AI HW3: Knapsack Hill Climbing Local Search

1. [5 pts.] Suppose at some iteration of simple hill climbing the current state is {A,E}.

What is the best neighbor of the state {A,E}?

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🧺 Current State = {A,E}
➕ Add C  : h({A,C,E}) = 2  # ✅ best neighbor
➕ Add D  : h({A,D,E}) = 2
➕ Add B  : h({A,B,E}) = 3
🔄 Swap E C: h({A,C}) = 4
🔄 Swap E B: h({A,B}) = 4
🔄 Swap A C: h({C,E}) = 9
🔄 Swap A D: h({D,E}) = 10
➖ Del A  : h({E}) = 16
🔄 Swap A B: h({B,E}) = 8
➖ Del E  : h({A}) = 10
🔄 Swap E D: h({A,D}) = 5
✨ New State = {A,C,E}
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What happens on the next iteration?

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🧺 Current State = {A,C,E}
🔄 Swap A B: h({C,E,B}) = 1  # ✅ best neighbor
🔄 Swap C D: h({A,E,D}) = 2
🔄 Swap A D: h({C,E,D}) = 3
🔄 Swap C B: h({A,E,B}) = 3
🔄 Swap E D: h({A,C,D}) = 4
➖ Del A  : h({C,E}) = 9
➖ Del E  : h({A,C}) = 4
➖ Del C  : h({A,E}) = 6
➕ Add D  : h({A,C,E,D}) = 5
➕ Add B  : h({A,C,E,B}) = 6
🔄 Swap E B: h({A,C,B}) = 5
✨ New State = {C,E,B}
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🧺 Current State = {C,E,B}
🔄 Swap E D: h({C,B,D}) = 0  # ✅ best neighbor
➕ Add D  : h({C,B,E,D}) = 1
🔄 Swap C D: h({B,E,D}) = 2
🔄 Swap E A: h({C,B,A}) = 5
🔄 Swap B A: h({C,E,A}) = 2
➖ Del C  : h({B,E}) = 8
🔄 Swap C A: h({B,E,A}) = 3
➕ Add A  : h({C,B,E,A}) = 6
➖ Del E  : h({C,B}) = 5
➖ Del B  : h({C,E}) = 9
🔄 Swap B D: h({C,E,D}) = 3
✨ New State = {C,B,D}

🎯 Solution: {C,B,D}
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2. [5 pts.] Consider now the general case where there are N objects.

What is the size of the state space?

$$\text{size of state space} = \sum_{k=1}^{N} C(N,k) = \sum_{k=1}^{N} \frac{N!}{k!(N-k)!} = \boxed{2^N - 1}$$

What is maximal number of neighbors of any state?

Let x be the number of objects in the knapsack

$$\begin{aligned} n_{\text{neighbors}} &= n_{\text{add}} + n_{\text{delete}} + n_{\text{swap}} \\ &= (N - x) + x + x(N - x) \\ &= N + xN - x^2 \end{aligned}$$

$$\max\{n_{\text{neighbors}}\} = \max\{N + xN - x^2\} = \boxed{\frac{N(N+4)}{4}} \quad \text{at} \quad x = \tfrac{N}{2}$$