CSC 212: Data Structures and Abstractions

Binary Search Trees

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Quick notes

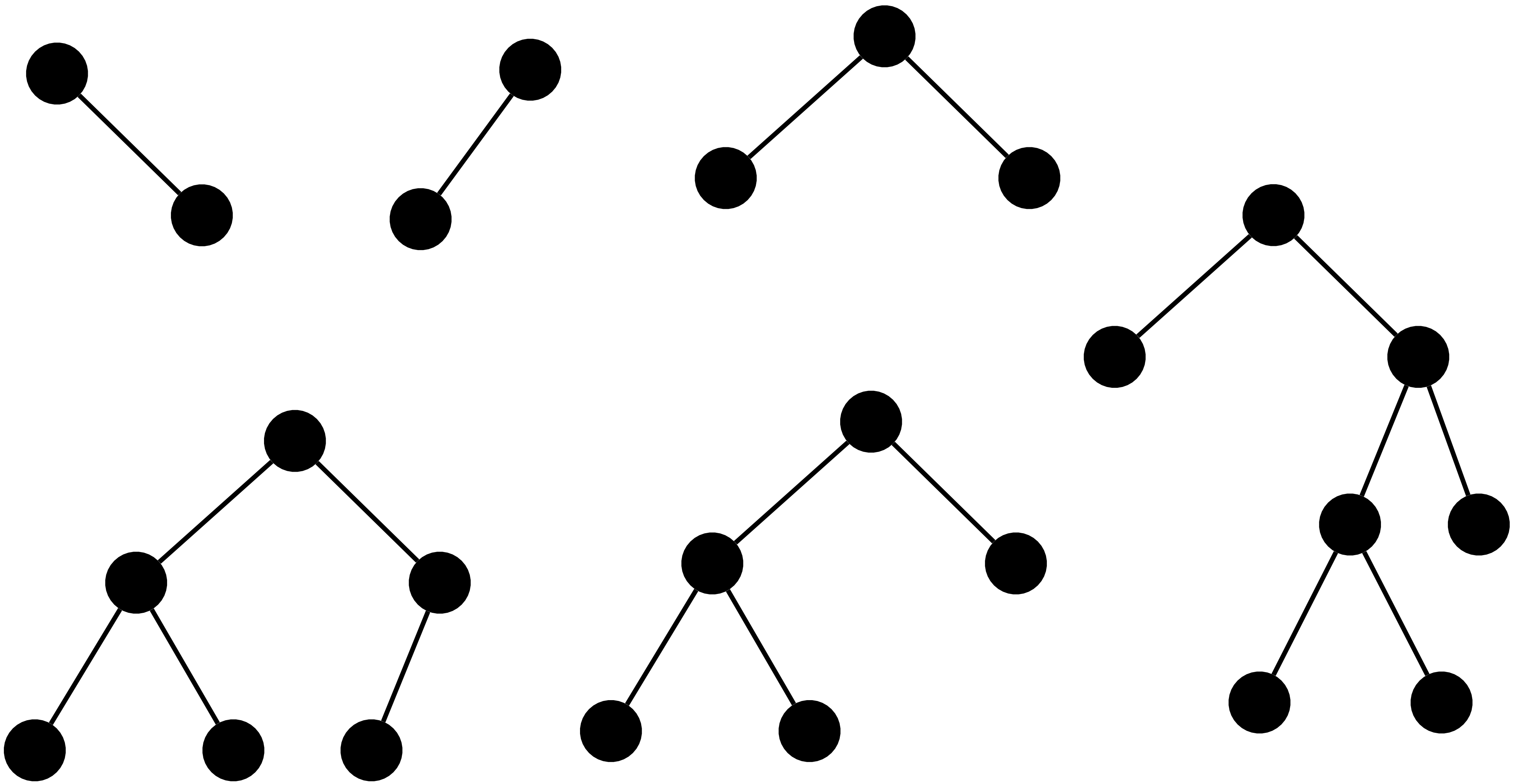
- Final Project (about 5 weeks)
  - requires planning and long coding hours
  - there is a lot to learn
- Team Work
  - motivate each other
  - all team members must understand the topic and code
    - a presentation to the class will follow by the end of the semester

k-ary Trees

k-ary Trees

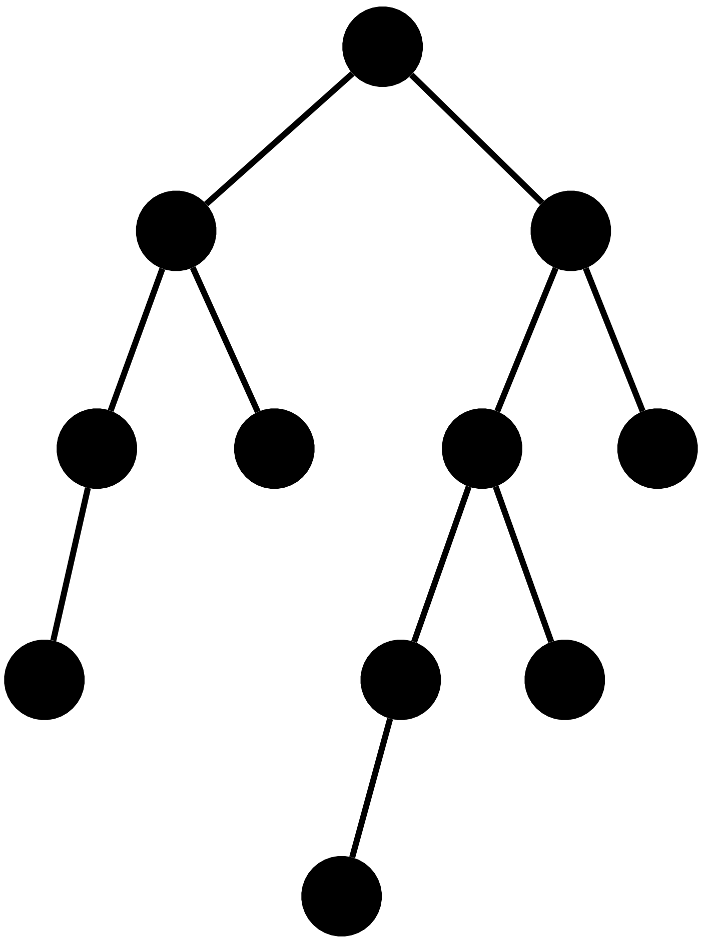
- In a **k-ary tree**, every node has between 0 and k children
- In a **full (proper)** k-ary tree, every node has exactly 0 or k children
- In a **complete** k-ary tree, every level is entirely filled, except possibly the deepest, where all nodes are as far left as possible
- In a **perfect** k-ary tree, every leaf has the same depth and the tree is full

Quiz (k = 2)

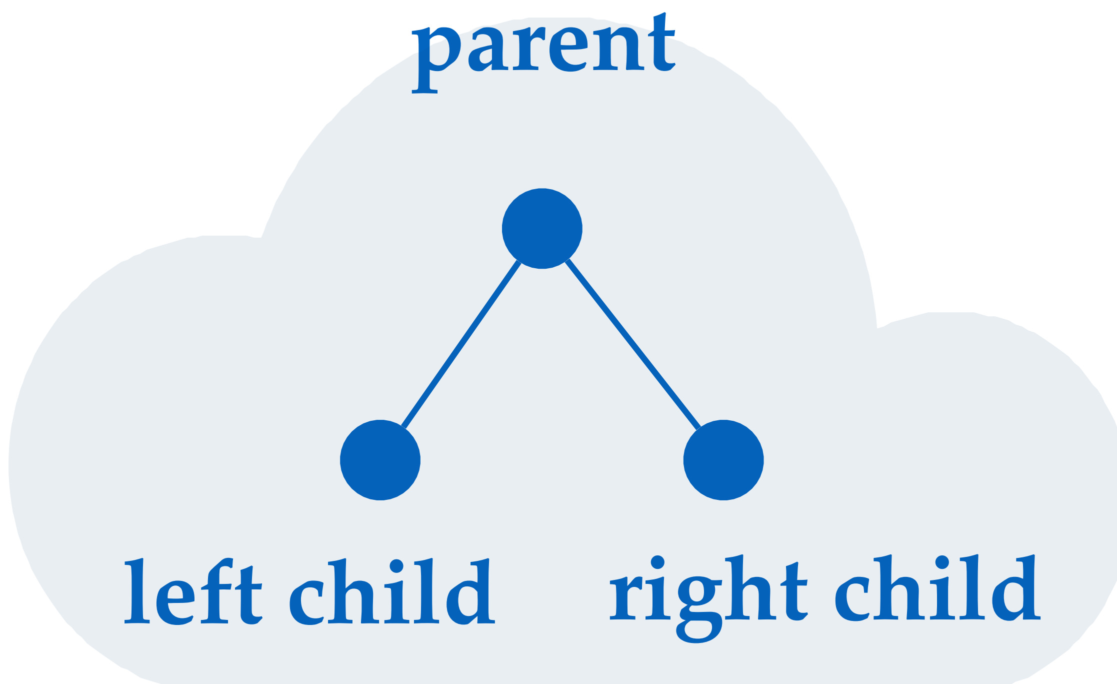


Full? Complete? Perfect?

Binary Tree



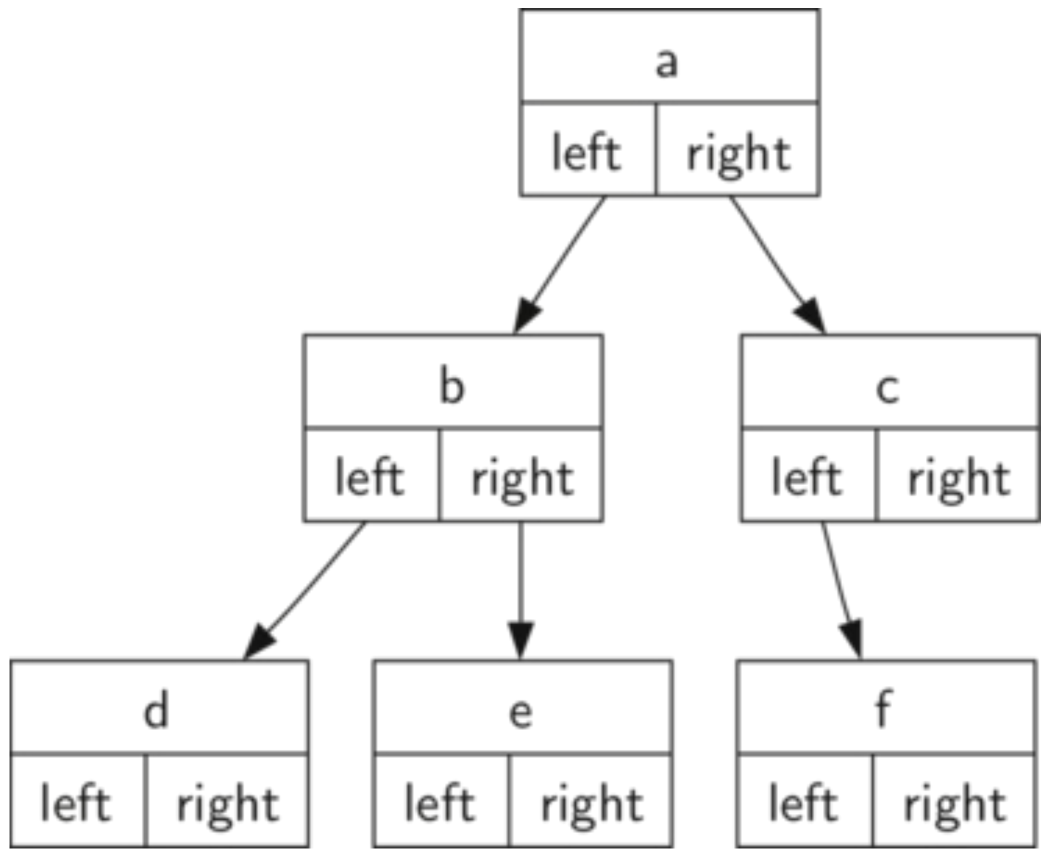
A k-ary tree where **k = 2**



How to implement binary trees?

Node:

data
left child
right child



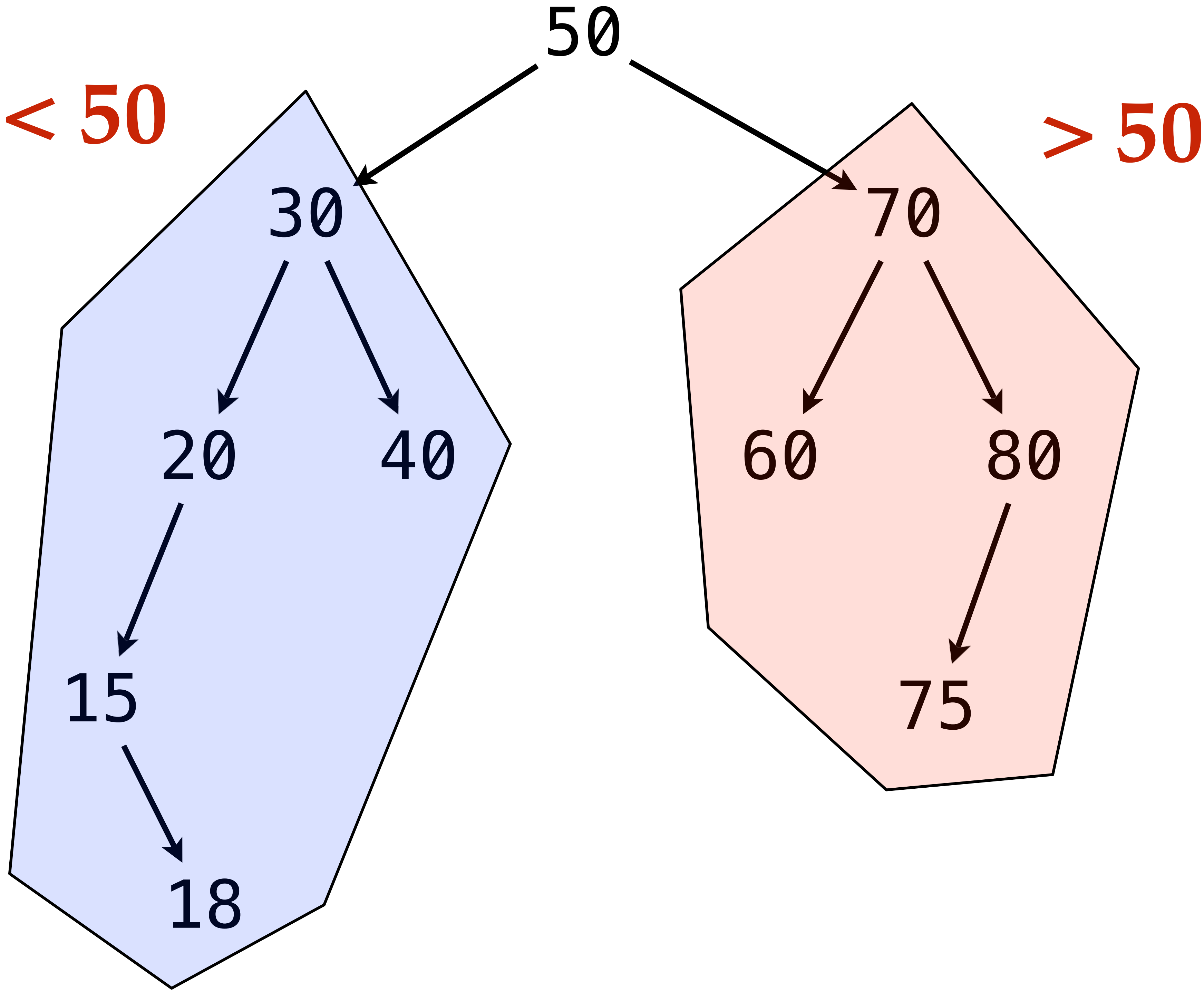
http://interactivepython.org/courselib/static/pythonds/Trees/NodesandReferences.html

Binary Search Trees

Binary Search Tree

- A BST is a binary tree
- A BST has symmetric order
  - each node **x** in a BST has a key **key(x)**
  - for all nodes **y** in the left subtree of **x**, **key(y) < key(x)** **
  - for all nodes **y** in the right subtree of **x**, **key(y) > key(x)** **

(**) assume that the keys of a BST are pairwise distinct


50
< 50
> 50
30
70
20
40
60
80
15
75
18

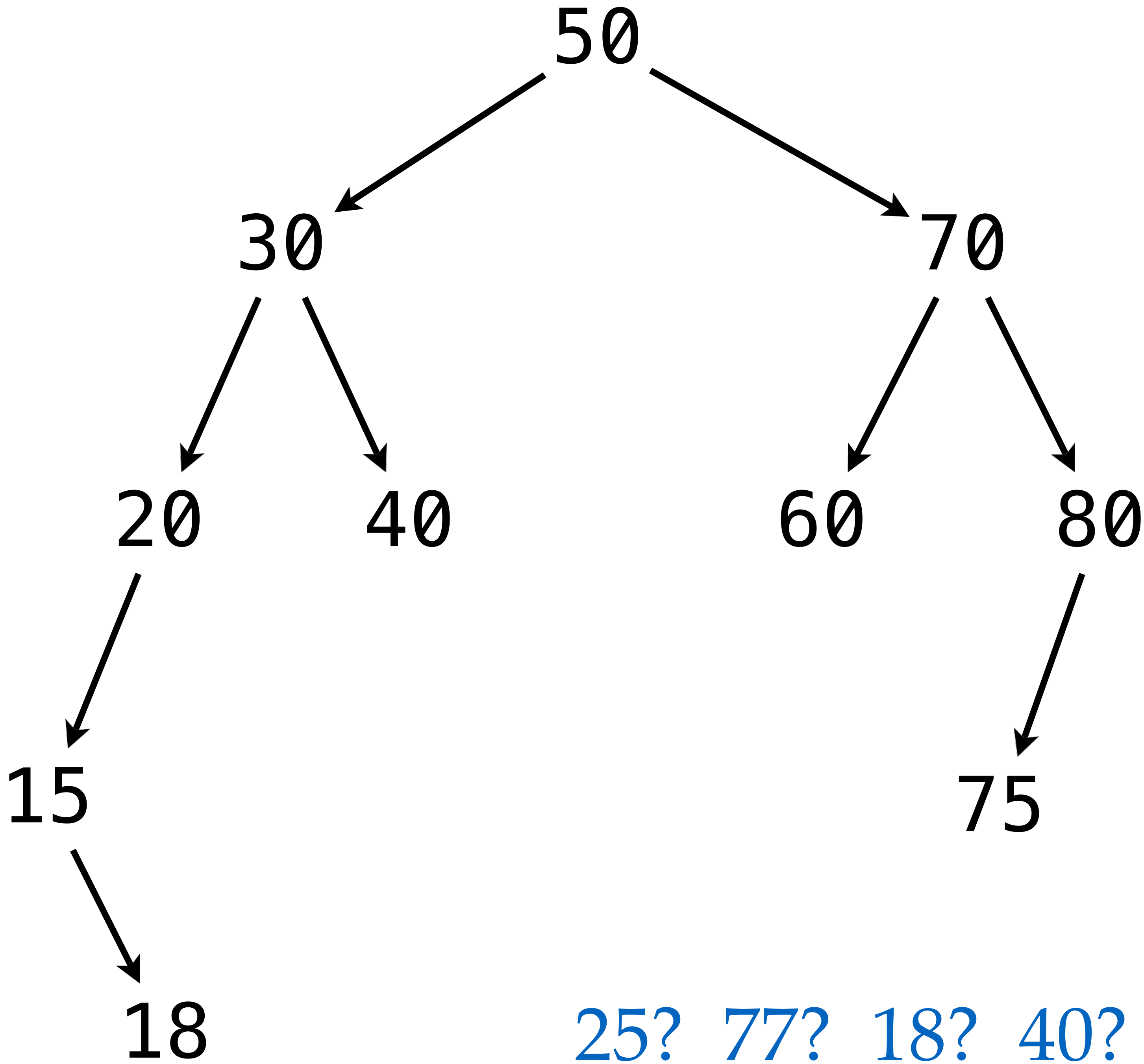
```cpp
class BSTNode {

    private:
        int data;
        BSTNode *left;
        BSTNode *right;

    public:
        BSTNode(int d);
        ~BSTNode();

    friend class BSTree;
};
```

```
class BSTree {

    private:
        BSTNode *root;
        void destroy(BSTNode *p);

    public:
        BSTree();
        ~BSTree();

        void insert(int d);
        void remove(int d);
        BSTNode *search(int d);

};
```

Search into BSTs

Search

- Start at root node
- If the search key matches the current node's key then **found**
- If search key is greater than current node's key
  - search on right child
- If search key is less than current node's
  - search on left child
- Stop when current node is NULL (**not found**)

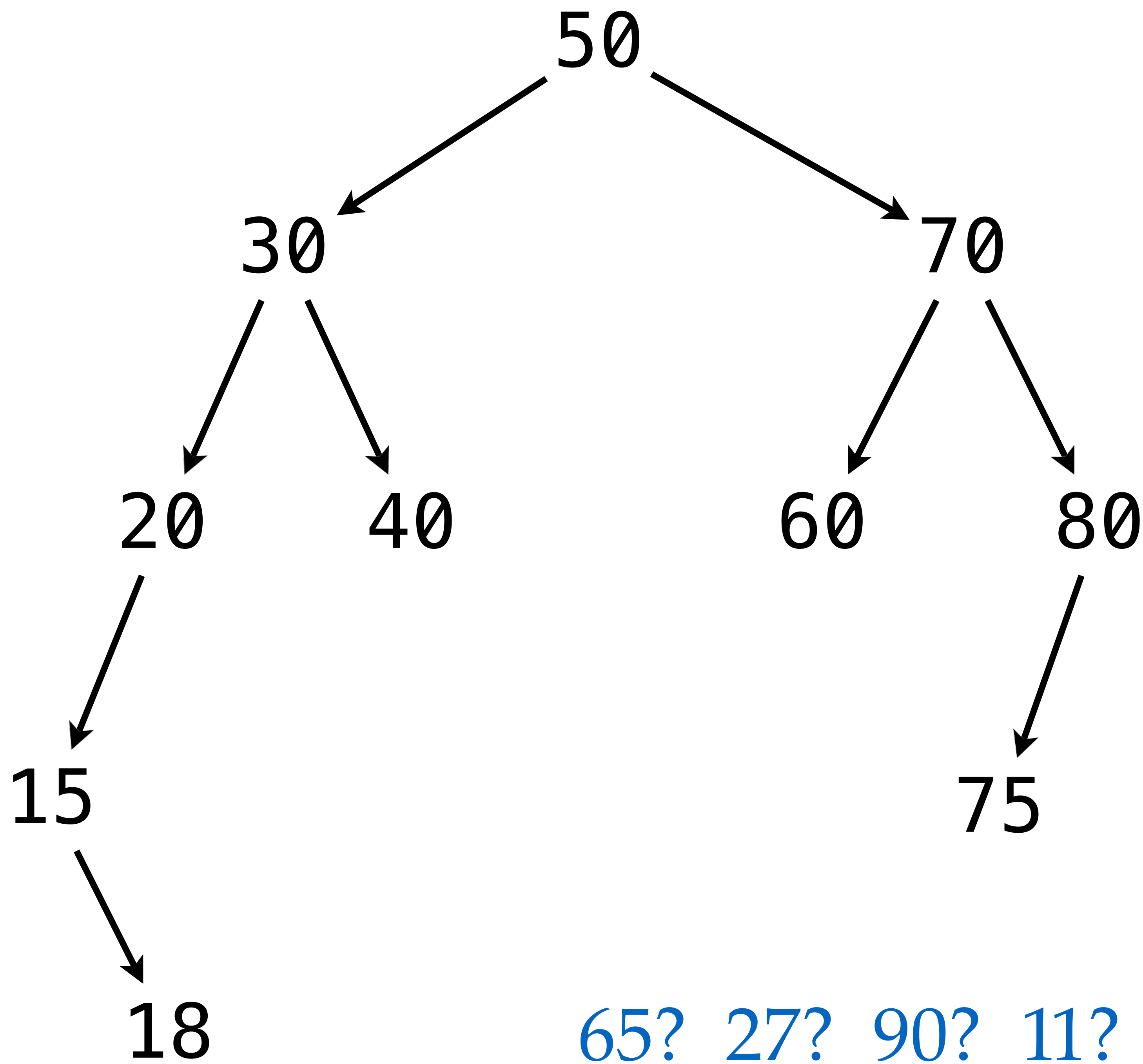


25? 77? 18? 40? 91?

Insert into BSTs

Insert

- Perform a Search operation
- If **found**, no need to insert (may increase counter)
- If **not found**, insert node where Search stopped



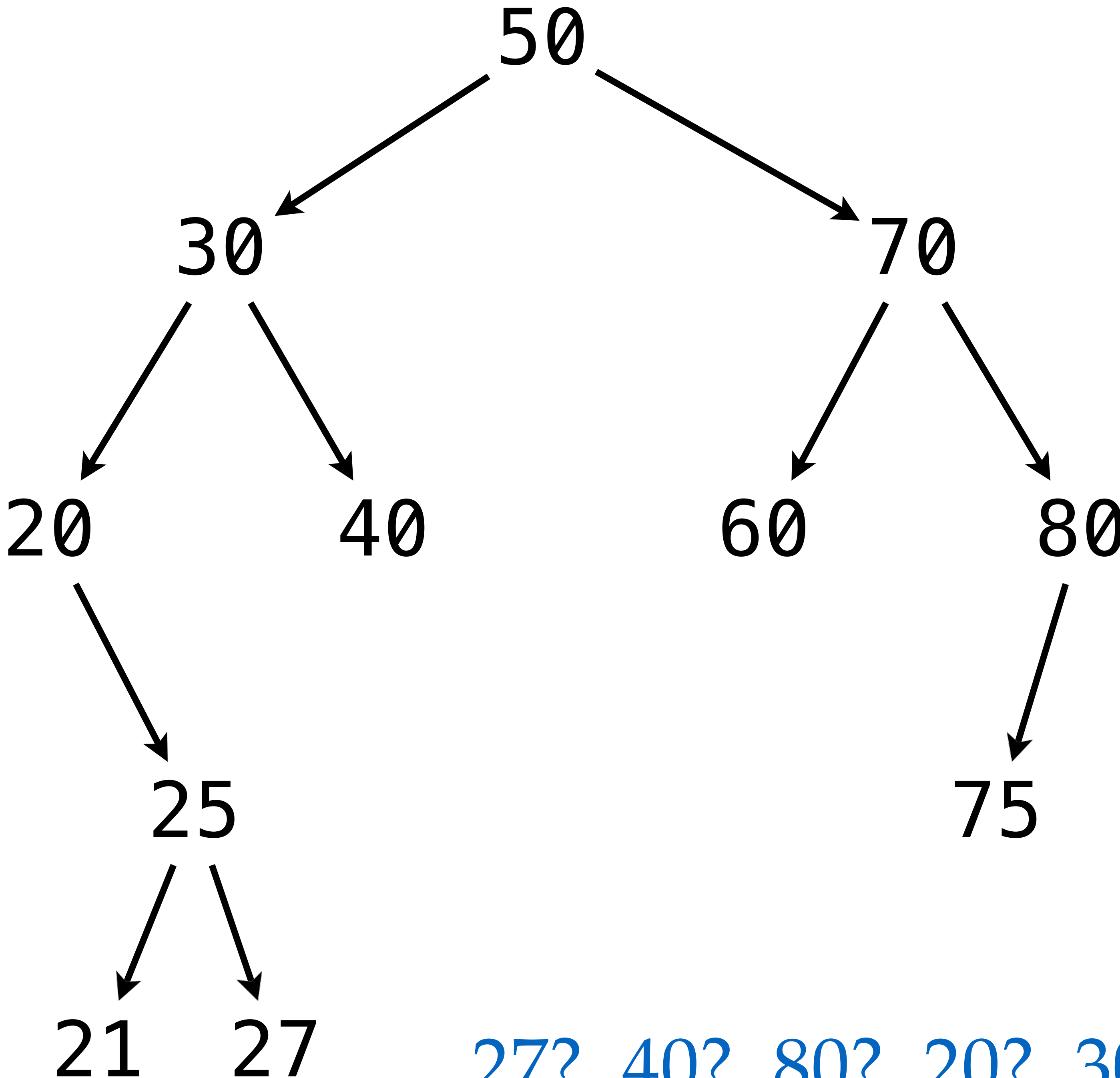
65? 27? 90? 11? 51?

Remove from BSTs

Remove

- **Case 1: node is a leaf**
  - trivial, delete node and set parent's pointer to NULL
- **Case 2: node has 1 child**
  - trivial, set parent's pointer to the only child and delete node
- **Case 3: node has 2 children**
  - find **successor**

    can also use predecessor
  - copy successor's data to node
  - delete successor



27? 40? 80? 20? 30? 50?

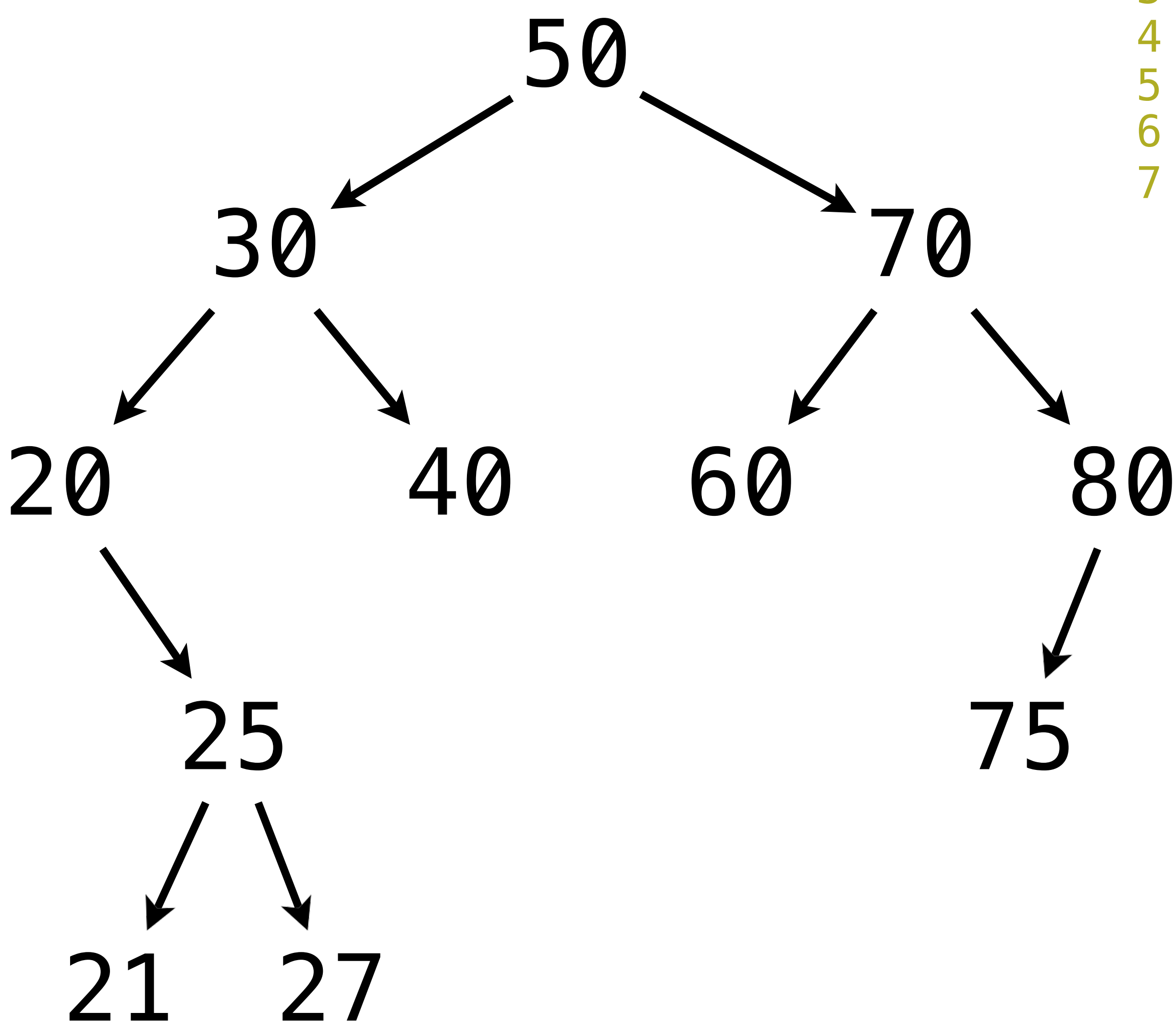
BST Traversals

Traversals

- Preorder traversal
- Inorder traversal
- Postorder traversal

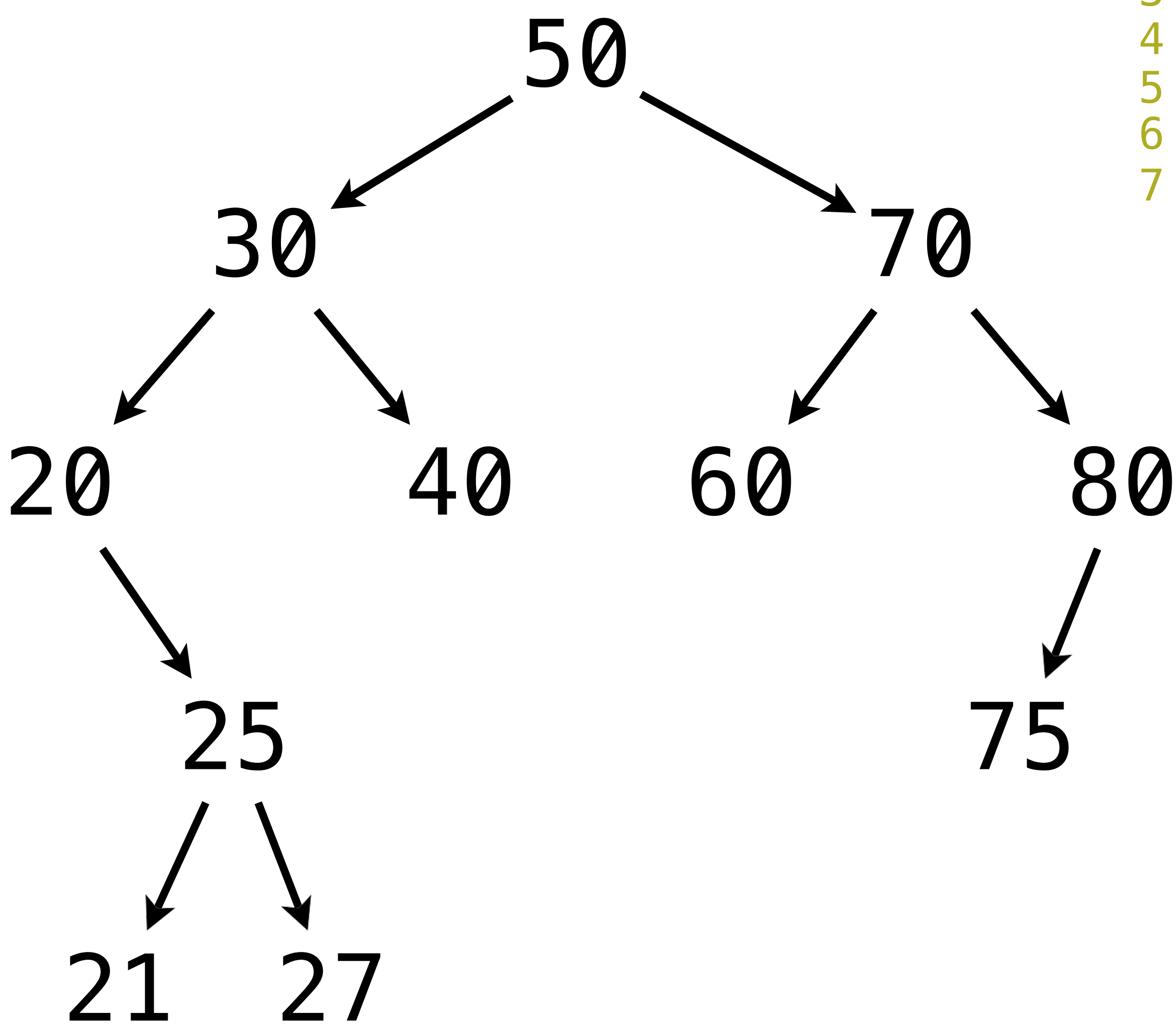
$\Theta(n$

Preorder traversal?



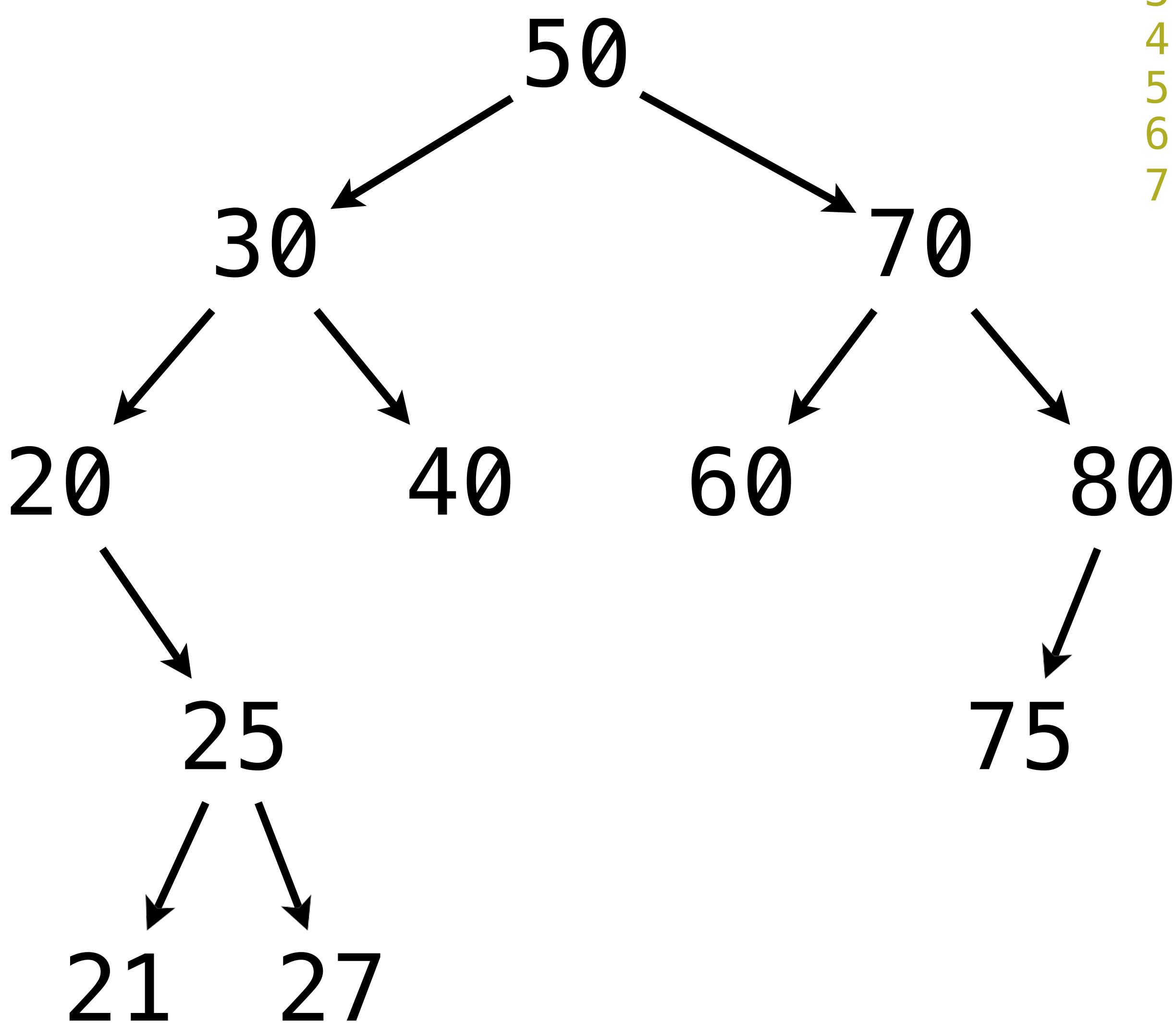

```
algorithm preorder(p) {
    if (p) {
        visit(p)
        preorder(p->left)
        preorder(p->right)
    }
}
```

Postorder traversal?




```
algorithm postorder(p) {
    if (p) {
        postorder(p->left)
        postorder(p->right)
        visit(p)
    }
}
```

Inorder traversal?




```
algorithm inorder(p) {
    if (p) {
        inorder(p->left)
        visit(p)
        inorder(p->right)
    }
}
```

How to destroy a binary tree?

How to print all elements in increasing order?

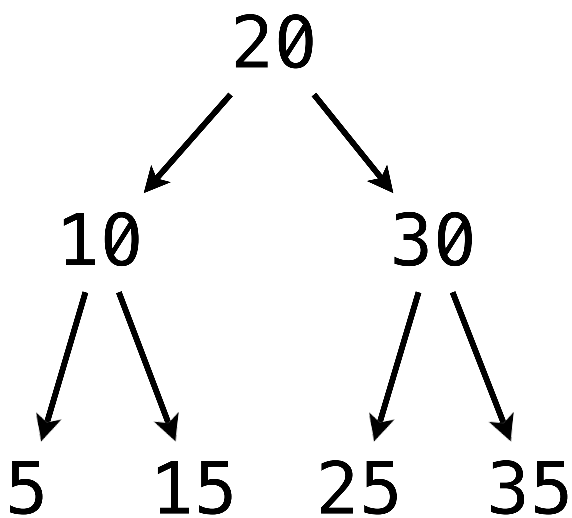
Analysis

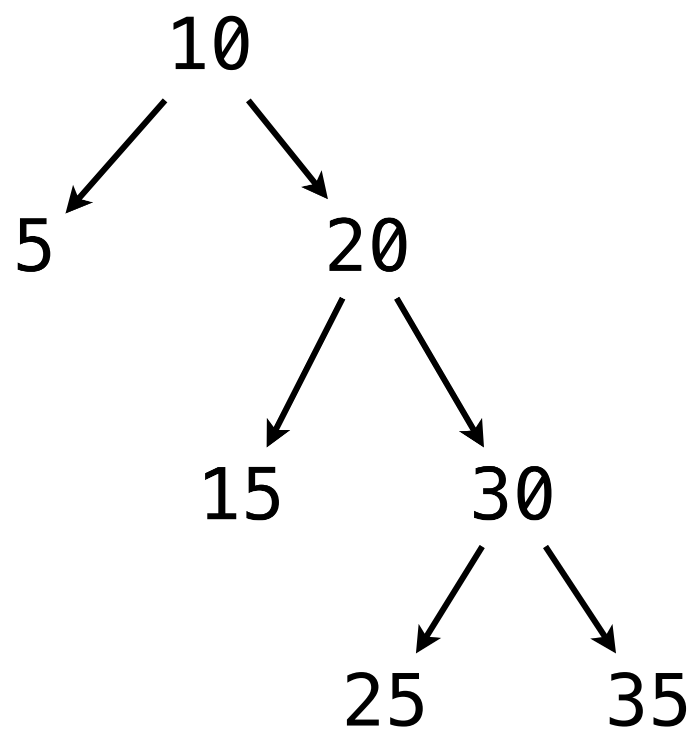
Tree Shape?

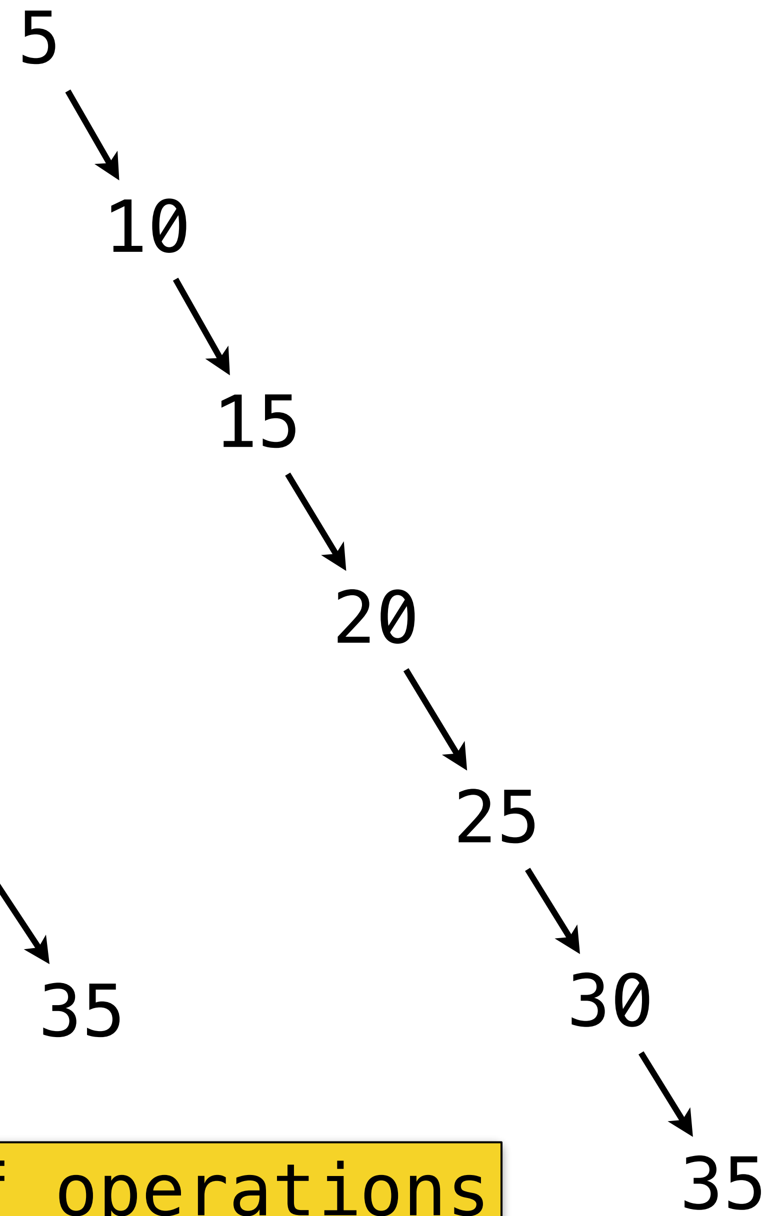
20 10 30 5 15 25 35

10 20 5 15 30 35 25

5 10 15 20 25 30 35







Depends on order of operations

Implications

Cost of basic Operations?

- ✓ Search
- ✓ Insert
- ✓ Remove

Worst-case?

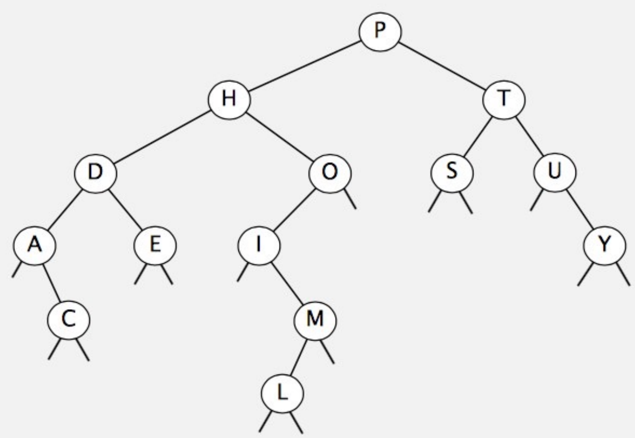
Best-case?

Average-case?

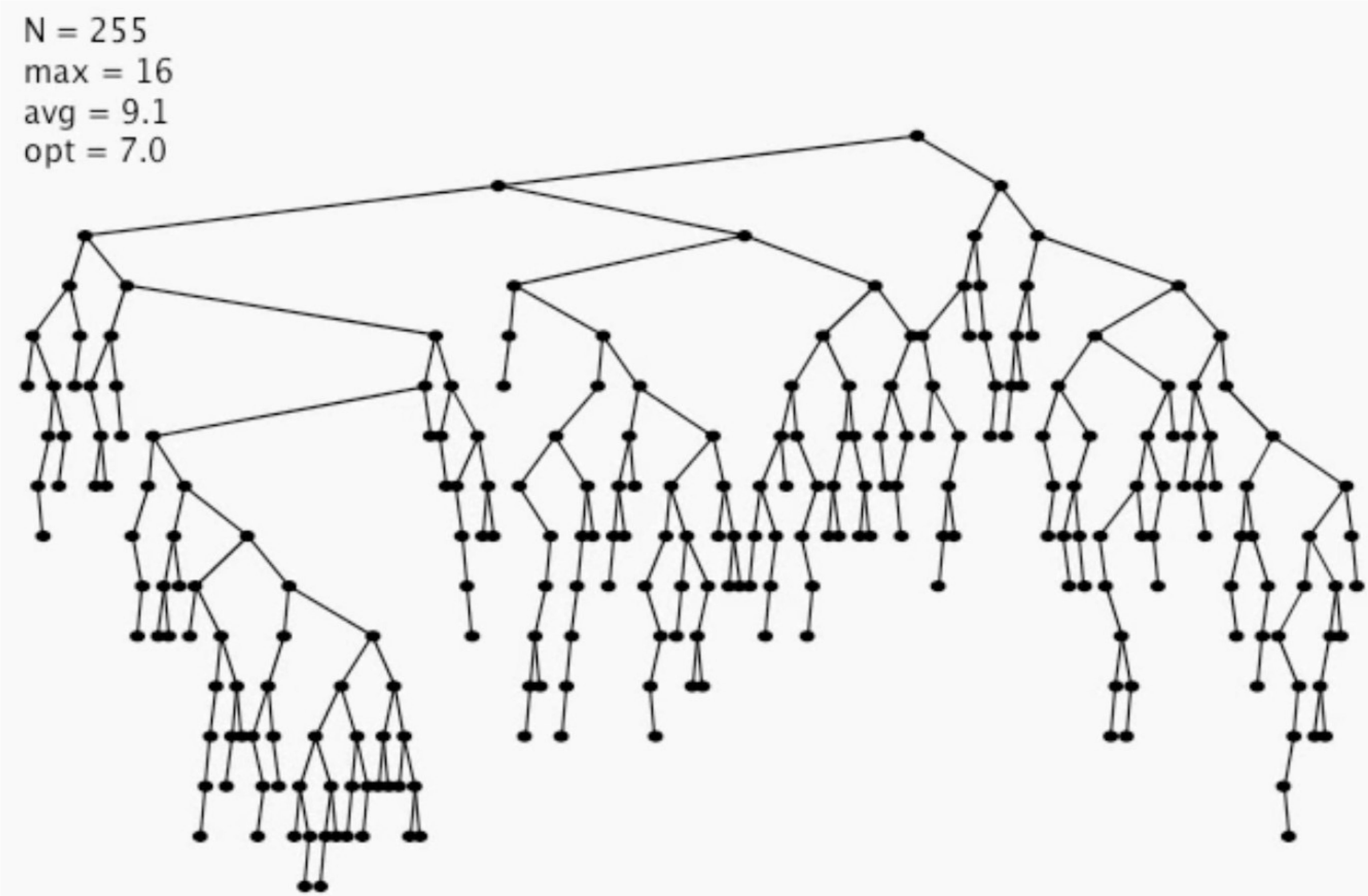
Average-case analysis

- If **n distinct keys** are inserted into a BST in random order, expected number of compares for basic operations is **~2 ln n ~= 1.39 log n**
  - **proof**: 1-1 correspondence with quick-sort partitioning

0	1	2	3	4	5	6	7	8	9	10	11	12	13
P	S	E	U	D	O	M	Y	T	H	I	C	A	L
P	S	E	U	D	O	M	Y	T	H	I	C	A	L
H	L	E	A	D	O	M	C	I	P	T	Y	U	S
D	C	E	A	H	O	M	L	I	P	T	Y	U	S
A	C	D	E	H	O	M	L	I	P	T	Y	U	S
A	C	D	E	H	O	M	L	I	P	T	Y	U	S
A	C	D	E	H	O	M	L	I	P	T	Y	U	S
A	C	D	E	H	O	M	L	I	P	T	Y	U	S
A	C	D	E	H	I	M	L	O	P	T	Y	U	S
A	C	D	E	H	I	M	L	O	P	T	Y	U	S
A	C	D	E	H	I	L	M	O	P	T	Y	U	S
A	C	D	E	H	I	L	M	O	P	T	Y	U	S
A	C	D	E	H	I	L	M	O	P	S	T	U	Y
A	C	D	E	H	I	L	M	O	P	S	T	U	Y
A	C	D	E	H	I	L	M	O	P	S	T	U	Y
A	C	D	E	H	I	L	M	O	P	S	T	U	Y
A	C	D	E	H	I	L	M	O	P	S	T	U	Y



$h = O(\log n)$


N = 255
max = 16
avg = 9.1
opt = 7.0

Collections / Dictionaries

	What?	Sequential (unordered)	Sequential (ordered)	BST
`search`	search for a key	`O(n)`	`O(log n)`	`O(h)`
`insert`	insert a key	`O(n)`	`O(n)`	`O(h)`
`delete`	delete a key	`O(n)`	`O(n)`	`O(h)`
`min/max`	smallest / largest key	`O(n)`	`O(1)`	`O(h)`
`floor/ceiling`	predecessor / successor	`O(n)`	`O(log n)`	`O(h)`
`rank`	number of keys less than key	`O(n)`	`O(log n)`	`O(h)`**

(**) requires the use of ‘size’ at every node