$$R_{i,i+1} = \frac{r_{i,i+1} + R_{i+1,i+2}\exp(\beta_{i+1})}{1 + r_{i,i+1} R_{i+1,i+2}\exp(\beta_{i+1})}$$

$$\beta_i = -2i\frac{2\pi}{\lambda} n_i \cos\theta_i \, t_i$$

DEFINE :

$$R_{i,i+1} \equiv R_i \qquad \rho_{i+1} = \exp\beta_{i+1}$$

$$r_{i,i+1} \equiv r_i$$

$$R_i = \frac{r_i + R_{i+1}\exp\beta_{i+1}}{1 + r_i R_{i+1}\exp\beta_{i+1}} = \frac{r_i + R_{i+1}\rho_{i+1}}{1 + r_i R_{i+1}\rho_{i+1}}$$

R_i as a function of r_i, R_{i+1} and ρ_{i+1} :

DERIVATIVES (partial)

$$\frac{\partial R_i}{\partial r_i} = \frac{1 - R_{i+1}^2 \rho_{i+1}^2}{den^2}$$

den is the denominator in the formula for R_i above

$$\frac{\partial R_i}{\partial \rho_{i+1}} = \frac{R_{i+1}(1 - r_i^2)}{den^2}$$

$$\frac{\partial R_i}{\partial R_{i+1}} = \frac{(1 - r_i^2)\rho_{i+1}}{den^2}$$

We call θ_0 the AOI (angle of incidence)

$$\frac{\partial R_i}{\partial \theta_0} = \frac{\partial R_i}{\partial r_i}\frac{\partial r_i}{\partial \theta_0} + \frac{\partial R_i}{\partial \rho_{i+1}}\frac{\partial \rho_{i+1}}{\partial \theta_0} + \frac{\partial R_i}{\partial R_{i+1}}\frac{\partial R_{i+1}}{\partial \theta_0}$$

Derivatives with θ_0 :

Preliminar results

$$\frac{\partial \cos\theta_i}{\partial \theta_0} = -\frac{n_0^2 \operatorname{sen}\theta_0 \cos\theta_0}{n_i^2 \cos\theta_i}$$

BECAUSE $\cos\theta_i = \sqrt{1 - \operatorname{sen}^2\theta_i} = \sqrt{1 - \frac{n_0^2 \operatorname{sen}^2\theta_0}{n_i^2}}$

SNELL'S LAW : $n_i \operatorname{sen}\theta_i = n_0 \operatorname{sen}\theta_0 \quad \forall i$

SEGUE :

$$\frac{\partial \beta_i}{\partial \theta_0} = 2i\frac{2\pi}{\lambda}\frac{n_0^2 \operatorname{sen}\theta_0 \cos\theta_0}{n_i \cos\theta_i} \cdot t_i$$

AND

$$\frac{\partial \rho_i}{\partial \theta_0} = \rho_i \frac{\partial \beta_i}{\partial \theta_0} \qquad \text{BECAUSE} \quad \rho_i = \exp(\beta_i)$$

DERIVATIVES OF r_i (FRESNEL COEFFS)

$$(r_i)_s \equiv (r_{i,i+1})_s = \frac{n_i \cos\theta_i - n_{i+1}\cos\theta_{i+1}}{n_i\cos\theta_i + n_{i+1}\cos\theta_{i+1}}$$

IT FOLLOWS

$$\frac{\partial (r_i)_s}{\partial\theta_0} = \frac{2 n_0^2 \operatorname{sen}\theta_0 \cos\theta_0}{(n_i\cos\theta_i + n_{i+1}\cos\theta_{i+1})^2}\left(\frac{n_i\cos\theta_i}{n_{i+1}\cos\theta_{i+1}} - \frac{n_{i+1}\cos\theta_{i+1}}{n_i\cos\theta_i}\right)$$

OBTAINED USING $\frac{\partial \cos\theta_i}{\partial\theta_0}$ (PAGE 1)

FOR THE P POLARIZATION:

$$(r_i)_p \equiv (r_{i,i+1})_p = \frac{n_{i+1}\cos\theta_i - n_i\cos\theta_{i+1}}{n_{i+1}\cos\theta_i + n_i\cos\theta_{i+1}}$$

$$\frac{\partial (r_i)_p}{\partial\theta_0} = \frac{2 n_0^2 \operatorname{sen}\theta_0 \cos\theta_0}{(n_{i+1}\cos\theta_i + n_i\cos\theta_{i+1})^2}\left(\frac{n_i\cos\theta_i}{n_{i+1}\cos\theta_{i+1}} - \frac{n_{i+1}\cos\theta_{i+1}}{n_i\cos\theta_i}\right)$$

NOTE, THE DERIVATIVES FOR S AND P ARE THE SAME ~~EXPT~~ EXCEPT FOR THE DENOMINATOR

FOR $\frac{\partial R_{i+1}}{\partial\theta_0}$ WE USE THE FACT THAT JUST ABOVE THE SUBSTRATE

$R_N = r_N$ WHERE $N - N+1$ IS THE INTERFACE BETWEEN THE FIRST FILM AND THE SUBSTRATE.

THEN $\frac{\partial R_i}{\partial\theta_0}$ is calculated by recursion from $\frac{\partial R_{i+1}}{\partial\theta_0}$ USING Total derivative formula on PAGE 1