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3.3 BALANCED SEARCH TREES

- *2–3 search trees*
- *red–black BSTs*
- *B-trees (see book or videos)*

Symbol table review

implementation	guarantee			average case			ordered ops?	key interface
	search	insert	delete	search	insert	delete		
sequential search (unordered list)	n	n	n	n	n	n		`equals()`
binary search (ordered array)	$\log n$	n	n	$\log n$	n	n	✔	`compareTo()`
BST	n	n	n	$\log n$	$\log n$	$\sqrt{n}$	✔	`compareTo()`
goal	$\log n$	$\log n$	$\log n$	$\log n$	$\log n$	$\log n$	✔	`compareTo()`

Challenge. Guarantee performance.

optimized for teaching and coding; introduced to the world in COS 226!

This lecture. 2–3 trees and left-leaning red–black BSTs.

co-invented by Bob Sedgewick

Algorithms

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3.3 BALANCED SEARCH TREES

- *2–3 search trees*
- *red–black BSTs*
- *B-trees*

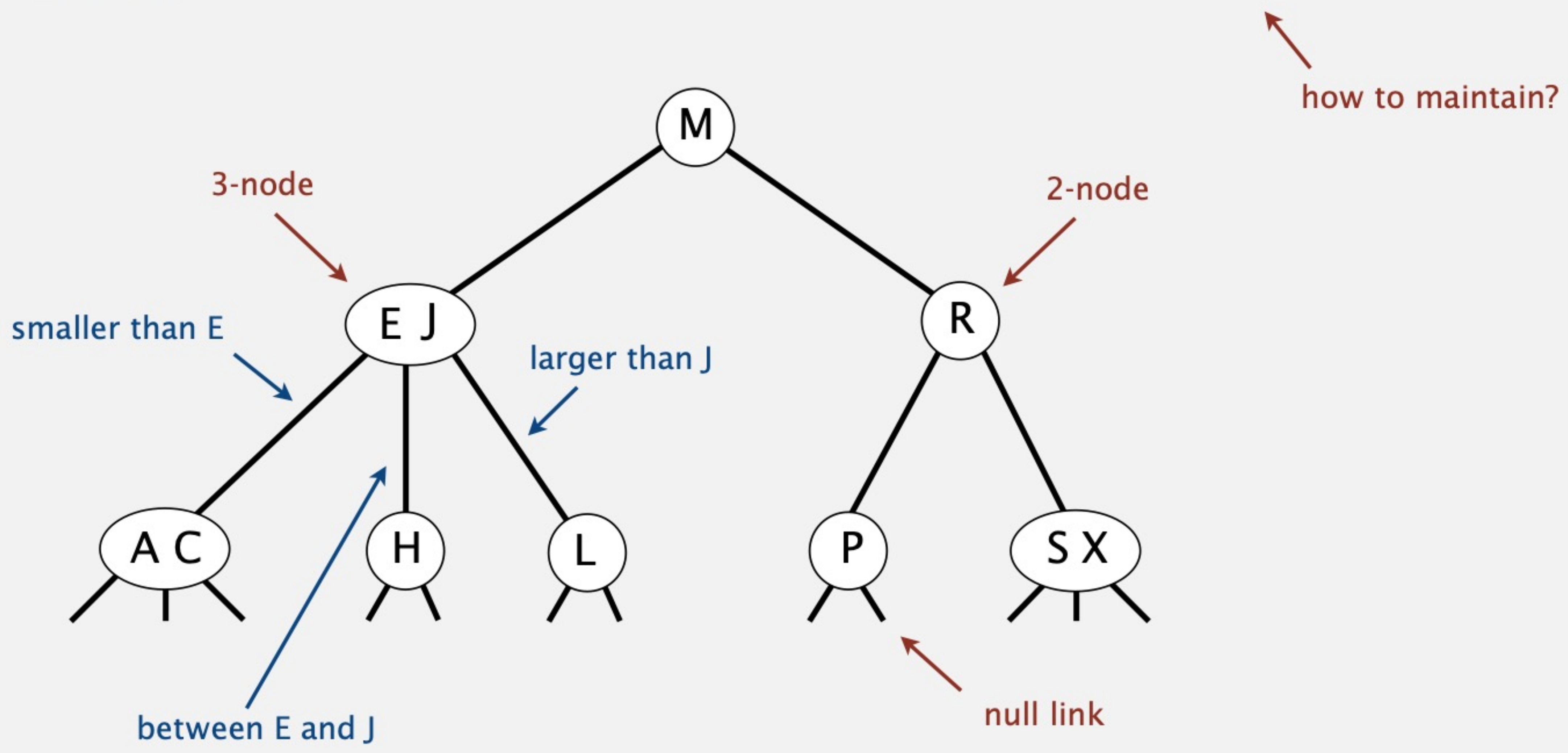
2-3 tree

Allow 1 or 2 keys per node.

- 2-node: one key, two children.
- 3-node: two keys, three children.

Symmetric order. Inorder traversal yields keys in ascending order.

Perfect balance. Every path from root to null link has same length.

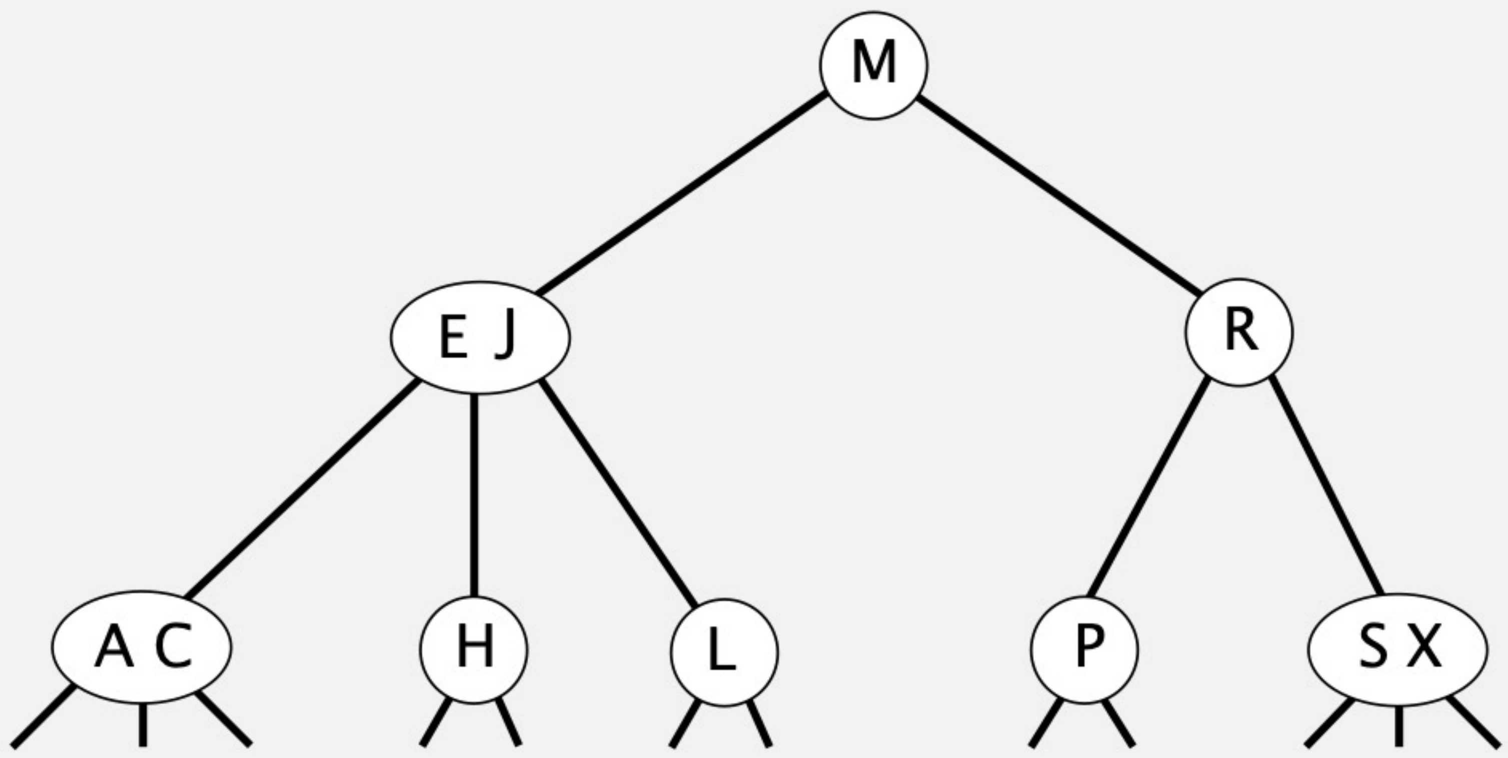


2-3 tree demo

Search.

- Compare search key against key(s) in node.
- Find interval containing search key.
- Follow associated link (recursively).

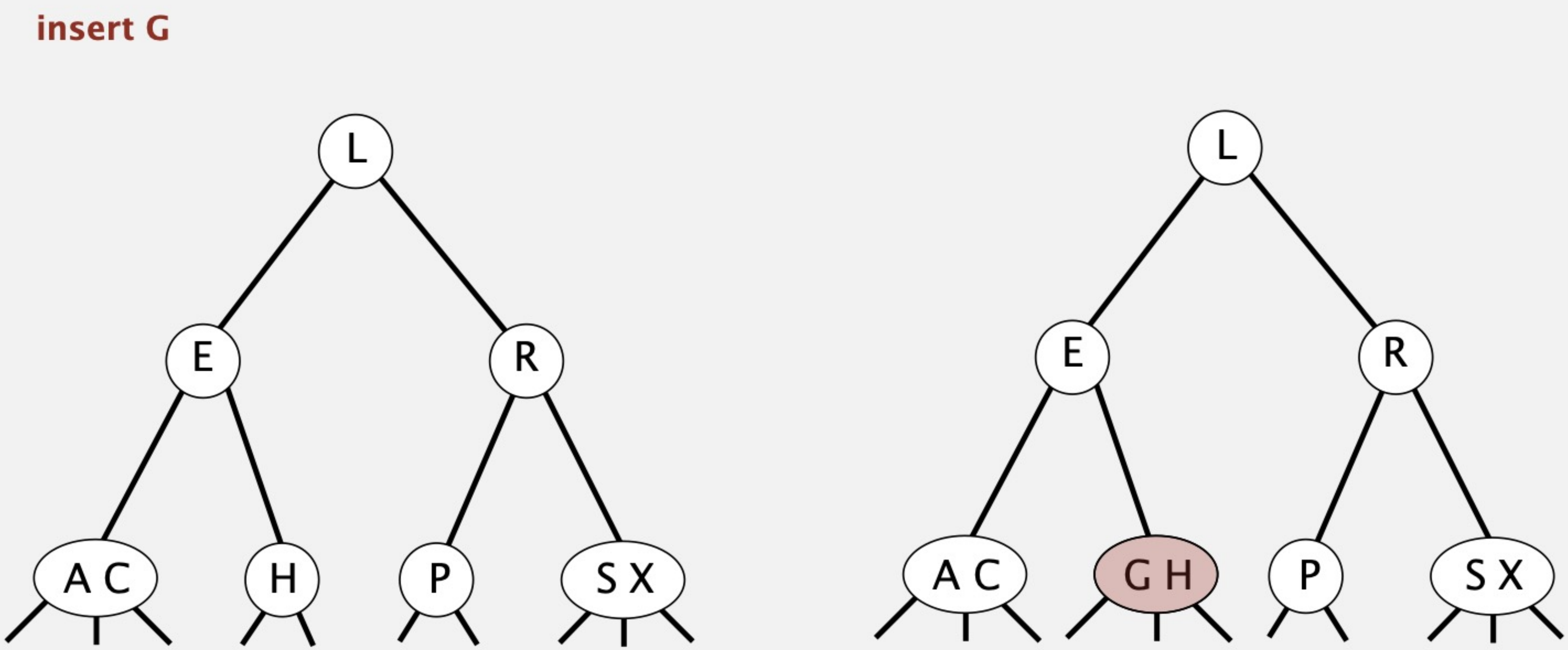
search for H



2–3 tree: insertion

Insertion into a 2-node at bottom.

- Add new key to 2-node to create a 3-node.

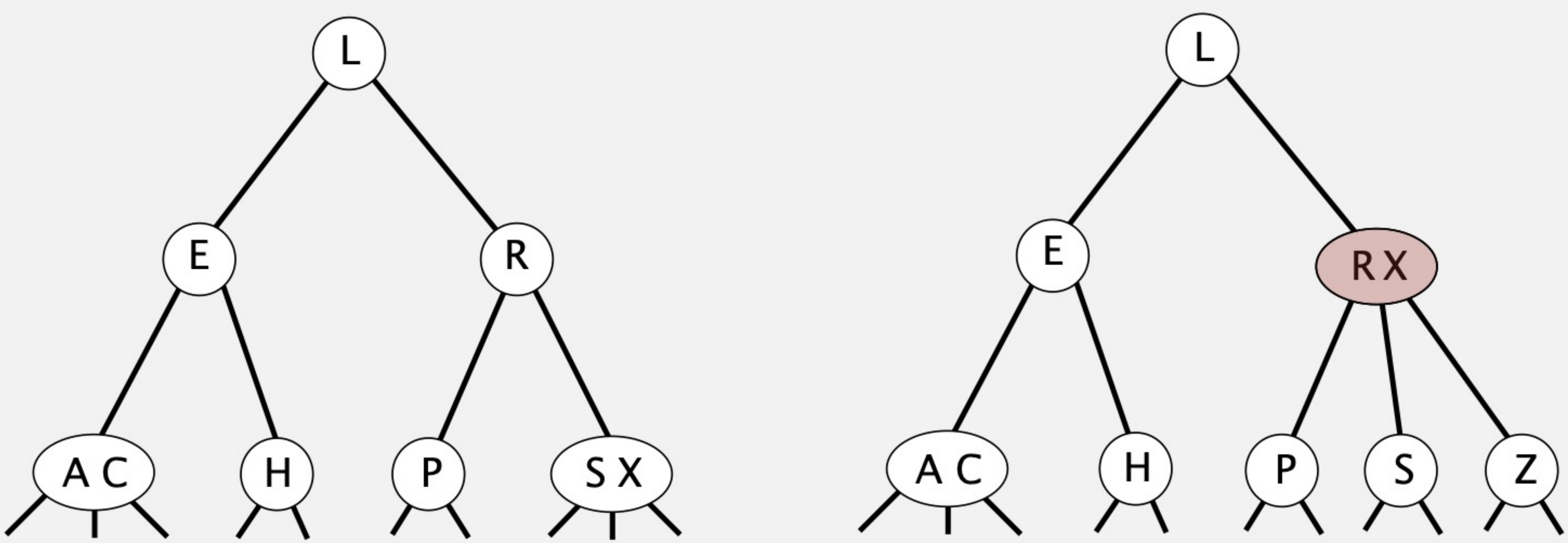


2–3 tree: insertion

Insertion into a 3-node at bottom.

- Add new key to 3-node to create temporary 4-node.
- Move middle key in 4-node into parent.
- Repeat up the tree, as necessary.
- If you reach the root and it's a 4-node, split it into three 2-nodes.

insert Z



2-3 tree construction demo

Balanced search trees: quiz 2

What is the maximum height of a 2-3 tree with n keys?

A. $\sim \log_3 n$

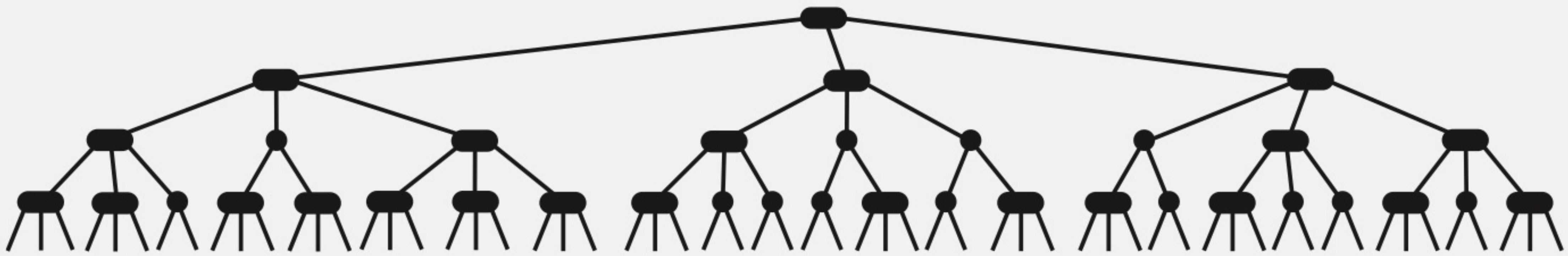
B. $\sim \log_2 n$

C. $\sim 2 \log_2 n$

D. $\sim n$

2–3 tree: performance

Perfect balance. Every path from root to null link has same length.



Tree height.

- Min: $\log_3 n \approx 0.631 \log_2 n$. [all 3-nodes]
- Max: $\log_2 n$. [all 2-nodes]
- Between 12 and 20 for a million nodes.
- Between 18 and 30 for a billion nodes.

Bottom line. Guaranteed logarithmic performance for search and insert.

ST implementations: summary

implementation	guarantee			average case			ordered ops?	key interface
	search	insert	delete	search	insert	delete		
sequential search (unordered list)	n	n	n	n	n	n		`equals()`
binary search (ordered array)	$\log n$	n	n	$\log n$	n	n	✔	`compareTo()`
BST	n	n	n	$\log n$	$\log n$	$\sqrt{n}$	✔	`compareTo()`
2-3 tree	$\log n$	$\log n$	$\log n$	$\log n$	$\log n$	$\log n$	✔	`compareTo()`

but hidden constant c is large
(depends upon implementation)

2-3 tree: implementation?

Direct implementation is complicated, because:

- Maintaining multiple node types is cumbersome.
- Need multiple compares to move down tree.
- Need to move back up the tree to split 4-nodes.
- Large number of cases for splitting.

fantasy code

```
public void put(Key key, Value val)
{
   Node x = root;
   while (x.getTheCorrectChild(key) != null)
   {
      x = x.getTheCorrectChildKey();
      if (x.is4Node()) x.split();
   }
   if      (x.is2Node()) x.make3Node(key, val);
   else if (x.is3Node()) x.make4Node(key, val);
}
```

Bottom line. Could do it, but there's a better way.

```
void BSTree::insert(int data){

        this->root = this->insert(data, this->root);

        }
```