Thermistor Error Analysis

Since the thermistors are our reference system and provide truth data, they need to be well characterized so that we can be confident in the data we receive. The thermistors are arranged in a resistor divider system with an $18k\Omega$ pullup resistor denoted as R_1. The voltage measurement is taken at the junction between the pullup resistor and the thermistor.

Error sources and propagation to R_{th}

The resistor divider equation gives the equation for voltage measured (V_{in}) as a function of supply voltage (V_{supply}) and temperature (T):

$$V_{in} = V_{supply} \frac{R_{th}}{R_{th} + R_1} \tag{1}$$

Where $R_{th} = f(T)$.
Rearranging for R_{th}:

$$R_{th} = \frac{R_1}{V_{in}/V_{supply} - 1} \tag{2}$$

Now we can solve for the thermistor temperature as a function of the measured voltage using the thermistor equation:

$$\frac{1}{T} = \frac{1}{T_{25}} + \frac{1}{B_{25}} ln(\frac{R_{th}}{R_{25}}) \tag{3}$$

Where $T_{25} = 25°C = 298.15K$, R_{25} is the thermistor's resistance at $25°C$, equivalent to $10k\Omega$, and B_{25} is an empirical parameter supplied by the thermistor's manufacturer.

While we can take T_{25} as a precise measurement, R_{25}, B_{25}, and R_{th} have associated uncertainty. Error in the temperature measurement creeps in from these sources, including noise in the supply voltage, additional noise on the input line, deviations in the thermistor response, and imperfections in R_1, which is only constructed to some certain tolerance (we have selected 1% precision resistors here).

Because R_{th} is a derived parameter, the uncertainty in a measurement of R_{th} is the root-square sum of the sensitivity of R_{th} to a given parameter times the error in that parameter.

$$U_{R_{th}} = \sqrt{\sum_x (\frac{\partial R_{th}}{\partial x} \delta x)^2} \tag{4}$$

Partial derivatives of equation 2 with respect to its input parameters:

$$\frac{\partial R_{th}}{\partial V_{in}} = -\frac{R_1}{V_{supply}(V_{in}/V_{supply} - 1)^2} \tag{5}$$

$$\frac{\partial R_{th}}{\partial V_{supply}} = \frac{R_1 V_{in}}{V_{supply}^2 (V_{in}/V_{supply} - 1)^2} \tag{6}$$

$$\frac{\partial R_{th}}{\partial R_1} = \frac{1}{V_{in}/V_{supply} - 1} \tag{7}$$

Each of these parameters has an associated error that either comes from biases (constant offsets) and random phenomena that need to be understood.

V_{in} has some bias error from the ADC that measures its value, about 0.4% of the reading. Is also vulnerable to noise. We estimate that the noise is about $1mV$.

The error in a direct measurement of V_{in} is:

$$\delta V_{in} = \frac{t}{\sqrt{N}} \sqrt{\sum_i (\delta S_i)^2} + \sqrt{\sum_j (\delta B_j)^2} \tag{8}$$

Where δS errors are precision errors and δB errors are bias errors. N is the number of measurements being taken and t comes from Student's t-distribution table for a given number of measurements.

N	t
1	12.71
3	3.182
10	2.228
100	1.984

Table 1: t distribution vs number of samples N for 95% confidence

δR_1 is much easier. Since we have 1% precision resistors with no randomness:

$$\delta R_1 = 1\% R_1 = 1\% 18k\Omega = 180\Omega \tag{9}$$

We'll also assume that δV_{supply} also has a random error of $1mV$ and no bias error, since it is easy to take a measurement of V_{supply} that is accurate to less than $1mV$.

In addition, the thermistor has its own error, δR_{th}. Nominally, this is only 1% of the thermistor's value but it increases as temperature moves away from $25°C$.

Total uncertainty in R_{th} is the root-square sum of all these terms.

$$U_{R_{th}} = \sqrt{(\delta R_{th})^2 + (\frac{\partial R_{th}}{\partial V_{in}} \delta V_{in})^2 + (\frac{\partial R_{th}}{\partial V_{supply}} \delta V_{supply})^2 + (\frac{\partial R_{th}}{\partial R_1} \delta R_1)^2} \tag{10}$$

Error propagation to T

Equation 3 is repeated here for clarity:

$$\frac{1}{T} = \frac{1}{T_{25}} + \frac{1}{B_{25}} ln(\frac{R_{th}}{R_{25}}) \tag{11}$$

Raising both sides to -1 to find a direct equation for T:

$$T = [\frac{1}{T_{25}} + \frac{1}{B_{25}} ln(\frac{R_{th}}{R_{25}})]^{-1} \tag{12}$$

Again, T is a derived parameter and the uncertainty in it depends on its sensitivity to its parameters, R_{th}, R_{25}, and B_{25}.

$$\frac{\partial T}{\partial B_{25}} = \frac{ln(R_{th}/R_{25})}{B_{25}^2(\frac{1}{T_{25}} + \frac{ln(R_{th}/R_{25})}{B_{25}})^2} \tag{13}$$

$$\frac{\partial T}{\partial R_{25}} = \frac{1}{B_{25}R_{25}(\frac{1}{T_{25}} + \frac{ln(R_{th}/R_{25})}{B_{25}})^2} \tag{14}$$

$$\frac{\partial T}{\partial R_{th}} = \frac{-1}{B_{25}R_{th}(\frac{1}{T_{25}} + \frac{ln(R_{th}/R_{25})}{B_{25}})^2} \tag{15}$$

The uncertainty in R_{th} was described in detail above, but it can be summarized as:

$$U_{R_{th}} = f(R_{th}, V_{in}, V_{supply}, R_1) \tag{16}$$

R_{25} is the thermistor's temperature at 25°. The manufacturer specifies it to have 1% uncertainty here:

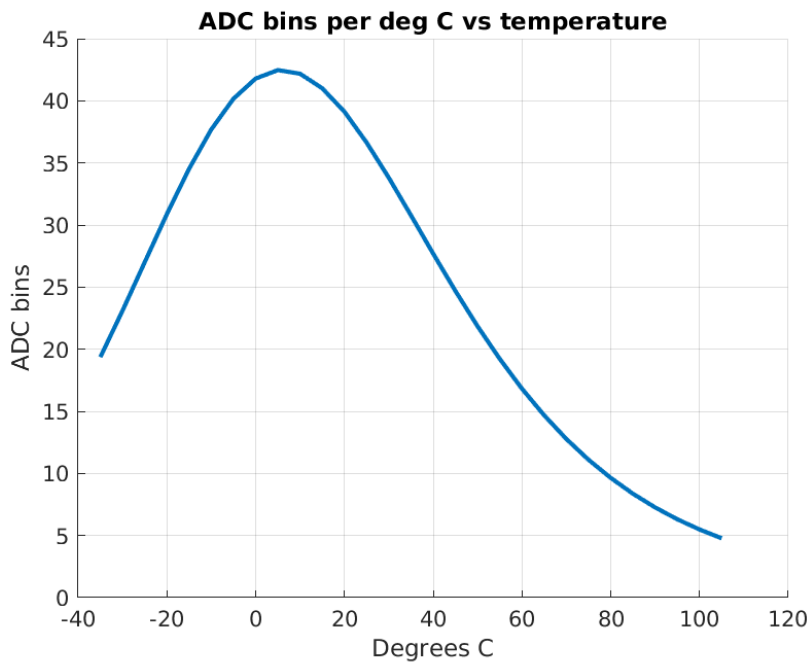
$$\delta R_{25} = 1\% \times 10k\Omega = 100\Omega \tag{17}$$

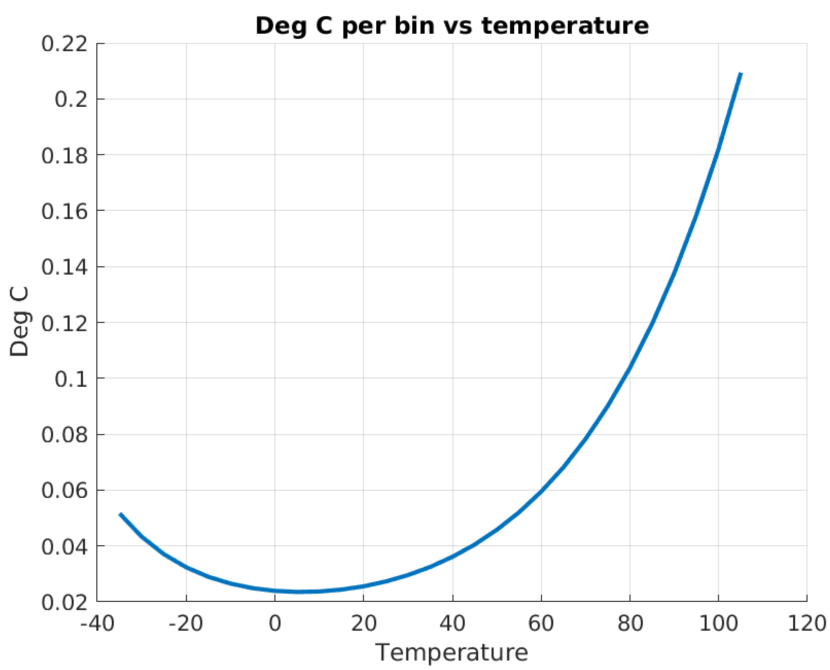
B_{25} is a parameter from the datasheet and the uncertainty is manufacturer specified:

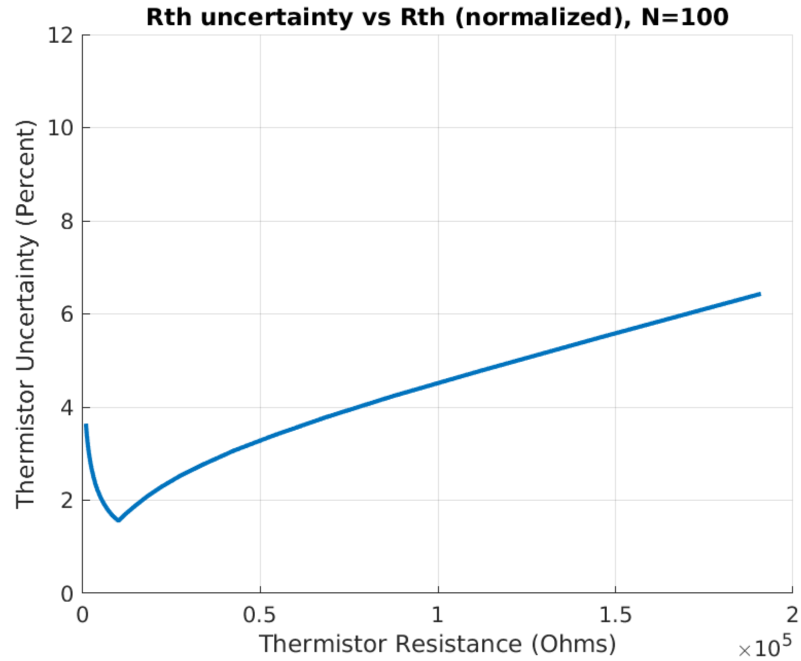
$$\delta B_{25} = 1\% \times 3435K = 34.35K \tag{18}$$

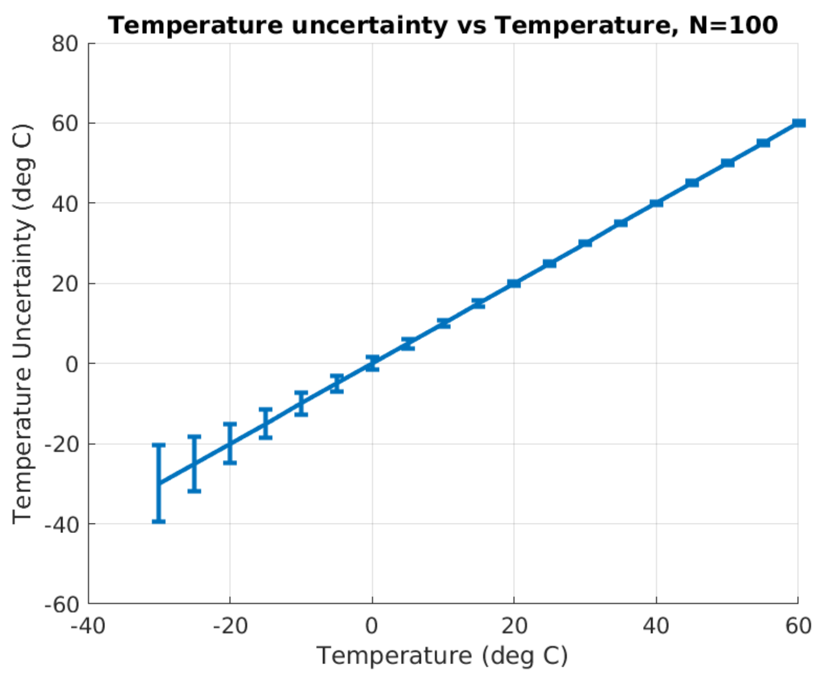
The uncertainty in T is again the root-square sum of these terms:

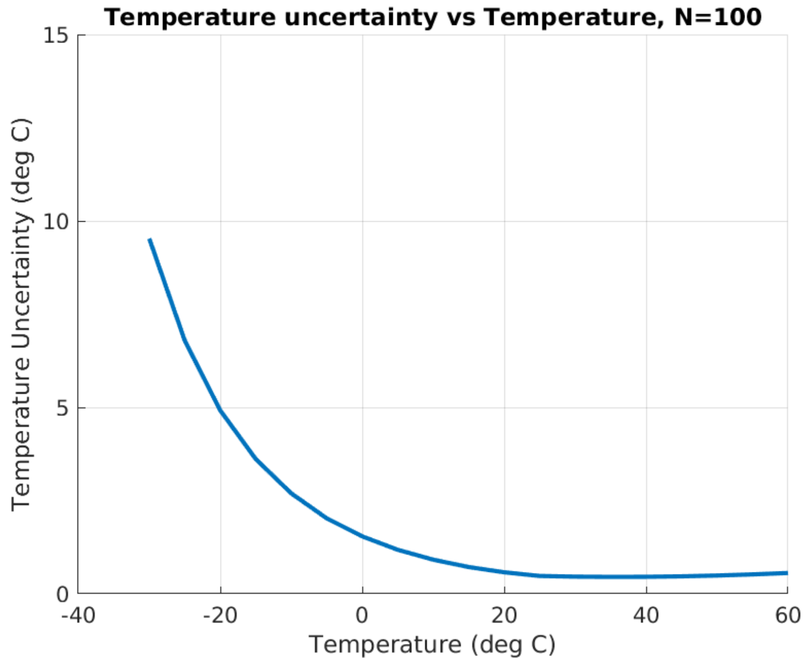
$$\delta T = \sqrt{(\frac{\partial T}{\partial R_{th}}\delta R_{th})^2 + (\frac{\partial T}{\partial R_{25}}\delta R_{25})^2 + (\frac{\partial T}{\partial B_{25}}\delta B_{25})^2} \tag{19}$$

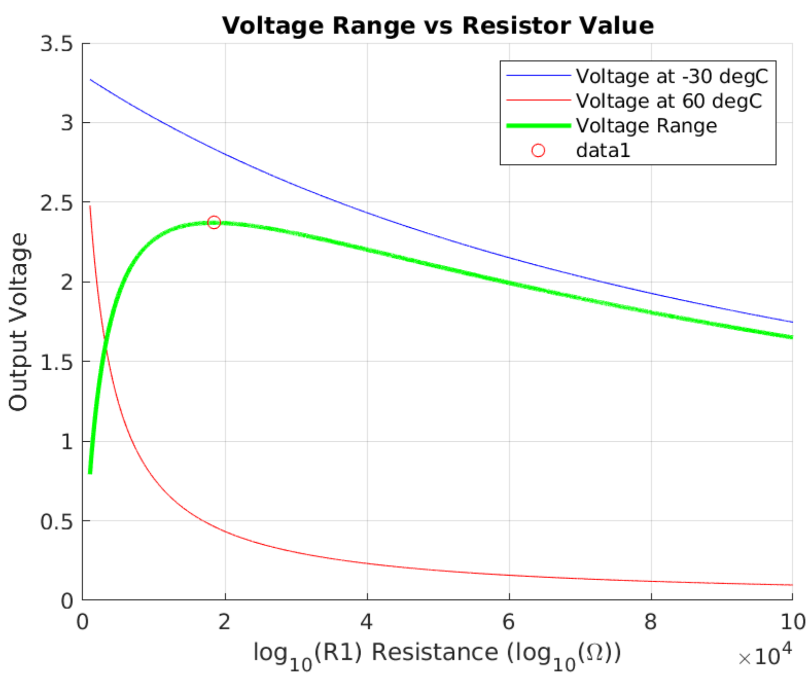

ADC bins per deg C vs temperature
ADC bins
Degrees C
45
40
35
30
25
20
15
10
5
0
-40
-20
0
20
40
60
80
100
120

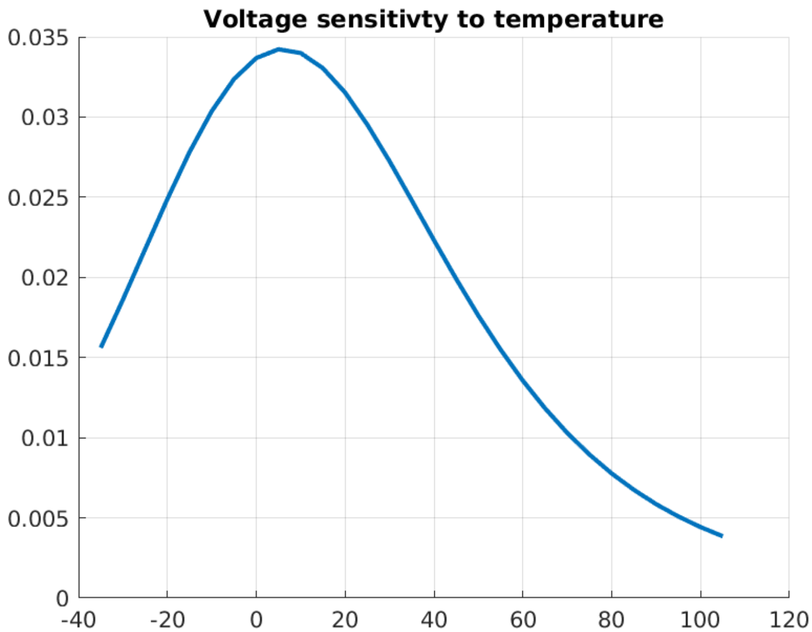

Deg C per bin vs temperature
Deg C
Temperature
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
-40
-20
0
20
40
60
80
100
120


Rth uncertainty vs Rth (normalized), N=100
Thermistor Uncertainty (Percent)
Thermistor Resistance (Ohms)
$\times 10^5$
0
2
4
6
8
10
12
0
0.5
1
1.5
2


Temperature uncertainty vs Temperature, N=100
Temperature Uncertainty (deg C)
Temperature (deg C)
-60
-40
-20
0
20
40
60
80
-40
-20
0
20
40
60


Temperature uncertainty vs Temperature, N=100
Temperature Uncertainty (deg C)
Temperature (deg C)
15
10
5
0
-40
-20
0
20
40
60


Voltage Range vs Resistor Value
Voltage at -30 degC
Voltage at 60 degC
Voltage Range
data1
Output Voltage
log10(R1) Resistance (log10(Ω))
×10^4
3.5
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10


Voltage sensitivty to temperature
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-40
-20
0
20
40
60
80
100
120