth close to the focal plane.

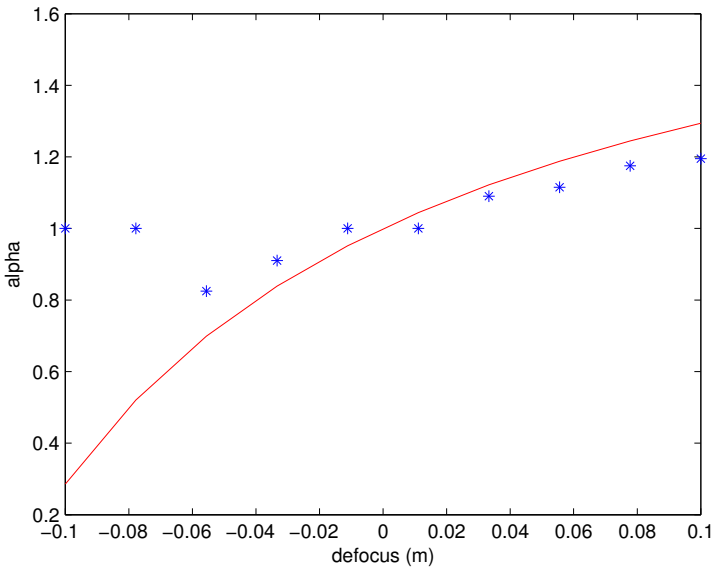

Figure 1.24: Estimated values of α represented by blea stars and the theoretical values represented with the blue line.

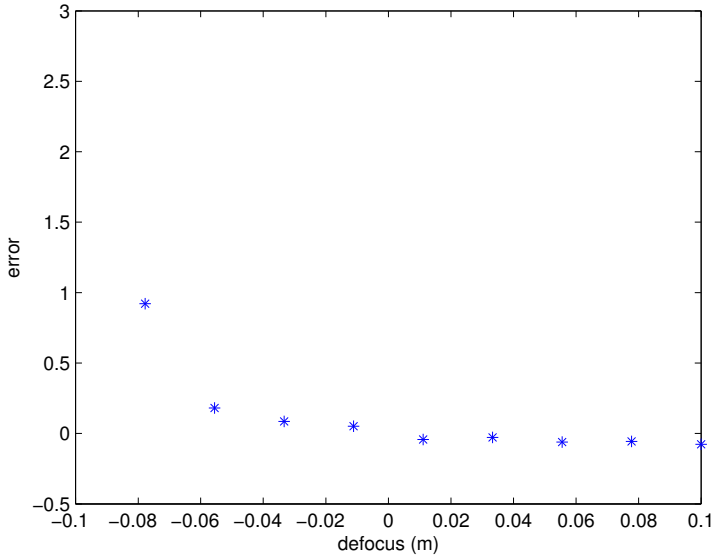

Figure 1.25: Distance between the estimated values of alpha and thje actual theoretical values.

1.7 Conclusion

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