Introduction to Machine Learning (CSCI-UA.473): Homework 0

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September 2, 2021

Submission Instructions

You must typeset the answers using LATEX and compile them into a single PDF file. Name the pdf file as: ⟨Your-NetID⟩_hw0.pdf. Submit the file on Brightspace. The due date is **September 13th, 2021, 11:59 PM**.

Questions

Probability and Calculus

Question 1 (10 points)

Two people take turns trying to sink a basketball into a net. Person 1 succeeds with probability 1/3 and Person 2 succeeds with the probability 1/4. Whoever succeeds first wins the game and the game is over. Assuming that Person 1 takes the first shot, what is the probability that Person 1 wins the game? Please derive your answer.

Question 2 (10 points)

You know that 1% of the population have COVID. You also know that 90% of the people who have COVID get a positive test result and 10% of people who do not have COVID also test positive. What is the probability that you have COVID given that you tested positive?

Question 3 (10 points)

Let the function $f(x)$ be defined as:

$$f(x) = \begin{cases} 0 & for\ x < 0 \\ \frac{1}{(1+x)} & otherwise. \end{cases} \tag{1}$$

Is $f(x)$ a PDF? If yes, then prove that it is a PDF. If no, then prove that it is not a PDF.

Question 4 (10 points)

Assume that X and Y are two independent random variables and both have the same density function:

$$f(x) = \begin{cases} 2x & if \quad\quad\quad 0 \leq x \leq 1 \\ 0 & otherwise \end{cases} \quad (2)$$

What is the value of $\mathbb{P}(X + Y \leq 1)$?

Question 5 (10 points)

Let X be a random variable which belongs to a Uniform distribution between 0 and 1: $X \sim Unif(0, 1)$. Let $Y = g(X) = e^X$. What is the value of $\mathbb{E}(Y)$?

Question 6 (10 points)

Suppose that the number of errors per computer program has a Poisson distribution with mean 5. We have 125 program submissions. Let $X_1, X_2, \ldots, X_{125}$ denote the number of errors in the programs. What is the value of $\mathbb{P}(\bar{X}_n < 5.5)$?

Question 7 (10 points)

Let $X_n = f(W_n, X_{n-1})$ for $n = 1, \ldots, P$, for some function $f()$. Let us define the value of variable E as

$$E = ||C - X_P||^2, \quad (3)$$

for some constant C. What is the value of the gradient $\frac{\partial E}{\partial X_0}$?

Question 8 (10 points)

Suppose

$$f(x, y) = \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} + \sqrt{\sqrt{x+y} - \sqrt{x-y}}. \quad (4)$$

What is the value of the expression

$$2y\frac{\partial^2 f}{\partial x^2} + 4x\frac{\partial^2 f}{\partial x \partial y} + 2y\frac{\partial^2 f}{\partial y^2} + 2\frac{\partial f}{\partial y} \quad (5)$$

at the point where $x = 5$ and $y = 4$?

Linear Algebra

Question 9 (10 points)

What is an Eigen value of a matrix? What is an Eigen Vector of a matrix? Describe one method (any method) you would use to compute both of them. Use the above described method to compute the Eigen Values of the matrix:

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & 0 \\ -2 & 2 & 1 \end{bmatrix} \quad (6)$$

Question 10 (10 points)

Let $X = (x_1, \ldots, x_k)$ for some fixed k, be a random variable whose probability density function is defined as:

$$f(x) = \left(\begin{array}{c} n \\ x_1, \ldots, x_k \end{array} \right) p_1^{x_1}, \ldots, p_k^{x_k}, \tag{7}$$

where

$$\left(\begin{array}{c} n \\ x_1, \ldots, x_k \end{array} \right) = \frac{n!}{x_1!, \ldots x_k!}. \tag{8}$$

Also $p_j \geq 0$ for all $j = \{1, \ldots, k\}$ and $\sum_{j=1}^{k} p_j = 1$. What is the value of $\mathbb{E}(X)$ and $\mathbb{V}(X)$?