Factor of Safety (a.k.a. Safety Factor)

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The factor of safety accounts for uncertainty in material and geometry properties and environment effects, among others, and leads to a simple design process. The factor of safety is defined as follows:

$$FOS = \frac{F_{\max}^{nominal}}{F_{design}}$$

FOS : factor of safety.

$F_{\max}^{nominal}$: nominal strength of material or joint.

F_{design} : design load -maximum internal load in structure, maximum force magnitude in bars.

The FOS is typically based on the following considerations:

- accuracy of predictions on the imposed loads (external and internal),
- variation of strength of material (inherent stochastic variations, manufacturing process, …),
- environmental effects leading to degradation of material strength over time and to changes in load conditions,
- the cost of over-engineering the component to achieve that factor of safety (increased weight, higher costs),
- consequences of failure.

In the aerospace industry, typical values for FOS are 1.5 to 2 with 1.7 being the standard value.

For lab 2 in ASEN 2001, we assume that only the strength of the joint and the predicted forces in the bars are random (uncertain). The following explains (a) how to obtain a FOS based on a desired failure probability, (b) how to use this FOS in the design process, and (c) how to compute the probability of failure of the final design.

1. Step:

First, let us consider that only the strength of the joints is random and the internal forces in the bars are deterministic:

- joint strength is Gaussian: $F_{\max} = N\left(\mu_F, \sigma_F\right)$

- design load is deterministic: F_{design}

To better understand what this model implies, consider the probability of a force F being larger than $F_{\max}$ for different values of F. For illustration purposes we set $F = \mu_F - n\sigma_F$ for $n = 0,1,2,3,4$:

$$F = \mu_F : \quad P\left(\mu_F > F_{\max}\right) = 0.5$$

$$F = \mu_F - \sigma_F : \quad P\left(\mu_F - \sigma_F > F_{\max}\right) = 0.32$$

$$F = \mu_F - 2\sigma_F : \quad P\left(\mu_F - 2\sigma_F > F_{\max}\right) = 0.045$$

$$F = \mu_F - 3\sigma_F : \quad P\left(\mu_F - 3\sigma_F > F_{\max}\right) = 0.0027$$

$$F = \mu_F - 4\sigma_F : \quad P\left(\mu_F - 4\sigma_F > F_{\max}\right) = 6.2 \cdot 10^{-5}$$

We see that the smaller the force F is, the smaller is the probability this force being larger than $F_{\max}$.

To compute the factor *n* for a desired failure probability P_{dsr} you can use the following procedure in MATLAB using the inverse cumulative function:

$$F_{dsr} = icdf('normal', P_{dsr}, \mu_F, \sigma_F)$$
$$n = \frac{\mu_F - F_{dsr}}{\sigma_F}$$

Considering $F = F_{design}$ we can now express the FOS as a function of Probability of failure, $P\left(F_{design} > F_{\max}\right)$, that is the probability of the design load F_{design} being larger than $F_{\max}$.

$$SOF = \frac{\mu_F}{\mu_F - n\sigma_F} = \frac{1}{1 - n \underbrace{\left(\sigma_F / \mu_F\right)}_{\text{coefficient of variation}}}$$

Procedure:

1. Pick a value for the desired probability of failure.
2. Determine the associated multiplier *n*
3. Compute the FOS.

Example:

Assume that $F_{\max} = N\left(5, 0.5\right)$.

1. set desired Probability of Failure $P\left(F_{design} > F_{\max}\right) = 0.01$
2. determine via lookup table for Gaussian distribution or calculation: $n = 3.84$
3. compute FOS:

$$FOS = \frac{\mu_F}{\mu_F - n\sigma_F} = \frac{1}{1 - 3.84(0.5/5.0)} = 1.62$$

2. Step

Using the above FOS, you can develop a nominal design which satisfies the following condition:

$$\max\left(\left|F_{bar}\right|\right) \le F_{design}$$

with $F_{design} = \dfrac{\mu_F}{FOS}$

$\max\left(\left|F_{bar}\right|\right)$: maximum bar force magnitude in truss.

3. Step

So far we have neglected to account for the fact that the forces in the bars are also uncertain given that the as-built design always deviates from the nominal design. To compute the true probability of failure we have to consider both the uncertainty in joint strength and the uncertainty in the truss design.

Let's assume that, due to manufacturing errors, the as-built design only deviates from the nominal design in the position of the joints $\mathbf{x}_j$. We further assume that the joint position can be described via Gaussian distribution:

$$\mathbf{x}_j = N\left(\mu_x^j, \sigma_x^j\right)$$

where μ_x^j is the nominal position of the joint *j* and σ_x^j is the standard deviation. For the sake of simplicity we assume that the stochastic variations in the joint positions are not correlated but the σ_x^j is the same for all joints. The uncertainty in the joint positions results in uncertainty in the bar forces. This relation is described implicitly through the truss model.

To compute the *true* probability of failure, $P\left(\max\left(\left|F_{bar}\right|\right) > F_{\max}\right)$, we need to sample the bar forces for a representative set of joint positions. We can do this via a so-called Monte Carlo simulation in which (a) the bar forces are computed for a set of random joint position via the truss analysis tool you have developed and (b) the maximum bar force is compared against a random joint strength. An example MATLAB code for a Monte Carlo simulation is posted on D2L: truss2dmcs.m.

Counting the number of samples where the $\max\left(\left|F_{bar}\right|\right)$ exceeds $F_{\max}$ relative to the total number of samples yields the *true* probability of failure. If the true probability of failure is much larger than your desired one you may have to increase your FOS by decreasing the failure probabilities in Step 1 and redesign your truss.