CSC 212: Data Structures and Abstractions

Recursion

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[credit Marco Alverez]

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Recursion

Recursion

- Solve a task by reducing it to smaller tasks (of the same structure)
- Technically, a recursive function is one that **calls itself**
- General form:
  - **base case**
    - solution for a **trivial case**
    - it can be used to stop the recursion (prevents "*stack overflow*")
    - every recursive algorithm needs at least one base case
  - **recursive call(s)**
    - divide problem into **smaller instance(s)** of the **same structure**

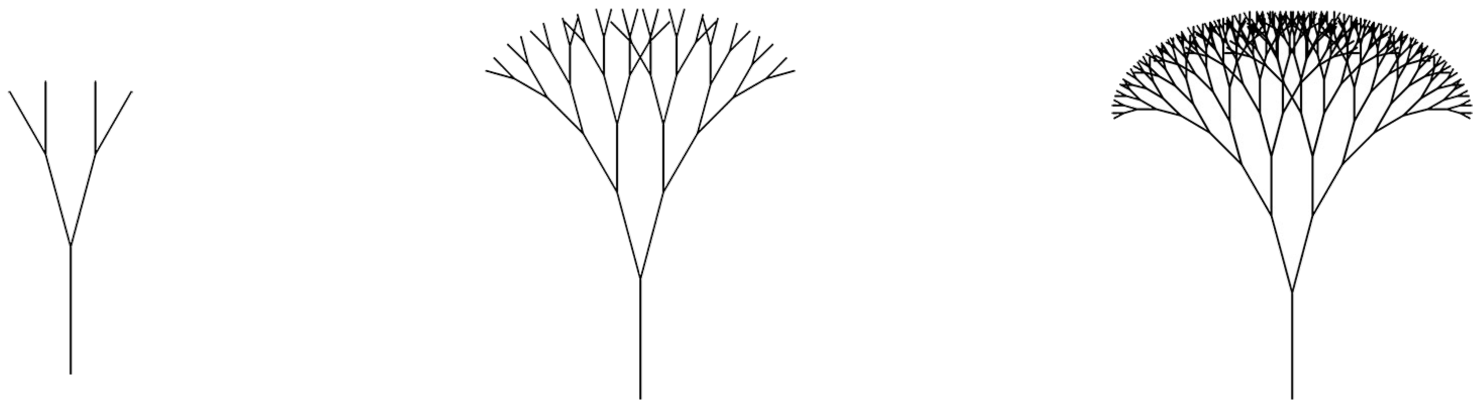
General Form

```
function() {
    if (this is the base case) {
        calculate trivialsolution

    } else {
        break task into subtasks
        solve each task recursively
        merge solutions if necessary
    }
}
```

Why recursion?

- Can we live without it?
  - yes, you can write "any program" with arrays, loops, and conditionals
- However …
  - some formulas are explicitly recursive
  - some problems exhibit a natural recursive solution



https://courses.cs.washington.edu/courses/cse120/17sp/labs/11/tree.html

```c
int sum_array(int *A, int n) {
    // base case
    if (n == 1) {
        return A[0];
    }

    // solve sub-task
    int sum = sum_array(A, n-1);

    // return sum
    return A[n-1] + sum;
}
```

Recursion call tree

```
int sum_array(int *A, int n) {
    // base case
    if (n == 1) {
        return A[0];
    }

    // solve sub-task
    int sum = sum_array(A, n-1);

    // return sum
    return A[n-1] + sum;
}
```

Example: power of a number

$$b^n = \underbrace{b \cdot b \cdot \ldots \cdot b}_{n \text{ times}}$$

```
double power(double x, int n) {
    // base case
    if (n == 0) {
        return 1;
    }
    // recursive call
    return x * power(x, n-1);
}
```

Recursion call tree

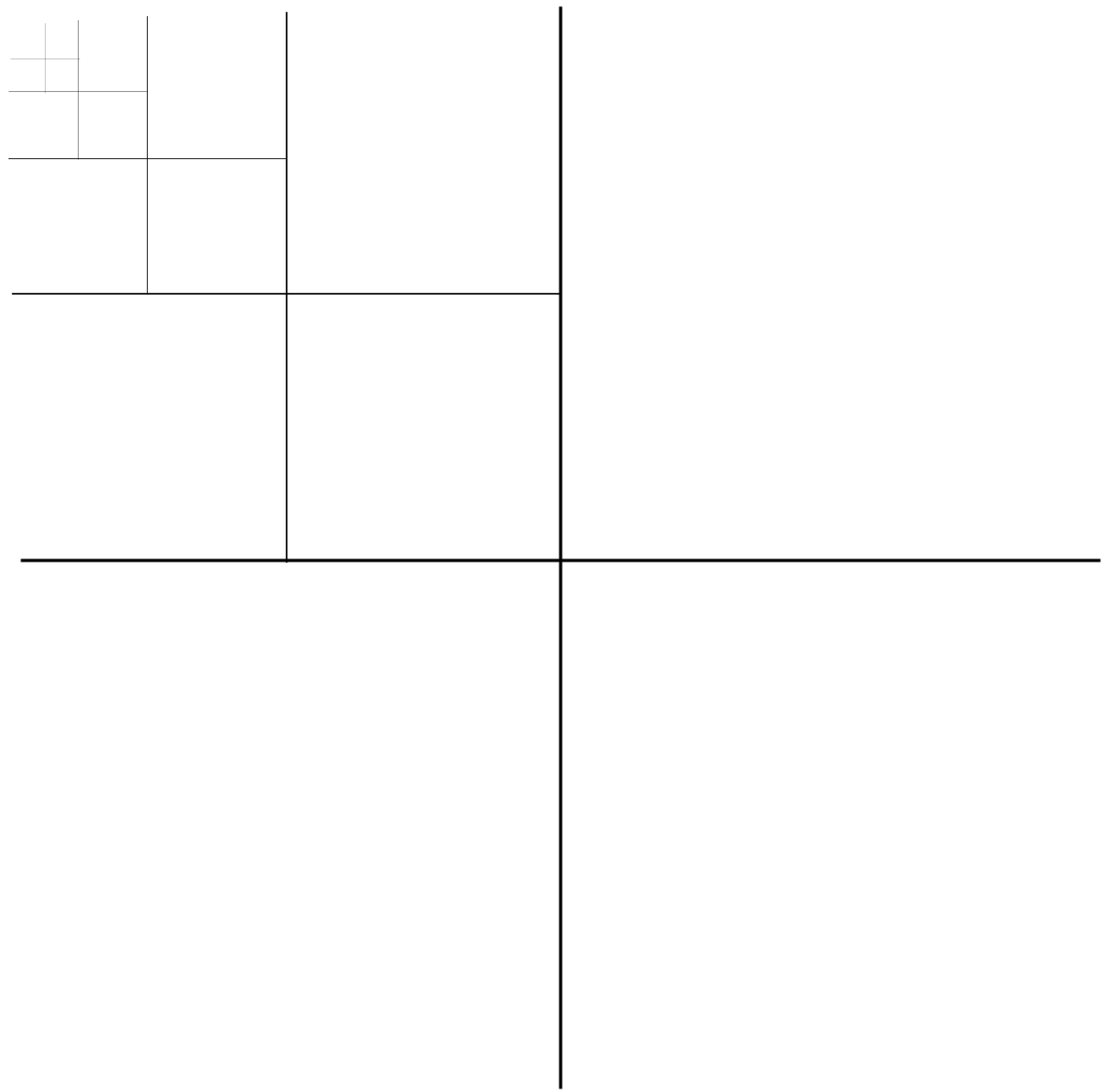
```
double power(double x, int n) {
    // base case
    if (n == 0) {
        return 1;
    }
    // recursive call
    return x * power(x, n-1);
}
```

Is this faster?

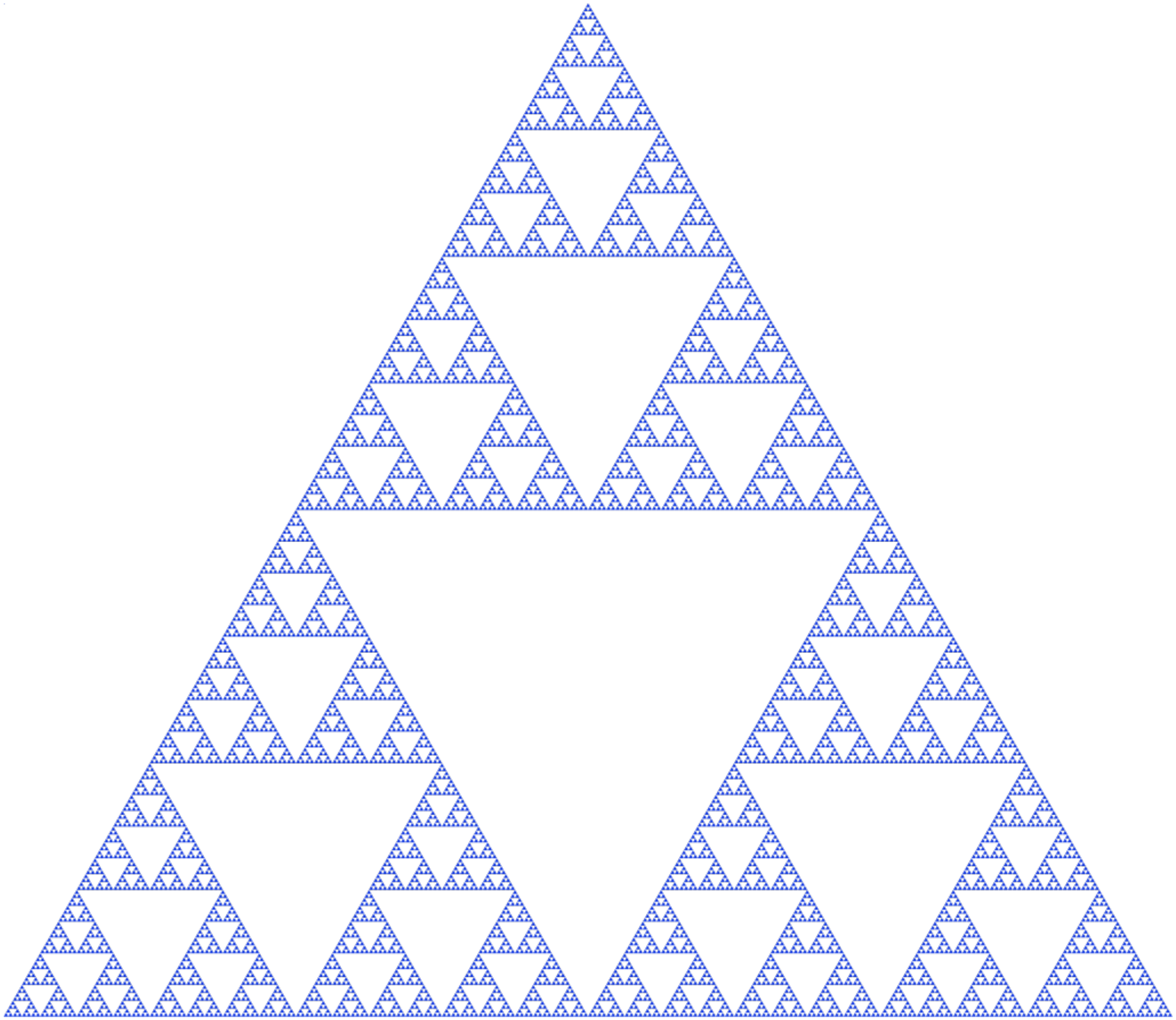
```
double power(double x, int n) {
    if (n == 0) {
        return 1;
    }
    double half = power(x, n/2);
    if (n % 2 == 0) {
        return half * half;
    } else {
        return x * half * half;
    }
}
```

Recursion call tree

```
double power(double x, int n) {
    if (n == 0) {
        return 1;
    }
    double half = power(x, n/2);
    if (n % 2 == 0) {
        return half * half;
    } else {
        return x * half * half;
    }
}
```

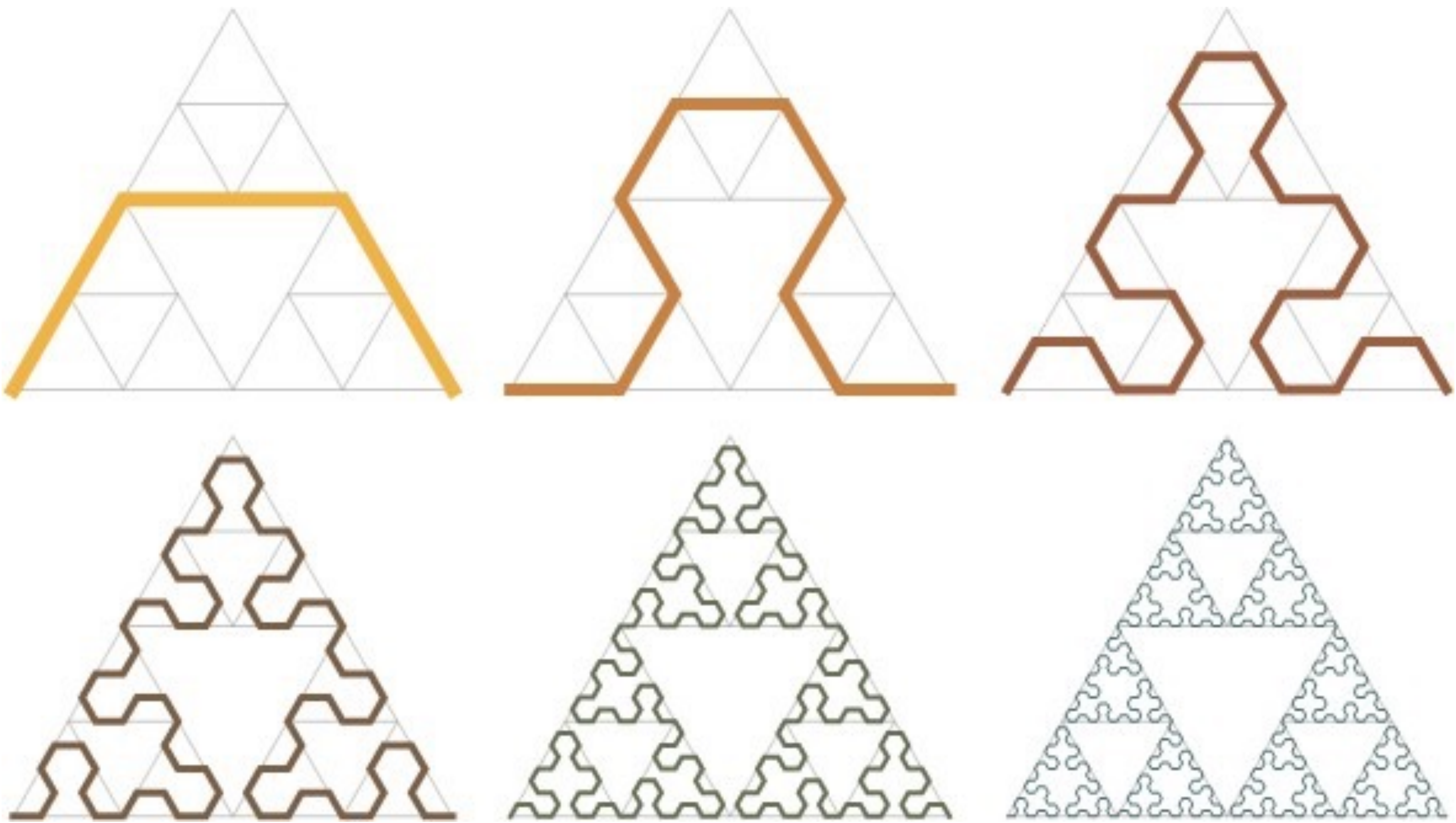


Sierpinski triangle



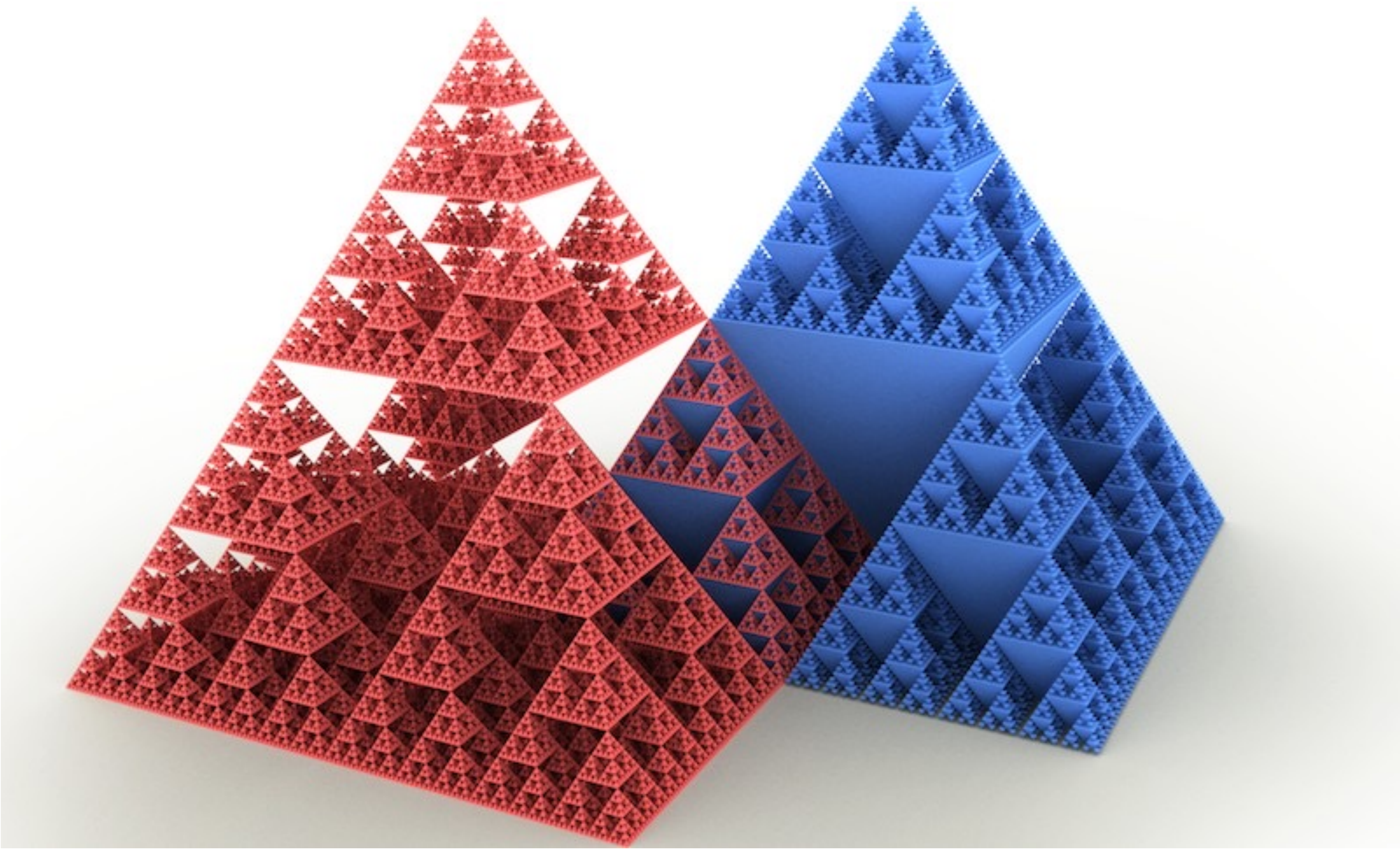
credit: wikipedia

Sierpinski arrowhead



credit: wikipedia

Sierpinski pyramid



credit: wikipedia

Binary Search

Binary Search

0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	5	10	15	20	22	30	35	40	43	48	51

`low` `high`

`k = 48?`

Binary Search

0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	5	10	15	20	22	30	35	40	43	48	51
low						mid						high

k = 48?

Binary Search

0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	5	10	15	20	22	30	35	40	43	48	51
							low					high

k = 48?

Binary Search

0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	5	10	15	20	22	30	35	40	43	48	51
							low		mid			high

k = 48?

Binary Search

0	1	2	3	4	5	6	7	8	9	10	11	12
1	2	5	10	15	20	22	30	35	40	43	48	51
										low	mid	high

k = 48?

```c
int bsearch(int *A, int lo, int hi, int k) {
    // basecase
    if (hi < lo) {
        return NOT_FOUND;
    }
    // calculate midpoint index
    int mid = lo + ((hi-lo)/2);
    // key found?
    if (A[mid] == k)
        return mid;
    // key in upper subarray?
    if (A[mid] < k)
        return bsearch(A, mid+1, hi, k);
    // key is in lowersubarray?
    return bsearch(A, lo, mid-1, k);
}
```

Recursion Tree (binary search)

```
int bsearch(int *A, int lo, int hi, int k) {
    // base case
    if (hi < lo) {
        return NOT_FOUND;
    }
    // calculate midpoint index
    int mid = lo + ((hi-lo)/2);
    // key found?
    if (A[mid] == k)
        return mid;
    // key in upper subarray?
    if (A[mid] < k)
        return bsearch(A, mid+1, hi, k);
    // key is in lower subarray?
    return bsearch(A, lo, mid-1, k);
}
```

Complexity? Best-case, Worst-case, Average-case?

Table 1

n	time (days)	~ log(n)
1	0.000	0
10	0.000	3
100	0.000	7
1000	0.000	10
10000	0.000	13
100000	0.000	17
1000000	0.000	20
10000000	0.000	23
100000000	0.000	27
1000000000	0.000	30
10000000000	0.000	33
100000000000	0.000	37
1000000000000	0.000	40
10000000000000	0.000	43
100000000000000	0.003	47
1000000000000000	0.028	50
10000000000000000	0.281	53
100000000000000000	2.809	56
1000000000000000000	28.086	60
10000000000000000000	280.863	63
100000000000000000000	2808.628	66

Intel Core i9-9900K	412,090 MIPS at 4.7 GHz

Example: find peak in unimodal arrays

Unimodal arrays

- An array is (**strongly**) **unimodal** if it can be split into an increasing part followed by a decreasing part

1	2	5	16	20	18	17	16	15	12	10	8	5

How to efficiently find the max?

1	2	5	16	20	18	17	16	15	12	10	8	5

Find the **max** (strongly unimodal)

1	2	5	8	15	20	22	20	15	12	10	8	5
1	2	5	8	15	20	22	20	15	12	10	8	5
1	2	5	8	15	20	22	20	15	12	10	8	5
1	2	5	8	15	20	22	20	15	12	10	8	5
1	2	5	8	15	20	22	20	15	12	10	8	5
1	2	5	8	15	20	22	20	15	12	10	8	5
1	2	5	8	15	20	22	20	15	12	10	8	5
1	2	5	8	15	20	22	20	15	12	10	8	5

Find the **max** (strongly unimodal)

- Recursion Tree?

- Complexity? Best-case, Worst-case, Average-case?

```
int max_s_unimodal(int *A, int low, int hi) {
    if (low == hi) {
        return A[low];
    }

    intmid  = (low + hi) / 2;

    if (A[mid]<A[mid+1]) {
        return max_s_unimodal(A, mid+1, hi);
    } else {
        return max_s_unimodal(A, low, mid);
    }
}
```

Unimodal arrays

- An array is (**weakly**) **unimodal** if it can be split into a nondecreasing part followed by a nonincreasing part

1	2	5	5	15	20	22	22	35	38	13	8	5

How to efficiently find the max?

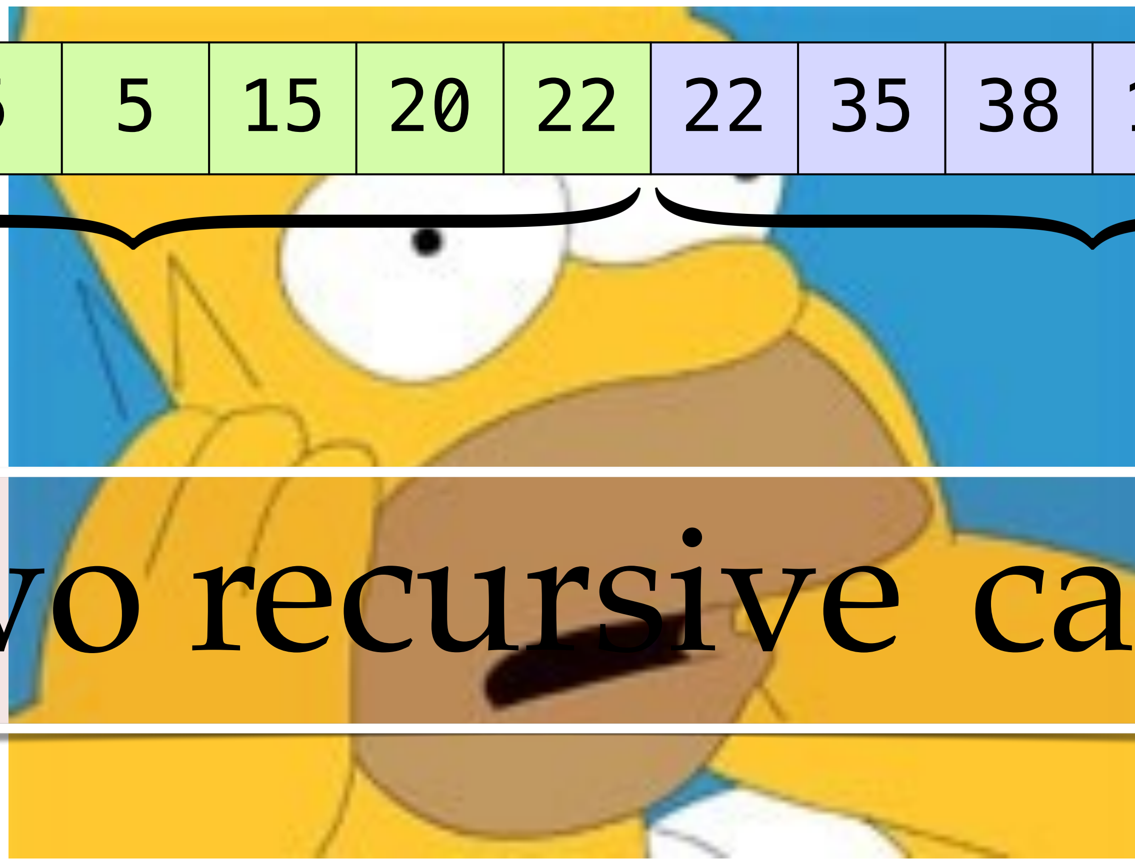
1	2	5	5	15	20	22	22	35	38	13	8	5

Find the **max** (weakly unimodal)

1	2	5	5	15	20	22	22	35	38	13	8	5

1	2	5	5	15	20	22	22	35	38	13	8	5

1	2	5	5	15	20	22	22	35	38	13	8	5



Two recursive calls

```
int max_w_unimodal(int *A, int low, int hi) {
    if (low == hi) {
        return A[low];
    }

    int mid = (low + hi) / 2;

    if (A[mid]<  A[mid+1]) {
        return max_w_unimodal(A, mid+1, hi);
    } else if (A[mid] > A[mid+1]) {
        return max_w_unimodal(A, low, mid);
    } else {
        int left = max_w_unimodal(A, mid+1, hi);
        int right = max_w_unimodal(A, low, mid);
        return std::max(left, right);
    }
}
```

- Recursion Tree?

- Complexity?