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Indian Institute of Information Technology, Nagpur

Applied Signals & System (ECE – 205)
Lab-3 Report
Batch- ECE- IoT-1

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Aim: Calculating and observing Fourier series and Gibb's phenomenon.

Theory:

Introduction
Certainly! Here's some theory on Fourier series and the Gibbs phenomenon:

Fourier Series:

A Fourier series is a mathematical representation of a periodic function as a sum of sinusoidal (sine and cosine) functions. It was developed by Jean-Baptiste Joseph Fourier in the early 19th century.

The Fourier coefficients are calculated using integrals over one period of the function:

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos\left(\frac{2\pi nt}{T}\right)dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin\left(\frac{2\pi nt}{T}\right)dt$$

The Fourier series allows us to represent a wide range of periodic signals in terms of their constituent sinusoidal components. It's a powerful tool in various fields, including signal processing, physics, and engineering.

Gibbs Phenomenon:

The Gibbs phenomenon is an inherent feature of the Fourier series when used to represent discontinuous functions. It refers to the overshoot and oscillations that occur near a discontinuity in the signal's representation. The overshoot means that the Fourier series representation extends beyond the discontinuity, resulting in a "gibb" or overshoot before converging to the correct value.

Key points about the Gibbs phenomenon:
- It is a result of the truncation of the Fourier series. When a finite number of terms are used in the series, they struggle to accurately represent discontinuities.
- The overshoot is typically about 9% of the discontinuity's magnitude.
- The Gibbs phenomenon occurs even when an infinite number of terms are used, but the overshoot decreases as more terms are included.

The Gibbs phenomenon is important to understand when working with Fourier series, especially in applications where accurate representation near discontinuities is crucial. Engineers and scientists often need to consider this phenomenon when analyzing signals or data.

In summary, Fourier series is a powerful tool for representing periodic functions in terms of sinusoidal components, but the Gibbs phenomenon serves as a reminder of its limitations, particularly near discontinuities in the signal. Understanding these concepts is vital for using Fourier series effectively in various fields of science and engineering.

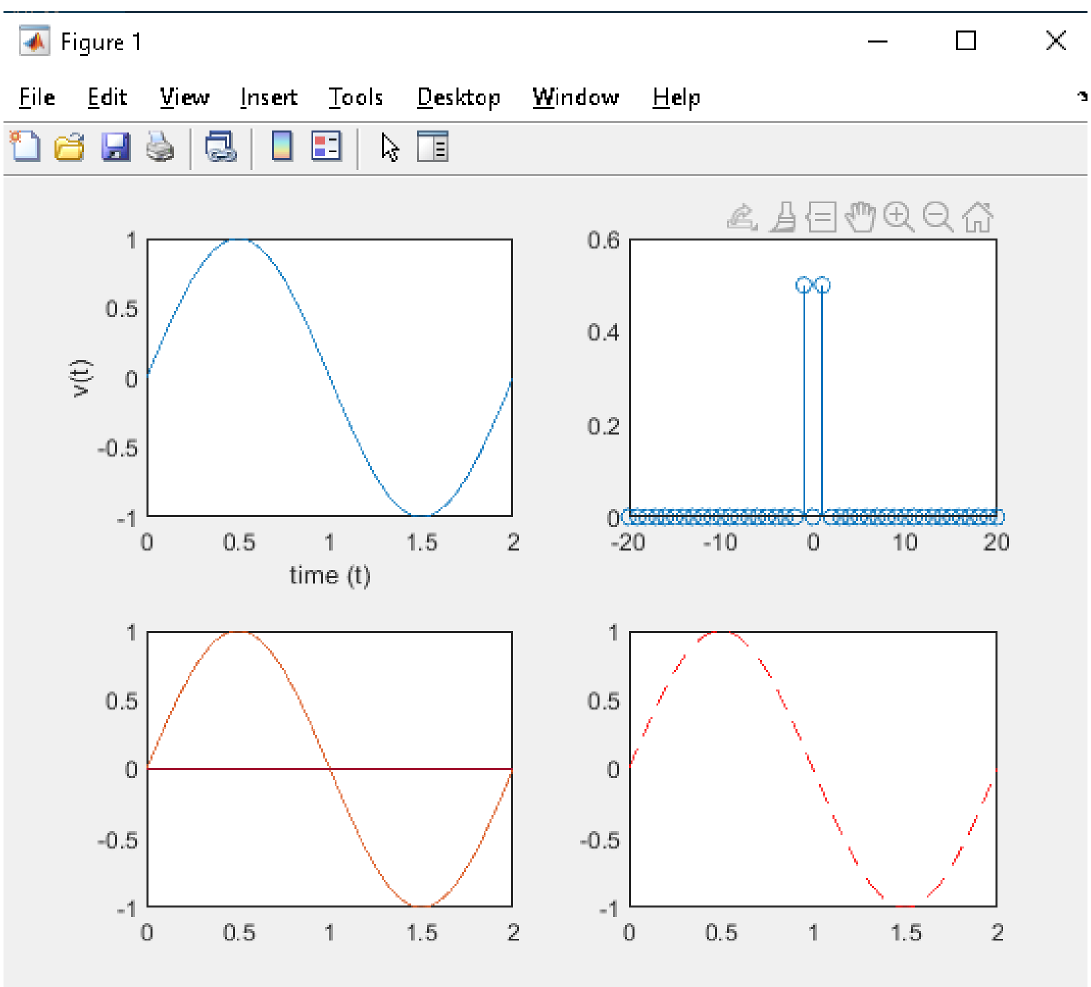
MATLAB Code

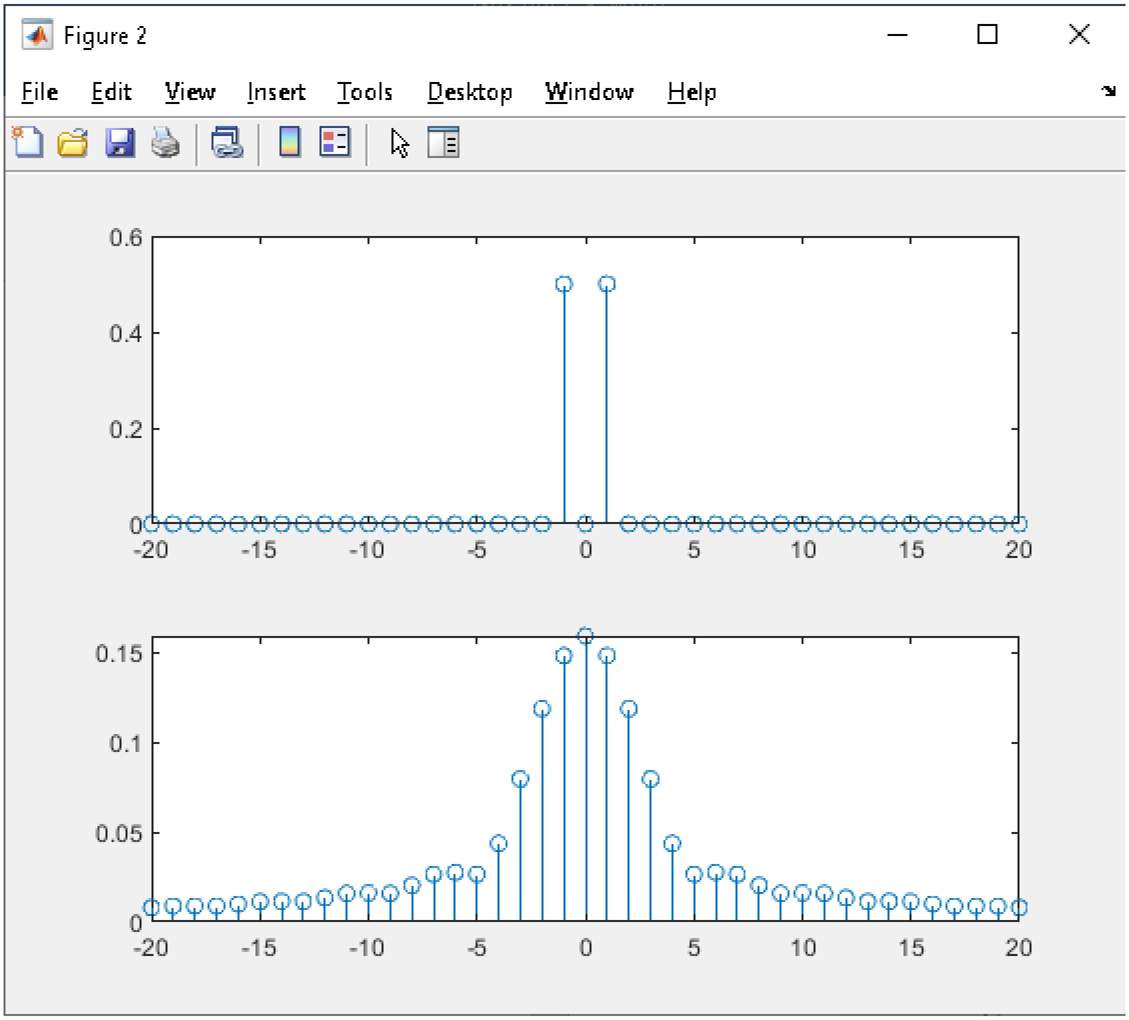
We have written a MATLAB code for this project. The code begins with the definition of the my_int_fun function, which is used for numerical integration. Then, it generates and observes various signals, including a half-wave rectified sine wave, a triangular wave, and a square wave. The code also calculates and plots the Fourier series coefficients for these signals.

```
function [int_ans] = my_int_fun(data, step_size)
%Integration function
%   Detailed explanation goes here

int_ans=(step_size/2)*...
    (data(1,1)+data(1,end)...
    +2*sum(data(1,2:end-1)));
end

clc
clear all
close all

time_period=2;
step_size_t=0.0001;
```

```
t=0:step_size_t:time_period;
tt=length(t);

omega_o=2*pi/time_period;

No_of_Fourier_coff=20;
kk=-No_of_Fourier_coff:1:No_of_Fourier_coff;

for ii=1:tt
    v_t(ii)=2*sin((pi/2)*t(ii));
     if(t(ii)<-0.25)
        v_t(ii)=0;
    elseif(t(ii)>=-0.25 && t(ii)<=0.25)
        v_t(ii)=1;
    else
        v_t(ii)=0;
    end
    v_t(ii)=(2/3)*t(ii);

    if(t(ii)<=0.5)
        v_t(ii)=5*cos(omega_o*t(ii));
    elseif(t(ii)>0.5 && t(ii)<1.5)
        v_t(ii)=0;
    else
        v_t(ii)=5*cos(omega_o*t(ii));
    end
    if(t(ii)<=2)
        v_t(ii)=1*sin(omega_o*t(ii));
    else
        v_t(ii)=0;
    end
end

%theortical values of a_k
for ii=1:1:(2*No_of_Fourier_coff+1)
    if(abs(kk(ii))==1)
        a_k_th(ii)=(1+1j*(pi/2))/(4*pi);
    else
    a_k_th(ii)=(1/((2*pi)*(1-kk(ii)^2)))*( 1-1j*kk(ii)*exp(-
1j*kk(ii)*pi/2) );
    end
end

subplot(2,2,1), plot(t, v_t)
xlabel("time (t)")
ylabel("v(t)")


for ii=1:1:(2*No_of_Fourier_coff+1)
    temp1=v_t.*exp(-1j*omega_o*kk(ii).*t);
    [int_ans] = my_int_fun(temp1, step_size_t);
    a_k(ii)=(1/time_period)*(int_ans);
```

```
end

subplot(2,2,2),stem(kk, abs(a_k))
pause
mid_term_of_fs=No_of_Fourier_coff+1;
t2=0:step_size_t:3*time_period;
for ii=1:mid_term_of_fs
    if(ii==1)
        harmonics(ii,:)=a_k(mid_term_of_fs)*...
            exp(1j*omega_o*kk(mid_term_of_fs).*t);
    else
        harmonics(ii,:)=(a_k(mid_term_of_fs-ii+1).*...
            exp(1j*omega_o*(kk(mid_term_of_fs-ii+1)).*t))...
            +(a_k(mid_term_of_fs+ii-1).*...
            exp(1j*omega_o*(kk(mid_term_of_fs+ii-1)).*t));
    en
end

subplot(2,2,3), plot(t, real(harmonics))
reconstructed_signal=sum(harmonics);
subplot(2,2,4),plot(t, real(reconstructed_signal), '--r' )
figure, subplot(2,1,1),stem(kk, abs(a_k))
subplot(2,1,2),stem(kk, abs(a_k_th))
```






```
clc
clear all
close all

time_period = 2;
step_size_t = 0.0001;
t = 0:step_size_t:time_period;
tt = length(t);

omega_o = 2 * pi / time_period;

No_of_Fourier_coff = 20;
kk = -No_of_Fourier_coff:1:No_of_Fourier_coff;

% Initialize v_t with zeros
v_t = zeros(size(t));

% Triangular wave signal
for ii = 1:tt
    % Periodic triangular wave (ramp signal)
    v_t(ii) = mod(t(ii), 0.5) / 0.5;
end

subplot(2, 2, 1), plot(t, v_t)
xlabel("time (t)")
```

```
ylabel("v(t) - Periodic Triangular Wave")

% Calculate Fourier coefficients for the periodic triangular wave signal
for ii = 1:1:(2 * No_of_Fourier_coff + 1)
    temp1 = v_t .* exp(-1j * omega_o * kk(ii) .* t);
    [int_ans] = my_int_fun(temp1, step_size_t);
    a_k(ii) = (1 / time_period) * (int_ans);
end

subplot(2, 2, 2), stem(kk, abs(a_k))

pause

mid_term_of_fs = No_of_Fourier_coff + 1;
t2 = 0:step_size_t:3 * time_period;
for ii = 1:mid_term_of_fs
    if (ii == 1)
        harmonics(ii, :) = a_k(mid_term_of_fs) * ...
            exp(1j * omega_o * kk(mid_term_of_fs) .* t);
    else
        harmonics(ii, :) = (a_k(mid_term_of_fs - ii + 1) .* ...
            exp(1j * omega_o * (kk(mid_term_of_fs - ii + 1)) .* t)) ...
            + (a_k(mid_term_of_fs + ii - 1) .* ...
            exp(1j * omega_o * (kk(mid_term_of_fs + ii - 1)) .* t));
    end
end

subplot(2, 2, 3), plot(t, real(harmonics))

reconstructed_signal = sum(harmonics);
subplot(2, 2, 4), plot(t, real(reconstructed_signal), '--r')

figure, subplot(2, 1, 1), stem(kk, abs(a_k))

% Theoretical values of a_k for the periodic triangular wave
for ii = 1:1:(2 * No_of_Fourier_coff + 1)
    if (kk(ii) == 0)
        a_k_th(ii) = 1/12;
    elseif (mod(kk(ii), 2) == 0)
        a_k_th(ii) = 0;
    else
        a_k_th(ii) = (-1)^((kk(ii) - 1) / 2) * (2 / (pi^2 * (kk(ii) -
1)^2));
    end
end

subplot(2, 1, 2), stem(kk, abs(a_k_th))
```

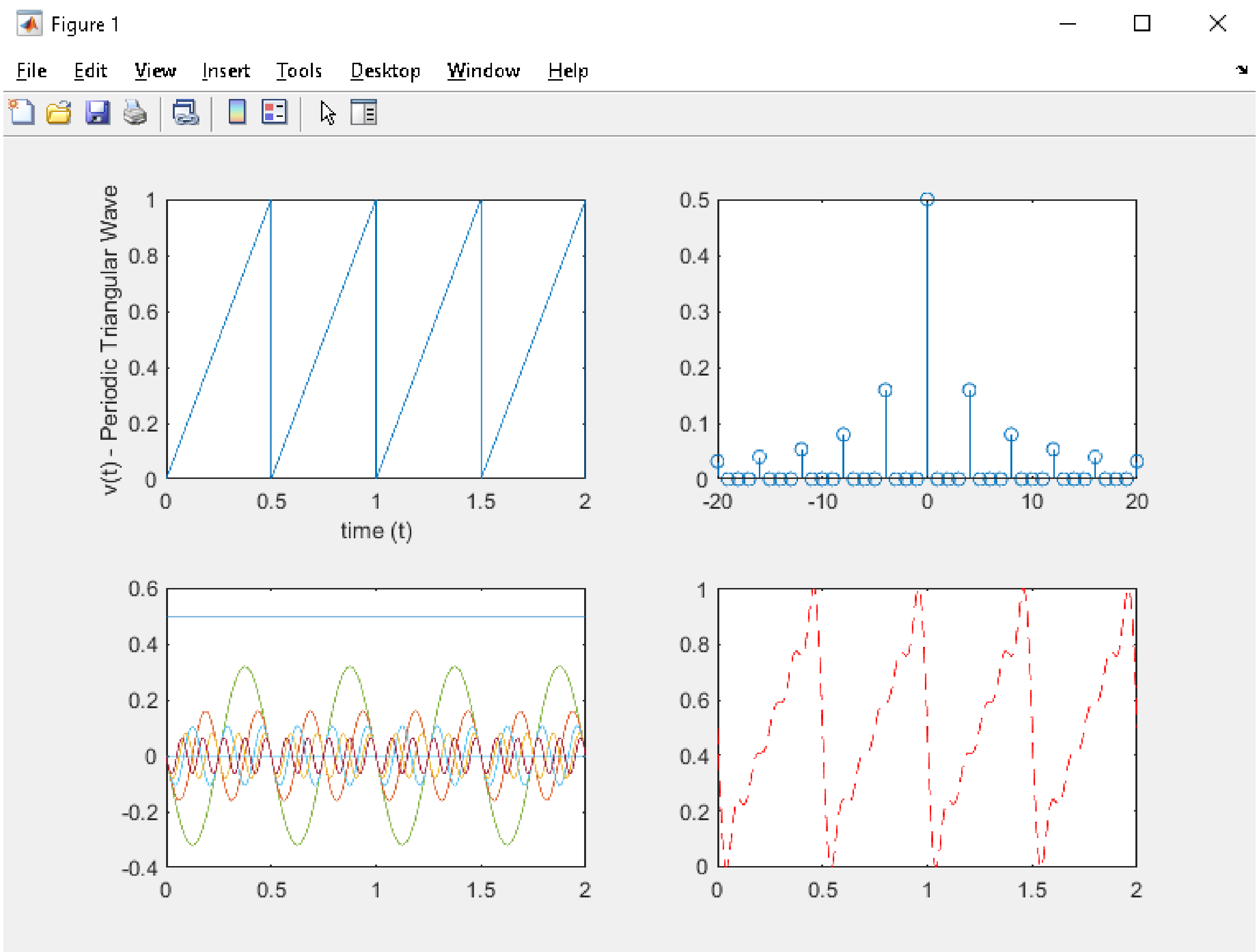


Figure 1
File Edit View Insert Tools Desktop Window Help
v(t) - Periodic Triangular Wave
time (t)

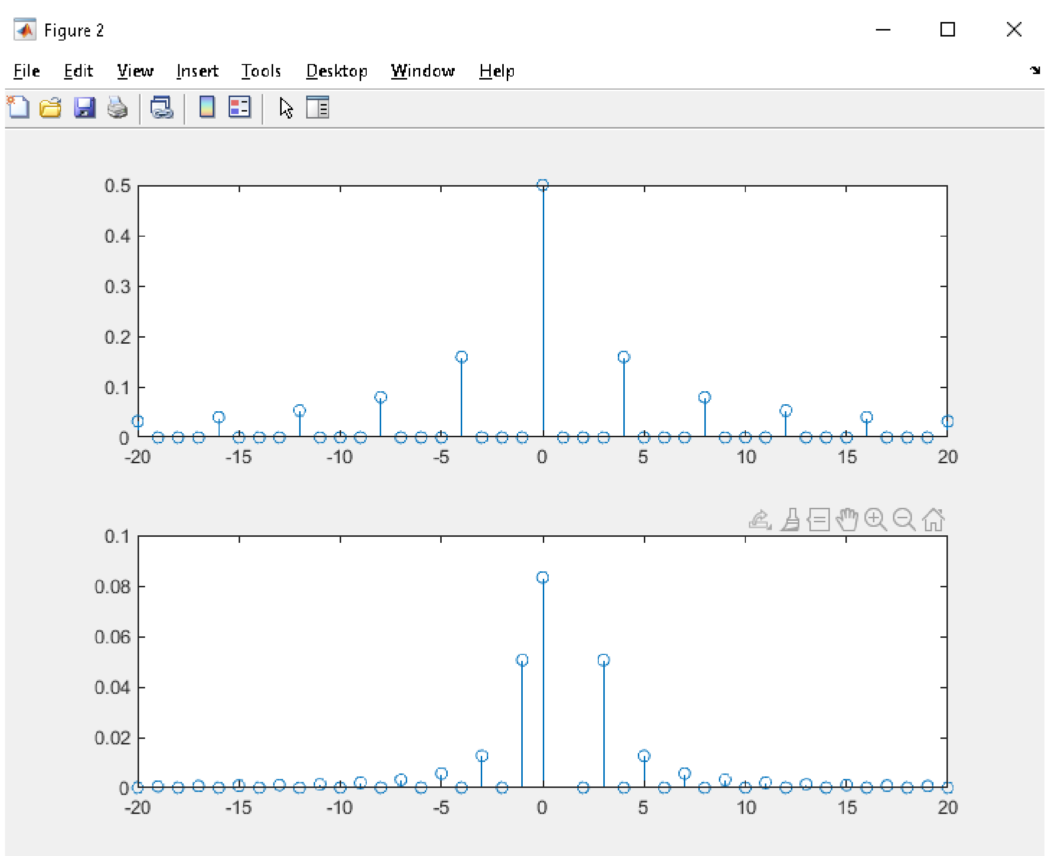


Figure 2
File Edit View Insert Tools Desktop Window Help

Key Components of the Code:

Signal Generation:
We generate different signals, such as a half-wave rectified sine wave, a triangular wave, and a square wave. These signals serve as our test cases.

Calculation of Fourier Series Coefficients:
We calculate the Fourier series coefficients (a_k) for each signal using numerical integration. These coefficients are essential for representing signals in the frequency domain.

Gibbs Phenomenon Observation:
We observe the Gibbs phenomenon by reconstructing the signal using a limited number of harmonics and comparing it to the original signal. The overshoots and oscillations near discontinuities can be observed in the reconstructed signal.

Results and Observations:
Fourier series coefficients (a_k) for the tested signals.
Plots of the original signals and reconstructed signals with a limited number of harmonics.
Observation of the Gibbs phenomenon and its effects on signal representation.

Conclusion:

This lab project provides a hands-on experience with Fourier series and the Gibbs phenomenon. We have generated, analyzed, and reconstructed various signals using Fourier series coefficients. Understanding the Gibbs phenomenon is crucial for accurately representing and analyzing signals with Fourier series.