

# CSC 212: Data Structures and Abstractions

## Big O Notation

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# Example Review

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```
for (int i = 0 ; i < n ; i ++ ) {  
    for (int j = 0 ; j < i*i ; j ++ ) {  
        for (int k = 0 ; k < j ; k ++ ) {  
            // count 1 instruction  
        }  
    }  
}
```

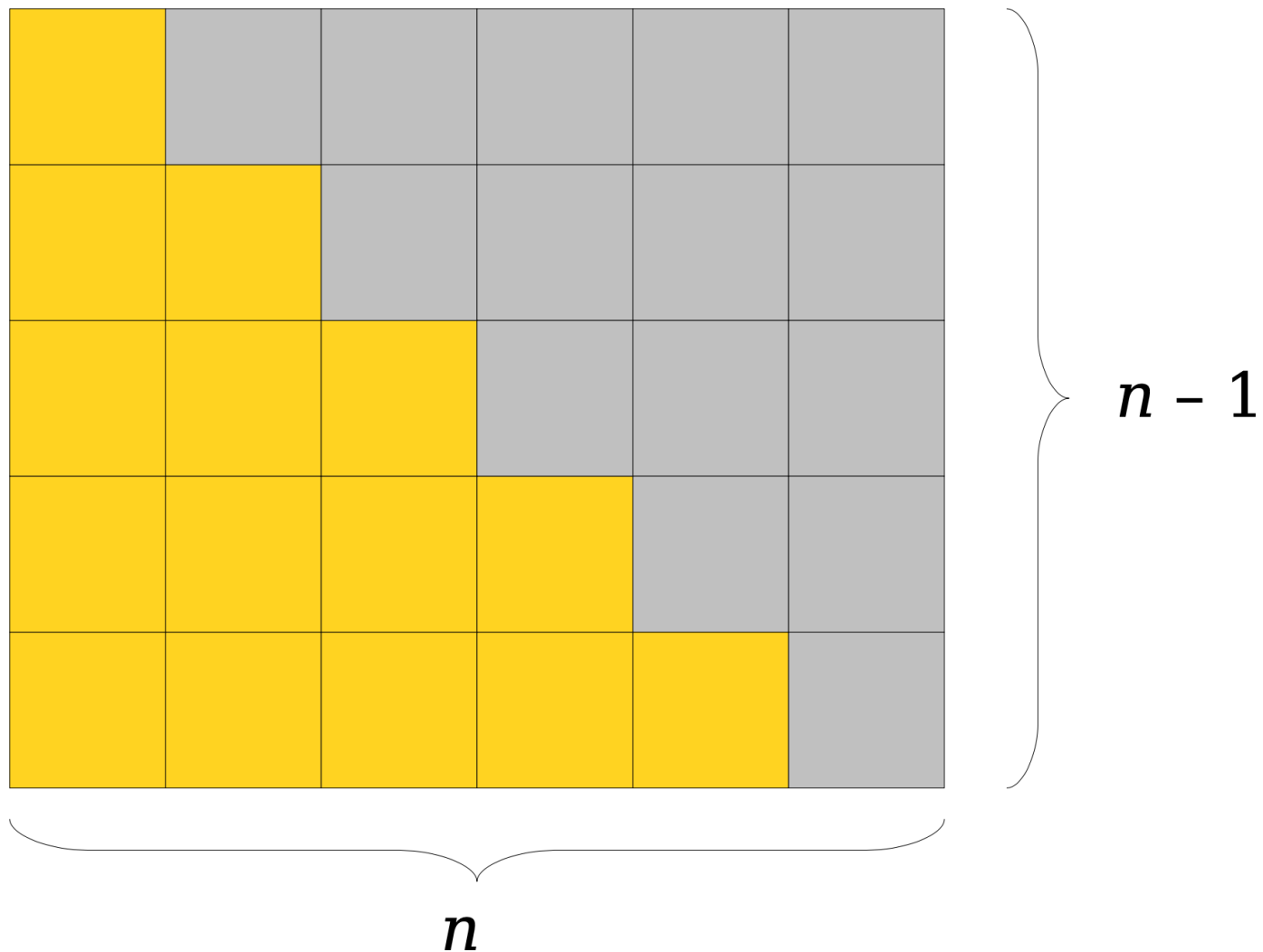
# The story so far ...

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- Can measure actual runtime to compare algorithms
  - ✓ however, runtime is noisy (highly sensitive to HW / SW and implementation details)
- Can count instructions to compare algorithms
  - ✓ can define  $T(n)$ , which depends on the input size
  - ✓ for large inputs, our focus should be on the dominant terms of  $T(n)$
  - ✓ we will now see formal ways for this approach

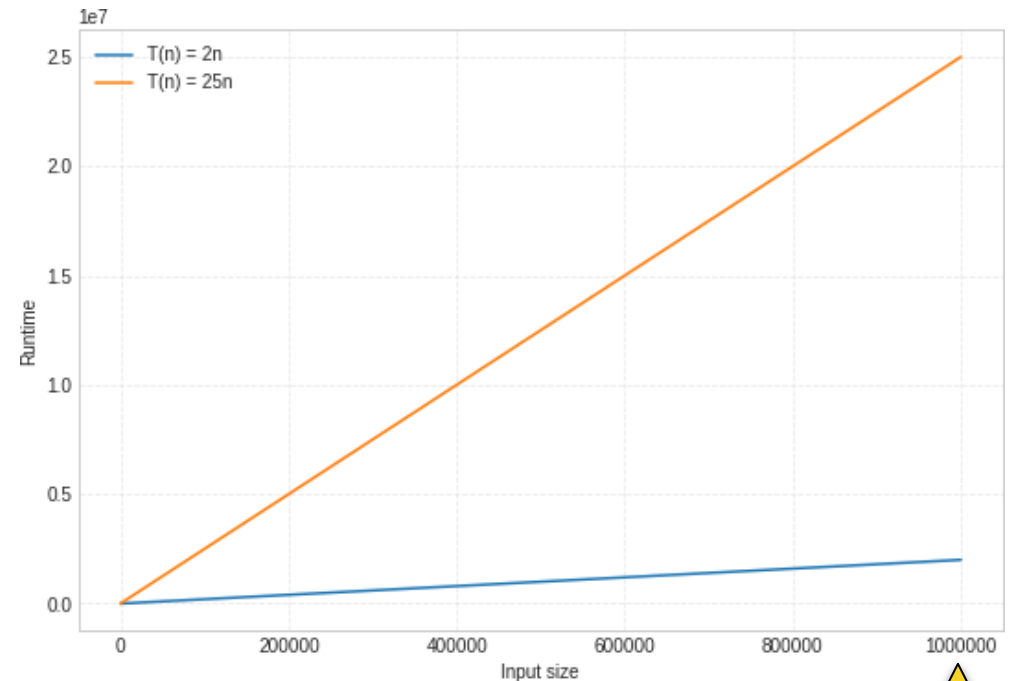
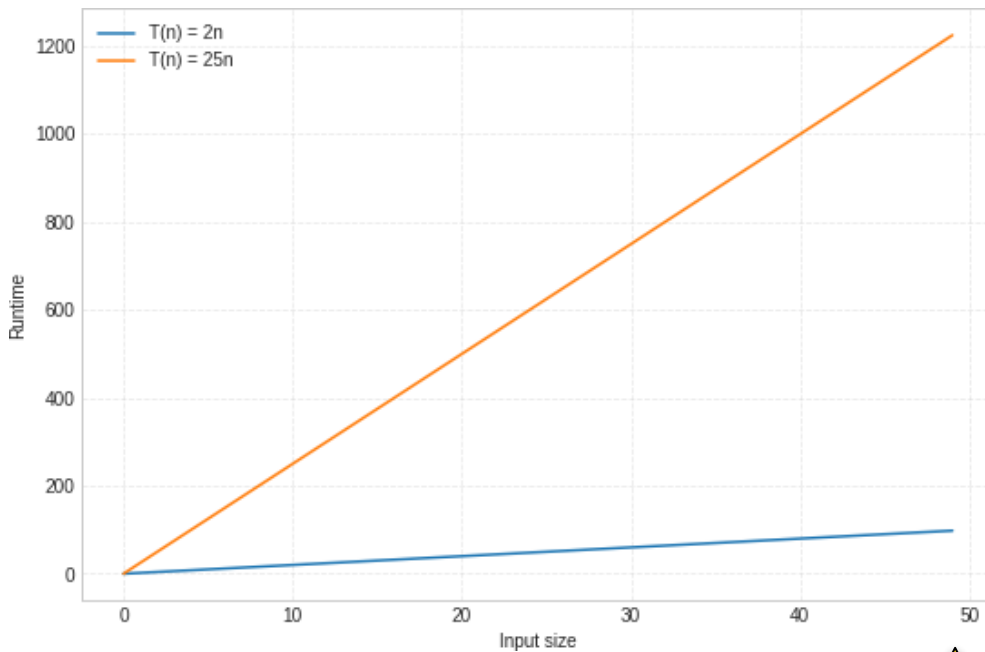
$$\sum_{i=1}^n i = 1 + 2 + 3 + \dots + (n - 1) + n$$

$$\sum_{i=1}^{n-1} i = 1 + 2 + 3 + \dots + (n-2) + (n-1)$$

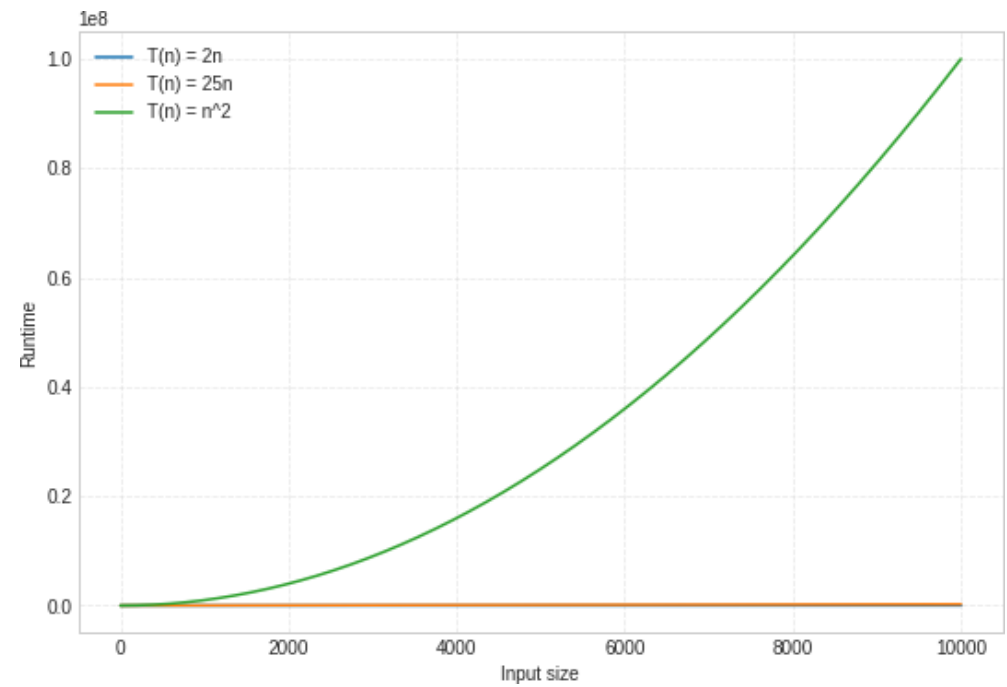
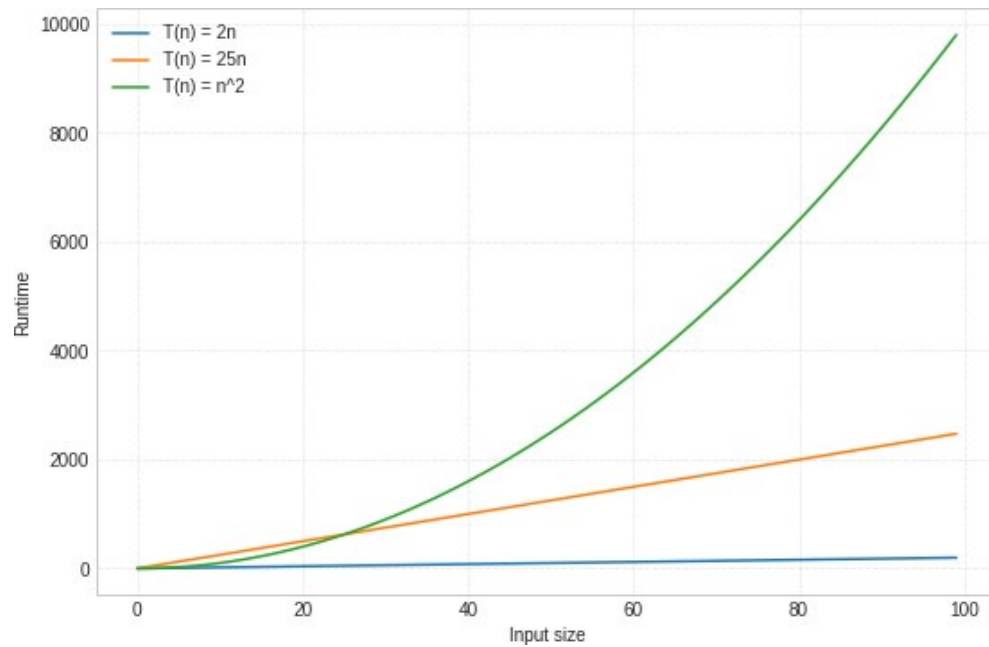


# Should we consider these the same?

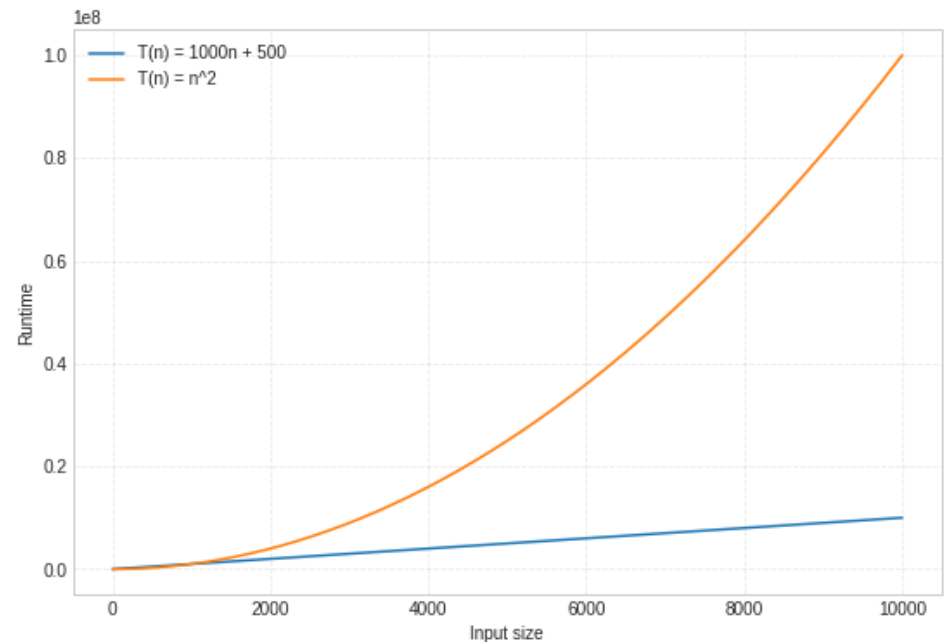
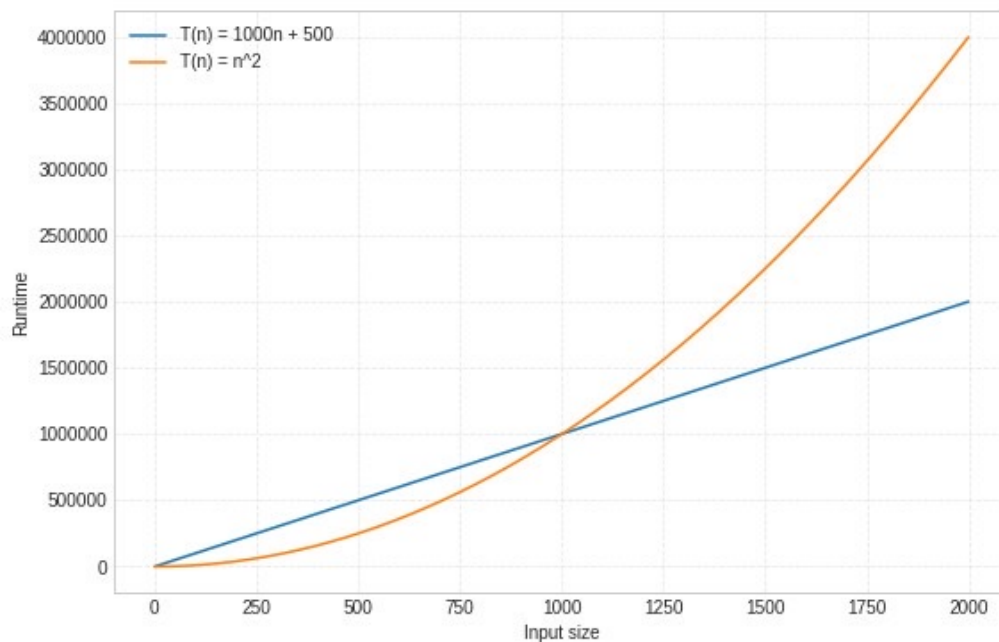
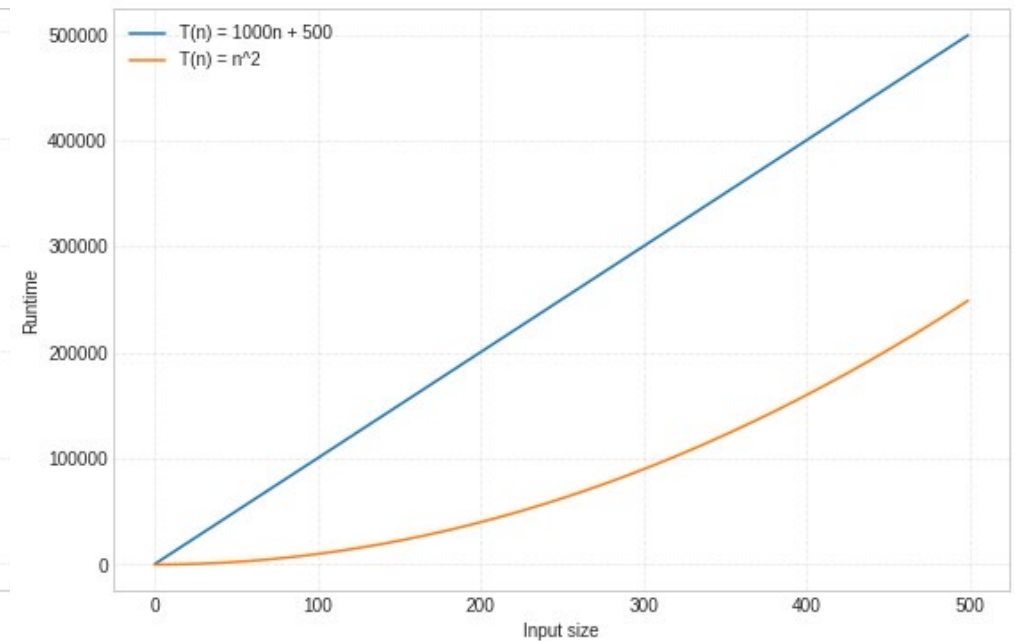
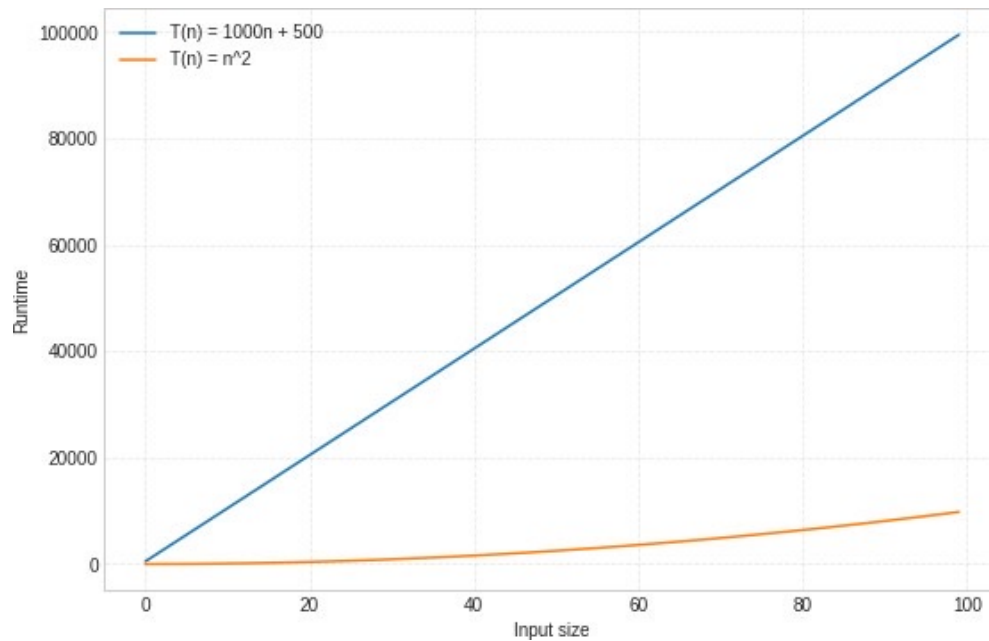
Comparing two algorithms with  $T(n) = 2n$  and  $T(n) = 25n$  respectively



# What about now?



# Now consider this example ...

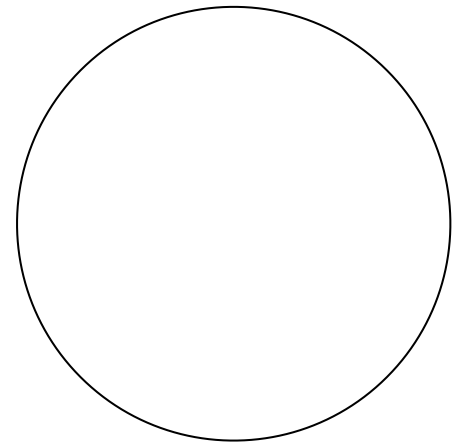
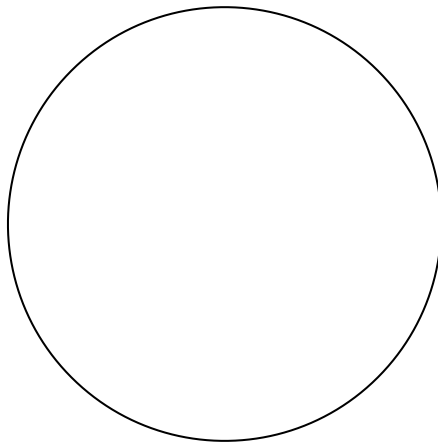
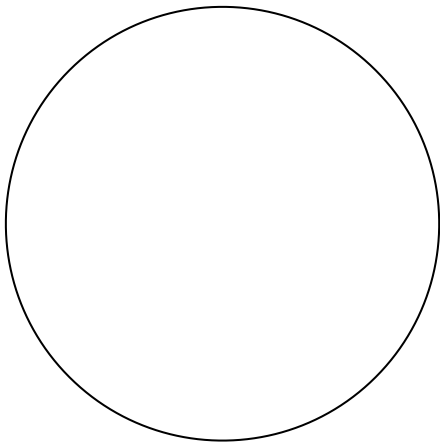




# Bottom line ...

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- We are trying to compare  $T(n)$  functions, but we also care about large values of  $n$
- Can we properly define ' $\leq$ ' for functions?
  - ✓ we can group functions into '**sets**' and make our lives easier



# Asymptotic Analysis

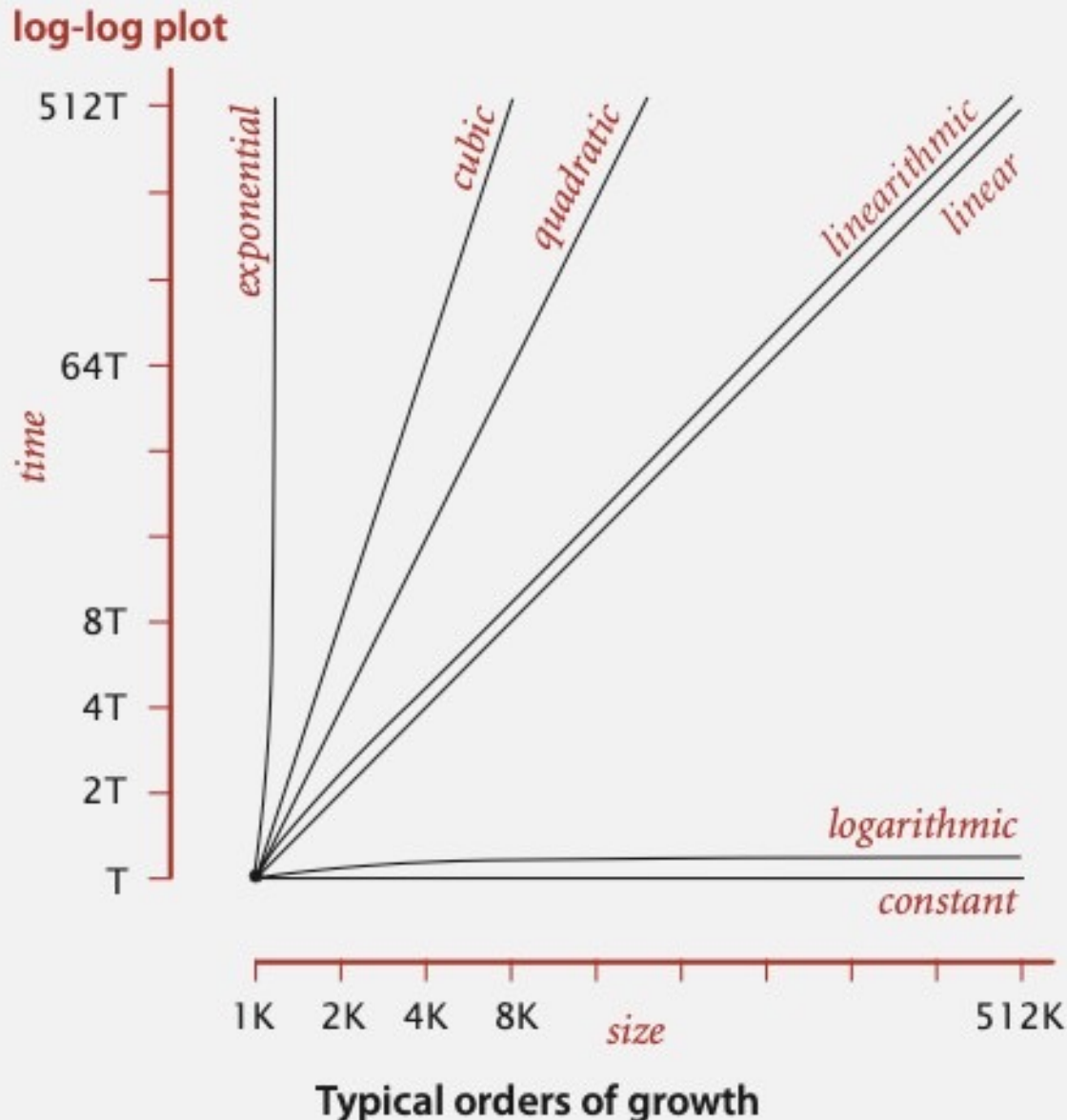
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- Refers to the study of an algorithm as the input size “gets big” or reaches a limit (in the calculus sense)
- **Growth rate**
  - ✓ rate at which the cost of an algorithm grows as the size of its input grows

$$c_1n$$

$$c_2n^2$$

# Common sets of functions



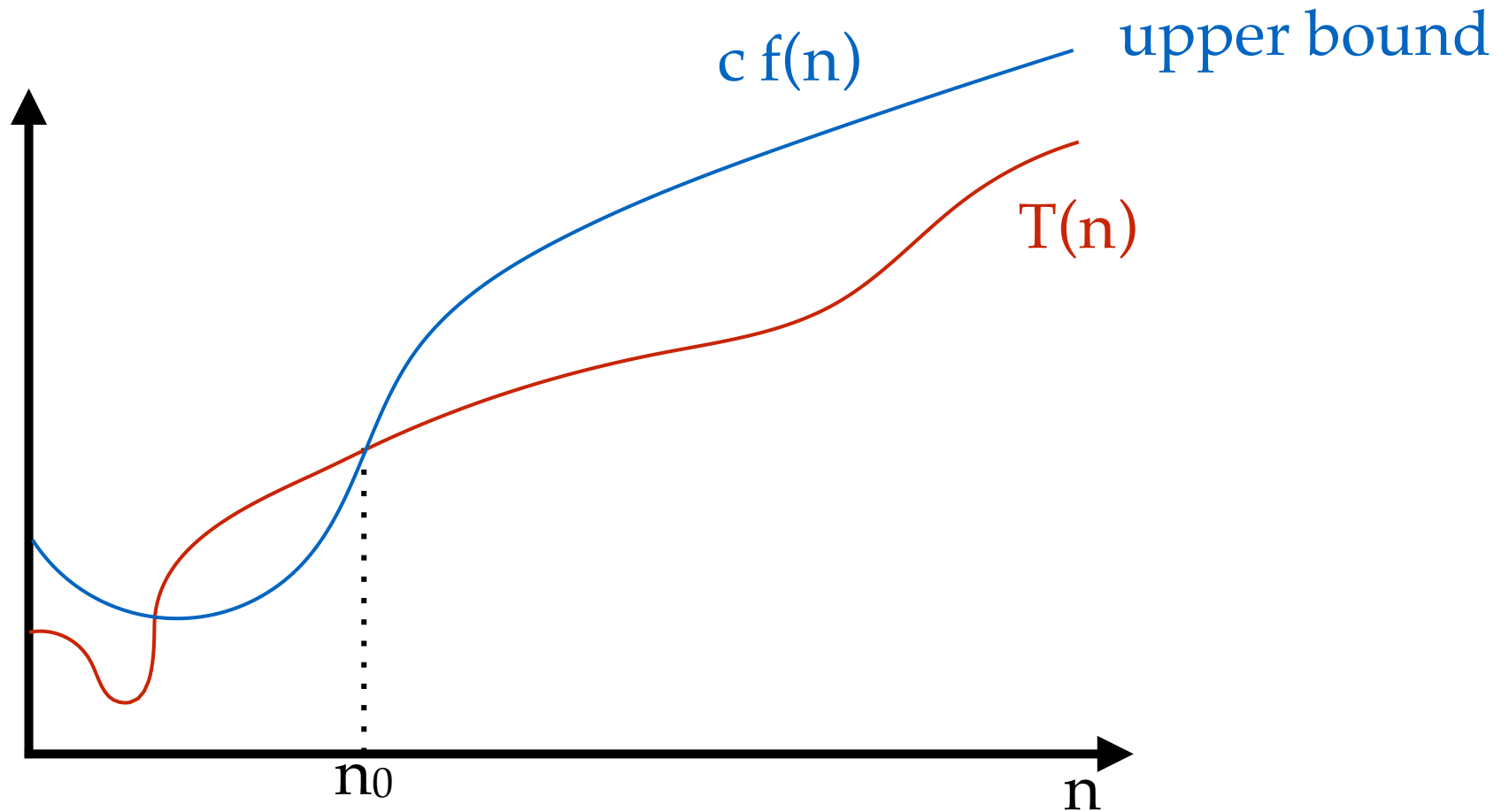
Faster  
growth rate  
... slower  
algorithm

Algorithm A is  
better than  
algorithm B if for  
large values of  $n$ ,  
 $T_A(n)$  grows  
slower than  $T_B(n)$

# A few examples ...

order of growth	name	typical code framework	description	example
1	constant	<code>a = b + c;</code>	statement	add two numbers
$\log n$	logarithmic	<code>while (n &gt; 1) { n = n/2; ... }</code>	divide in half	binary search
$n$	linear	<code>for (int i = 0; i &lt; n; i++) { ... }</code>	single loop	find the maximum
$n \log n$	linearithmic	<i>see mergesort lecture</i>	divide and conquer	mergesort
$n^2$	quadratic	<code>for (int i = 0; i &lt; n; i++)   for (int j = 0; j &lt; n; j++)   { ... }</code>	double loop	check all pairs
$n^3$	cubic	<code>for (int i = 0; i &lt; n; i++)   for (int j = 0; j &lt; n; j++)     for (int k = 0; k &lt; n; k++)     { ... }</code>	triple loop	check all triples
$2^n$	exponential	<i>see combinatorial search lecture</i>	exhaustive search	check all subsets

# Big O



$$T(n) \text{ is } O(f(n)) \iff \exists \text{ positive } c, n_0 \mid 0 \leq T(n) \leq cf(n), \forall n \geq n_0$$

set of functions

# Examples

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$$7n - 2 = O(n)$$

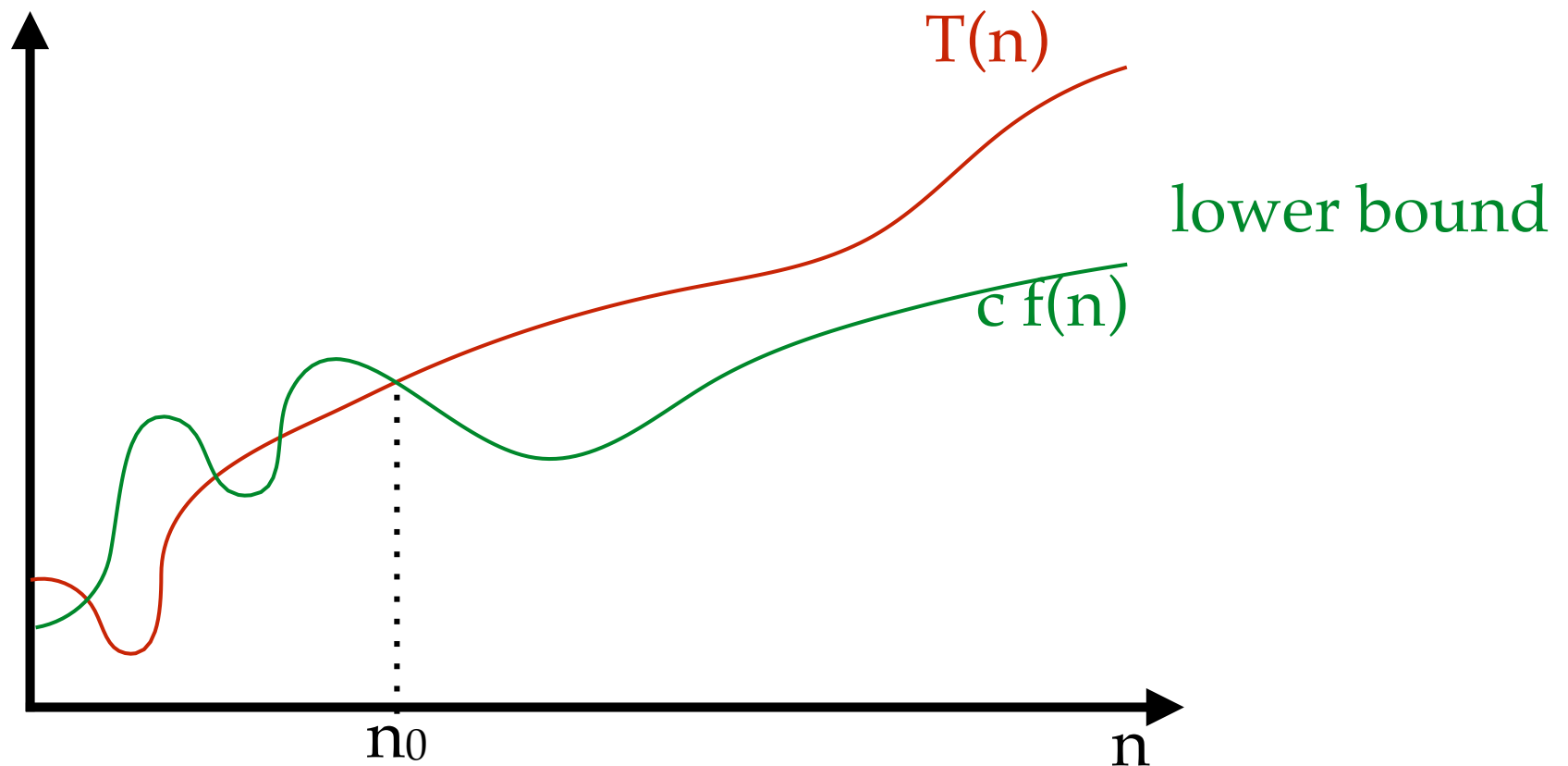
$$20n^3 + 10n \log n + 5 = O(n^3)$$

$$3 \log n + \log \log n = O(\log n)$$

$$2^{100} = O(1)$$

$T(n)$  is  $O(f(n)) \iff \exists$  positive  $c, n_0 \mid 0 \leq T(n) \leq cf(n), \forall n \geq n_0$

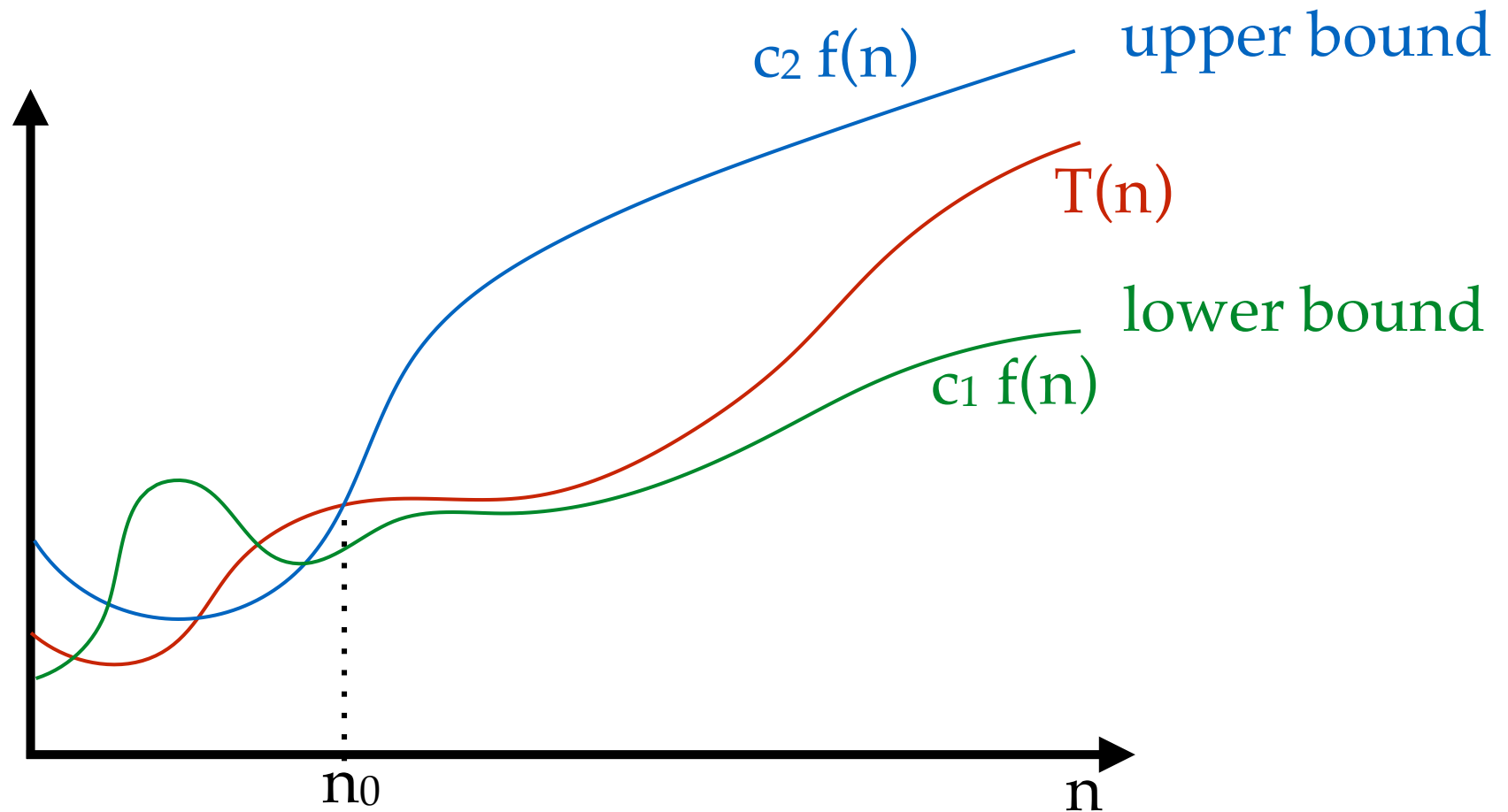
# Big Omega



$$T(n) \text{ is } \Omega(f(n)) \iff \exists \text{ positive } c, n_0 \mid 0 \leq cf(n) \leq T(n), \forall n \geq n_0$$

set of functions

# Big Theta



$$T(n) \text{ is } \Theta(f(n)) \iff T(n) \text{ is } O(f(n)) \text{ and } T(n) \text{ is } \Omega(f(n))$$



# Prove that ...

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$$3 \log n + \log \log n = \Omega(\log n)$$

$$3 \log n + \log \log n = \Theta(\log n)$$

$$T(n) \text{ is } O(f(n)) \iff \exists \text{ positive } c, n_0 \mid 0 \leq T(n) \leq cf(n), \forall n \geq n_0$$

$$T(n) \text{ is } \Omega(f(n)) \iff \exists \text{ positive } c, n_0 \mid 0 \leq cf(n) \leq T(n), \forall n \geq n_0$$

# In practice you can ...

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“ignore constants and drop lower order terms”

# True or False?

$\{n^2, n^4, 2^n, \log n, \dots\}$

	Big O	Big Omega	Big Theta
$10^2 + 3000n + 10$			
$21 \log n$			
$500 \log n + n^4$			
$\sqrt{n} + \log n^{50}$			
$4^n + n^{5000}$			
$3000n^3 + n^{3.5}$			
$2^5 + n!$			

# Asymptotic Performance

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- For **large** values of  $n$ , a  $\Theta(n^2)$  algorithm always beats a  $\Theta(n^3)$  algorithm

However, we shouldn't completely ignore asymptotically slower algorithms